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Research Article

The Alternating Direction Implicit Body of Revolution Multiresolution Time Domain Method with Convolution Perfect Matched Layer

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Overmuch memory and time of CPU have been taken by multiresolution time domain (MRTD) method in three-dimension issues. In order to solve this problem, the alternating direction implicit body of revolution multiresolution time domain (ADI-BOR-MRTD) scheme is presented. Firstly, based on body of revolution finite difference time domain (BOR-FDTD) method, equations of body of revolution multiresolution time domain (BOR-MRTD) method are implemented. Then alternating direction implicit (ADI) is introduced into BOR-MRTD method. Lastly, convolution perfect matched layer (CPML) is applied for ADI-BOR-MRTD method. Numerical results demonstrate that ADI-BOR-MRTD method saves more memory and time of CPU than FDTD and MRTD methods.

1. Introduction

As an efficient numerical algorithm, the multiresolution time domain (MRTD) method was applied in electromagnetic field computation in 1996 by Krumpholz and Katehi [1] firstly. Compared with the finite difference time domain (FDTD) method, the MRTD method has lower numerical dispersion and saves more memory and time of CPU [1, 2].

The time index and the calculating efficiency of the MRTD method are generally limited by the Courant-Friedrich-Levy (CFL) stability condition. However, the alternating direction implicit (ADI) technique can overcome the CFL limitation [3]. Chen and Zhang had published the ADI-MRTD scheme in 2001 [4]. The time step size for the ADI-MRTD is only limited by modeling accuracy of the calculation. Then, the study on the numerical dispersion, absorbing boundary conditions, and the application in the one-dimension photoelectronic band-gap of the ADI-MRTD scheme are developed gradually [5–7].

The body of revolution is an important target in electromagnetic field computation. In order to calculate the body of revolution with less memory and time of CPU, the ADI-BOR-MRTD scheme is presented. At the end of the work, the convolution perfect matched layer (CPML) formulations are derived for the ADI-BOR-MRTD scheme.

2. Equations of BOR-MRTD

In cylindrical coordinates, Maxwell's equations should be written as

$$\varepsilon \frac{\partial E_\rho}{\partial t} + \sigma E_\rho = \frac{1}{\rho}\frac{\partial H_z}{\partial \varphi} - \frac{\partial H_\varphi}{\partial z} \tag{1a}$$

$$\varepsilon \frac{\partial E_\varphi}{\partial t} + \sigma E_\varphi = \frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \tag{1b}$$

$$\varepsilon \frac{\partial E_z}{\partial t} + \sigma E_z = \frac{1}{\rho}\frac{\partial \left(\rho H_\varphi\right)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \varphi} \tag{1c}$$

$$-\mu \frac{\partial H_\rho}{\partial t} - \sigma_m H_\rho = \frac{1}{\rho}\frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z} \quad (1d)$$

$$-\mu \frac{\partial H_\varphi}{\partial t} - \sigma_m H_\varphi = \frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} \quad (1e)$$

$$-\mu \frac{\partial H_z}{\partial t} - \sigma_m H_z = \frac{1}{\rho}\frac{\partial (\rho E_\varphi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial E_\rho}{\partial \varphi}. \quad (1f)$$

The electric and magnetic fields are expanded by Fourier series as

$$\mathbf{E} = \sum_{m=0}^{\infty} (\mathbf{e}_u \cos m\phi + \mathbf{e}_v \sin m\phi) \quad (2a)$$

$$\mathbf{H} = \sum_{m=0}^{\infty} (\mathbf{h}_u \cos m\phi + \mathbf{h}_v \sin m\phi), \quad (2b)$$

where $\mathbf{e}_u, \mathbf{e}_v, \mathbf{h}_u, \mathbf{h}_v$ are Fourier coefficients and $\mathbf{e} = \mathbf{E}_m$, $\mathbf{h} = \mathbf{H}_m$. ϕ is azimuth angle; m is modulus. u is related to $\cos m\phi$; v is related to $\sin m\phi$.

Substituting (2a) and (2b) to (1a)–(1f), (1a)–(1f) are rewritten as

$$\varepsilon \frac{\partial e_\rho}{\partial t} = \pm \frac{m}{\rho} h_z - \frac{\partial h_\varphi}{\partial z} \quad (3a)$$

$$\varepsilon \frac{\partial e_\varphi}{\partial t} = \frac{\partial h_\rho}{\partial z} - \frac{\partial h_z}{\partial \rho} \quad (3b)$$

$$\varepsilon \frac{\partial e_z}{\partial t} = \frac{1}{\rho}\frac{\partial (\rho h_\varphi)}{\partial \rho} \mp \frac{m}{\rho} h_\rho \quad (3c)$$

$$\mu \frac{\partial h_\rho}{\partial t} = \pm \frac{m}{\rho} e_z + \frac{\partial e_\varphi}{\partial z} \quad (3d)$$

$$\mu \frac{\partial h_\varphi}{\partial t} = -\frac{\partial e_\rho}{\partial z} + \frac{\partial e_z}{\partial \rho} \quad (3e)$$

$$\mu \frac{\partial h_z}{\partial t} = -\frac{1}{\rho}\frac{\partial (\rho e_\varphi)}{\partial \rho} \mp \frac{m}{\rho} e_\rho. \quad (3f)$$

The electric and magnetic fields are expanded by Daubechies' scaling function in space domain and by Haar's scaling function in time domain.

$$e_\rho(\vec{r}, t) = \sum_{i,j,k,n=-\infty}^{+\infty} e^{\alpha\rho,n}_{i+1/2,j,k} \Phi_n(t)\, \alpha_{i+1/2}(\rho)\, \alpha_j(\varphi) \cdot \alpha_k(z) \quad (4a)$$

$$e_\varphi(\vec{r}, t) = \sum_{i,j,k,n=-\infty}^{+\infty} e^{\alpha\varphi,n}_{i,j+1/2,k} \Phi_n(t)\, \alpha_i(\rho)\, \alpha_{j+1/2}(\varphi) \cdot \alpha_k(z) \quad (4b)$$

$$e_z(\vec{r}, t) = \sum_{i,j,k,n=-\infty}^{+\infty} e^{\alpha z,n}_{i,j,k+1/2} \Phi_n(t)\, \alpha_i(\rho)\, \alpha_j(\varphi) \cdot \alpha_{k+1/2}(z) \quad (4c)$$

