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Research Article

Theta Function Solutions of the 3 + 1-Dimensional Jimbo-Miwa Equation

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The 3 + 1-dimensional Jimbo-Miwa equation can be written into a Hirota bilinear form by the dependent variable transformation. We give its one-periodic wave solution and two-periodic wave solution by utilizing multidimensional elliptic Θ-function. With the help of the solution curves, the asymptotic properties of the periodic waves are analyzed in detail.

1. Introduction

Nonlinear phenomena arise in many physical problems in a variety of fields. Solutions of the governing nonlinear equations can shed light on these phenomena. There are various systematical methods for constructing solutions, for example, nonlinearization method of Lax pairs [1–3], extended F-expansion method [4–6] and homogeneous balance method [7–10], and dressing method and generalized dressing method [11–16]. It is well known that the Hirota method with the aid of Riemann-theta function is a good method to obtain periodic and quasiperiodic solutions. Nakamura [17, 18] used this method to study some famous equations such as KdV, KP, Boussinesq, and Toda. By extending the approach adopted by Nakamura, Dai et al. obtained the graphic quasiperiodic solutions for the KP equation for the first time [19] and later they also gave the quasiperiodic solutions for Toda lattice [20]. Recently, a lot of researchers have used this method to study various soliton equations [21–24].

In the present paper, we consider the 3 + 1-dimensional Jimbo-Miwa equation

$$u_{xxxy} + 3u_{xy}u_x + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0, \quad (1)$$

which is the second member of a KP hierarchy [25]. It has important physics to describe 3 + 1-dimensional waves. In the last decade or so, many researchers have studied this equation. Multiple-soliton solutions of (1) and its extended version were given in Wazwaz [26]. Tang in [27] obtained its Pfaffian solution and extended Pfaffian solutions with the aid of the Hirota bilinear form. Su et al. [28] constructed its Wronskian and Grammian solutions. Multiple-front solutions for (1) were obtained by employing the Hirota bilinear method in [29]. Ozixs and Aslan in [30] derived exact solutions of (1) via Exp-function method. In [31], Ma and Lee have obtained rational solutions of (1) including travelling wave solutions, variable separated solutions, and polynomial solutions by using rational function transformation and Bäcklund transformation. Li et al. have utilized generalized three-wave method to derive explicit three-wave solutions, such as doubly periodic solitary wave solutions and breather type of two-solitary wave solutions for (1) in [32]. Zhang et al. have obtained generalized Wronskian solution in [33]. Dai et al. obtained new periodic kink-wave and kinky periodic wave solutions for (1) in [34]. Ma in [35] has derived exact solutions by using Lie point symmetry groups of (1). Tang and Liang in [36] have obtained two types of variable separation solutions and abundant nonlinear coherent structure by using multilinear variable separation approach. Ma et al. obtained new exact solutions for (1) by utilizing improved mapping approach [37]. Liu and Jiang by applying the extended homogeneous balance method have obtained new solutions of (1) in [38].

Authors in [39] obtained special solutions by using extended homogeneous balance method. Hu et al. [40] discussed 3-soliton and 4-soliton solutions with the aid of bilinear form of (1). In [41], some complexion type solutions of (1) are presented by using two kinds of transcendental functions. Authors presented rational solutions of (1) with the aid of the generalized Riccati equation mapping method [42]. By utilizing two-soliton method and bilinear method, cross kink-wave and periodic solitary solutions of (1) are given in [43]. With the help of the bilinear form, here we construct one-periodic and two-periodic solutions of this equation (1) by utilizing the method of Dai et al. [19].

The paper is organized as follows. In Section 2, we obtain the formula of one-periodic wave solutions and discuss its asymptotic behavior. Further, in order to analyze the solution, some solution curves are plotted. In the third section, two-periodic wave solutions, their asymptotic behaviors and the solution curves are given.

2. One-Periodic Wave Solution of the 3 + 1-Dimensional Jimbo-Miwa Equation and Its Asymptotic Form

We consider the 3 + 1-dimensional Jimbo-Miwa equation

$$u_{xxxy} + 3u_{xy}u_x + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0. \quad (2)$$

Substituting the transformation

$$u = u_0 + 2(\ln f)_x \quad (3)$$

into (2) and integrating once again, we derive the following bilinear form:

$$\frac{D_x^3 D_y f\cdot f}{f^2} + 2\frac{D_t D_y f\cdot f}{f^2} - 3\frac{D_x D_z f\cdot f}{f^2} + c = 0; \quad (4)$$

namely,

$$\begin{aligned} &G(D_x, D_t, D_y, D_z) f\cdot f \\ &\quad = (D_x^3 D_y + 2D_t D_y - 3D_x D_z + c) f\cdot f = 0, \end{aligned} \quad (5)$$

where c is an integration constant.

The Hirota bilinear differential operator is defined by [44]

$$\begin{aligned} &D_x^m D_t^n D_y^{m_1} D_z^{n_1} f\cdot g \\ &\quad = (\partial_x - \partial_{x'})^m (\partial_t - \partial_{t'})^n (\partial_y - \partial_{y'})^{m_1} (\partial_z - \partial_{z'})^{n_1} f \\ &\quad \cdot g|_{x=x', t=t', y=y', z=z'}. \end{aligned} \quad (6)$$

