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Research Article

Analytical Analysis on Nonlinear Parametric Vibration of an Axially Moving String with Fractional Viscoelastic Damping

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The nonlinear parametric vibration of an axially moving string made by rubber-like materials is studied in the paper. The fractional viscoelastic model is used to describe the damping of the string. Then, a new nonlinear fractional mathematical model governing transverse motion of the string is derived based on Newton's second law, the Euler beam theory, and the Lagrangian strain. Taking into consideration the fractional calculus law of Riemann-Liouville form, the principal parametric resonance is analytically investigated via applying the direct multiscale method. Numerical results are presented to show the influences of the fractional order, the stiffness constant, the viscosity coefficient, and the axial-speed fluctuation amplitude on steady-state responses. It is noticeable that the amplitudes and existing intervals of steady-state responses predicted by Kirchhoff's fractional material model are much larger than those predicted by Mote's fractional material model.

1. Introduction

Axially moving structures are one of common elements in many mechanical systems, which is extensively used in engineering fields, such as paper sheets, magnetic tapes, power transmission belts, chains, fiber textiles, aerial tramways, pipes conveying fluids, and thread lines. In many cases, due to initial, parametric, and external excitations, the generation of unwanted transverse vibrations may limit their applications. Therefore, the dynamic behaviors of such devices have been widely investigated by numerous scholars for the past decades and are still of interesting today [1–10]. Moreover, according to the demand of the actual engineering problems, some researchers have applied their theoretical results to design and optimize the axially moving structures [11, 12].

In general, small imperfections induced by either geometrical or dynamic sources may bring about the occurrence of an unsteady axial speed or tension. For example, when an axially moving belt is installed on rotating pulleys, the torsional vibration of the pulleys would lead to a small fluctuation in the axial moving velocity, and consequently this system may exhibit more complicated dynamics behaviors like the parametric resonances. Thus, the parametric resonances of the axially moving structures caused by the pulsatile transport speed or tension have received extensive attention. Fung et al. [13] applied the Galerkin method and the numerical integration technique to investigate the parametric vibration of a viscoelastic string with the nonuniform transport speed. Pellicano et al. [14] used the approximate analytical and the experimental methods to study the primary and the parametric resonances of a power transmission belt with the fluctuation of the tension. Chen et al. [15] employed the averaging method to investigate the stability problems of an axially accelerating tensioned beam under the condition of the subharmonic and combination resonances. Pakdemirli and Öz [16] adopted a perturbation technique to discuss stable regions of a simply supported axially moving beams subjected to sum- and difference-type combination resonances. Ghayesh [17] utilized the method of multiple scales to study the stability characteristics for principal and combination parametric resonances of an axially moving string with the partial elastic support.

With the development of engineering technique, the various complicated materials are utilized to fabricate the axially moving structures. In order to better understand energy dissipation mechanism of such materials, some viscoelastic constitutive relations like the Kelvin-Voigt form have been used to describe mechanical behaviors of the materials, which is widely applied in axially moving continuums such as strings [18, 19], beams [20, 21], belts [22, 23], and plates [24, 25]. In addition, some structures made of more complex viscoelastic materials are explored. Marynowski and Kapitaniak [26] presented a mathematical model of an axially moving viscoelastic beam with the three-parameter Zener element and investigated both regular and chaos motions using the Galerkin method. Chen et al. [27] analytically studied nonlinear parametric responses of an axially moving string composed of the complicated viscoelastic material based on the Boltzmann superposition principle. Wang and Chen [28] developed the differential quadrature scheme to determine the stable boundary of an axially moving viscoelastic beam with the standard linear solid in the case of the principal resonance. Ding and Chen [29] employed the material time derivative to characterize the viscoelastic property of an axially moving viscoelastic beam. They investigated the stability in principle resonance of the system by using analytical and numerical methods.

On the other hand, the axially moving structures made by rubber-like materials are widely applied in some engineering fields. To exactly describe the viscoelastic features, some scholars have adopted the fractional derivative theory to model the structures. For example, Chen et al. [30] established the fractional dynamic model of an axially moving string and analyzed the transient responses using the Galerkin and numerical methods. Yang et al. [31, 32] employed the multiscale method to investigate the nonlinear free and parametric vibrations of an axially moving viscoelastic string with a fractional order damping.

The above-mentioned studies are concentrated on axially moving strings based on Mote's model. Besides, the nonlinear integro-partial-differential equation called Kirchhoff's model is also used for describing the transverse motion of axially moving strings [33–35]. Nevertheless, the applications of Kirchhoff's model in axially moving structures obeying the fractional differentiation law are rather limited, and the difference of fractional parametric resonances between the Mote's model and the Kirchhoff's model remains unclear. Therefore, the present paper further explores the nonlinear parametric vibration of an axially moving string with the fractional viscoelastic damping based on the literature [32].

