# Stabilization for Damping Multimachine Power System with Time-Varying Delays and Sector Saturating Actuator 

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#### Abstract

This paper studies the stabilization problem for damping multimachine power system with time-varying delays and sector saturating actuator. The multivariable proportional plus derivative (PD) type stabilizer is designed by transforming the problem of PD controller design to that of state feedback stabilizer design for a system in descriptor form. A new sufficient condition of closedloop multimachine power system asymptomatic stability is derived based on the Lyapunov theory. Computer simulation of a twomachine power system has verified the effectiveness and efficiency of the proposed approach.


## 1. Introduction

To cope with the increasing demand for quality electric power, excitation control, power system stabilizer (PSS), and other power system controllers are playing important roles in power system stability and maintaining dynamic performance. Conventional PSS is mainly designed based on a linear model and considered one operating point. Recently, to interconnect large energy pools connecting neighboring electric grids together and transmit bulk energy during peak times of load demand can satisfy the growing demand for energy [1]. But it introduces some modes of electromechanical oscillations and frequency deviations within the range of $0.2-2 \mathrm{~Hz}$ in the power system which will make power system more complicated [2, 3]. A conventional PSS cannot guarantee to have the best performance. Hence, a variety of control strategies have been used to obtain PSS, such as lead-lag controller [4], variable structure controller [57], robust controller [8], PID controller [9-11], and fuzzy logic controller [10]. Most of the controllers are nonlinear. Some researchers have designed the PSS by using searching algorithms such as genetic algorithms [10, 12], particle swarm optimization [13, 14], and chaotic optimization algorithm $[15,16]$. But these algorithms are hard to program and are not sure to find the optimum solutions.

It is well known that the amplitude of the controller is always bounded in the real world [17]. So it is very necessary that the actuator saturation is taken into consideration. Timedelay is very common in power systems which can be a source of instability of performance degradation [18]. Multimachine power system with time-varying delay and sector saturating actuator [19] is a complex interconnected large-scale system that is composed of many electric devices and mechanical components with a better description of real world. The state feedback control problem for such a system is addressed by [19] based on the LMI methods. However, the conditions in [19] are conservative because of the amplifying technique to deal with the nonlinear terms in the conditions. Moreover, we usually cannot find the state feedback controller to satisfy demand when system becomes more complex.

The purpose of this paper is to design a PD controller for damping multimachine power systems with time-varying delay and sector saturating actuator. Under a descriptor transformation, the problem of PD type controller design is transformed into the state feedback controller design for a descriptor system. Then, a new sufficient condition is derived for the admissible of the descriptor system based on the Lyapunov theory. Compared with the existing LMI methods in [19], our method introduces more relax matrix variables. Therefore, it is less conservative. Compared with some of the
nondeterministic methods, our method has the advantages of low complexity.

This paper is organized as follows. Section 2 is the problem formulation and preliminaries. Section 3 gives the main results. Section 4 provides an example to show the merits and effectiveness of the results and Section 5 concludes this paper.

Notation. $R^{n}$ denotes $n$-dimensional Euclidean space; the superscripts -1 and $T$ denote the matrix inverse and transpose, respectively; $X>0(X \geq 0)$ means that $X$ is positive definite (positive semidefinite); the star $*$ denotes the symmetric term in a matrix.

## 2. Problem Formulation and Preliminaries

Consider N -machine power system with time-varying delays and input constraints which is described by the interconnection of $N$ subsystems as follows:

$$
\begin{align*}
\dot{x}_{i}(t)= & A_{i} x_{i}(t)+B_{i} u_{s i}(t) \\
& +\sum_{j=1}^{N} p_{i j} G_{i j} g_{i j}\left(x_{i}(t), x_{j}\left(t-\tau_{i j}(t)\right)\right),  \tag{1}\\
u_{s i}(t)= & \operatorname{sat}\left(u_{i}(t)\right), \\
x_{i}(t)= & \phi_{i}(t), \quad t \in[-\tau, 0],
\end{align*}
$$

where $x_{i}(t)=\left[\begin{array}{lll}\Delta \delta_{i}(t) & \Delta W_{i}(t) & \Delta E_{q i}^{\prime}(t)\end{array} \Delta E_{f d i}(t) \Delta V_{D C i}(t)\right]^{T}$, $u_{i}(t)=\left[\Delta m_{i}\right], u_{i}(t)$ is the control input vector to the actuator, and $u_{s i}(t)$ is the control input vector to the plant.
$g_{i j}\left(x_{i}(t), x_{j}\left(t-\tau_{i j}(t)\right)\right.$ is the nonlinear function vector characterizing the interconnection between $i$ th generator and $j$ th generator with

$$
\begin{align*}
g_{i j} & \left(x_{i}(t)\right), x_{j}\left(t-\tau_{i j}(t)\right) \\
& =-\cos \beta_{i}(\delta)\left(\Delta \delta_{i}(t)-\Delta \delta_{j}\left(t-\tau_{i j}(t)\right)\right)  \tag{2}\\
& =-\cos \beta_{i}(\delta) W\left(x_{i}(t)-x_{j}\left(t-\tau_{i j}(t)\right)\right),
\end{align*}
$$

where $\tau_{i j}(t)$ is the time-varying delay and satisfied $0 \leqslant \tau_{i j}(t) \leqslant$ $\tau \leqslant \infty, \dot{\tau}_{i j}(t) \leqslant \dot{\tau}^{*}<1$, and $\beta_{i}(\delta)=\left(\delta_{i}(t)-\delta_{j}\left(t-\tau_{i j}(t)\right)+\right.$ $\left.\delta_{i 0}-\delta_{j 0}\right) / 2, W=\left[\begin{array}{lllll}1 & 0 & 0 & 0 & 0\end{array}\right]$.

The nominal system matrices are represented as follows:
$A_{i}$

$$
=\left[\begin{array}{ccccc}
0 & \omega_{0} & 0 & 0 & 0 \\
-\frac{K_{1 i}}{M_{i}} & -\frac{P_{i}}{M_{i}} & -\frac{K_{2 i}}{M_{i}} & 0 & -\frac{K_{p d c i}}{M_{i}} \\
-\frac{K_{4 i}}{T_{d 0 i}^{\prime}} & 0 & -\frac{K_{3 i}}{T_{d 0 i}^{\prime}} & \frac{1}{T_{d 0 i}^{\prime}} & -\frac{K_{p d c i}}{T_{d 0 i}^{\prime}} \\
-\frac{K_{A i} K_{5 i}}{T_{A i}} & 0 & -\frac{K_{A i} K_{6 i}}{T_{A i}} & -\frac{1}{T_{A i}} & -\frac{K_{A i} K_{V D C i}}{T_{A i}} \\
K_{7 i} & 0 & K_{8 i} & 0 & K_{9 i}
\end{array}\right],
$$

$$
\begin{align*}
& B_{i}=\left[\begin{array}{lllll}
0 & -K_{p m i} & -K_{q m i} & -K_{v m i} & -K_{D C m i}
\end{array}\right]^{T}, \\
& G_{i j}=\left[\begin{array}{llll}
0 \frac{-\left(\omega_{0} E_{q i}^{\prime} E_{q j}^{\prime} B_{i j}\right)}{M_{i}} & 0 & 0 & 0
\end{array}\right]^{T} . \tag{3}
\end{align*}
$$

In this modeling, the single-machine infinite bus is modeled by Heffron-Phillips model which is shown in Figure 1.