The D-operator possesses the good property when acting on exponential functions:

$$\begin{aligned} &D_x^m D_t^n D_y^{m_1} D_z^{n_1} e^{\xi_1}\cdot e^{\xi_2} \\ &\quad = (p_1 - p_2)^m (\rho_1 - \rho_2)^n (l_1 - l_2)^{m_1} (\mu_1 - \mu_2)^{n_1} e^{\xi_1+\xi_2}, \end{aligned} \quad (7)$$

where $\xi_j = p_j x + l_j y + \mu_j z + \rho_j t + \xi_{0_j}$, $j = 1, 2$. More generally, we have

$$\begin{aligned} &G(D_x, D_t, D_y, D_z) e^{\xi_1}\cdot e^{\xi_2} \\ &\quad = G(p_1 - p_2, l_1 - l_2, \rho_1 - \rho_2, \mu_1 - \mu_2) e^{\xi_1+\xi_2}. \end{aligned} \quad (8)$$

2.1. One-Periodic Wave Solution. In what follows, we consider Riemann-theta function solution of (5):

$$f = \sum_{n\in Z^N} e^{\pi i\langle\tau n, n\rangle + 2\pi i\langle\xi, n\rangle}, \quad (9)$$

where $n = (n_1, n_2, \ldots, n_N)^T$, $\xi = (\xi_1, \xi_2, \ldots, \xi_N)^T$, τ is a symmetric matrix, and $\operatorname{Im}\tau > 0$, $\xi_j = p_j x + l_j y + \mu_j z + \rho_j t + \xi_{oj}$, $j = 1, 2, \ldots, N$.

First, we consider one-periodic solution of (5) when $N = 1$; then (9) is

$$f = \sum_{n=-\infty}^{\infty} e^{2\pi i n\xi + \pi i n^2\tau}. \quad (10)$$

Substituting (10) into (5) yields that

$$\begin{aligned} &G(D_x, D_y, D_z, D_t) f\cdot f = G(D_x, D_y, D_z, D_t) \\ &\quad\cdot \sum_{n=-\infty}^{\infty} e^{2\pi i n\xi + \pi i n^2\tau} \cdot \sum_{m=-\infty}^{\infty} e^{2\pi i m\xi + \pi i m^2\tau} \\ &\quad = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} G(D_x, D_y, D_z, D_t) e^{2\pi i n\xi + \pi i n^2\tau} \\ &\quad\cdot e^{2\pi i m\xi + \pi i m^2\tau} = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} G(2\pi i(n-m)p, \\ &\quad 2\pi i(n-m)l, 2\pi i(n-m)\mu, 2\pi i(n-m)\rho) \\ &\quad\cdot e^{2\pi i(n+m)\xi + \pi i(n^2+m^2)\tau} = \sum_{m'=-\infty}^{\infty}\left\{\sum_{n=-\infty}^{\infty} G\right. \\ &\quad\cdot (2\pi i(2n-m')p, 2\pi i(2n-m')l, 2\pi i(2n-m')\mu, \\ &\quad\left. 2\pi i(2n-m')\rho) e^{\pi i[n^2+(n-m')^2\tau]}\right\} e^{2\pi i m'\xi} \\ &\quad = \sum_{m'=-\infty}^{\infty} \overline{G}(m') e^{2\pi i m'\xi} = 0, \end{aligned} \quad (11)$$

where $m' = n + m$.

$$\begin{aligned} &\overline{G}(m') = \sum_{n=-\infty}^{\infty} G(2\pi i(2n-m')p, 2\pi i(2n-m') \\ &\quad\cdot l, 2\pi i(2n-m')\mu, 2\pi i(2n-m')\rho) e^{\pi i[n^2+(n-m')^2\tau]} \\ &\quad = \sum_{n'=-\infty}^{\infty} G(2\pi i[2n'-(m'-2)] \\ &\quad\cdot p, 2\pi i[2n'-(m'-2)] l, 2\pi i[2n'-(m'-2)] \end{aligned}$$

$$\begin{aligned}&\cdot \mu, 2\pi i\left[2n' - \left(m' - 2\right)\right]\rho)\, e^{\pi i[n'^2+(n'-(m'-2))^2]\tau}\\ &\cdot e^{2\pi i(m'-1)\tau} = \overline{G}\left(m' - 2\right) e^{2\pi i(m'-1)\tau} = \cdots\\ &= \begin{cases}\overline{G}(0)\, e^{\pi i m'(m'-1)\tau}, & m' \text{ is even,}\\ \overline{G}(1)\, e^{\pi i(m'+1)(m'-2)\tau}, & m' \text{ is odd,}\end{cases}\end{aligned} \tag{12}$$

where $n' = n - 1$, which implies that if $\overline{G}(0) = \overline{G}(1) = 0$, then $\overline{G}(m') = 0$, $m' \in N$.

First, we consider

$$\begin{aligned}\overline{G}(0) &= \sum_{n=-\infty}^{\infty}\left[256\pi^4 n^4 p^3 l - 32\pi^2 n^2 \rho l + 48\pi^2 n^2 p\mu\right.\\ &\quad \left.+ c\right] e^{2\pi i n^2\tau} = 0,\\ \overline{G}(1) &= \sum_{n=-\infty}^{\infty}\left[16\pi^4 (2n-1)^4 p^3 l - 8\pi^2 (2n-1)^2 \rho l\right.\\ &\quad \left.+ 12\pi^2 (2n-1)^2 p\mu + c\right] e^{\pi i[n^2+(n-1)^2]\tau} = 0;\end{aligned} \tag{13}$$

let

$$\begin{aligned}&\delta_1(n) = e^{2\pi i n^2\tau},\\ &\delta_2(n) = e^{\pi i[n^2+(n-1)^2]\tau},\\ &b_{11} = \sum_{n=-\infty}^{\infty}\left[256\pi^4 n^3 p^3 - 32\pi^2 n^2 \rho\right]\delta_1(n),\\ &b_{12} = \sum_{n=-\infty}^{\infty} 48\pi^2 n^2 p\delta_1(n),\\ &\Delta_1 = \sum_{n=-\infty}^{\infty}\delta_1(n),\\ &b_{22} = \sum_{n=-\infty}^{\infty} 12\pi^2 (2n-1)^2 p\delta_2(n),\\ &\Delta_2 = \sum_{n=-\infty}^{\infty}\delta_2(n),\\ &b_{21}\\ &\quad = \sum_{n=-\infty}^{\infty}\left[16\pi^4 (2n-1)^4 p^3 - 8\pi^2 (2n-1)^2 \rho\right]\delta_2(n);\end{aligned} \tag{14}$$

then (13) can be written as

$$\begin{aligned}l b_{11} + b_{12}\mu + c\Delta_1 &= 0,\\ l b_{21} + b_{22}\mu + c\Delta_2 &= 0.\end{aligned} \tag{15}$$