2. Equations of Motion

Consider a uniform, axially moving viscoelastic string made by rubber-like materials, with linear density ρ, the length L, and the length cross-sectional area A, traveling at time-dependent axially speed $V(\hat{t})$ between two fixed supports at both ends, shown in Figure 1. The symbol $\hat{u}(\hat{x},\hat{t})$ represents the transverse displacement, $\hat{x}$ is the coordinate along the axial direction, and $\hat{t}$ is the time. On the basis of the Euler beam theory and the Newton second law [34], the Kirchhoff's mathematical model governing transverse motion can be obtained as

$$\begin{aligned}&\rho A\left(\frac{\partial^2\hat{u}}{\partial\hat{t}^2}+2V\frac{\partial^2\hat{u}}{\partial\hat{x}\partial\hat{t}}+\frac{dV}{d\hat{t}}\frac{\partial\hat{u}}{\partial\hat{x}}+V^2\frac{\partial^2\hat{u}}{\partial\hat{x}^2}\right)-P_0\frac{\partial^2\hat{u}}{\partial\hat{x}^2}\\&\quad-\frac{\partial^2\hat{u}}{\partial\hat{x}^2}\frac{1}{L}\int_0^L A\hat{\sigma}\left(\hat{x},\hat{t}\right)d\hat{x}=0,\end{aligned}\tag{1}$$

where P_0 represents the initial tension and $\hat{\sigma}(\hat{x},\hat{t})$ denotes the axial disturbed stress. According to the Lagrangian strain [35], the disturbed strain accounted for the geometric nonlinearity can be expressed as

$$\varepsilon_L\left(\hat{x},\hat{t}\right)=\frac{1}{2}\left(\frac{\partial\hat{u}}{\partial\hat{x}}\right)^2.\tag{2}$$

In the work, the viscoelastic damping of the string is supposed as obeying the fractional derivative Kelvin-Voigt model [32]. Thus, the stress-strain relation can be given as

$$\hat{\sigma}\left(\hat{x},\hat{t}\right)=E_0\varepsilon_L\left(\hat{x},\hat{t}\right)+E_1\frac{\partial^\alpha}{\partial\hat{t}^\alpha}{}_{RL+}\varepsilon_L\left(\hat{x},\hat{t}\right),\tag{3}$$

where E_0 and E_1 represent the stiffness constant and viscosity coefficient of the string, respectively, and α is a constant which is used to describe the viscosity characteristic. Considering the fractional derivative operator defined by the Riemann-Liouville form [36, 37], we write the expression $\partial^\alpha/\partial\hat{t}^\alpha{}_{RL+}(\ )$ in (3) as follows:

$$\frac{\partial^\alpha}{\partial\hat{t}^\alpha}{}_{RL+}\varepsilon_L=\frac{1}{\Gamma(1-\alpha)}\frac{d}{d\hat{t}}\int_0^{\hat{t}}\left(\hat{t}-\tau\right)^{-\alpha}\varepsilon_L(\tau)\,d\tau,\tag{4}$$

where Γ is the Gamma function. The fractional derivative operator is applied to investigate an intermediate viscoelastic characteristic between the elastic string ($\alpha=0$) and Kelvin-Voigt viscoelastic string ($\alpha=1$). For calculating the fractional differentiation operator, we introduce the following property as

$$\frac{d^\alpha}{d\hat{t}^\alpha}{}_{RL+}e^{i\omega\hat{t}}=(i\omega)^\alpha e^{i\omega\hat{t}}+\frac{\sin(\pi\alpha)}{\pi}\int_0^\infty\frac{u^\alpha}{u+i\omega}e^{-u\hat{t}}du,\tag{5}$$

where i denotes complex number $\sqrt{-1}$.

Substituting (2), (3), and (4) into (1), the mathematical model of transverse motion for the axially moving viscoelastic string can be obtained as

$$\begin{aligned}&\rho A\left(\frac{\partial^2\hat{u}}{\partial\hat{t}^2}+2V\frac{\partial^2\hat{u}}{\partial\hat{x}\partial\hat{t}}+\frac{dV}{dT}\frac{\partial\hat{u}}{\partial\hat{x}}+V^2\frac{\partial^2\hat{u}}{\partial\hat{x}^2}\right)-P_0\frac{\partial^2\hat{u}}{\partial\hat{x}^2}\\&\quad=\frac{E_0A}{2L}\frac{\partial^2\hat{u}}{\partial\hat{x}^2}\int_0^L\left(\frac{\partial\hat{u}}{\partial\hat{x}}\right)^2d\hat{x}\\&\qquad+\frac{E_1A}{2L}\frac{\partial^2\hat{u}}{\partial\hat{x}^2}\int_0^L\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial\hat{u}}{\partial\hat{x}}\right)^2d\hat{x}.\end{aligned}\tag{6}$$

