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Research Article

Capacity Allocation and Revenue Sharing in Airline Alliances: A Combinatorial Auction-Based Modeling

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This paper attempts to establish a framework to help airline alliances effectively allocate their seat capacity with the purpose of maximizing alliances' revenue. By assuming the airline alliance as the auctioneer and seat capacity in an itinerary as lots, the combinatorial auction model is constructed to optimize the allocation of the seat, and the revenue sharing method is established to share revenue between partners by Vickrey-Clarke-Groves (VCG) mechanism. The result of the numerical study shows that the seat capacity allocation is effective even without information exchanging completely and the twofold revenue shares method shows more excitation for the airlines.

1. Introduction

1.1. Motivation of Research. With the globalization of the economy, masses of airline companies usually chose to join airline alliances to extend their aviation networks and increase load factors. In an airline alliance, airlines could combine the legs they operate separately and create additional itineraries through code-sharing agreement. In this paper, code-sharing allows the product of an airline offered to the cooperated airline, and the ticket for the product may be sold by both airlines regardless of which one operates it. The airline revenue management problem is concerned with the decision of which fare classes to make available for sale during the booking period. In an airline alliance, because of the antitrust law, the cooperated airlines cannot use the others' revenue management, how many tickets of different classes should be allocated to the airlines to make the revenue of the airline alliance maximize and how to share the revenue not only fair but also excitation are the mainly problems to research.

In this study, we focus on the development of a method to allocate the seat capacity efficiency and a motivated revenue sharing mechanism for alliances. We assume that the airline alliance is the third party, as it could knowledge the referred information but member airlines parts could not. With the combination of combinatorial auction and the EMSR method [1], we establish combinatorial auction-based seat capacity allocation model in the airline alliances and a twofold revenue sharing model under VCG mechanism. To a certain extent, it may be regarded as allocation centrally but be in line with antitrust law.

1.2. Literature Review of Related Topics. Compared with central decision of a network of a single airline, the decisions of the airlines in an airline alliance are multiple. Boyd [2] considered that using the marginal seat benefit balance principle can maximize the alliances revenue in the discrete revenue management system. He mentioned that the most ideal method to maximize the airline alliance revenue is regarding the alliance as an independent company, but such a centralized decision-making system seems to be impossible for the airline alliances on account of the legal barrier and technical requirements. Similarly, Vinod [3] pointed out the present research situation of the revenue management of airline alliances and the Network Equilibrium Conditions. On this condition, if the airline alliances want maximize the revenue management, they must ask their members to share the seats on the itineraries and then redistribute those shipping spaces. He also thought that the airline alliances should treat the buying price as a control measure of the shipping spaces. At the 2011 AGIFORS Cargo and Revenue Management Study

Group meeting in Taipei, Ratliff and Weatherford summarized some issues about code sharing and airline alliance revenue management [4]. They drew a conclusion that members in the airline alliances have a huge effect on the cooperative partner's revenue management and analyzed the present research issues and future research direction of the revenue management in an airline alliance. Houghtalen et al. [5] consider carrier collaborations. They develop models revealing transfer prices between the partners so that the decisions of the individual carriers coinciding in the alliance could be optimal decisions. Then, they analyze whether the allocations obtained by using the models are desirable by the alliance partners. In order to research the seat capacity allocation and control problem of a multiple segment alliance network, Fernandez de la Torre [6] discussed the space allocation methods under the code-sharing principle, one of which is the fixed or unfixed numbers of shipping space controlled by operators and this region space code-sharing failed to be wildly used. The other is the free sales agreement which is the most commonly used at present. He also came up with the likely potential loss in the airlines caused by overestimating the code sharing, and he advised to distribute the revenue in proportion to avoid the individual losses. Oum and Park [7] list further incentives for airlines to join strategic alliances. Capacity control procedures are employed to allocate seat capacity. For individual airlines not part of an alliance, capacity control has attracted a lot of attention. Netessine and Shumsky [8] analyze the seat allocation decisions of the airlines in horizontal and vertical competition. Topaloglu [9] assumed that the airlines make the seat capacity control decision independently in a multiple segment alliance network, and only those airlines who sell tickets can decide whether to accept passengers' arrival demand or not. He established a linear programming model of inventory control of a single airline, working out the airlines' amount of the revenue sharing by using the duality approach.