Solving this system, we get

$$\begin{aligned}l &= \frac{\mu\left(b_{22}\Delta_1 - b_{12}\Delta_2\right)}{b_{11}\Delta_2 - b_{21}\Delta_1},\\ c &= \frac{\mu\left(b_{12}b_{21} - b_{22}b_{11}\right)}{b_{11}\Delta_2 - b_{21}\Delta_1}.\end{aligned} \tag{16}$$

Then we get one-periodic wave solution

$$u = u_0 + 2\left(\ln f\right)_x = u_0 + 2\frac{f_x}{f}, \tag{17}$$

where f and l, p, μ, and ρ are given by (10) and (16).

We see that the well known soliton solution of the 3 + 1-dimensional Jimbo-Miwa equation can be obtained as limit of the periodic solution (17).

Theorem 1. *Under the condition* ($\operatorname{Im}\tau \to \infty$)*, the solution (10) of (5) tends to the following exact solution of (2) via (3):*

$$u = u_0 + 4\pi i p\frac{e^{2\pi i(px+ly+\mu z+\rho t+\widetilde{\xi_0})}}{1 + e^{2\pi i(px+ly+\mu z+\rho t+\widetilde{\xi_0})}}, \tag{18}$$

with $l \to -3p\mu/(4\pi^2 p^3 - 2\rho)$*, as* $\alpha = e^{\pi i\tau} \to 0$*, where* $\widetilde{\xi_0}$ *is an arbitrary constant.*

Proof. We write f as

$$f = 1 + \alpha\left(e^{2\pi i\xi} + e^{-2\pi i\xi}\right) + \alpha^4\left(e^{4\pi i\xi} + e^{-4\pi i\xi}\right) + \cdots, \tag{19}$$

with $\xi = px + ly + \mu z + \rho t + \xi_0$.

If we make an arbitrary phase constant slightly as $\xi_0 = \widetilde{\xi_0} - (1/2)\tau$ and have small amplitude limit of $\alpha = e^{\pi i\tau} \to 0$, then we derive proper limit

$$f \longrightarrow 1 + e^{2\pi i(px+ly+\mu z+\rho t+\widetilde{\xi_0})}. \tag{20}$$

It is easy to obtain the (18). In fact, we have

$$\begin{aligned}b_{11} &= \sum_{n=-\infty}^{\infty}\left(256\pi^4 n^4 p^3 - 32\pi^2 n^2 \rho\right) e^{2\pi i n^2\tau}\\ &= \left(512\pi^4 p^3 - 64\pi^2 \rho\right)\alpha^2 + o\left(\alpha^2\right),\\ \Delta_1 &= \sum_{n=-\infty}^{\infty} e^{2\pi i n^2\tau} = 1 + 2\alpha^2 + o\left(\alpha^2\right),\\ b_{12} &= \sum_{n=-\infty}^{\infty} 48\pi^2 n^2 p e^{2\pi i n^2\tau} = 96\pi^2 p\alpha^2 + o\left(\alpha^2\right),\\ \Delta_2 &= \sum_{n=-\infty}^{\infty} e^{\pi i(2n^2-2n+1)\tau} = 2\alpha + 2\alpha^5 + o\left(\alpha^5\right),\end{aligned}$$

$$b_{22} = \sum_{n=-\infty}^{\infty} 12\pi^2 (2n-1)^2 p e^{\pi i (n^2 + (n-1)^2)\tau} = 24\pi^2 p\alpha + o(\alpha),$$
$$b_{21} = \sum_{n=-\infty}^{\infty} \left[16\pi^4 (2n-1)^4 p^3 - 8\pi^2 (2n-1)^2 \rho\right] \cdot e^{\pi i (2n^2-2n+1)\tau} = \left(32\pi^4 p^3 - 16\pi^2 \rho\right)\alpha + o\left(\alpha^5\right). \tag{21}$$

By utilizing (16), it is easy to deduce that

$$l \longrightarrow \frac{-3p\mu}{4\pi^2 p^3 - 2\rho}, \quad c \longrightarrow 0. \tag{22}$$

The one-periodic solution curves are presented in Figures 1 and 2 for $u_0 = 0$, respectively, in two-dimensional and three-dimensional space. It is obvious that the solution is periodic and cuspon from the above solution graphs. It is different to the Pfaffian solutions and extended Pfaffian solutions derived by Tang in [27]. The results are different to one-soliton solution and two-soliton solutions represented by researchers in [28, 30–33]. There are some difference between new types of exact periodic solitary wave and kinky periodic wave solutions in [34] by Dai et al. and the solutions in the paper. □