For the axially moving string with two fixed supports, the boundary conditions are satisfied by

$$u\left(0,\hat{t}\right)=u\left(L,\hat{t}\right)=0.\tag{7}$$

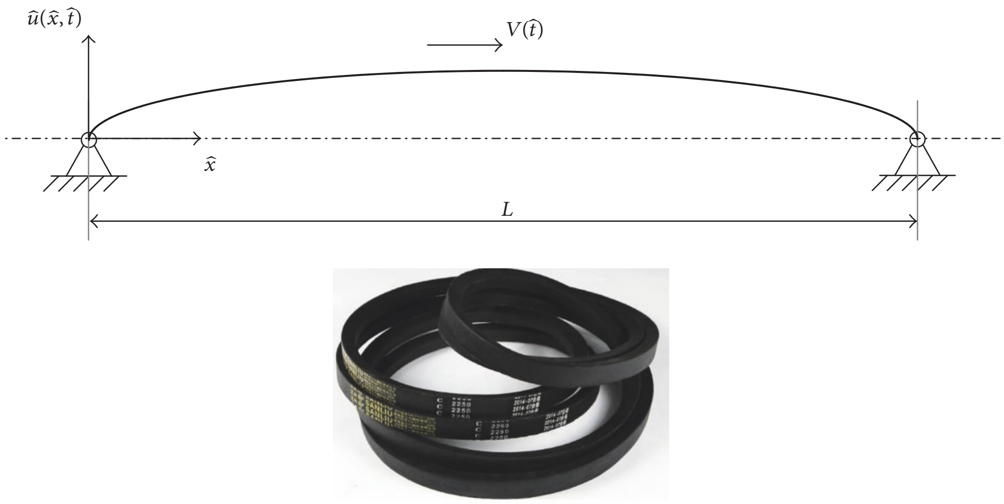


Figure 1: The physical model of the axially moving strings made by rubber-like materials.

Introduce the dimensionless variables

$$\begin{aligned} u &= \frac{\hat{u}}{L}, \\ x &= \frac{\hat{x}}{L}, \\ t &= \frac{\hat{t}}{L}\sqrt{\frac{P_0}{\rho A}}, \\ \gamma &= V\sqrt{\frac{\rho A}{P_0}}, \\ \varepsilon e_0 &= \frac{E_0 A}{P_0}, \\ \varepsilon \eta &= \frac{E_1 A P^{\alpha/2-1}}{\sqrt{(\rho A L^2)^{\alpha}}}, \end{aligned} \tag{8}$$

where the book-keeping device ε is a small parameter, indicating that both the stiffness constant and the viscosity coefficient are very small. Substituting (8) into (6) and (7), one can obtain the following dimensionless dynamic equation,

$$\begin{aligned} &\frac{\partial^2 u}{\partial t^2} + 2\gamma\frac{\partial^2 u}{\partial x \partial t} + \frac{d\gamma}{dt}\frac{\partial u}{\partial x} + \left(\gamma^2 - 1\right)\frac{\partial^2 u}{\partial x^2} \\ &\quad = \frac{1}{2}\varepsilon e_0 \frac{\partial^2 u}{\partial x^2}\int_0^1 \left(\frac{\partial u}{\partial x}\right)^2 dx \\ &\quad + \frac{1}{2}\varepsilon\eta\frac{\partial^2 u}{\partial x^2}\int_0^1 \frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial u}{\partial x}\right)^2 dx, \end{aligned} \tag{9}$$

and boundary conditions,

$$u(0,t) = u(1,t) = 0. \tag{10}$$

In order to study parametric vibrations, we suppose that the axial moving speed is a small harmonically varying about the constant mean speed γ_0:

$$\gamma = \gamma_0 + \varepsilon\gamma_1 \sin(\Omega t), \tag{11}$$

where γ_1 and Ω are the fluctuation amplitude and frequency, respectively. Substituting (11) into (9) yields

$$\begin{aligned} &\frac{\partial^2 u}{\partial t^2} + 2\gamma_0\frac{\partial^2 u}{\partial x \partial t} + \frac{d\gamma}{dt}\frac{\partial u}{\partial x} + \left({\gamma_0}^2 - 1\right)\frac{\partial^2 u}{\partial x^2} \\ &\quad = \varepsilon\left[\frac{1}{2}e_0\frac{\partial^2 u}{\partial x^2}\int_0^1\left(\frac{\partial u}{\partial x}\right)^2 dx\right. \\ &\quad + \frac{1}{2}\eta\frac{\partial^2 u}{\partial x^2}\int_0^1\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial u}{\partial x}\right)^2 dx - \gamma_1\Omega\cos(\Omega t)\frac{\partial u}{\partial x} \\ &\quad \left. - 2\gamma_1\sin(\Omega t)\frac{\partial^2 u}{\partial x \partial t} - 2\gamma_0\gamma_1\sin(\Omega t)\frac{\partial^2 u}{\partial x^2}\right]. \end{aligned} \tag{12}$$