The nonlinear saturation function $u_{s i}(t)$ is considered to be inside sector $\left(a_{i}, 1\right)$ and is shown in Figure 1, where $0 \leqslant$ $a_{i} \leqslant 1$.

From Figure 2, it is obvious that $u_{s i}(t)-0.5\left(1+a_{i}\right) u_{i}(t)=$ $\Delta t_{i} u_{i}(t) \Rightarrow u_{s i}(t)=\psi_{i} u_{i}(t)$, where $\psi_{i}=0.5\left(1+a_{i}\right)+\Delta t_{i}$, $\Delta t_{i}$ is a real number which varying between $-0.5\left(1-\bar{a}_{i}\right)$ and $0.5\left(1+\bar{a}_{i}\right), a_{i}<\bar{a}_{i}<1$.

The control law for a PD controller is

$$
\begin{equation*}
u_{i}(t)=K_{p i} x_{i}(t)+K_{D i} \dot{x}_{i}(t) . \tag{4}
\end{equation*}
$$

Substituting (4) into (1), we have

$$
\begin{align*}
& \left(I-\psi_{i} B_{i} K_{D i}\right) \dot{x}_{i}(t) \\
& \quad=\left(A_{i}+\psi_{i} B_{i} K_{p i}\right) x_{i}(t) \\
& \quad+\sum_{j=1}^{N} p_{i j} G_{i j} g_{i j}\left(x_{i}(t), x_{j}\left(t-\tau_{i j}(t)\right)\right),  \tag{5}\\
& x_{i}(t)=\phi_{i}(t), \quad t \in[-\tau, 0] .
\end{align*}
$$

Taking the inverse of the left-hand side of (5), we obtain

$$
\begin{align*}
& \dot{x}_{i}(t)=\left(I-\psi_{i} B_{i} K_{D i}\right)^{-1}\left(\left(A_{i}+\psi_{i} B_{i} K_{p i}\right) x_{i}(t)\right. \\
& \left.\quad+\sum_{j=1}^{N} p_{i j} G_{i j} g_{i j}\left(x_{i}(t), x_{j}\left(t-\tau_{i j}(t)\right)\right)\right),  \tag{6}\\
& x_{i}(t)=\phi_{i}(t), \quad t \in[-\tau, 0] .
\end{align*}
$$

Properly selecting the controller gains $K_{p i}$ and $K_{D i}$, so that the closed-loop systems are stable, then we have the PD controller design. It is obvious from (6) that the PD controller is nonlinear. Some researchers have designed such a controller with the aid of searching algorithms [10]. A huge amount of computation burden is foreseeable. In the following, we introduce a new state variable $\bar{x}_{i}(t)=\left[\begin{array}{ll}x_{i}^{T}(t) & \dot{x}_{i}^{T}(t)\end{array}\right]^{T}$, then system (1) with controller (4) is transformed into the following PD control system:

$$
\begin{align*}
E \dot{\bar{x}}_{i}(t)= & \bar{A}_{i} \bar{x}_{i}(t)+\bar{B}_{i} \bar{u}_{s i}(t) \\
& +\sum_{j=1}^{N} p_{i j} \bar{G}_{i j} \bar{g}_{i j}\left(\bar{x}_{i}(t), \bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right),  \tag{7}\\
\bar{u}_{s i}(t)= & \operatorname{sat}\left(\bar{u}_{i}(t)\right), \\
\bar{u}_{i}(t)= & \bar{K}_{i} \bar{x}_{i}(t),
\end{align*}
$$



FIgure 1: Heffron-Phillips model for single-machine power system connected to infinite bus along with SSSC series in the transmission line.


Figure 2: Sector saturation function.
where

$$
\begin{align*}
& \bar{A}_{i}=\left[\begin{array}{cc}
0 & I \\
A_{i} & -I
\end{array}\right],  \tag{8}\\
& \bar{B}_{i}=\left[\begin{array}{c}
0 \\
B_{i}
\end{array}\right] \\
& \bar{G}_{i j}=\left[\begin{array}{c}
0 \\
G_{i j}
\end{array}\right], \\
& E=\left[\begin{array}{cc}
I_{n} & 0 \\
0 & 0
\end{array}\right]
\end{align*}
$$

$$
\begin{aligned}
& \bar{g}_{i j}\left(\bar{x}_{i}(t), \bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)=g_{i j}\left(x_{i}(t), x_{j}\left(t-\tau_{i j}(t)\right)\right) \\
& \quad=-\cos \beta_{i}(\delta) \bar{W}\left(\bar{x}_{i}(t)-\bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)
\end{aligned}
$$

$$
\bar{K}_{i}=\left[\begin{array}{ll}
K_{p i} & K_{D i}
\end{array}\right] \text {, and } \bar{W}=\left[\begin{array}{ll}
W & 0
\end{array}\right] .
$$

$$
\text { System (7) is a descriptor system as } \operatorname{rank}(E)=n<
$$ $\operatorname{dim}(E), \bar{u}_{i}(t)=K_{p i} x_{i}(t)+K_{D i} \dot{x}_{i}(t)=\bar{K}_{i} \bar{x}_{i}(t)$; then, we have

$$
\begin{aligned}
& E \dot{\bar{x}}_{i}(t) \\
& \quad=\left(\bar{A}_{i}+\psi_{i} \bar{B}_{i} \bar{K}_{i}\right) \bar{x}_{i}(t) \\
& \quad+\sum_{j=1}^{N} p_{i j} \bar{G}_{i j} \bar{g}_{i j}\left(\bar{x}_{i}(t), \bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)
\end{aligned}
$$

$$
\begin{align*}
= & \left(\bar{A}_{i}+\psi_{i} \bar{B}_{i} \bar{K}_{i}\right) \bar{x}_{i}(t) \\
& -\cos \beta_{i}(\delta) \sum_{j=1}^{N} p_{i j} \bar{G}_{i j} \bar{W}\left(\bar{x}_{i}(t)-\bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right) \\
= & \left(\bar{A}_{c i}-\sum_{j=1}^{N} A_{d i j}\right) \bar{x}_{i}(t)+\sum_{j=1}^{N} A_{d i j} \bar{x}_{j}\left(t-\tau_{i j}(t)\right), \tag{9}
\end{align*}
$$

where $A_{d i j}=\cos \beta_{i}(\delta) p_{i j} \bar{G}_{i j} \bar{W}$ and $\bar{A}_{c i}=\bar{A}_{i}+\psi_{i} \bar{B}_{i} \bar{K}_{i}$.
The following definition and lemmas will be useful in this paper.