In the research field of revenue sharing, Wright et al. [10] analyzed the revenue sharing mechanism between two airlines which operate multilegs in the same airline alliance and find out the static and dynamic allocation mechanism influence airlines' revenue and the total revenue of the airline alliances. He emphasized the complexity in using the space allocation mechanism based on discrete dynamic allocation and used a similar method, in which he firstly assumed, in a period, besides release, the transfer price of every flight the airlines should know the class level, future passenger arrival rate, and revenue sharing of their cooperative airlines. And then he eased restrictions; airlines just need to consider their own Internet booking situation with the transfer price serving as the only one linkage information. The advantage of this method is having a reduced calculation and more accurate result. Wright [11] implied incomplete information between the alliance partners for that they were unable or unwilling to share certain information concerning code sharing. He introduced a decomposition rule for central dynamic program to determine approximate bid prices in an airline alliance for the individual partners. Çetiner and Kimms [12] analyzed the revenue sharing mechanism in the airline alliances with the method of cooperative game theory; they use the proposed nucleolus allocations as a benchmark to evaluate the different mechanisms applied in a decentralized setting. The evaluation of the mechanisms is accomplished through using the fairness measure. Graf and Kimms [13] used the option method to study the internal income distribution of airline alliances and established the revenue sharing linear programming model, with introducing the Option Price and Striking Price, but the Recovery Price in the option method was not shown in this model. Grauberger and Kimms [14] analyzed the competition model in the Parallel Alliance and Complementary Alliance with the Nash equilibrium method, and he was the first person who considered the multigrades of seats in a model.

Besides, Shumsky [15] considers that major traditional carriers are forced by low-cost competitors to process an increasing amount of their traffic in airline alliances. The passengers recognize strategic alliances if they book a code-sharing flight. A code-sharing agreement allows an airline to sell flight tickets under its own brand that are provided by its partners. Airlines have incentives to cooperate with other airlines within a strategic alliance due to new expected revenue potentials founded by greater airline networks, coordinated flight schedules, and access to protected markets. Moreover, there are cost-cutting potentials justified by a higher load factor. Another motivation for building strategic alliances could be the generation of market entry barriers. Transchel and Shumsky [16] introduced a closed-loop dynamic pricing game for alliance partners that operate a parallel and substitutable flight. On the one hand, the competitors are assumed to compete horizontally on this flight, while, on the other hand, they have to set prices for their local and for their code-shared products. Belobaba and Jain [17] described the technical difficulties in the process of information sharing faced by alliance revenue management and proposed information sharing mechanisms to overcome the difficulties.

Auctions in which bidders are allowed to bid on bundles of items and the bidder gets either each item in the bundle if the bid wins or no item at all if the bid loses are called combinatorial auctions (CA). In recent years, many auctions involve the sale of a variety of distinct assets. Examples are airport time, network routing, and delivery routes. Ledyard et al. [18] describe the design and use of a combinatorial auction to select carriers. Here, the objects bid upon were delivery routes, and it was profitable for bidders to have their trucks full on the return journey. CA is always applied in allocating resources. Kuo and Miller-Hooks [19] solved the problem of allocating residual track capacity among multiple competing carriers where infrastructure ownership and train operations are vertically separated to facilitate delivery by train.

1.3. Structure. The organization of this paper is as follows. In Section 2, we establish a combinatorial auction model to allocate the seat capacity to the airlines. In Section 3, we formulate a twofold revenue sharing method for the airlines of the airline alliance. In Section 4, we consider the related concept and procedure to compute. In Section 5, we use an example to test the model. Finally, several conclusions from this research are presented in Section 6.