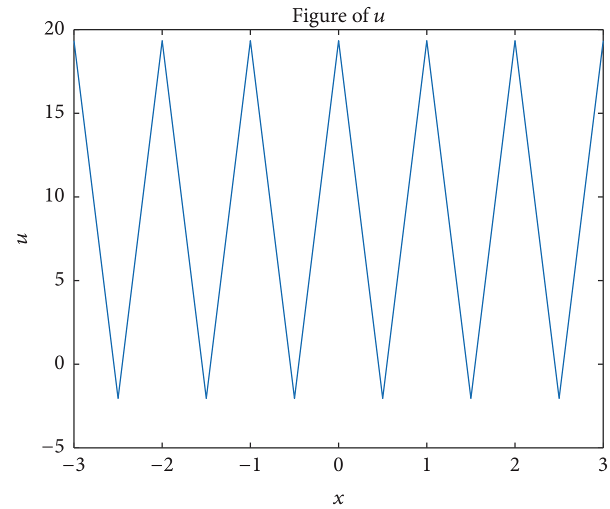

FIGURE 1: $z = 0.6$, $t = 2$, $y = 2$, $p = 0.1$, $\mu = 2$, $\rho = 4$, $\tilde{\zeta} = 0.8$, and $x \in [-3, 3]$.

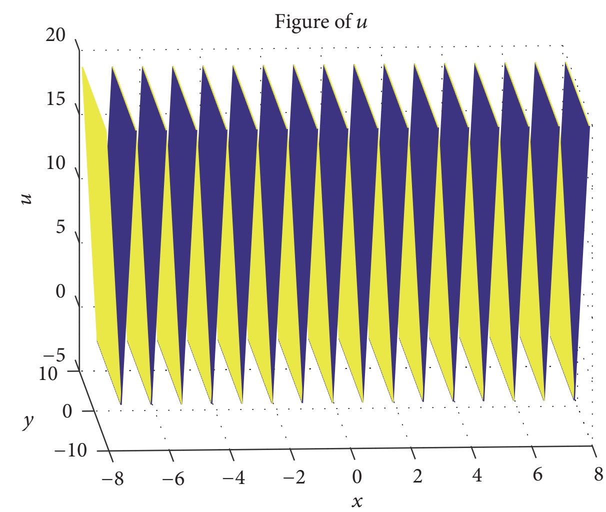

FIGURE 2: $z = 0.6$, $t = 2$, $p = 0.1$, $\mu = 2$, $\rho = 4$, $\tilde{\zeta} = 0.8$, $x \in [-6, 6]$, and $y \in [5, 20]$.

3. Two-Periodic Wave Solution of the 3 + 1-Dimensional Jimbo-Miwa Equation and Its Asymptotic Behavior

We consider the two-periodic wave solution of the 3 + 1-dimensional Jimbo-Miwa equation (2). Substituting (9) ($N = 2$) into (5), we obtain

$$\begin{aligned} Gf \cdot f &= \sum_{m,n \in Z^2} G\left(D_x, D_t, D_y, D_z\right) e^{2\pi i \langle \zeta, n\rangle + \pi i \langle \tau n, n\rangle} \cdot e^{2\pi i \langle \zeta, m\rangle + \pi i \langle \tau m, m\rangle} = \sum_{m,n\in Z^2} G(2\pi i \langle n-m, p\rangle, \\ &\quad 2\pi i \langle n-m, l\rangle, 2\pi i \langle n-m, \mu\rangle, 2\pi i \langle n-m, \rho\rangle) \cdot e^{2\pi i \langle \zeta, n+m\rangle + \pi i (\langle \tau m, m\rangle + \langle \tau n, n\rangle)} = \sum_{m' \in Z^2} \sum_{n_1, n_2 = -\infty}^{\infty} G \\ &\quad \cdot \left(2\pi i \left\langle 2n - m', p\right\rangle, 2\pi i \left\langle 2n-m', l\right\rangle, 2\pi i \left\langle 2n - m', \mu\right\rangle, 2\pi i \left\langle 2n-m', \rho\right\rangle\right) \\ &\quad \cdot \exp\left(\pi i \left(\left\langle \tau\left(n-m'\right), n-m'\right\rangle + \langle \tau n, n\rangle\right)\right) \cdot \exp\left(2\pi i \left\langle \xi, m'\right\rangle\right) \equiv \sum_{m' \in Z^2} \overline{G}\left(m_1', m_2'\right) \\ &\quad \cdot \exp\left(2\pi i \left\langle \xi, m'\right\rangle\right) = 0, \end{aligned} \tag{23}$$

where $m' = n + m$. It is easy to calculate that

$$\begin{aligned} \overline{G}\left(m_1', m_2'\right) &= \sum_{n_1, n_2 = -\infty}^{\infty} G\left(2\pi i \left\langle 2n - m', p\right\rangle, 2\pi i \left\langle 2n-m', l\right\rangle, 2\pi i \left\langle 2n-m', \mu\right\rangle, 2\pi i \left\langle 2n - m', \rho\right\rangle\right) e^{\pi i (\langle \tau (n-m'), n-m'\rangle + \langle \tau n, n\rangle)} \\ &= \sum_{n_1, n_2 = -\infty}^{\infty} G\Bigg(2\pi i \sum_{j=1}^{2} \left[2n_j' - \left(m_j' - 2\delta_{jl}\right)\right] p_j, \\ &\quad 2\pi i \sum_{j=1}^{2} \left[2n_j' - \left(m_j' - 2\delta_{jl}\right)\right] l_j, \\ &\quad 2\pi i \sum_{j=1}^{2} \left[2n_j' - \left(m_j' - 2\delta_{jl}\right)\right] \mu_j, \end{aligned}$$