Definition 1 (see [20]). (i) Descriptor system

$$
\begin{equation*}
E \dot{x}(t)=A x(t)+\sum_{j=1}^{N} A_{d j} x\left(t-d_{j}\right) \tag{10}
\end{equation*}
$$

is said to be regular and impulse-free, if pair $(E, A)$ is regular and impulse-free.
(ii) System (10) is said to be stable if for any scalar $\varepsilon>0$, there exists a scalar $\delta(\varepsilon)>0$ such that, for any compatible initial conditions $\phi(k)$ satisfying $\sup _{-d \leq t \leq 0}\|\phi(t)\| \leq \delta(\varepsilon)$, solution $x(t)$ of system (10) satisfies $\|x(t)\| \leq \varepsilon$ for any $t \geq 0$; moreover, $\lim _{t \rightarrow \infty} x(t)=0$.
(iii) System (10) is said to be admissible if it is regular, impulse-free, and stable.

Lemma 2 (see [18]). For any constant matrix $M>0$, scalar $\gamma>0$, and vector function $W:[0, \gamma] \rightarrow R^{n}$, such that integrations concerned are well defined, the following inequality holds:

$$
\begin{align*}
& \left(\int_{0}^{\gamma} W(s) d s\right)^{T} M\left(\int_{0}^{\gamma} W(s) d s\right)  \tag{11}\\
& \quad \leqslant \gamma\left(\int_{0}^{\gamma} W^{T}(s) M W(s) d s\right)
\end{align*}
$$

Lemma 3. Given any matrices $D$ and $E$ with appropriate dimensions, the inequality

$$
\begin{equation*}
D E+E^{T} D^{T} \leqslant D T D^{T}+E^{T} T^{-1} E \tag{12}
\end{equation*}
$$

holds for any matrix $T>0$.

## 3. Main Results

In this section, we will give the following condition for system (9).

Theorem 4. The delay descriptor system (9) is admissible with $\bar{K}_{i}=Y_{i} X_{i}^{-T}$ if there exist matrices $X_{i}=\left[\begin{array}{cc}X_{1 i} & X_{2 i} \\ 0 & X_{3 i}\end{array}\right], Q_{i j}>0$, $T>0, Y_{i}, U_{i j}>0,(i, j=1,2, \ldots, N)$, such that the following inequalities hold:

$$
\begin{equation*}
E X_{i}^{T}=X_{i} E^{T} \geqslant 0 \tag{13}
\end{equation*}
$$

$$
\left[\begin{array}{cccc}
\operatorname{sym}\left\{\bar{A}_{i} X_{i}^{T}+\psi_{i} \bar{B}_{i} Y_{i}\right\} & * & \cdots & * \\
+\tau \sum_{j=1}^{N} p_{j i} E^{T} Q_{j i} E-\frac{1-\dot{\tau}^{*}}{\tau} \sum_{j=1}^{N} p_{j i} E^{T} Q_{j i} E & & & * \\
+\sum_{j=1}^{N} p_{j i} U_{j i}+\sum_{j=1}^{N} p_{i j} \bar{G}_{i j} \bar{W} T \bar{W}^{T} \bar{G}_{i j}^{T} & & & \\
\frac{1-\dot{\tau}^{*}}{\tau} E^{T} Q_{1 i} E & -\alpha\left(1-\dot{\tau}^{*}\right) U_{1 i} & & \\
\vdots & -\frac{1-\dot{\tau}^{*}}{\tau} E^{T} Q_{1 i} E & & \\
\frac{1-\dot{\tau}^{*}}{\tau} E^{T} Q_{N i} E & \ddots & \\
& & & -\frac{1-\dot{\tau}^{*}}{\tau} E^{T} Q_{N i} E
\end{array}\right]<0,
$$

$$
\left[\begin{array}{cc}
-(1-\alpha)\left(1-\dot{\tau}^{*}\right) U_{i j} & X_{i}  \tag{15}\\
X_{i}^{T} & -T
\end{array}\right]<0
$$

where $\alpha$ is a fixed scalar which satisfies $0<\alpha<1$.

Proof. Firstly, we prove that system (9) with PD gain matrices $\bar{K}_{i}$ is regular and impulse-free. System (9) can be rewritten as

$$
\begin{align*}
\bar{E} \dot{X}(t)= & \left(\widetilde{A}+\psi \widetilde{B} \widetilde{K}-\sum_{i=1}^{N} \sum_{j=1}^{N} \widetilde{A}_{d i j}\right) X(t)  \tag{16}\\
& +\sum_{i=1}^{N} \sum_{j=1}^{N} \widetilde{A}_{d i j} X\left(t-\tau_{i j}(t)\right)
\end{align*}
$$

where $X(t)=\left[\bar{x}_{1}^{T}(t), \bar{x}_{2}^{T}(t), \ldots, \bar{x}_{N}^{T}(t)\right]^{T}$, matrices $\bar{E}=\operatorname{diag}\{E$, $E, \ldots, E\}, \widetilde{A}=\operatorname{diag}\left\{\bar{A}_{1}, \bar{A}_{2}, \ldots, \bar{A}_{N}\right\}, \psi=\operatorname{diag}\left\{\psi_{1}, \psi_{2}, \ldots\right.$, $\left.\psi_{N}\right\}, \widetilde{B}=\operatorname{diag}\left\{\bar{B}_{1}, \bar{B}_{2}, \ldots, \bar{B}_{N}\right\}, \widetilde{K}=\operatorname{diag}\left\{\bar{K}_{1}, \bar{K}_{2}, \ldots, \bar{K}_{N}\right\}$, and

$$
\widetilde{A}_{d i j}=\left[\begin{array}{ccccc}
0 & \cdots & 0 & \cdots & 0  \tag{17}\\
0 & \cdots & \cos \beta_{i}(\delta) p_{i j} \bar{G}_{i j} \bar{W} & \cdots & 0 \\
0 & \cdots & 0 & \cdots & 0
\end{array}\right] \quad i
$$