3. Analytical Treatments

Equation (12) is a fractional, nonlinear integro-partial-differential equation; the direct multiscale method is applied to solve this equation for finding analytical solutions. Considering a first-order uniform approximation, one gives

$$u(x,t,\varepsilon) = u_0(x,T_0,T_1) + \varepsilon u_1(x,T_0,T_1) + \cdots, \tag{13}$$

where u_0 and u_1 are the displacement functions at orders 1 and ε, $T_0 = t$ and $T_1 = \varepsilon t$ are the fast and slow time scales, respectively. Substituting (13) into (12) and then equalizing the coefficients ε^0 and ε lead to

$$\varepsilon^0: \frac{\partial^2 u_0}{\partial {T_0}^2} + 2\gamma_0\frac{\partial^2 u_0}{\partial T_0 \partial x} + \left({\gamma_0}^2 - 1\right)\frac{\partial^2 u_0}{\partial x^2} = 0, \tag{14}$$

$$\varepsilon^1: \frac{\partial^2 u_1}{\partial {T_0}^2} + 2\gamma_0\frac{\partial^2 u_1}{\partial T_0 \partial x} + \left({\gamma_0}^2 - 1\right)\frac{\partial^2 u_1}{\partial x^2}$$

$$
\begin{aligned}
&= -2\frac{\partial^2 u_0}{\partial T_0 \partial T_1} - 2\gamma_0 \frac{\partial^2 u_0}{\partial x \partial T_1} \\
&\quad + \frac{1}{2} e_0 \frac{\partial^2 u_0}{\partial x^2} \int_0^1 \left(\frac{\partial u}{\partial x}\right)^2 dx \\
&\quad + \frac{1}{2}\eta \frac{\partial^2 u_0}{\partial x^2} \int_0^1 \left(\frac{\partial}{\partial T_0}\right)^{\alpha}_{RL+} \left(\frac{\partial u}{\partial x}\right)^2 dx \\
&\quad - \gamma_1 \Omega \cos(\Omega T_0) \frac{\partial u_0}{\partial x} - 2\gamma_1 \sin(\Omega T_0) \frac{\partial^2 u_0}{\partial x \partial T_0} \\
&\quad - 2\gamma_0 \gamma_1 \sin(\Omega T_0) \frac{\partial^2 u_0}{\partial x^2}.
\end{aligned} \tag{15}
$$

At ε^0, the solution can be written as follows:

$$
\begin{aligned}
u_0(x, T_0, T_1) &= A_n(T_1)\phi_n(x) e^{i\omega_n T_0} \\
&\quad + \overline{A}_n(T_1)\overline{\phi}_n(x) e^{-i\omega_n T_0},
\end{aligned} \tag{16}
$$

where A_n is a complex function which is only related to the slow time scale T_1, the overbar represents the complex conjugate, and ω_n and $\phi_n(x)$ are the nth natural frequency and mode function of the corresponding linear system, which is given by the following [1]:

$$
\begin{aligned}
\omega_n &= n\pi\left(1 - \gamma_0^{\,2}\right), \\
\phi_n(x) &= \sqrt{2}\sin(n\pi x) e^{in\pi\gamma_0 x}.
\end{aligned} \tag{17}
$$

It is widely known that the principal parametric resonance may occur if Ω approaches $2\omega_n$. Thus, the nearness relation can be supposed as follows:

$$
\Omega = 2\omega_n + \varepsilon\sigma, \tag{18}
$$

in which σ is a detuning parameter.

Substituting (16), (17), and (18) into (15), one yields the following solvability condition [38, 39]:

$$
\frac{dA_n}{dT_1} + \chi_1 A_n^{\,2}\overline{A}_n + \chi_2 \overline{A}_n e^{i\sigma T_1} = 0, \tag{19}
$$

where

$$
\begin{aligned}
\chi_1 &= \frac{-2e_0 \int_0^1 \overline{\phi}_n'' \overline{\phi}_n dx \int_0^1 \phi_n' \overline{\phi}_n' dx + \left[e_0 + \eta(2i\omega_n)^{\alpha}\right] \int_0^1 \overline{\phi}_n'' \overline{\phi}_n dx \int_0^1 \phi_n'^2 dx}{4\left[i\omega_n \int_0^1 \phi_n \overline{\phi}_n dx + \gamma_0 \int_0^1 \phi_n' \overline{\phi}_n dx\right]}, \\
\chi_2 &= -\frac{(\omega_n - \Omega/2)\int_0^1 \overline{\phi}_n' \overline{\phi}_n dx + 2\gamma_0 \int_0^1 \overline{\phi}_n'' \overline{\phi}_n dx}{2\left[i\omega_n \int_0^1 \phi_n \overline{\phi}_n dx + \gamma_0 \int_0^1 \phi_n' \overline{\phi}_n dx\right]}\gamma_1,
\end{aligned} \tag{20}
$$