2. Combinatorial Auction-Based Capacity Allocation

2.1. Problem Description. The cooperation among the airline alliance members is usually based on bilateral and multilateral agreements. From the perspective of revenue management, how to allocate the seat capacity among the airline alliance members in order to maximize the total revenue of the airline alliance is one of the most important problems during the cooperation. For the antitrust laws and lack of technical support, the airlines are not aware of the cabin classes and demands of their cooperative partners.

In the method of airline alliance seat capacity allocation based on combinatorial auction, the total amount of all airlines' seat capacity is seen as the resources (commodities), and each member airline can buy the group of most optimal combined commodities seen as the seat capacity distribution for different O-Ds, so that airline alliance could acquire the maximization revenue.

This study is based on the following assumptions:

(1) Each member airline in the airline alliance only operates one leg.

(2) The airline alliance implements code-sharing principle, so each member airline that can "sell" seat capacity belongs to the airlines.

(3) Each airline only knows its own company's fare classes and passenger demands.

(4) According to the antitrust laws, the airline alliance, as a third party, cannot disclose the airlines' sales quantities before joining the alliance to any other airlines.

(5) The passenger demands are independent and comply with the normal distribution.

2.2. Parameter Meaning. N is the quantity of airlines in an airline alliance (bidders); n is the number order of an airline in the airline alliance, $n \in N$; C_n is the numbers of seat capacity belonging to the nth airline; $N+1$ is the numbers of nodes of the itinerary which is operated cooperatively; (i, j) is symbol of an O-D pair, i represents the origin and j represents the destination, $i = 1, \ldots, N$, $j = 2, \ldots, N+1$, $j > i$; m is the mth combinatorial bidding strategy; A_n^m is the mth combinatorial bidding strategy of the nth airline; $a_n^{m(i,j)}$ is the demand quantity in the nth airline's the mth combinatorial bidding strategy. And the vector quantity is

$$A_n^m = \begin{bmatrix} a_n^{m(1,2)} & a_n^{m(1,3)} & \cdots & a_n^{m(1,N+1)} \\ & a_n^{m(2,3)} & \cdots & a_n^{m(2,N+1)} \\ & & \vdots & \\ & & & a_n^{m(N,N+1)} \end{bmatrix}, \quad (1)$$

$$i = 1, N, \quad j = 2, N+1, \quad j > i;$$

$\gamma_n^{(i,j)}$: if airline n operates the O-D pair (i, j), then $i \le n < j$, $\gamma_n^{(i,j)} = 1$, otherwise $\gamma_n^{(i,j)} = 0$; γ_n represents the bidding segment in a strategy whether it uses the nth airline's seat capacity or not:

$$\gamma_n = \begin{bmatrix} \gamma_n^{(1,2)} & \gamma_n^{(1,3)} & \cdots & \gamma_n^{(1,N+1)} \\ & \gamma_n^{(2,3)} & \cdots & \gamma_n^{(2,N+1)} \\ & & \vdots & \\ & & & \gamma_n^{(N,N+1)} \end{bmatrix}; \quad (2)$$

$b(A_n^m)$ is the bid paid for the strategy A_n^m; R is the total revenue of the airline alliance by seat capacity auction; u_n^m: let $u_n^m = 1$ if strategy A_n^m is selected and zero otherwise.

2.3. Model Establishment

$$\max \quad R = \sum_{n=1}^{N} b\left(A_n^m\right) \times u_n^m \quad (3)$$

$$\begin{aligned} \text{Subject to} \quad & \sum_{n=1}^{N} A_n^m * \gamma_1 * u_n^m \le C_1 \\ & \vdots \\ & \sum_{n=1}^{N} A_n^m * \gamma_N * u_n^m \le C_N \end{aligned} \quad (4)$$

$$\sum_{m=1} u_n^m = 1, \quad n \in N \quad (5)$$

$$u_n^m = \{0, 1\}. \quad (6)$$

The objective function (3) maximizes the expected revenue of the airline alliance; constraint (4) force the sum of the seat capacity on different O-Ds that the member airlines bid for cannot exceed the operating airlines' transport capacity; constrain (5) ensures that the each airline must win a bidding and one biding only; in constrain (6), $u(A_n^m) = 1$ if the combinatorial bidding strategy is selected and zero otherwise.