From (14), it is easy to see that

$$
\begin{align*}
& \operatorname{sym}\left\{\bar{A}_{i} X_{i}^{T}+\psi_{i} B_{i} Y_{i}\right\}-\frac{1-\dot{\tau}^{*}}{\tau} \sum_{j=1}^{N} p_{j i} E^{T} Q_{j i} E \\
& \quad=\operatorname{sym}\left\{\left(\bar{A}_{i}+\psi_{i} \bar{B}_{i} K_{i}\right) X_{i}^{T}\right\}-\frac{1-\dot{\tau}^{*}}{\tau} \sum_{j=1}^{N} p_{j i} E^{T} Q_{j i} E \tag{18}
\end{align*}
$$

$$
\begin{align*}
\operatorname{sym}\{ & \left.(\widetilde{A}+\psi \widetilde{B} \widetilde{K}) X^{T}\right\} \\
& -\frac{1-\dot{\tau}^{*}}{\tau} E^{T} \operatorname{diag}\left\{\sum_{j=1}^{N} p_{j i} Q_{j i}\right\} \bar{E}<0, \tag{19}
\end{align*}
$$

$$
<0
$$

That is
where $X=\operatorname{diag}\left\{X_{1}, \ldots, X_{N}\right\}$. Since $\bar{E}$ is descriptor, there exist nonsingular matrices $G$ and $H$ such that

$$
G \bar{E} H=\left[\begin{array}{cc}
I_{n N} & 0  \tag{20}\\
0 & 0
\end{array}\right]
$$

## Suppose

$$
\begin{align*}
G(\widetilde{A}+\psi \widetilde{B} \widetilde{K}) H & =\left[\begin{array}{ll}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{array}\right] \\
G X H & =\left[\begin{array}{cc}
X_{11} & X_{12} \\
0 & X_{22}
\end{array}\right] \tag{21}
\end{align*}
$$

Equation (19) yields $\operatorname{sym}\left\{A_{22} X_{22}^{T}\right\}<0$, which implies that the pair $(\bar{E}, \widetilde{A}+\psi \widetilde{B} \widetilde{K})$ is regular and impulse-free. By Definition 1 , system (16) is regular and impulse-free. It also shows that system (9) is regular and impulse-free.

Next, we will show that system (9) is stable. Consider a Lyapunov-Krasovskii functional as

$$
\begin{align*}
V(t) & =\sum_{i=1}^{N}\left\{x_{i}^{T}(t) P_{11 i} x_{i}(t)+\sum_{j=1}^{N} p_{i j}\right. \\
& \cdot\left[\int_{t-\tau_{i j}(t)}^{t} \bar{x}_{j}^{T}(\xi) U_{i j} \bar{x}_{j}(\xi) d \xi\right.  \tag{22}\\
& \left.\left.+\int_{-\tau_{i j}(t)}^{0} \int_{t+\alpha}^{t} \dot{\bar{x}}_{j}^{T}(\xi) E^{T} Q_{i j} E \dot{\bar{x}}_{j}^{T}(\xi) d \xi d \alpha\right]\right\}
\end{align*}
$$

Taking the time derivative of $V(t)$ along the solution of system (9) yields

$$
\dot{V}(t)=\sum_{i=1}^{N}\left\{\dot{x}_{i}^{T}(t) P_{11 i} x_{i}(t)+x_{i}^{T}(t) P_{11 i} \dot{x}_{i}(t)+\sum_{j=1}^{N} p_{i j}\right.
$$

$$
\cdot\left[\bar{x}_{j}^{T}(t) U_{i j} \bar{x}_{j}(t)\right.
$$

$$
\begin{equation*}
-\left(1-\dot{\tau}_{i j}(t)\right) \bar{x}_{j}^{T}(t)\left(t-\tau_{i j}(t)\right) U_{i j} \bar{x}_{j}\left(t-\tau_{i j}(t)\right) \tag{23}
\end{equation*}
$$

$$
+\tau_{i j}(t) \bar{x}_{j}^{T}(t) E^{T} Q_{i j} E \bar{x}_{j}(t)
$$

$$
\left.\left.-\left(1-\dot{\tau}_{i j}(t)\right) \int_{t-\tau_{i j}(t)}^{t} \dot{\bar{x}}_{j}(\alpha) E^{T} Q_{i j} E \bar{x}_{j}(\alpha) d \alpha\right]\right\}
$$

By Lemma 2, we obtain

$$
\begin{align*}
& -\left(1-\dot{\tau}_{i j}(t)\right) \int_{t-\tau_{i j}(t)}^{t} \dot{\bar{x}}_{j}(\alpha) E^{T} Q_{i j} E \dot{\bar{x}}_{j}(\alpha) d \alpha \\
& \quad \leqslant-\frac{1-\dot{\tau}_{i j}(t)}{\tau_{i j}(t)}\left(\bar{x}_{j}(t)-x_{j}\left(t-\tau_{i j}(t)\right)\right)^{T}  \tag{24}\\
& \quad \cdot E^{T} Q_{i j} E\left(\bar{x}_{j}(t)-\bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)
\end{align*}
$$

Then,

$$
\begin{align*}
& \dot{V}(t) \leqslant \sum_{i=1}^{N}\left\{\dot{\bar{x}}_{i}^{T}(t) E^{T} P_{i} \bar{x}_{i}(t)+\bar{x}_{i}^{T}(t) P_{i}^{T} E \bar{x}_{i}(t)\right. \\
& +\sum_{j=1}^{N} p_{i j}\left[\bar{x}_{j}^{T}(t) U_{i j} \bar{x}_{j}(t)-\left(1-\dot{\tau}^{*}\right) \bar{x}_{j}^{T}\left(t-\tau_{i j}(t)\right)\right. \\
& \cdot U_{i j} \bar{x}_{j}\left(t-\tau_{i j}(t)\right)+\tau \bar{x}_{j}^{T}(t) E^{T} Q_{i j} E \bar{x}_{j}(t) \\
& -\frac{1-\dot{\tau}^{*}}{\tau}\left(\bar{x}_{j}(t)-\bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)^{T} \\
& \left.\left.\cdot E^{T} Q_{i j} E\left(\bar{x}_{j}(t)-\bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)\right]\right\}=\sum_{i=1}^{N}\left\{\bar{x}_{i}^{T}(t)\right. \\
& \cdot\left[\left(\bar{A}_{i}+\psi_{i} \bar{B}_{i} \bar{K}_{i}\right)^{T} P_{i}+P_{i}^{T}\left(\bar{A}_{i}+\psi_{i} \bar{B}_{i} \bar{K}_{i}\right)\right] \bar{x}_{i}(t) \\
& +\operatorname{sym}\left\{\bar{x}_{i}^{T}(t) P_{i}^{T}\left(-\cos \beta_{i}(\delta)\right) \sum_{j=1}^{N} p_{i j}\right.  \tag{25}\\
& \left.\cdot \bar{G}_{i j} \bar{W}\left(\bar{x}_{i}(t)-\bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)\right\}+\sum_{j=1}^{N} p_{i j} \bar{x}_{j}^{T}(t) \\
& \cdot U_{i j} \bar{x}_{j}(t)-\sum_{j=1}^{N}\left(1-i^{*}\right) p_{i j} \bar{x}_{j}^{T}\left(t-\tau_{i j}(t)\right) U_{i j} \bar{x}_{j}(t \\
& \left.-\tau_{i j}(t)\right)+\sum_{j=1}^{N} p_{i j} \tau \bar{x}_{j}^{T}(t) E^{T} Q_{i j} E \bar{x}_{j}(t)-\sum_{j=1}^{N} p_{i j} \\
& \cdot \frac{1-\dot{\tau}^{*}}{\tau}\left(\bar{x}_{j}(t)-\bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)^{T} E^{T} Q_{i j} E\left(\bar{x}_{j}(t)\right. \\
& \left.\left.-\bar{x}_{j}\left(t-\tau_{i j}(t)\right)\right)\right\},
\end{align*}
$$