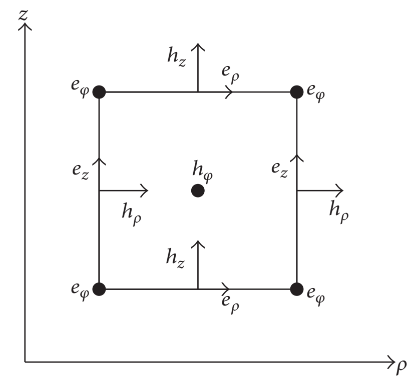

Figure 1: Distribution of field components for BOR-MRTD.

$$h_\rho(\vec{r}, t) = \sum_{i,j,k,n=-\infty}^{+\infty} h^{\alpha\rho,n+1/2}_{i,j+1/2,k+1/2} \Phi_{n+1/2}(t)\, \alpha_i(\rho) \cdot \alpha_{j+1/2}(\varphi)\, \alpha_{k+1/2}(z) \quad (4d)$$

$$h_\varphi(\vec{r}, t) = \sum_{i,j,k,n=-\infty}^{+\infty} h^{\alpha\varphi,n+1/2}_{i+1/2,j,k+1/2} \Phi_{n+1/2}(t)\, \alpha_{i+1/2}(\rho) \cdot \alpha_j(\varphi)\, \alpha_{k+1/2}(z) \quad (4e)$$

$$h_z(\vec{r}, t) = \sum_{i,j,k,n=-\infty}^{+\infty} h^{\alpha z,n+1/2}_{i+1/2,j+1/2,k} \Phi_{n+1/2}(t)\, \alpha_{i+1/2}(\rho) \cdot \alpha_{j+1/2}(\varphi)\, \alpha_k(z). \quad (4f)$$

$e^{\alpha\zeta,n}_{i,j,k}$ and $h^{\alpha\zeta,n}_{i,j,k}$ are the field coefficients, with $\zeta = \rho, \varphi, z$. The indexes i, j, k, and n are the space indices and time indices as $\rho = i\Delta\rho$, $\varphi = j\Delta\varphi$, $z = k\Delta z$, and $t = n\Delta t$, where $\Delta\rho$, $\Delta\varphi$, Δz, and Δt represent the space and time discretization intervals in ρ-, φ-, z-, and t-direction. The function $\Phi(t)$ is Haar's scaling function [8], and α is Daubechies' scaling function [9].

The distribution of field components is shown in Figure 1.

The equations of BOR-MRTD method are presented as follows:

$$e^{n+1}_\rho\left(i+\frac{1}{2}, k\right) = e^n_\rho\left(i+\frac{1}{2}, k\right) \pm \frac{m\Delta t}{(i+1/2)\,\varepsilon\Delta\rho} \cdot h^{n+1/2}_z\left(i+\frac{1}{2}, k\right) - \frac{\Delta t}{\varepsilon\Delta z} \cdot \sum_l a(l)\, h^{n+1/2}_\varphi\left(i+\frac{1}{2}, k+l+\frac{1}{2}\right) \quad (5a)$$

$$e^{n+1}_\varphi(i, k) = e^n_\varphi(i, k) + \frac{\Delta t}{\varepsilon\Delta z} \cdot \sum_l a(l)\, h^{n+1/2}_\rho\left(i, k+l+\frac{1}{2}\right) - \frac{\Delta t}{\varepsilon\Delta\rho} \cdot \sum_l a(l)\, h^{n+1/2}_z\left(i+l+\frac{1}{2}, k\right). \quad (5b)$$

Table 1: The coefficients $a(l)$ of D_2.

l	$a(l)$
0	1.22916661202745
1	−0.09374997764746
2	0.01041666418309

$$e_z^{n+1}\left(i,k+\frac{1}{2}\right)=e_z^{n}\left(i,k+\frac{1}{2}\right)\mp\frac{m\Delta t}{\varepsilon i\Delta\rho}h_\rho^{n+1/2}\left(i,k+\frac{1}{2}\right)+\frac{\Delta t}{i\varepsilon\Delta\rho}\sum_l a(l)\left(i+l+\frac{1}{2}\right)h_\varphi^{n+1/2}\left(i+l+\frac{1}{2},k+\frac{1}{2}\right) \quad (5c)$$

$$h_\rho^{n+1/2}\left(i,k+\frac{1}{2}\right)=h_\rho^{n-1/2}\left(i,k+\frac{1}{2}\right)+\frac{\Delta t}{\mu}\left(\frac{1}{\Delta z}\sum_l a(l)\,e_\varphi^n(i,k+l+1)\pm\frac{m}{i\Delta\rho}e_z^n\left(i,k+\frac{1}{2}\right)\right) \quad (5d)$$

$$h_\varphi^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right)=h_\varphi^{n-1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right)+\frac{\Delta t}{\mu\Delta\rho}\cdot\sum_l a(l)\,e_z^n\left(i,k+l+\frac{1}{2}\right)-\frac{\Delta t}{\mu\Delta z}\cdot\sum_l a(l)\,e_\rho^n\left(i+\frac{1}{2},k+l+1\right) \quad (5e)$$

$$h_z^{n+1/2}\left(i+\frac{1}{2},k\right)=h_z^{n-1/2}\left(i+\frac{1}{2},k\right)+\frac{\Delta t}{\mu(i+1/2)\Delta\rho}\left(\mp m e_\rho^n\left(i+\frac{1}{2},k\right)-\sum_l a(l)(i+l+1)\,e_\varphi^n(i+l+1,k)\right). \quad (5f)$$

The coefficient $a(l)$ is equal to

$$a(l)\equiv\left\langle\frac{\partial\alpha_{m'+1/2}(x)}{\partial x},\alpha_{m-1}(x)\right\rangle=\frac{1}{\pi}\int_0^\infty|\widehat{\alpha}(\omega)|^2\sin[\omega(l+0.5)]\,d\omega. \quad (6)$$

For Daubechies' scaling function with two vanishing moments (D_2), the coefficients are shown in Table 1; for $l>2$, $a(l)$ are zeros due to the compact support of Daubechies' scaling function; for $l<0$, $a(l)$ are given by the symmetry relation $a(-1-l)=-a(l)$.