where χ_1 and χ_2 are the constant coefficients depending on the fraction order, viscosity, stiffness constant, axial moving speed, and the linear frequency. Separating these coefficients into real and image parts, one yields

$$
\begin{aligned}
\chi_1 &= \mathrm{Re}(\chi_1) + i\,\mathrm{Im}(\chi_1), \\
\chi_2 &= \mathrm{Re}(\chi_2) + i\,\mathrm{Im}(\chi_2).
\end{aligned} \tag{21}
$$

Express the solution to (19) in the polar form:

$$
A_n = \frac{1}{2} a_n(T_1) e^{i\phi_n(T_1)}, \tag{22}
$$

in which real functions a_n and ϕ_n are the amplitude and phase of the nth resonance, respectively. Substituting of (21) and (22) in (19) and then separating real and imaginary parts, we have

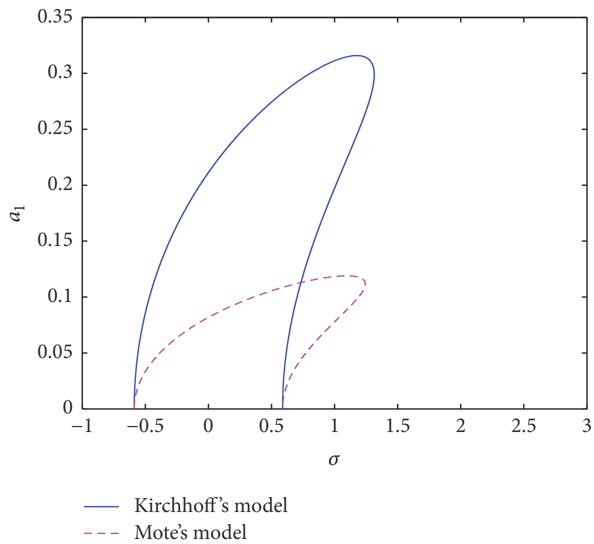
$$
\begin{aligned}
\frac{da_n}{dT_1} &= a_n\left[\mathrm{Im}(\chi_2)\sin\theta_n - \mathrm{Re}(\chi_2)\cos\theta_n\right] \\
&\quad - \frac{1}{4}\mathrm{Re}(\chi_1) a_n^{\,3}, \\
\frac{d\theta_n}{dT_1} &= \sigma + 2\left[\mathrm{Re}(\chi_2)\sin\theta_n - \mathrm{Im}(\chi_2)\cos\theta_n\right] \\
&\quad + \frac{1}{2}\mathrm{Im}(\chi_1) a_n^{\,2},
\end{aligned} \tag{23}
$$

where

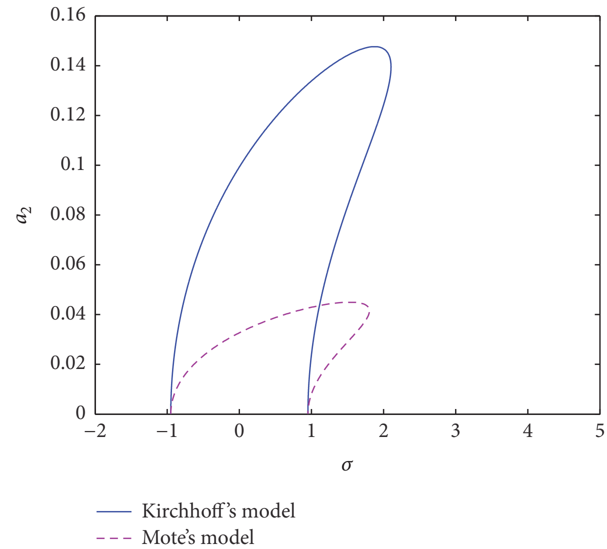
$$
\theta_n = \sigma T_1 - 2\phi_n. \tag{24}
$$

For steady-state responses, the amplitude a_n and the new phase θ_n should be constants; thus,

$$
\begin{aligned}
& a_n\left[\mathrm{Im}(\chi_2)\sin\theta_n - \mathrm{Re}(\chi_2)\cos\theta_n\right] \\
&\quad - \frac{1}{4}\mathrm{Re}(\chi_1) a_n^{\,3} = 0, \\
& \sigma + 2\left[\mathrm{Re}(\chi_2)\sin\theta_n - \mathrm{Im}(\chi_2)\cos\theta_n\right] \\
&\quad + \frac{1}{2}\mathrm{Im}(\chi_1) a_n^{\,2} = 0.
\end{aligned} \tag{25}
$$



(a) The first-order mode



(b) The second-order mode

FIGURE 2: Comparison the steady-state responses between Kirchhoff's and Mote's fractional model.