where

$$
P_{i}=\left[\begin{array}{cc}
P_{11 i} & P_{12 i}  \tag{26}\\
0 & P_{22 i}
\end{array}\right] .
$$

It is obvious that $\dot{V}(t)<0$ can be obtained by the following equation:

$$
\begin{align*}
\dot{\bar{V}} & =\operatorname{diag}\left\{\dot{V}_{1}(t), \ldots, \dot{V}_{N}(t)\right\}=X^{T}(t)\left[(\widetilde{A}+\psi \widetilde{B} \widetilde{K})^{T} P\right. \\
& \left.+P^{T}(\widetilde{A}+\psi \widetilde{B} \widetilde{K})\right] X(t)+\operatorname{sym}\left\{-X^{T}(t) P^{T} \sum_{i=1}^{N} \sum_{j=1}^{N} \widetilde{A}_{d i j} X(t)\right. \\
& \left.+X^{T}(t) P^{T} \sum_{i=1}^{N} \sum_{j=1}^{N} \widetilde{A}_{d i j} X\left(t-\tau_{i j}(t)\right)\right\} \\
& +X^{T}(t) \operatorname{diag}\left\{\sum_{j=1}^{N} p_{j 1} U_{j 1}, \ldots, \sum_{j=1}^{N} p_{j N} U_{j N}\right\} X(t)  \tag{27}\\
& -\sum_{i=1}^{N} \sum_{j=1}^{N}\left(1-\dot{\tau}^{*}\right) X^{T}\left(t-\tau_{i j}(t)\right) \mathscr{A}_{i j} X\left(t-\tau_{i j}(t)\right) \\
& +\tau X^{T}(t) \operatorname{diag}\left\{\sum_{j=1}^{N} p_{j 1} E^{T} Q_{j 1} E, \ldots, \sum_{j=1}^{N} p_{j N} E^{T} Q_{j N} E\right\} X(t) \\
& -\sum_{i=1}^{N} \sum_{j=1}^{N} p_{i j} \frac{1-\dot{\tau}^{*}}{\tau}\left(X(t)-X\left(t-\tau_{i j}(t)\right)\right)^{T} \mathfrak{Q}_{i j}(X(t) \\
& \left.-X\left(t-\tau_{i j}(t)\right)\right),
\end{align*}
$$

where

$$
\begin{align*}
P & =\operatorname{diag}\left\{P_{1}, P_{2}, \ldots, P_{N}\right\}, \\
\mathfrak{U}_{i j} & =\left[\begin{array}{ccccc}
0 & \cdots & 0 & \cdots & 0 \\
0 & \cdots & p_{j i} U_{j i} & \cdots & 0 \\
0 & \cdots & 0 & \cdots & 0
\end{array}\right], i  \tag{28}\\
\mathfrak{Q}_{i j} & =\left[\begin{array}{ccccc}
0 & \cdots & 0 & \cdots & 0 \\
0 & \cdots & E^{T} Q_{j i} E & \cdots & 0 \\
0 & \cdots & 0 & \cdots & 0
\end{array}\right] i
\end{align*}
$$

It is obtained from Lemma 3 that

$$
\begin{aligned}
& \operatorname{sym}\left\{-X^{T}(t) P^{T} \sum_{i=1}^{N} \sum_{j=1}^{N} \widetilde{A}_{d i j} X(t)\right\}+X^{T}(t) P^{T} \sum_{i=1}^{N} \sum_{j=1}^{N} \widetilde{A}_{d i j} X\left(t-\tau_{i j}(t)\right) \\
& \quad \leqslant X^{T}(t) \operatorname{diag}\left\{P_{1}^{T} \sum_{j=1}^{N} p_{1 j} \bar{G}_{1 j} \bar{W} T \bar{W}^{T} G_{1 j}^{T} P_{1}, \ldots, P_{N}^{T} \sum_{j=1}^{N} p_{N j} \bar{G}_{N j} \bar{W} T \bar{W}^{T} G_{N j}^{T} P_{N}\right\} X(t)
\end{aligned}
$$

$$
\begin{equation*}
+X^{T}(t) \operatorname{diag}\left\{\sum_{j=1}^{N} p_{1 j} T^{-1}, \ldots, \sum_{j=1}^{N} p_{N j} T^{-1}\right\} X(t)+\sum_{i=1}^{N} \sum_{j=1}^{N} p_{i j} X^{T}\left(t-\tau_{i j}(t)\right) \mathfrak{T} X\left(t-\tau_{i j}(t)\right) \tag{29}
\end{equation*}
$$

where $\mathfrak{T}=\operatorname{diag}\left\{0, \ldots, 0, T^{-1}, 0, \ldots, 0\right\}$.
Substituting (29) into (27), we obtain

$$
+\sum_{i=1}^{N} \sum_{j=1}^{N} p_{i j} \frac{1-\dot{\tau}^{*}}{\tau} \operatorname{sym}\left\{X^{T}(t) \mathfrak{Q}_{i j} X\left(t-\tau_{i j}(t)\right)\right\}
$$

$$
\begin{align*}
\dot{\bar{V}} & \leqslant X^{T}(t) \Phi X(t)+\sum_{i=1}^{N} \sum_{j=1}^{N} p_{i j} X^{T}\left(t-\tau_{i j}(t)\right)  \tag{30}\\
& \cdot\left(\mathfrak{T}-\left(1-\dot{\tau}^{*}\right) \mathfrak{U}_{i j}-\frac{1-\dot{\tau}^{*}}{\tau} \mathfrak{Q}_{i j}\right) X\left(t-\tau_{i j}(t)\right)
\end{align*}
$$

$$
=\bar{\xi}^{T}(t) \Xi \bar{\xi}(t)
$$

where $\bar{\xi}(t)=\left[X^{T}(t)\left[X^{T}\left(t-\tau_{i j}\right)\right]_{i, j=1, \ldots, N, i \neq j}\right]^{T}$,