When $\rho=0$, namely, $i=0$, the values of e_ϕ, e_z, h_ρ are singular. It can be solved via Ampere's law:

$$\oint_c \mathbf{H}\cdot d\mathbf{l}=\varepsilon\frac{\partial}{\partial t}\iint_s \mathbf{E}\cdot d\mathbf{s}+\iint_s \sigma\mathbf{E}\cdot d\mathbf{s}. \quad (7)$$

Taking the integral of (7),

$$e_z^{n+1}\left(0,k+\frac{1}{2}\right)=e_z^{n}\left(0,k+\frac{1}{2}\right)+\frac{4\Delta t}{\varepsilon\Delta\rho}h_\varphi^{n+1/2}\left(\frac{1}{2},k+\frac{1}{2}\right),\quad m=0. \quad (8)$$

According to distribution of e_φ, e_z, h_ρ, when $m\neq 1$, $h_\rho(0,k+1/2)=e_\varphi(0,k)=0$, and when $m>0$, $e_z(0,k+1/2)=0$. When $\rho=0$, it is not necessary to calculate e_φ, because the coefficient of $e_\varphi(0,k)$ is 0 in (5f). $h_\rho(0,k+1/2)$ is just useful to calculate $e_\varphi(0,k)$, so it is needless too. The conclusion is that e_z with $m=0$ is the only field component to be calculated when $\rho=0$.

3. Equations of ADI-BOR-MRTD

In alternating direction implicit method, field components have been calculated at $t=n\Delta t$ and $t=(n+1/2)\Delta t$. The time has been discretized in two steps, namely, $n\to n+1/2$ and $n+1/2\to n+1$ [10]. The ADI-BOR-MRTD equations are presented as follows.

First Step ($n\to n+1/2$)

$$e_\rho^{n+1/2}\left(i+\frac{1}{2},k\right)=e_\rho^{n}\left(i+\frac{1}{2},k\right)+\frac{\Delta t}{2\varepsilon}\left(\pm\frac{m}{(i+1/2)\Delta\rho}h_z^{n+1/2}\left(i+\frac{1}{2},k\right)-\frac{1}{\Delta z}\sum_l a(l)\,h_\varphi^n\left(i+\frac{1}{2},k+l+\frac{1}{2}\right)\right) \quad (9a)$$

$$e_\varphi^{n+1/2}(i,k)=e_\varphi^{n}(i,k)+\frac{\Delta t}{2\varepsilon}\left(\frac{1}{\Delta z}\sum_l a(l)\,h_\rho^{n+1/2}\left(i,k+l+\frac{1}{2}\right)-\frac{1}{\Delta\rho}\sum_l a(l)\,h_z^n\left(i+l+\frac{1}{2},k\right)\right) \quad (9b)$$

$$e_z^{n+1/2}\left(i,k+\frac{1}{2}\right)=e_z^{n}\left(i,k+\frac{1}{2}\right)+\frac{\Delta t}{2\varepsilon}\left(\frac{\sum_l a(l)(i+l+1/2)\,h_\varphi^{n+1/2}(i+l+1/2,k+1/2)}{i\Delta\rho}\mp\frac{m}{i\Delta\rho}h_\rho^n\left(i,k+\frac{1}{2}\right)\right) \quad (9c)$$

$$h_\rho^{n+1/2}\left(i,k+\frac{1}{2}\right)=h_\rho^{n}\left(i,k+\frac{1}{2}\right)+\frac{\Delta t}{2\mu}\left(\frac{\sum_l a(l)\,e_\varphi^{n+1/2}(i,k+l+1)}{\Delta z}\pm\frac{m}{i\Delta\rho}e_z^{n}\left(i,k+\frac{1}{2}\right)\right) \tag{9d}$$

$$h_\varphi^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right)=h_\varphi^{n}\left(i+\frac{1}{2},k+\frac{1}{2}\right)+\frac{\Delta t}{2\mu}\left(\frac{\sum_l a(l)\,e_z^{n+1/2}(i+l+1,k+1/2)}{\Delta\rho}-\frac{\sum_l a(l)\,e_\rho^{n}(i+1/2,k+l+1)}{\Delta z}\right) \tag{9e}$$

$$h_z^{n+1/2}\left(i+\frac{1}{2},k\right)=h_z^{n}\left(i+\frac{1}{2},k\right)+\frac{\Delta t}{2(i+1/2)\mu\Delta\rho}\left(\mp m e_\rho^{n+1/2}\left(i+\frac{1}{2},k\right)-\sum_l a(l)(i+l+1)\,e_\varphi^{n}(i+l+1,k)\right). \tag{9f}$$