$$
\begin{aligned}
&2\pi i\sum_{j=1}^{2}\left[2n_j'-\left(m_j'-2\delta_{jl}\right)\right]\rho_j\Bigg) \\
&\cdot\exp\left\{\pi i\sum_{j,k=1}^{2}\left[\left(n_j'+\delta_{jl}\right)\tau_{jk}\left(n_k'+\delta_{kl}\right)\right]\right. \\
&+\left[\left(m_j'-2\delta_{jl}-n_j'\right)+\delta_{jl}\right] \\
&\left.\cdot\tau_{jk}\left[\left(m_k'-2\delta_{kl}-n_k'\right)+\delta_{kl}\right]\right\} \\
&=\begin{cases}\overline{G}\left(m_1'-2,m_2'\right)e^{2\pi i(m_1'-1)\tau_{11}+2\pi i m_2'\tau_{12}}, & l=1,\\ \overline{G}\left(m_1',m_2'-2\right)e^{2\pi i(m_2'-1)\tau_{22}+2\pi i m_1'\tau_{21}}, & l=2,\end{cases}
\end{aligned} \tag{24}
$$

which implies that if $\overline{G}(0,0)=\overline{G}(0,1)=\overline{G}(1,0)=\overline{G}(1,1)=0$, then $\overline{G}(m_1',m_2')=0$ and f is an exact solution of (5).

$$
\begin{aligned}
\overline{G}(0,0)&=\sum_{n_1,n_2=-\infty}^{\infty}\Big[\left(2n_1l_1+2n_2l_2\right) \\
&\cdot\left(16\pi^4\left\langle 2n-m^0,p\right\rangle^3-8\pi^2\left\langle 2n-m^0,\rho\right\rangle\right) \\
&+12\pi^2\left(2n_1\mu_1+2n_2\mu_2\right)\left\langle 2n-m^0,p\right\rangle+c\Big]\delta_0(n), \\
\overline{G}(0,1)&=\sum_{n_1,n_2=-\infty}^{\infty}\Big[\left(2n_1l_1+\left(2n_2-1\right)l_2\right) \\
&\cdot\left(16\pi^4\left\langle 2n-m^1,p\right\rangle^3-8\pi^2\left\langle 2n-m^1,\rho\right\rangle\right) \\
&+12\pi^2\left(2n_1\mu_1+\left(2n_2-1\right)\mu_2\right)\left\langle 2n-m^1,p\right\rangle+c\Big] \\
&\cdot\delta_1(n), \\
\overline{G}(1,0)&=\sum_{n_1,n_2=-\infty}^{\infty}\Big[\left(\left(2n_1-1\right)l_1+2n_2l_2\right) \\
&\cdot\left(16\pi^4\left\langle 2n-m^2,p\right\rangle^3-8\pi^2\left\langle 2n-m^2,\rho\right\rangle\right) \\
&+12\pi^2\left(\left(2n_1-1\right)\mu_1+2n_2\mu_2\right)\left\langle 2n-m^2,p\right\rangle+c\Big] \\
&\cdot\delta_2(n), \\
\overline{G}(1,1)&=\sum_{n_1,n_2=-\infty}^{\infty}\Big[\left(\left(2n_1-1\right)l_1+\left(2n_2-1\right)l_2\right) \\
&\cdot\left(16\pi^4\left\langle 2n-m^3,p\right\rangle^3-8\pi^2\left\langle 2n-m^3,\rho\right\rangle\right) \\
&+12\pi^2\left(\left(2n_1-1\right)\mu_1+\left(2n_2-1\right)\mu_2\right)\left\langle 2n-m^3,p\right\rangle \\
&+c\Big]\delta_3(n).
\end{aligned} \tag{25}
$$

Letting

$$
\begin{aligned}
\delta_j(n)&=e^{\pi i\langle\tau n-m^j,n-m^j\rangle+\pi i\langle\tau n,n\rangle}, \\
m^0&=(0,0)^T, \\
m^1&=(0,1)^T, \\
m^2&=(1,0)^T, \\
m^3&=(1,1)^T,
\end{aligned} \tag{26}
$$

we have

$$
A\begin{pmatrix}l_1\\ l_2\\ \mu_1\\ \mu_2\end{pmatrix}=\vec{b}, \tag{27}
$$

where

$$
\begin{aligned}
a_{j1}&=\sum_{n_1,n_2=-\infty}^{\infty}\Big[\left(2n_1-m_1^j\right) \\
&\cdot\left(16\pi^4\left\langle 2n-m^j,p\right\rangle^3-8\pi^2\left\langle 2n-m^j,p\right\rangle\right)\Big] \\
&\cdot\delta_j(n), \\
a_{j2}&=\sum_{n_1,n_2=-\infty}^{\infty}\Big[\left(2n_2-m_2^j\right) \\
&\cdot\left(16\pi^4\left\langle 2n-m^j,p\right\rangle^3-8\pi^2\left\langle 2n-m^j,p\right\rangle\right)\Big] \\
&\cdot\delta_j(n), \\
a_{j3}&=\sum_{n_1,n_2=-\infty}^{\infty}\left[\left(2n_1-m_1^j\right)\left(12\pi^2\right)\left\langle 2n-m^j,p\right\rangle\right] \\
&\cdot\delta_j(n), \\
a_{j4}&=\sum_{n_1,n_2=-\infty}^{\infty}\left[\left(2n_2-m_2^j\right)\left(12\pi^2\right)\left\langle 2n-m^j,p\right\rangle\right] \\
&\cdot\delta_j(n), \\
b_j&=-c\sum_{n_1,n_2=-\infty}^{\infty}\delta_j(n), \\
&\qquad j=0,1,2,3.
\end{aligned} \tag{28}
$$

From this, we have

$$l_1 = \frac{\Delta_1}{\Delta},$$
$$l_2 = \frac{\Delta_2}{\Delta},$$
$$\mu_1 = \frac{\Delta_3}{\Delta},$$
$$\mu_2 = \frac{\Delta_4}{\Delta}, \tag{29}$$

where $\Delta = A$ and $\Delta_1, \Delta_2, \Delta_3$, and Δ_4 are from Δ by replacing 1st, 2nd, 3rd, and 4th column with $\vec{b}$, respectively.