From (30), we obtain that

$$
\begin{equation*}
\left[T^{-1}-(1-\alpha)\left(1-\dot{\tau}^{*}\right) U_{j i}\right]<0, \quad i \neq j \tag{34}
\end{equation*}
$$

$$
\begin{equation*}
\dot{V}(t) \leqslant \sum_{i=1}^{N} \xi_{i}^{T}(t) \Xi_{i} \xi_{i}(t) \tag{33}
\end{equation*}
$$

where $\xi_{i}(t)=\left[\bar{x}_{i}^{T}(t), \bar{x}_{i}^{T}\left(t-\tau_{1 i}(t)\right), \ldots, \bar{x}_{i}^{T}\left(t-\tau_{(i-1) i}(t)\right), \bar{x}_{i}^{T}(t-\right.$ $\left.\left.\tau_{(i+1) i}(t)\right), \ldots, \bar{x}_{i}^{T}\left(t-\tau_{N i}(t)\right)\right]^{T}$ and

$$
\begin{align*}
& \Phi=(\widetilde{A}+\psi \widetilde{B} \widetilde{K})^{T} P+P^{T}(\widetilde{A}+\psi \widetilde{B} \widetilde{K})+\operatorname{diag}\left\{P_{1}^{T} \sum_{j=1}^{N} p_{1 j} \bar{G}_{1 j} \bar{W} T \bar{W}^{T} G_{1 j}^{T} P_{1}, \ldots, P_{N}^{T} \sum_{j=1}^{N} p_{N j} \bar{G}_{N j} \bar{W} T \bar{W}^{T} G_{N j}^{T} P_{N}\right\} \\
& +\operatorname{diag}\left\{\sum_{j=1}^{N} p_{1 j} T^{-1}, \ldots, \sum_{j=1}^{N} p_{N j} T^{-1}\right\}+\operatorname{diag}\left\{\sum_{j=1}^{N} p_{j 1} U_{j 1}, \ldots, \sum_{j=1}^{N} p_{j N} U_{j N}\right\}  \tag{31}\\
& -\frac{1-\dot{\tau}^{*}}{\tau} \operatorname{diag}\left\{\sum_{j=1}^{N} p_{j 1} E^{T} Q_{j 1} E, \ldots, \sum_{j=1}^{N} p_{j N} E^{T} Q_{j N} E\right\}+\tau \operatorname{diag}\left\{\sum_{j=1}^{N} p_{j 1} E^{T} Q_{j 1} E, \ldots, \sum_{j=1}^{N} p_{j N} E^{T} Q_{j N} E\right\}, \\
& \bar{\Xi}=\left[\begin{array}{ccc}
\Phi & * & \cdots \\
\sum_{i=1}^{N} \frac{1-\dot{\tau}^{*}}{\tau} \mathfrak{Q}_{1 i} & \sum_{i=1}^{N}\left(\mathfrak{T}-\left(1-\dot{\tau}^{*}\right) \mathfrak{U}_{1 i}-\frac{1-\dot{\tau}^{*}}{\tau} \mathfrak{Q}_{1 i}\right) & \\
\vdots & \ddots & * \\
\sum_{i=1}^{N} \frac{1-\dot{\tau}^{*}}{\tau} \mathfrak{Q}_{N i} & & \sum_{i=1}^{N}\left(\mathfrak{T}-\left(1-\dot{\tau}^{*}\right) \mathfrak{U}_{N i}-\frac{1-\dot{\tau}^{*}}{\tau} \mathfrak{Q}_{N i}\right)
\end{array}\right]<0 . \tag{32}
\end{align*}
$$

$$
\left[\begin{array}{ccll}
\operatorname{sym}\left\{\left(\bar{A}_{i}+\psi_{i} \bar{B}_{i} \bar{K}_{i}\right)^{T} P_{i}\right\} & * & \cdots & *  \tag{35}\\
+P_{i}^{T} \sum_{j=1}^{N} p_{i j} \bar{G}_{i j} \bar{W} T \bar{W}^{T} \bar{G}_{i j}^{T} P_{i} & & & \\
+\sum_{j=1}^{N} p_{i j} T^{-1}+\sum_{j=1}^{N} p_{j i} U_{j i} & & & \\
\quad+\tau \sum_{j=1}^{N} p_{j i} E^{T} Q_{j i} E & & \\
-\frac{1-i^{*}}{\tau} \sum_{j=1}^{N} p_{j i} E^{T} Q_{j i} E & & \\
\frac{1-i^{*}}{\tau} \sum_{j=1}^{N} E^{T} Q_{1 i} E & -\alpha\left(1-i^{*}\right) U_{1 i} & \\
& -\frac{1-i^{*}}{\tau} \sum_{j=1}^{N} E^{T} Q_{1 i} E & \\
\vdots & & & -\alpha\left(1-i^{*}\right) U_{N i} \\
\frac{1-i^{*}}{\tau} \sum_{j=1}^{N} E^{T} Q_{N i} E & & -\frac{1-i^{*}}{\tau} \sum_{j=1}^{N} E^{T} Q_{N i} E
\end{array}\right]<0 .
$$

Pre- and postmultiply (45) by $\operatorname{diag}\left\{X_{i}, X_{i}, \ldots, X_{i}\right\}$ and its transpose, respectively, where $X_{i}=P_{i}^{-T}$, and define $Y_{i}=\bar{K}_{i} X_{i}^{T}, Q_{j i}=X_{i} Q_{j i} X_{i}^{T}$, and $U_{j i}=X_{i} U_{j i} X_{i}^{T}$; we arrive at

$$
\left[\begin{array}{cccc}
\operatorname{sym}\left\{\bar{A}_{i} X_{i}^{T}+\psi_{i} \bar{B}_{i} Y_{i}\right\}+\sum_{j=1}^{N} p_{j i} U_{j i} & * & \cdots & *  \tag{36}\\
+\sum_{j=1}^{N} \tau p_{j i} E^{T} Q_{j i} E-\sum_{j=1}^{N} \frac{1-\dot{\tau}^{*}}{\tau} p_{j i} E^{T} Q_{j i} E & & & \\
+\sum_{j=1}^{N} p_{i j} \bar{G}_{i j} \bar{W} T \bar{W}^{T} G_{i j}^{T}+X_{i} \sum_{j=1}^{N} p_{j i} T^{-1} X_{i}^{T} & & & \\
\frac{1-\dot{\tau}^{*}}{\tau} E^{T} Q_{1 i} E & -\alpha\left(1-\dot{\tau}^{*}\right) U_{1 i} & & \\
\vdots & -\frac{1-\dot{\tau}^{*}}{\tau} E^{T} Q_{1 i} E & & \\
\frac{1-\dot{\tau}^{*}}{\tau} E^{T} Q_{N i} E & \ddots & \\
& & & -\alpha\left(1-\dot{\tau}^{*}\right) U_{N i} \\
& & 1-\dot{\tau}^{*} \\
\tau & E^{T} Q_{N i} E
\end{array}\right]
$$

By Schur complement, (44) is equivalent to (14).