Second Step $(n+1/2\rightarrow n+1)$

$$e_\rho^{n+1}\left(i+\frac{1}{2},k\right)=e_\rho^{n+1/2}\left(i+\frac{1}{2},k\right)+\frac{\Delta t}{2\varepsilon}\left(\pm\frac{m}{(i+1/2)\Delta\rho}h_z^{n+1/2}\left(i+\frac{1}{2},k\right)-\frac{1}{\Delta z}\sum_l a(l)\,h_\varphi^{n+1}\left(i+\frac{1}{2},k+l+\frac{1}{2}\right)\right) \tag{10a}$$

$$e_\varphi^{n+1}(i,k)=e_\varphi^{n+1/2}(i,k)+\frac{\Delta t}{2\varepsilon}\left(\frac{1}{\Delta z}\sum_l a(l)\,h_\rho^{n+1/2}\left(i,k+l+\frac{1}{2}\right)-\frac{1}{\Delta\rho}\sum_l a(l)\,h_z^{n+1}\left(i+l+\frac{1}{2},k\right)\right) \tag{10b}$$

$$e_z^{n+1}\left(i,k+\frac{1}{2}\right)=e_z^{n+1/2}\left(i,k+\frac{1}{2}\right)+\frac{\Delta t}{2\varepsilon}\left(\frac{\sum_l a(l)(i+l+1/2)\,h_\varphi^{n+1/2}(i+l+1/2,k+1/2)}{i\Delta\rho}\mp\frac{m}{i\Delta\rho}h_\rho^{n+1}\left(i,k+\frac{1}{2}\right)\right) \tag{10c}$$

$$h_\rho^{n+1}\left(i,k+\frac{1}{2}\right)=h_\rho^{n+1/2}\left(i,k+\frac{1}{2}\right)+\frac{\Delta t}{2\mu}\left(\frac{\sum_l a(l)\,e_\varphi^{n+1/2}(i,k+l+1)}{\Delta z}\pm\frac{m}{i\Delta\rho}e_z^{n+1}\left(i,k+\frac{1}{2}\right)\right) \tag{10d}$$

$$h_\varphi^{n+1}\left(i+\frac{1}{2},k+\frac{1}{2}\right)=h_\varphi^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right)+\frac{\Delta t}{2\mu}\left(\frac{\sum_l a(l)\,e_z^{n+1/2}(i+l+1,k+1/2)}{\Delta\rho}-\frac{\sum_l a(l)\,e_\rho^{n+1}(i+1/2,k+l+1)}{\Delta z}\right) \tag{10e}$$

$$h_z^{n+1}\left(i+\frac{1}{2},k\right)=h_z^{n+1/2}\left(i+\frac{1}{2},k\right)+\frac{\Delta t}{2(i+1/2)\mu\Delta\rho}\left(\mp m e_\rho^{n+1/2}\left(i+\frac{1}{2},k\right)-\sum_l a(l)(i+l+1)\,e_\varphi^{n+1}(i+l+1,k)\right). \tag{10f}$$

Equations (9a)–(9f) and (10a)–(10f) can be solved by the generalized Thomas method [5].

When $\rho=0$, namely, $i=0$, the values of e_z are singular as BOR-MRTD scheme. So e_z is calculated as follows:

$(n\rightarrow n+1/2)$

$$e_z^{n+1/2}\left(0,k+\frac{1}{2}\right)-\frac{\Delta t^2}{\mu\varepsilon\Delta\rho^2}\sum_l a(l)\,e_z^{n+1/2}\left(l+1,k+\frac{1}{2}\right)=e_z^{n}\left(0,k+\frac{1}{2}\right)+\frac{2\Delta t}{\varepsilon\Delta\rho}h_\varphi^{n}\left(\frac{1}{2},k+\frac{1}{2}\right)-\frac{\Delta t^2}{\mu\varepsilon\Delta\rho\Delta z}\sum_l a(l)\,e_\rho^{n}\left(\frac{1}{2},k+l+1\right). \tag{11}$$

$(n+1/2\rightarrow n+1)$

$$e_z^{n+1}\left(0,k+\frac{1}{2}\right)=e_z^{n+1/2}\left(0,k+\frac{1}{2}\right)+\frac{2\Delta t}{\varepsilon\Delta\rho}h_\varphi^{n+1/2}\left(\frac{1}{2},k+\frac{1}{2}\right). \tag{12}$$

4. Convolution Perfect Matched Layer

Based on equations of ADI-BOR-MRTD scheme, we can present equations of CPML with consulting paper [11].

$(n\rightarrow n+1/2)$

$$e_\rho^{n+1/2}\left(i+\frac{1}{2},k\right)=CA_\rho\left(i+\frac{1}{2},k\right)e_\rho^{n}\left(i+\frac{1}{2},k\right)+CB_\rho\left(i+\frac{1}{2},k\right)\left(\pm\frac{m}{(i+1/2)\Delta\rho}h_z^{n+1/2}\left(i+\frac{1}{2},k\right)-\frac{1}{\kappa_k\Delta z}\sum_l a(l)\,h_\varphi^{n}\left(i+\frac{1}{2},k+l+\frac{1}{2}\right)-\psi_{e_{\rho z}}^{n}\left(i+\frac{1}{2},k\right)\right) \tag{13a}$$

$$\psi_{e_{\rho z}}^{n+1/2}\left(i+\frac{1}{2},k\right)=Q_k\psi_{e_{\rho z}}^{n}\left(i+\frac{1}{2},k\right)+\frac{P_k}{\Delta z}\cdot\sum_l a(l)\,h_\varphi^n\left(i+\frac{1}{2},k+l+\frac{1}{2}\right) \tag{13ai}$$

$$\begin{aligned}e_\phi^{n+1/2}(i,k)&=CA_\varphi(i,k)\,e_\varphi^n(i,k)+CB_\varphi(i,k)\\&\cdot\left(\frac{1}{\kappa_k\Delta z}\sum_l a(l)\,h_\rho^{n+1/2}\left(i,k+l+\frac{1}{2}\right)\right.\\&-\frac{1}{\kappa_i\Delta\rho}\sum_l a(l)\,h_z^n\left(i+l+\frac{1}{2},k\right)+\psi_{e_{\varphi z}}^{n+1/2}(i,k)\\&\left.-\psi_{e_{\varphi\rho}}^n(i,k)\right)\end{aligned} \tag{13b}$$

$$\psi_{e_{\varphi z}}^{n+1/2}(i,k)=Q_k\psi_{e_{\varphi z}}^n(i,k)+\frac{P_k}{\Delta z}\cdot\sum_l a(l)\,h_\rho^{n+1/2}\left(i,k+l+\frac{1}{2}\right) \tag{13bi}$$