Theorem 2. *Under the condition $\alpha_1 \to 0, \alpha_2 \to 0$, the solution (9) ($N = 2$) of (5) tends to the two-periodic solution of (2) via (3).*

$$u = u_0 + 4\pi i \frac{p_1 e^{\tilde{\xi}_1} + p_2 e^{\tilde{\xi}_2} + (p_1 + p_2) e^{\tilde{\xi}_1 + \tilde{\xi}_2 + A}}{1 + e^{\tilde{\xi}_1} + e^{\tilde{\xi}_2} + e^{\tilde{\xi}_1 + \tilde{\xi}_2 + A}}, \tag{30}$$

where

$$e^{2\pi i \tau_{12}} = e^A = -\frac{(l_1 - l_2)\left[4\pi^4 (p_1 - p_2)^3 - 2\pi^2 (\rho_1 - \rho_2)\right] + 3\pi^2 (\mu_1 - \mu_2)(p_1 - p_2)}{(l_1 + l_2)\left[4\pi^4 (p_1 + p_2)^3 - 2\pi^2 (\rho_1 + \rho_2)\right] + 3\pi^2 (\mu_1 + \mu_2)(p_1 + p_2)},$$

$$\begin{aligned} &l_2 \left(4\pi^2 p_2^3 - 2\rho_2\right) + 3\mu_2 p_2 = 0, \\ &l_1 \left(4\pi^2 p_1^3 - 2\rho_1\right) + 3\mu_1 p_1 = 0, \end{aligned} \tag{31}$$

$$\tilde{\xi}_1 = 2\pi i \left(p_1 x + l_1 y + \mu_1 z + \rho_1 t + \tilde{\xi}_{01}\right),$$
$$\tilde{\xi}_2 = 2\pi i \left(p_2 x + l_2 y + \mu_2 z + \rho_2 t + \tilde{\xi}_{02}\right),$$

and $\tilde{\xi}_{01}$ and $\tilde{\xi}_{02}$ are arbitrary constants.

Proof. We write f as

$$\begin{aligned} f = 1 &+ \left(e^{2\pi i \xi_1} + e^{-2\pi i \xi_1}\right) e^{\pi i \tau_{11}} \\ &+ \left(e^{2\pi i \xi_2} + e^{-2\pi i \xi_2}\right) e^{\pi i \tau_{22}} \\ &+ \left(e^{2\pi i (\xi_1 + \xi_2)} + e^{-2\pi i (\xi_1 + \xi_2)}\right) e^{\pi i (\tau_{11} + 2\tau_{12} + \tau_{22})} + \cdots . \end{aligned} \tag{32}$$

Set

$$\begin{aligned} \xi_1 &= p_1 x + l_1 y + \mu_1 z + \rho_1 t + \xi_{01}, \\ \xi_2 &= p_2 x + l_2 y + \mu_2 z + \rho_2 t + \xi_{02}, \\ \xi_{01} &= \tilde{\xi}_{01} - \frac{\tau_{11}}{2}, \\ \xi_{02} &= \tilde{\xi}_{02} - \frac{\tau_{22}}{2}, \\ \alpha_1 &= e^{\pi i \tau_{11}}, \\ \alpha_2 &= e^{\pi i \tau_{22}}. \end{aligned} \tag{33}$$

Thus

$$f = 1 + e^{\tilde{\xi}_1} + e^{\tilde{\xi}_2} + e^{\tilde{\xi}_1 + \tilde{\xi}_2 + 2\pi i \tau_{12}} + o\left(\alpha_1^{s_1} \alpha_2^{s_2}\right). \tag{34}$$

As $\alpha_1 \to 0, \alpha_2 \to 0$, and $s_1 + s_2 \geq 3$, then $f \to 1 + e^{\tilde{\xi}_1} + e^{\tilde{\xi}_2} + e^{\tilde{\xi}_1 + \tilde{\xi}_2 + 2\pi i \tau_{12}}$.

In view of transformation (3), we have derived the solution (30) of (2). In what follows, we will certify (31).

$$\begin{aligned} \overline{G}(0,0) = &\sum_{n_1, n_2 = -\infty}^{\infty} \left[(2n_1 l_1 + 2n_2 l_2)\right. \\ &\cdot \left(16\pi^4 \left\langle 2n - m^0, p \right\rangle^3 - 8\pi^2 \left\langle 2n - m^0, \rho \right\rangle\right) \\ &\left. + 12\pi^2 (2n_1 \mu_1 + 2n_2 \mu_2) \left\langle 2n - m^0, p \right\rangle + c\right] \delta_0(n) \\ &= c + \left[2(l_1 + l_2)\left(128\pi^4 (p_1 + p_2)^3\right.\right. \\ &\left.\left. - 16\pi^2 (\rho_1 + \rho_2)\right) + c\right] e^{2\pi i (\tau_{11} + 2\tau_{12} + \tau_{22})} + \left[2(l_1 + l_2)\right. \\ &\left. \cdot \left(128\pi^4 (p_1 + p_2)^3 - 16\pi^2 (\rho_1 + \rho_2)\right) + c\right] \\ &\cdot e^{2\pi i (\tau_{11} + 2\tau_{12} + \tau_{22})} + \left[32 l_2 \pi^2 \left(8\pi^2 p_2^3 - \rho_2\right)\right. \\ &\left. + 48\pi^2 \mu_2 p_2 + c\right] e^{2\pi i \tau_{22}} + \left[32 l_1 \pi^2 \left(8\pi^2 p_1^3 - \rho_1\right)\right. \\ &\left. + 48\pi^2 \mu_1 p_1 + c\right] e^{2\pi i \tau_{11}} + o\left(\alpha_1^{s_1} \alpha_1^{s_2}\right) = c + 2\left[2(l_1\right. \\ &\left. + l_2)\left(128\pi^4 (p_1 + p_2)^3 - 16\pi^2 (\rho_1 + \rho_2)\right) + c\right] \\ &\cdot \alpha_1^2 \alpha_2^2 e^{4\pi i \tau_{12}} + \left[32 l_2 \pi^2 \left(8\pi^2 p_2^3 - \rho_2\right) + 48\pi^2 \mu_2 p_2\right. \end{aligned}$$