Pre-and postmultiplying (34) by $X_{i}$ and its transpose, respectively, and by Schur complement, (34) is equivalent to (15).

Since $A_{22}$ is nonsingular, there exist two nonsingular matrices $\bar{G}$ and $\bar{H}$ such that

$$
\begin{gathered}
\bar{G} \bar{E} \bar{H}=\left[\begin{array}{cc}
I_{n N} & 0 \\
0 & 0
\end{array}\right], \\
\bar{G}(A+\psi B K) \bar{H}=\left[\begin{array}{cc}
A_{1} & 0 \\
0 & I_{n N}
\end{array}\right], \\
\bar{H}^{T} \bar{E}^{T} P \bar{E} \bar{H}=\left[\begin{array}{cc}
\bar{P}_{1} & 0 \\
0 & 0
\end{array}\right], \\
\bar{G} A_{i j} \bar{H}=\left[\begin{array}{cc}
A_{d i j 11} & A_{d i j 12} \\
A_{d i j 21} & A_{d i 22}
\end{array}\right], \\
\bar{H}^{T} P \bar{H}=\left[\begin{array}{cc}
\bar{P}_{11} & \bar{P}_{12} \\
0 & \bar{P}_{22}
\end{array}\right], \\
\bar{H}^{T} \mathfrak{u}_{j i} \bar{H}=\left[\begin{array}{cc}
U_{j i 11} & U_{j i 12} \\
U_{j i 21} & U_{j i 22}
\end{array}\right] .
\end{gathered}
$$

Denote $V(t)=\bar{H}^{-1} X(t)=\left[\begin{array}{l}v_{1}(t) \\ v_{2}(t)\end{array}\right]$, where $Q_{1}(t) \in \mathbb{R}^{n N}$ and $v_{2}(t) \in \mathbb{R}^{n N}$. We obtain

$$
\begin{align*}
-\|X(t)\|^{2} & \leqslant-\left\|\bar{H}^{-1}\right\|^{-2}\|V(t)\|^{2}  \tag{38}\\
& \leqslant-\left\|\bar{H}^{-1}\right\|^{-2}\left\|v_{1}(t)\right\|^{2}
\end{align*}
$$

By (30), we obtain that there exists scalar $\varepsilon>0$ such that

$$
\begin{equation*}
\dot{V}(t)<-\varepsilon\|X(t)\|^{2}, \tag{39}
\end{equation*}
$$

which implies

$$
\begin{align*}
& \lambda \min \left(\bar{P}_{1}\right)\left\|v_{1}(t)\right\|^{2}-V(0) \leqslant \int_{0}^{t} \dot{V}(s) d s \\
& \quad \leqslant-\varepsilon\left\|\bar{H}^{-1}\right\|^{-2} \int_{0}^{t}\left\|v_{1}(s)\right\|^{2} d s \tag{40}
\end{align*}
$$

Then, $\sum_{i=1}^{N}\left\|v_{1}(t)\right\|^{2} \leqslant V(0) / \lambda \min \left(\bar{P}_{1}\right)$, and $\int_{0}^{t}\left\|v_{1}(s)\right\|^{2} d s \leqslant$ $(1 / \varepsilon) V(0)\left\|\bar{H}^{-1}\right\|^{-2}$.

Thus,

$$
\begin{equation*}
\lim _{t \rightarrow \infty} v_{1}(t)=0 \tag{41}
\end{equation*}
$$

Next, we will prove that $\lim _{t \rightarrow \infty} v_{2}(t)=0$, which is equivalent to the stability of system (5).

Pre-and postmultiplying (32) by $\operatorname{diag}\left\{\bar{H}^{T}, \bar{H}^{T}, \ldots, \bar{H}^{T}\right\}$ and its transpose, respectively, we obtain

$$
\left[\begin{array}{cccc}
\bar{P}_{22}+\bar{P}_{22}^{T}+\sum_{i=1}^{N} \sum_{j=1}^{N} U_{j i 22} & * & \cdots & *  \tag{42}\\
\sum_{i=1}^{N} \bar{P}_{22}^{T} A_{d i 122} & -\sum_{i=1}^{N}\left(1-\tau^{*}\right) U_{1 i 22} & & \\
\vdots & \ddots & \\
\sum_{i=1}^{N} \bar{P}_{22}^{T} A_{d i N 22} & & & -\sum_{i=1}^{N}\left(1-\tau^{*}\right) U_{N i 22}
\end{array}\right]<0
$$

Pre- and postmultiplying (42) by $\left[-\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 22}^{T} I \cdots I\right]$ and its transpose, respectively, we obtain $\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 22}^{T}$ - $\sum_{i=1}^{N} \sum_{j=1}^{N} U_{j i 22} \sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 22}-\sum_{i=1}^{N} \sum_{j=1}^{N}\left(1-\tau^{*}\right) U_{j i 22}<0$, which implies that

$$
\begin{equation*}
\rho\left(\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 22}\right)<1 \tag{43}
\end{equation*}
$$

Using the expression in (37), the singular delay system (16) can be decomposed as

$$
\begin{align*}
\dot{v}_{1}(t)= & A_{1} v_{1}(t)+\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 11} v_{1}\left(t-\tau_{i j}(t)\right) \\
& +\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 12} v_{2}\left(t-\tau_{i j}(t)\right) \tag{44}
\end{align*}
$$

$$
\begin{gather*}
0=v_{2}(t)+\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 21} v_{1}\left(t-\tau_{i j}(t)\right) \\
+\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 22} v_{2}\left(t-\tau_{i j}(t)\right) . \tag{45}
\end{gather*}
$$

Noting that, for any $t>0$, there exists positive integer $k$.
Such that $k \tau-\tau \leqslant t \leqslant k \tau$, and considering (44), we have $v_{2}(t)$

$$
\begin{align*}
= & \left(-\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 22}\right)^{k} v_{2}(t-k \tau)  \tag{46}\\
& -\sum_{l=1}^{k}\left(\sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 22}\right)^{l-1} \sum_{i=1}^{N} \sum_{j=1}^{N} A_{d i j 21} v_{1}(t-k \tau) .
\end{align*}
$$



Figure 3: Connecting diagram of two-machine system.

This, together with (43) and (41), we obtain $\lim _{t \rightarrow \infty} v_{2}(t)=0$, and this completes the proof.