Eliminating phase θ_n from (25), we can yield the amplitude of steady-state responses:

$$a_n = \frac{\sqrt{-2\sigma \operatorname{Im}(\chi_1) \pm 2\sqrt{4\chi_1^2\chi_2^2 - \sigma^2 [\operatorname{Re}(\chi_1)]^2}}}{|\chi_1|}, \tag{26}$$

where the positive and the minus signs represent the bigger and the smaller amplitudes, respectively. From [27, 28], it is proved that the bigger (smaller) response amplitude is stable (unstable). In addition, (26) is the analytical expression for steady-state responses of the string in principle parametric resonances, where the existing condition of the steady-state responses should be satisfied by

$$\mp 2|\chi_1| \le \sigma \le \frac{2|\chi_1||\chi_2|}{\operatorname{Re}(\chi_2)}. \tag{27}$$

4. Numerical Results and Discussion

In the section, based on (26) and (27), the numerical results are presented to discuss the influences of physical parameters like the fractional order on the amplitudes and the existing conditions of steady-state responses in the principle parametric resonances of the first two modes.

Here we choose the parameters as $\gamma_0 = 0.2$, $\gamma_1 = 0.5$, $e_0 = 0.5$, $\eta = 1$, and $\alpha = 0.5$; the differences of steady-state responses for the string based on Kirchhoff's and Mote's fractional model are shown in Figures 2(a) and 2(b). It can be seen that the response amplitudes and the upper boundaries of exiting conditions predicted by Kirchhoff's model are much larger than those predicted by Mote's model. This implies that the application of the Kirchhoff's fractional model in axially moving strings could yield greater security prediction in the engineering. Therefore, we study the parametric resonances of the fractional damping string based on Kirchhoff's model in the next section.

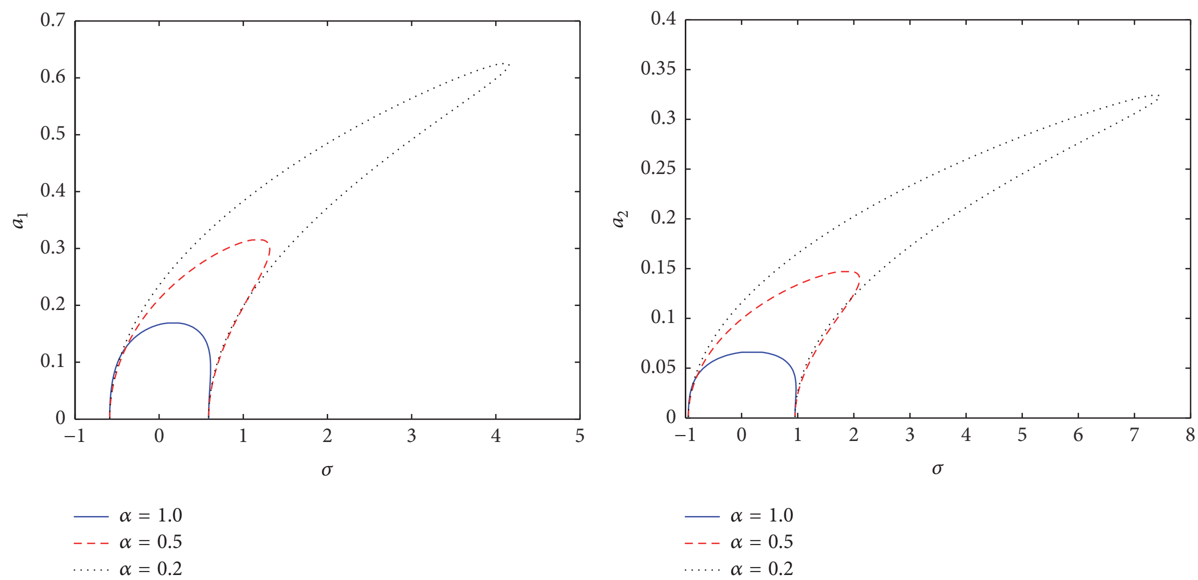
The influence of the fractional order on steady-state responses for Kirchhoff's model is illustrated in Figures 3(a) and 3(b). It is revealed that the response amplitudes decrease with the fractional order increases. Moreover, decreasing the fractional order leads to a larger upper boundary of the existing conditions, but the lower boundary is independent of the fractional order. It is indicated that the larger fractional order would lead to the more evident influence of the damping and the decreasing in existing intervals of steady-state responses. Besides, the response amplitude in the first parametric resonance is much larger than that in the second one.