$$+ c\big] \alpha_2^2 + \left[32 l_1 \pi^2 \left(8\pi^2 p_1^3 - \rho_1\right) + 48\pi^2 \mu_1 p_1 + c\right] \alpha_1^2 + o\left(\alpha_1^{s_1} \alpha_1^{s_2}\right). \tag{35}$$

As $\alpha_1 \to 0$ and $\alpha_2 \to 0$, then $\overline{G}(0,0) \to 0$, with $s_1 + s_2 > 3$. Thus, we derive $c \to 0$.

$$\begin{aligned}\overline{G}(0,1) &= \sum_{n_1,n_2=-\infty}^{\infty} \Big[\left(2n_1 l_1 + (2n_2-1) l_2\right) \cdot \left(16\pi^4 \left\langle 2n - m^1, p\right\rangle^3 - 8\pi^2 \left\langle 2n - m^1, \rho\right\rangle\right) \\ &\quad + 12\pi^2 \left(2n_1\mu_1 + (2n_2-1)\mu_2\right) \left\langle 2n - m^1, p\right\rangle + c\Big] \cdot \delta_1(n) = 2\left[l_2\left(16\pi^4 p_2^3 - 8\pi^2 \rho_2\right) + 12\pi^2 \mu_2 p_2 + c\right] \\ &\quad \cdot e^{\pi i \tau_{22}} + \left[(2l_1 - l_2)\left[16\pi^4 (2p_1 - p_2)^3 - 8\pi^2 (2\rho_1 - \rho_2)\right] + 12\pi^2 (2\mu_1 - \mu_2)(2p_1 - p_2)\right] \\ &\quad \cdot e^{\pi i (2\tau_{11} + \tau_{22} - 2\tau_{12})} + o\left(\alpha_1^{s_1}\alpha_2^{s_2}\right) \quad (s_1 + s_2 \geq 3).\end{aligned} \tag{36}$$

From $\overline{G}(0,1) \to 0$, we have

$$l_2\left(4\pi^2 p_2^3 - 2\rho_2\right) + 3\mu_2 p_2 = 0. \tag{37}$$

Similarly,

$$\begin{aligned}\overline{G}(1,0) &= \sum_{n_1,n_2=-\infty}^{\infty} \Big[\left(2n_2 l_2 + (2n_1-1) l_1\right) \cdot \left(16\pi^4 \left\langle 2n - m^2, p\right\rangle^3 - 8\pi^2 \left\langle 2n - m^2, \rho\right\rangle\right) \\ &\quad + 12\pi^2 \left(2n_2\mu_2 + (2n_1-1)\mu_1\right) \left\langle 2n - m^2, p\right\rangle + c\Big] \cdot \delta_2(n) = 2\left[l_1\left(16\pi^4 p_1^3 - 8\pi^2 \rho_1\right) + 12\pi^2 \mu_1 p_1 + c\right] \\ &\quad \cdot e^{\pi i \tau_{11}} + \left[(2l_2 - l_1)\left[16\pi^4 (2p_2 - p_1)^3 - 8\pi^2 (2\rho_2 - \rho_1)\right] + 12\pi^2 (2\mu_2 - \mu_1)(2p_2 - p_1)\right] \\ &\quad \cdot e^{\pi i (2\tau_{22} + \tau_{11} - 2\tau_{12})} + o\left(\alpha_1^{s_1}\alpha_2^{s_2}\right) \quad (s_1 + s_2 \geq 3).\end{aligned} \tag{38}$$