$$\psi_{e_{\varphi\rho}}^{n+1/2}(i,k)=Q_i\psi_{e_{\varphi\rho}}^n(i,k)+\frac{P_i}{\Delta\rho}\cdot\sum_l a(l)\,h_z^n\left(i+l+\frac{1}{2},k\right) \tag{13bii}$$

$$\begin{aligned}e_z^{n+1/2}\left(i,k+\frac{1}{2}\right)&=CA_z\left(i,k+\frac{1}{2}\right)e_z^n\left(i,k+\frac{1}{2}\right)\\&+CB_z\left(i,k+\frac{1}{2}\right)\\&\cdot\left(\frac{\sum_l a(l)(i+l+1/2)\,h_\varphi^{n+1/2}(i+l+1/2,k+1/2)}{i\kappa_i\Delta\rho}\right.\\&\left.\mp\frac{m}{i\Delta\rho}h_\rho^n\left(i,k+\frac{1}{2}\right)+\psi_{e_{z\rho}}^{n+1/2}\left(i,k+\frac{1}{2}\right)\right)\end{aligned} \tag{13c}$$

$$\psi_{e_{z\rho}}^{n+1/2}\left(i,k+\frac{1}{2}\right)=Q_i\psi_{e_{z\rho}}^n\left(i,k+\frac{1}{2}\right)+\frac{P_i}{\Delta\rho}\sum_l a(l)\cdot h_\varphi^{n+1/2}\left(i+l+\frac{1}{2},k+\frac{1}{2}\right) \tag{13ci}$$

$$\begin{aligned}h_\rho^{n+1/2}\left(i,k+\frac{1}{2}\right)&=CP_x\left(i,k+\frac{1}{2}\right)h_\rho^n\left(i,k+\frac{1}{2}\right)\\&+CQ_x\left(i,k+\frac{1}{2}\right)\left(\frac{\sum_l a(l)\,e_\varphi^{n+1/2}(i,k+l+1)}{\kappa_{k+1/2}\Delta z}\right.\\&\left.\pm\frac{m}{i\Delta\rho}e_z^n\left(i,k+\frac{1}{2}\right)+\psi_{h_{\rho z}}^{n+1/2}\left(i,k+\frac{1}{2}\right)\right)\end{aligned} \tag{13d}$$

$$\psi_{h_{\rho z}}^{n+1/2}\left(i,k+\frac{1}{2}\right)=Q_{k+1/2}\psi_{h_{\rho z}}^n\left(i,k+\frac{1}{2}\right)+\frac{P_{k+1/2}}{\Delta z}\cdot\sum_l a(l)\,e_\varphi^{n+1/2}(i,k+l+1) \tag{13di}$$

$$\begin{aligned}h_\varphi^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right)&=CP_\varphi\left(i+\frac{1}{2},k+\frac{1}{2}\right)h_\varphi^n\left(i+\frac{1}{2},k+\frac{1}{2}\right)+CQ_\varphi\left(i+\frac{1}{2},k+\frac{1}{2}\right)\\&\cdot\left(\frac{\sum_l a(l)\,e_z^{n+1/2}(i+l+1,k+1/2)}{\kappa_{i+1/2}\Delta\rho}\right.\\&-\frac{\sum_l a(l)\,e_\rho^n(i+1/2,k+l+1)}{\kappa_{i+1/2}\Delta z}\\&\left.+\psi_{h_{\varphi\rho}}^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right)-\psi_{h_{\varphi z}}^n\left(i+\frac{1}{2},k+\frac{1}{2}\right)\right)\end{aligned} \tag{13e}$$

$$\psi_{h_{\varphi z}}^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right)=Q_{k+1/2}\psi_{h_{\varphi z}}^n\left(i+\frac{1}{2},k+\frac{1}{2}\right)+\frac{P_{k+1/2}}{\Delta z}\sum_l a(l)\,e_\rho^n\left(i+\frac{1}{2},k+l+1\right) \tag{13ei}$$

$$\psi_{h_{\varphi\rho}}^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right)=Q_{i+1/2}\psi_{h_{\varphi\rho}}^n\left(i+\frac{1}{2},k+\frac{1}{2}\right)+\frac{P_{i+1/2}}{\Delta\rho}\sum_l a(l)\,e_z^{n+1/2}\left(i+l+1,k+\frac{1}{2}\right) \tag{13eii}$$

$$\begin{aligned}h_z^{n+1/2}\left(i+\frac{1}{2},k\right)&=CP_z\left(i+\frac{1}{2},k\right)h_z^n\left(i+\frac{1}{2},k\right)\\&+CQ_z\left(i+\frac{1}{2},k\right)\left(\mp\frac{me_\rho^{n+1/2}(i+1/2,k)}{(i+1/2)\Delta\rho}\right.\\&-\frac{\sum_l a(l)(i+l+1)\,e_\varphi^n(i+l+1,k)}{\kappa_{i+1/2}(i+1/2)\Delta\rho}\\&\left.-\psi_{h_{z\rho}}^n\left(i+\frac{1}{2},k\right)\right)\end{aligned} \tag{13f}$$

$$\psi_{h_{z\rho}}^{n+1/2}\left(i+\frac{1}{2},k\right)=Q_{i+1/2}\psi_{h_{z\rho}}^n\left(i+\frac{1}{2},k\right)+\frac{P_{i+1/2}}{\Delta\rho}\cdot\sum_l a(l)\,E_y^n(i+l+1,k). \tag{13fi}$$