Remark 5. Theorem 4 provides a PD control method for system (1). An LMI based criterion is obtained by transforming a regular system into the state feedback stabilizer design for a descriptor system. It is worth noting that if $\bar{A}_{i}, \bar{B}_{i}, \bar{G}_{i j}$,
$\bar{W}$, and $E$ are replaced with $A_{i}, B_{i}, G_{i j}, W$, and $I_{n}$, the state feedback controller can be solved by the following corollary. It is obvious that Theorem 4 has wider range of application.

Corollary 6. The delay system (5) with $K_{D i}=0$ is stable with $K_{p i}=Y_{i} X_{i}^{-T}$ if there exist matrices $X_{i}>0, Q_{i j}>0$, $T>0, Y_{i}, U_{i j}>0,(i, j=1,2, \ldots, N)$, such that the following inequalities hold:

$$
\begin{aligned}
& {\left[\begin{array}{cc}
-(1-\alpha)\left(1-\dot{\tau}^{*}\right) U_{i j} & X_{i} \\
X_{i}^{T} & -T
\end{array}\right]<0,}
\end{aligned}
$$

where $\alpha$ is a fixed scalar which satisfies $0<\alpha<1$.
Remark 7. Both Theorem 4 and Corollary 6 are LMIs. The solutions of $X_{i}, Y_{i}$ are obtained, and the corresponding controller gain matrices are derived as $\bar{K}_{i}=Y_{i} X_{i}^{-T}$. Our method is a deterministic method which can be solved easier than some of the nondeterministic methods, such as the genetic algorithm [10] and particle swarm optimization [13].

Remark 8. Compared with Theorem 2 in [19], Corollary 6 is less conservative in two aspects. Firstly, By using constraint (48), $U_{i j}$, and $T_{i}$ in Corollary 6 can be variables matrices, while relatives $u_{i j}, v_{i}$ in Theorem 2 in [19] are fixed scalars. Secondly, the integral term $-\left(\dot{\tau}_{i j}(t)\right) \int_{t-\tau_{i j}(t)}^{t} \dot{\bar{x}}_{j}(\alpha) E^{T} Q_{i j} E \dot{\bar{x}}_{j}(\alpha) d \alpha$ in the proof is enlarged by using Lemma 2 instead of being removed in the proof in [19].


Figure 4: Continued.


Figure 4: The close-loop state trajectories.

## 4. Simulation

In this section, a two-machine infinite bus example system is chosen to show the effectiveness of the proposed method, which is shown in Figure 3. The system parameters used in the simulation are as follows:

$$
\begin{aligned}
& p_{11}=p_{22}=0, \\
& p_{12}=p_{21}=1, \\
& D_{1}=5, \\
& D_{2}=3, \\
& A_{1} \\
& =\left[\begin{array}{lllll}
0 & 379.2000 & 0 & 0 & 0 \\
-0.3169 & -0.8333 & -0.1123 & 0 & -0.0041 \\
-0.0099 & 0 & -0.2266 & 0.1983 & -0.0048 \\
12.7000 & 0 & -951.7000 & -200.0000 & -24.4000 \\
-0.1759 & 0 & 0.0302 & 0 & 0.0257
\end{array}\right], \\
& A_{2} \\
& =\left[\begin{array}{lllll}
0 & 379.2000 & 0 & 0 & 0 \\
-0.4054 & -0.5000 & -0.0463 & 0 & -0.0041 \\
-0.0495 & 0 & -0.0283 & 0.1983 & -0.0048 \\
-12.7000 & 0 & -551.7000 & -200.0000 & -24.4000 \\
-0.2759 & 0 & 0 & 0 & 0.0257
\end{array}\right], \\
& B_{1}=B_{2}=\left[\begin{array}{lllll}
0 & 1.0000 & -3.0000 & 0.8000 & 4.0000
\end{array}\right]^{T}, \\
& G_{12}=G_{13}=\left[\begin{array}{llll}
0 & 0 & -2.7 & 0
\end{array}\right]^{T}, \\
& G_{21}=G_{23}=\left[\begin{array}{llll}
0 & 0 & -2.3 & 0 \\
0
\end{array}\right]^{T} .
\end{aligned}
$$

If we set $a_{1}=a_{2}=0.3$ and $\dot{a}_{1}=\dot{a}_{2}=0.5$, so $\psi_{1}=\psi_{2}$ varies between 0.4 and 0.9. The upper bound of delay $\tau=5$ and $\dot{\tau}^{*}=0.5$. The method in [19] fails to find a state feedback
controller for this system. According to Theorem 4, the PD controller can be solved as

$$
\begin{align*}
& K_{D 1}=\left[\begin{array}{lllll}
0.2057 & 0.0837 & -0.3127 & -0.1429 & 0.3615
\end{array}\right] \\
& K_{p 1} \\
& =\left[\begin{array}{lllll}
-645.2015 & -107.9008 & -156.5715 & -33.0835 & -94.0722
\end{array}\right]  \tag{50}\\
& K_{D 2}=\left[\begin{array}{llllll}
0.1030 & 0.0874 & -0.3113 & -0.2068 & 0.3635
\end{array}\right] \\
& K_{p 2}=\left[\begin{array}{lllll}
-559.7891 & 5.6352 & -95.1653 & -35.7473 & -74.1156
\end{array}\right]
\end{align*}
$$

The close-loop state trajectories of generator 1-2 are shown in Figure 4.

## 5. Conclusion

In this paper, a decentralized PD control scheme has been proposed to deal with the time-delay multimachine power system with sector saturating actuator. A sufficient condition of closed-loop system asymptomatic stability is presented in terms of LMIs, which can be solved easily by LMI toolbox. Then, a sufficient condition of state feedback control is also obtained which is less conservative than that in [19]. A twomachine infinite bus system is considered as an example, and the simulation result shows the effectiveness of proposed method.

## Nomenclature

$p_{i j}$ : Constant of either 1 or 0 and $p_{i j}=0$ means that $j$ th generator has no connection with $i$ th generator
$B_{i j}: i$ th row and $j$ th column element of nodal susceptance matrix at the internal nodes after eliminated all physical buses, in pu
$H_{i}$ : Inertia constant for $i$ th generator, in seconds
$D_{i}: \quad$ Damping coefficient for $i$ th generator, in pu
$\omega_{i}: \quad$ Relative speed for $i$ th machine, in radian/s
$\delta_{i}: \quad$ Rotor angle for $i$ th machine, in radian
$\omega_{0}$ : $\quad$ The synchronous machine speed
$E_{q i}^{\prime}$ and $E_{q j}^{\prime}:$ Internal transient voltage for $i$ th and $j$ th machine, in pu, which are assumed to be constant
$\delta_{i 0}: \quad$ The initial values of $\delta_{i}$
$\Delta P_{e i}: \quad$ The generator power for $i$ th machine
$\Delta E_{f d i}: \quad$ The generator stimulus voltage for $i$ th machine.

## Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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