From $\overline{G}(1,0) \to 0$, we have

$$l_1\left(4\pi^2 p_1^3 - 2\rho_1\right) + 3\mu_1 p_1 = 0. \tag{39}$$

Then

$$\begin{aligned}\overline{G}(1,1) &= \sum_{n_1,n_2=-\infty}^{\infty} \Big[\left((2n_1-1) l_1 + (2n_2-1) l_2\right) \cdot \left(16\pi^4 \left\langle 2n - m^3, p\right\rangle^3 - 8\pi^2 \left\langle 2n - m^3, \rho\right\rangle\right) \\ &\quad + 12\pi^2 \left((2n_1-1)\mu_1 + (2n_2-1)\mu_2\right) \left\langle 2n - m^3, p\right\rangle + c\Big] \delta_3(n) = 2\left[(l_1 + l_2)\left(16\pi^4 (p_1+p_2)^3 \right.\right. \\ &\quad \left.\left. - 8\pi^2 (\rho_1 + \rho_2)\right) + 12\pi^2 (\mu_1 + \mu_2)(p_1 + p_2) + c\right] \cdot e^{\pi i (\tau_{11} + \tau_{22} + 2\tau_{12})} + 2\left[(l_1 - l_2)\left(16\pi^4 (p_1 - p_2)^3 \right.\right. \\ &\quad \left.\left. - 8\pi^2 (\rho_1 - \rho_2)\right) + 12\pi^2 (\mu_1 + \mu_2)(p_1 + p_2) + c\right] \cdot e^{\pi i (\tau_{11} + \tau_{22})} + \left[(3l_2 - l_1)\left(16\pi^4 (3p_2 - p_1)^3 \right.\right. \\ &\quad \left.\left. - 8\pi^2 (3\rho_2 - \rho_1)\right) + 12\pi^2 (3\mu_2 - \mu_1)(3p_2 - p_1) + c\right] e^{\pi i (\tau_{11} - \tau_{12} + 3\tau_{22})} + o\left(\alpha_1^{s_1}\alpha_2^{s_2}\right) = \Big[2\Big[(l_1 + l_2) \\ &\quad \cdot \left(16\pi^4 (p_1 + p_2)^3 - 8\pi^2 (\rho_1 + \rho_2)\right) + 12\pi^2 (\mu_1 + \mu_2)(p_1 + p_2) + c\Big] e^{\pi i 2\tau_{12}} \\ &\quad + 2\left[(l_1 - l_2)\left(16\pi^4 (p_1 - p_2)^3 - 8\pi^2 (\rho_1 - \rho_2)\right) + 12\pi^2 (\mu_1 + \mu_2)(p_1 + p_2) + c\right]\Big] e^{\pi i (\tau_{11} + \tau_{22})} + \Big[(3l_2 \\ &\quad - l_1)\left(16\pi^4 (3p_2 - p_1)^3 - 8\pi^2 (3\rho_2 - \rho_1)\right) + 12\pi^2 (3\mu_2 - \mu_1)(3p_2 - p_1) + c\Big] e^{\pi i (\tau_{11} - \tau_{12} + 3\tau_{22})} \\ &\quad + o\left(\alpha_1^{s_1}\alpha_2^{s_2}\right), \quad (s_1 + s_2 \geq 3).\end{aligned} \tag{40}$$

Using $\overline{G}(1,1) \to 0$, we obtain

$$e^{2\pi i \tau_{12}} = e^{A} = -\frac{(l_1 - l_2)\left[4\pi^4 (p_1 - p_2)^3 - 2\pi^2 (\rho_1 - \rho_2)\right] + 3\pi^2 (\mu_1 - \mu_2)(p_1 - p_2)}{(l_1 + l_2)\left[4\pi^4 (p_1 + p_2)^3 - 2\pi^2 (\rho_1 + \rho_2)\right] + 3\pi^2 (\mu_1 + \mu_2)(p_1 + p_2)}. \tag{41}$$

□

In order to show the solution character, we drop the solution curves of real u and imaginary u. Figures 3 and 4 plot the real and imaginary of u, respectively, in three-dimensional space. From the solution graphs, we can see that the solutions are periodic and cuspon. The derived two-periodic solutions in the paper are different to the

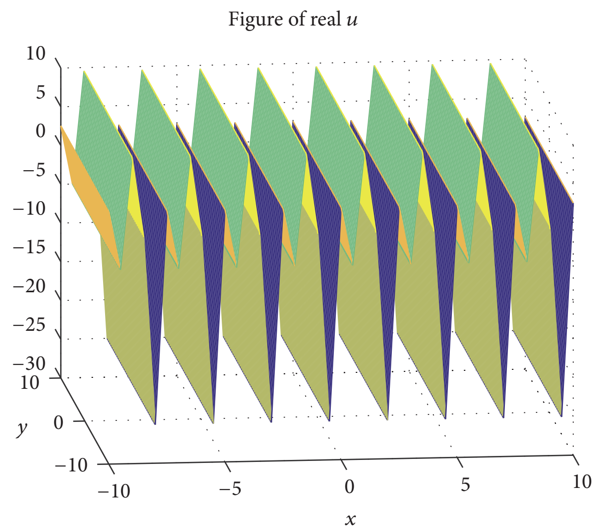

FIGURE 3: $z = 0.1$, $t = 0.5$, $p1 = 0.2$, $\mu_1 = 0.35$, $\rho_1 = 0.9$, $\tilde{\zeta}_{01} = 1$, $p2 = 0.2$, $\mu_2 = 0.3$, $\rho_2 = 0.6$, $\tilde{\zeta}_{02} = 0.9$, $x \in [-10, 10]$, and $y \in [-10, 10]$.

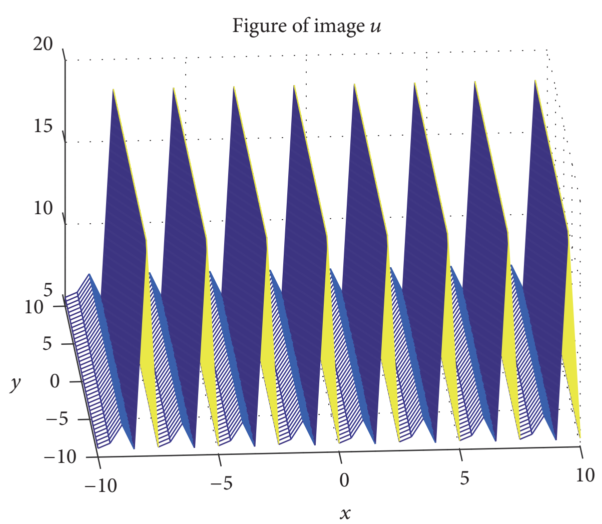

FIGURE 4: $z = 0.1$, $t = 0.5$, $p1 = 0.2$, $\mu_1 = 0.35$, $\rho_1 = 0.9$, $\tilde{\zeta}_{01} = 1$, $p2 = 0.2$, $\mu_2 = 0.3$, $\rho_2 = 0.6$, $\tilde{\zeta}_{02} = 0.9$, $x \in [-10, 10]$, and $y \in [-10, 10]$.

two-solitary solutions in [32] and rational solutions presented by Ma and Lee in [31].

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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