$(n+1/2\rightarrow n+1)$

$$\begin{aligned}e_\rho^{n+1}\left(i+\frac{1}{2},k\right)&=CA_x\left(i+\frac{1}{2},k\right)e_\rho^{n+1/2}\left(i+\frac{1}{2},k\right)\\&+CB_x\left(i+\frac{1}{2},k\right)\left(\pm\frac{m}{(i+1/2)\Delta\rho}h_z^{n+1/2}\left(i+\frac{1}{2},k\right)\right.\\&-\frac{1}{\kappa_k\Delta z}\sum_l a(l)\,h_\varphi^{n+1}\left(i+\frac{1}{2},k+l+\frac{1}{2}\right)\\&\left.-\psi_{e_{\rho z}}^{n+1}\left(i+\frac{1}{2},k\right)\right)\end{aligned} \tag{14a}$$

$$\psi_{e_{\rho z}}^{n+1}\left(i+\frac{1}{2},k\right)=Q_k\psi_{e_{\rho z}}^{n+1/2}\left(i+\frac{1}{2},k\right)+\frac{P_k}{\Delta z}\sum_l a(l)\cdot h_\varphi^{n+1}\left(i+\frac{1}{2},k+l+\frac{1}{2}\right) \tag{14ai}$$

$$e_{\varphi}^{n+1}(i,k) = CA_{\varphi}(i,k)\, e_{\varphi}^{n+1/2}(i,k) + CB_{\varphi}(i,k) \cdot \left(\frac{1}{\kappa_k \Delta z} \sum_l a(l)\, h_{\rho}^{n+1/2}\left(i, k+l+\frac{1}{2}\right) - \frac{1}{\kappa_i \Delta\rho} \sum_l a(l)\, h_z^{n+1}\left(i+l+\frac{1}{2}, k\right) + \psi_{e_{\varphi z}}^{n+1/2}(i,k) - \psi_{e_{\varphi\rho}}^{n+1}(i,k) \right) \tag{14b}$$

$$\psi_{e_{\varphi z}}^{n+1}(i,k) = Q_k \psi_{e_{\varphi z}}^{n+1/2}(i,k) + \frac{P_k}{\Delta z} \sum_l a(l)\, h_{\rho}^{n+1/2}\left(i, k+l+\frac{1}{2}\right) \tag{14bi}$$

$$\psi_{e_{\varphi\rho}}^{n+1}(i,k) = Q_i \psi_{e_{\varphi\rho}}^{n+1/2}(i,k) + \frac{P_i}{\Delta\rho} \sum_l a(l)\, h_z^{n+1}\left(i+l+\frac{1}{2}, k\right) \tag{14bii}$$

$$e_z^{n+1}\left(i,k+\frac{1}{2}\right) = CA_z\left(i,k+\frac{1}{2}\right) e_z^{n+1/2}\left(i,k+\frac{1}{2}\right) + CB_z\left(i,k+\frac{1}{2}\right) \cdot \left(\frac{\sum_l a(l)(i+l+1/2)\, h_{\varphi}^{n+1/2}(i+l+1/2, k+1/2)}{i\kappa_i \Delta\rho} \mp \frac{m}{i\Delta\rho} h_{\rho}^{n+1}\left(i,k+\frac{1}{2}\right) + \psi_{e_{z\rho}}^{n+1/2}\left(i,k+\frac{1}{2}\right) \right) \tag{14c}$$

$$\psi_{e_{z\rho}}^{n+1}\left(i,k+\frac{1}{2}\right) = Q_i \psi_{e_{z\rho}}^{n+1/2}\left(i,k+\frac{1}{2}\right) + \frac{P_i}{\Delta\rho} \sum_l a(l) \cdot h_{\varphi}^{n+1/2}\left(i+l+\frac{1}{2}, k+\frac{1}{2}\right) \tag{14ci}$$

$$h_{\rho}^{n+1}\left(i,k+\frac{1}{2}\right) = CP_{\rho}\left(i,k+\frac{1}{2}\right) h_{\rho}^{n+1/2}\left(i,k+\frac{1}{2}\right) + CQ_{\rho}\left(i,k+\frac{1}{2}\right) \left(\frac{\sum_l a(l)\, e_{\varphi}^{n+1/2}(i,k+l+1)}{\kappa_{k+1/2}\Delta z} \pm \frac{m}{i\Delta\rho} e_z^{n+1}\left(i,k+\frac{1}{2}\right) + \psi_{h_{\rho z}}^{n+1/2}\left(i,k+\frac{1}{2}\right) \right) \tag{14d}$$

$$\psi_{h_{\rho z}}^{n+1}\left(i,k+\frac{1}{2}\right) = Q_{k+1/2} \psi_{h_{\rho z}}^{n+1/2}\left(i,k+\frac{1}{2}\right) + \frac{P_{k+1/2}}{\Delta z} \cdot \sum_l a(l)\, e_{\varphi}^{n+1/2}(i,k+l+1) \tag{14di}$$

$$h_{\varphi}^{n+1}\left(i+\frac{1}{2},k+\frac{1}{2}\right) = CP_{\varphi}\left(i+\frac{1}{2},k+\frac{1}{2}\right) h_{\varphi}^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right) + CQ_{\varphi}\left(i+\frac{1}{2},k+\frac{1}{2}\right) \cdot \left(\frac{\sum_l a(l)\, e_z^{n+1/2}(i+l+1, k+1/2)}{\Delta\rho} - \frac{\sum_l a(l)\, e_{\rho}^{n+1}(i+1/2, k+l+1)}{\Delta z} + \psi_{h_{\varphi\rho}}^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right) - \psi_{h_{\varphi z}}^{n+1}\left(i+\frac{1}{2},k+\frac{1}{2}\right) \right) \tag{14e}$$

$$\psi_{h_{\varphi z}}^{n+1}\left(i+\frac{1}{2},k+\frac{1}{2}\right) = Q_{k+1/2}\psi_{h_{\varphi z}}^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right) + \frac{P_{k+1/2}}{\Delta z}\sum_l a(l)\, e_{\rho}^{n+1}\left(i+\frac{1}{2},k+l+1\right) \tag{14ei}$$