The changes of steady-state responses with the stiffness constant and the viscosity coefficient of the string for the Kirchhoff's fractional model are plotted in Figures 4 and 5, respectively. It can be demonstrated that the response amplitudes decrease dramatically with the increasing of the stiffness constant and the viscosity coefficient. Moreover, the larger the stiffness constant, the larger the existing intervals of steady-state responses. However, a smaller viscosity coefficient can increase the existing intervals, which is in accordance with in the previous viscoelastic string.

Figure 6 depicts the effect of the axial-speed fluctuation amplitude on steady-state responses for Kirchhoff's fractional string. It is observed that an increase in the axial-speed fluctuation amplitude causes the higher in both the response amplitudes and the existing intervals of steady-state responses. Therefore, we can conclude that the fractional damping string with a smaller axial-speed fluctuation amplitude has a smaller steady-state response.

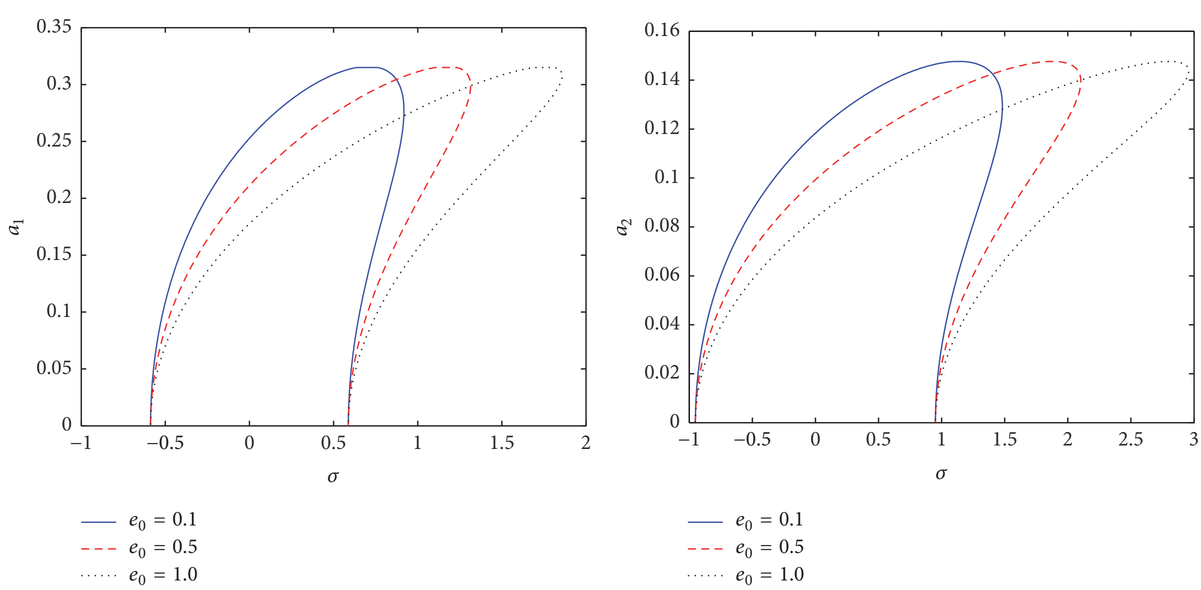
5. Conclusions

In this study, on the basis of the fractional definition and calculus law of the Riemann-Liouville form, the nonlinear parametric vibration of an axially moving string with the fractional damping is investigated based on Kirchhoff's



(a) The first-order mode (b) The second-order mode

Figure 3: Influence of the fractional order on the steady-state responses based on Kirchhoff's model.



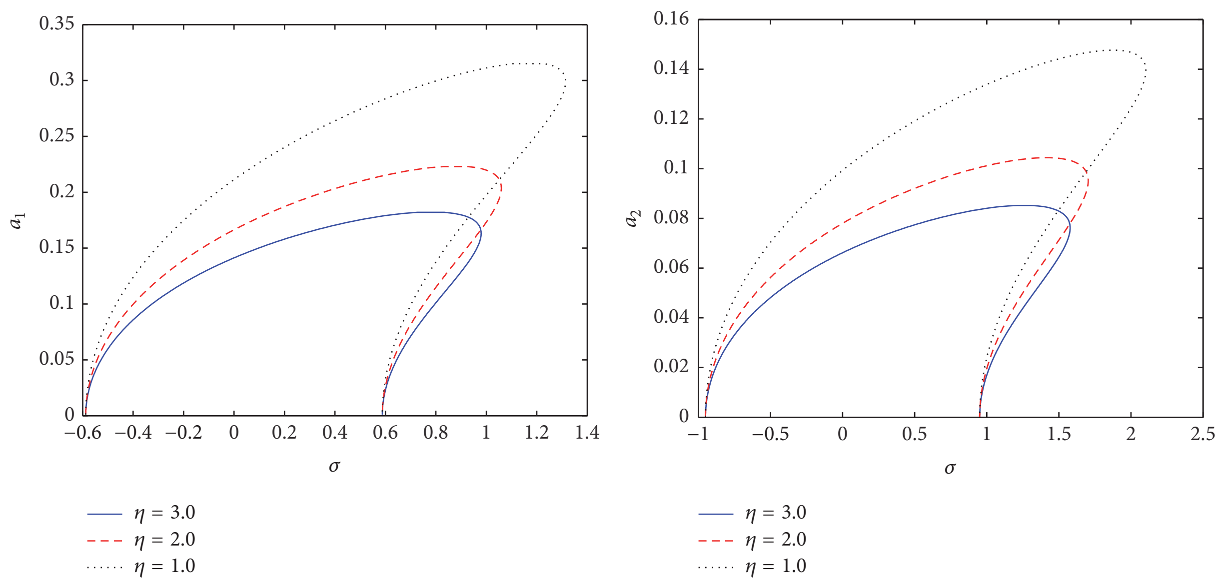
(a) The first-order mode (b) The second-order mode