$$\psi_{h_{\varphi\rho}}^{n+1}\left(i+\frac{1}{2},k+\frac{1}{2}\right) = Q_{i+1/2}\psi_{h_{\varphi\rho}}^{n+1/2}\left(i+\frac{1}{2},k+\frac{1}{2}\right) + \frac{P_{i+1/2}}{\Delta\rho}\sum_l a(l)\, e_z^{n+1/2}\left(i+l+1,k+\frac{1}{2}\right) \tag{14eii}$$

$$h_z^{n+1}\left(i+\frac{1}{2},k\right) = CP_z\left(i+\frac{1}{2},k\right) h_z^{n+1/2}\left(i+\frac{1}{2},k\right) + CQ_z\left(i+\frac{1}{2},k\right)\left(\mp\frac{m e_{\rho}^{n+1/2}(i+1/2,k)}{(i+1/2)\Delta\rho} - \frac{\sum_l a(l)(i+l+1)\, e_{\varphi}^{n+1}(i+l+1,k)}{\kappa_{i+1/2}(i+1/2)\Delta\rho} - \psi_{h_{z\rho}}^{n+1}\left(i+\frac{1}{2},k\right) \right) \tag{14f}$$

$$\psi_{h_{z\rho}}^{n+1}\left(i+\frac{1}{2},k\right) = Q_{i+1/2}\psi_{h_{z\rho}}^{n+1/2}\left(i+\frac{1}{2},k\right) + \frac{P_{i+1/2}}{\Delta\rho} \cdot \sum_l a(l)\, e_{\varphi}^{n+1}(i+l+1,k), \tag{14fi}$$

where

$$CA_{\rho}\left(i+\frac{1}{2},k\right) = \frac{4\varepsilon - \sigma\Delta t}{4\varepsilon + \sigma\Delta t}$$
$$CP_{\rho}\left(i,k+\frac{1}{2}\right) = \frac{4\mu - \sigma_m\Delta t}{4\mu + \sigma_m\Delta t}$$
$$CB_{\rho}\left(i+\frac{1}{2},k\right) = \frac{2\Delta t}{4\varepsilon + \sigma\Delta t}$$
$$CQ_{\rho}\left(i,k+\frac{1}{2}\right) = \frac{2\Delta t}{4\mu + \sigma_m\Delta t} \tag{15a}$$

$$CA_{\varphi}(i,k) = \frac{4\varepsilon - \sigma\Delta t}{4\varepsilon + \sigma\Delta t}$$
$$CP_{\varphi}\left(i+\frac{1}{2},k+\frac{1}{2}\right) = \frac{4\mu - \sigma_m\Delta t}{4\mu + \sigma_m\Delta t}$$
$$CB_{\varphi}(i,k) = \frac{2\Delta t}{4\varepsilon + \sigma\Delta t}$$
$$CQ_{\varphi}\left(i+\frac{1}{2},k+\frac{1}{2}\right) = \frac{2\Delta t}{4\mu + \sigma_m\Delta t} \tag{15b}$$

$$CA_z\left(i,k+\frac{1}{2}\right)=\frac{4\varepsilon-\sigma\Delta t}{4\varepsilon+\sigma\Delta t}$$
$$CP_z\left(i+\frac{1}{2},k\right)=\frac{4\mu-\sigma_m\Delta t}{4\mu+\sigma_m\Delta t}$$
$$CB_z\left(i,k+\frac{1}{2}\right)=\frac{2\Delta t}{4\varepsilon+\sigma\Delta t} \tag{15c}$$
$$CQ_z\left(i+\frac{1}{2},k\right)=\frac{2\Delta t}{4\mu+\sigma_m\Delta t}$$

$$P_\xi=\frac{\sigma_\xi}{\sigma_\xi\kappa_\xi+\kappa_\xi^2\alpha_\xi}\left(e^{-(\sigma_\xi/\kappa_\xi+\alpha_\xi)(\Delta t/2\varepsilon_0)}-1\right),$$
$$Q_\xi=e^{-(\sigma_\xi/\kappa_\xi+\alpha_\xi)(\Delta t/2\varepsilon_0)}, \tag{15d}$$
$$\xi=i,k.$$

Equations (13a)–(13fi) and (14a)–(14fi) can be also solved by the generalized Thomas method [5]. The value of e_z is calculated the same as (11) and (12).

In the matched layer, the coefficients σ_i and κ_i are defined as follows [12, 13]:

$$\sigma(\rho)=\sigma_{\max}\left(\frac{\rho}{d}\right)^m$$
$$\kappa(\rho)=1+(\kappa_{\max}-1)\left(\frac{\rho}{d}\right)^m, \tag{16}$$

where ρ is the distance from the spot in the matched layer to the interface between computational domain and matched layer, d is the thickness of matched layer, and m is a polynomial coefficient. $\sigma_{\max}$ is defined as follows:

$$\sigma_{\max}=k\sigma_{\text{opt}}$$
$$\sigma_{\text{opt}}=\frac{(m+1)}{150\pi\sqrt{\varepsilon_r}\Delta}, \tag{17}$$

where $k=\sigma_{\max}/\sigma_{\text{opt}}$ is positive and α is positive too.

5. Numerical Results

ADI-BOR-MRTD method has been tested by a metal ball and a metal cylinder with half-ball-hat. For comparison, they have been also calculated by FDTD and MRTD methods.

CPU is Intel(R) Core(TM) i3 2.93 GHz; the memory bank is 1.93 GB; the Mac OS is Microsoft Windows XP Professional; the operating system is Fortran 90 Complier.

5.1. The Ball. The radius of metal ball is 1 meter. The results are shown in Figure 2 and Table 2.

(1) FDTD: $\Delta x\times\Delta y\times\Delta z=2\,\text{cm}\times2\,\text{cm}\times2\,\text{cm}$, $\Delta t=3.33\times10^{-11}$ s, and the cell lattice is $138\times138\times138$ with eight-cell-thick matched layer.

(2) MRTD: $\Delta x\times\Delta y\times\Delta z=10\,\text{cm}\times10\,\text{cm}\times10\,\text{cm}$, $\Delta t=11.11\times10^{-11}$ s, and the cell lattice is $56\times56\times56$ with eight-cell-thick matched layer.

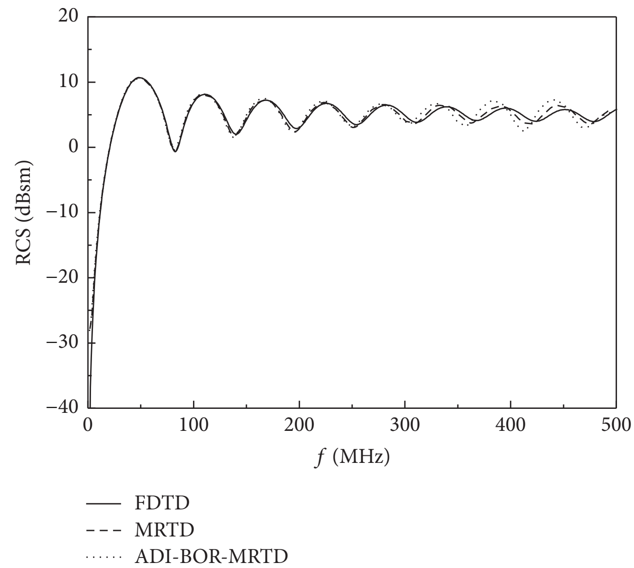

Figure 2: Single station RCS of metal ball.

Table 2: Comparison of the time and memory of CPU.

	FDTD	MRTD	ADI-BOR-MRTD
CPU time/s	1652	5	0.4
Memory/MB	172.3	13.4	2.6

Table 3: Comparison of the time and memory of CPU.

	FDTD	MRTD	ADI-BOR-MRTD
CPU time/s	1550	37	5
Memory/MB	161	13	0.23

(3) ADI-BOR-MRTD: $\Delta\rho\times\Delta z=10\,\text{cm}\times10\,\text{cm}$, $\Delta t=22.22\times10^{-11}$ s, the cell lattice is 28×56 with eight-cell-thick matched layer, and the modulus range is $m=0\sim16$.

Figure 2 shows that when the frequency is less than 500 MHz, the differences among three numerical results are less than 2 dB, which validate the feasibility of the ADI-BOR-MRTD method. Moreover, Table 2 demonstrates that the ADI-BOR-MRTD method has taken less time and memory of CPU than the other two methods.

5.2. The Cylinder with Half-Ball-Hat. The metal cylinder with half-ball-hat is designed as Figure 3.

The results are shown as Figure 4 and Table 3.

(1) FDTD: $\Delta x\times\Delta y\times\Delta z=1\,\text{cm}\times1\,\text{cm}\times1\,\text{cm}$, $\Delta t=1.67\times10^{-11}$ s, and the cell lattice is $96\times96\times266$ with eight-cell-thick matched layer.

(2) MRTD: $\Delta x\times\Delta y\times\Delta z=5\,\text{cm}\times5\,\text{cm}\times5\,\text{cm}$, $\Delta t=5.56\times10^{-11}$ s, and the cell lattice is $48\times48\times82$ with eight-cell-thick matched layer.

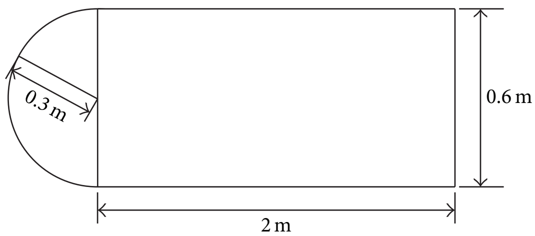

Figure 3: Structure of cylinder with half-ball-hat.

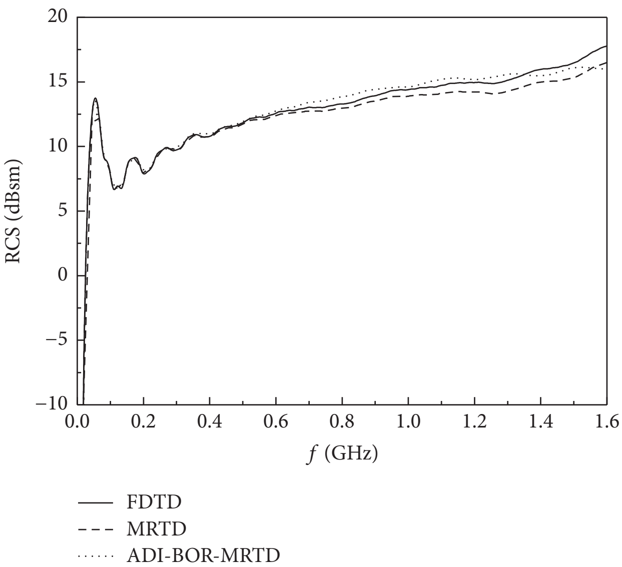

Figure 4: Single station RCS of metal cylinder with half-ball-hat.

(3) ADI-BOR-MRTD: $\Delta\rho \times \Delta z = 5\,\text{cm} \times 5\,\text{cm}$, $\Delta t = 11.11 \times 10^{-11}$ s, the cell lattice is 24×82 with eight-cell-thick matched layer, and the modulus range is $m = 0 \sim 16$.

From Figure 4 we can see that when the frequency is less than 1.5 GHz, the differences among three numerical results are less than 3 dB and the curves are similar. The results in Table 3 have also supported that ADI-BOR-MRTD method has taken less time and memory of CPU than the other two methods.

6. Conclusion

This paper has developed an ADI-BOR-MRTD algorithm. Furthermore, the CPML absorbing boundary condition is derived for ADI-BOR-MRTD algorithm. The simulated results suggest that the ADI-BOR-MRTD scheme can save more CPU time and memory than the FDTD and MRTD methods, which proves that the ADI-BOR-MRTD scheme is practicable, especially in the body of revolution case. In the next work, the method would be improved feasible for the frequency more than 500 MHz.

Competing Interests

The authors declare that they have no competing interests.

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