Figure 4: Influence of the stiffness constant on the steady-state responses based on Kirchhoff's fractional model.

model. Consider that the viscoelastic behavior of the string is described by the fractional Kelvin-Voigt constitutive relation; a new nonlinear fractional mathematical model governing transverse vibration of the string is derived from the Euler beam theory and Newton's second law. The direct multiscale method is applied to investigate the steady-state responses of the fractional damping string undergoing the principle parametric resonances. The investigation yields the following conclusions:

(1) The amplitudes and existing intervals of steady-state responses predicted by Kirchhoff's fractional model are much larger than those predicted by Mote's fractional model.

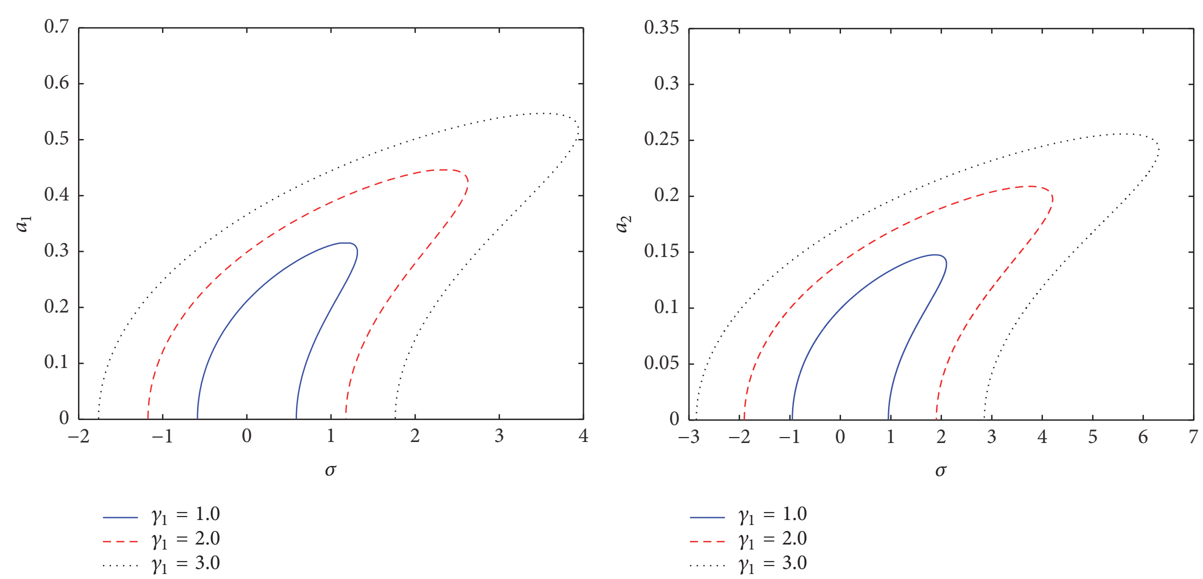
(2) In Kirchhoff's model, the fractional derivative has a significant effect on steady-state responses; the smaller fractional order can cause more rapid increasing in the response amplitudes and existing intervals.



(a) The first-order mode

(b) The second-order mode

FIGURE 5: Effect of the viscosity coefficient on the steady-state responses based on Kirchhoff's fractional model.



(a) The first-order mode

(b) The second-order mode

FIGURE 6: Effect of the axial-speed fluctuation amplitude on the steady-state responses based on Kirchhoff's fractional model.

(3) In Kirchhoff's fractional model, the response amplitudes decrease with the increasing of the stiffness constant and viscosity coefficient, while they increase as the axial-speed fluctuation amplitude increases. In addition, the larger the stiffness constant, the higher the axial-speed fluctuation amplitude; and the smaller viscosity coefficient tends to enlarge existing intervals of steady-state responses.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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