3. Combinatorial Auction-Based Twofold Revenue Sharing Model

The theories about revenue sharing of the airline alliances by most writers are not used; at present, the most mainly used method is the proportional distribution in accordance with the airline alliance agreement. In the agreement, the increased revenue coming from those cooperative airlines after the airline companies joining the alliance shall be distributed in proportion which is stipulated in the agreement. However, this allocation method is not completely reasonable, because distributing in the fixed proportion would decrease the airlines' enthusiasm, for member airlines, no matter who sales a ticket, they will share the same percentage of the fare form the ticket.

Therefore, we make references to the above-mentioned combinatorial auction space allocation model and establish a twofold revenue allocation for airlines. In the new model, the airlines in the alliances can obtain revenue in two ways: one is from the traditional sharing method in proportion corresponding to the agreement and the other is from selling tickets of seat capacity. So, the more expensive tickets airlines sell, the more they earn. The method is a kind of encouragement for all the airlines, and the model is as follows:

$$r_n = \alpha_n \left(\sum_{n=1}^{N} h_n^{\prime m} - \sum_{n=1}^{N} r_n^{\prime} \right) + r_n^{\prime} + p_n - h_n^{\prime m} \quad (7)$$

$$\text{Subject to} \quad \sum_{n=1}^{N} \alpha_n = 1 \quad (8)$$

$$\alpha_n \in (0,1). \quad (9)$$

In formula (7) of this model, α_n denotes the proportion of the increased revenue sharing for each member airline, r_n denotes the airlines' revenue of seats by combinatorial auction allocation after joining the alliances, $h_n^{\prime m}$ denotes the payment price for the selected strategy of the member airline, $r_n^{\prime}$ denotes each airline's revenue before joining the airline alliance, p_n represents each airline's revenue for selling tickets of the seat capacity allocated, that is, the expected revenue which equals the bid price in this paper. Constraints (8) and (9) make sure that the proportions are between 0 and 1, and the sum of them is 1.

The increased revenue of airlines is

$$\Delta r_n = \alpha_n \left(\sum_{n=1}^{N} h_n^{\prime m} - \sum_{n=1}^{N} r_n^{\prime} \right) + p_n - h_n^{\prime m}, \quad (10)$$

$\Delta r_n > 0$; that is to say, only the airlines' revenue increases when they participate in the alliances and the stability of the airline alliances can be ensured.

4. Model Calculations

4.1. Determination of the Bid Price. When airlines bid for the combinatorial strategy of seat capacity, they do not know the other airline's bidding prices for different combinations of seat capacity. Incomplete information is among buyers for the two reasons: ① because the various brand value, fare levels, and passengers' demands of airlines are different and ② the existence of the antitrust laws.

In the model, we take the predictive value of the seat capacity strategy as the largest willing price of bidders. After participating in the airline alliances, the airlines can forecast the passengers' various demands for multiclass seats on the different itineraries. Through the EMSR method, we can figure out this airline's expected value of the seat capacity and regard it as the bidding price.

4.2. Realization Process of the Seat Capacity Allocation Model

Step 1. The combinatorial auction is a kind of NP-hard problem, so it is impossible for an airline to bid for every strategy in a real world application. In order to correspond to reality and be convenient to calculate, airlines use an individual value which acts as the minimum reference variable unit to generate all the viable strategies. Taking two airline companies of one airline, for example, the form of a random strategy is

$$A_1^m = \begin{pmatrix} a_1^{m(1,2)} \\ a_1^{m(1,3)} \\ a_1^{m(2,3)} \end{pmatrix}. \quad (11)$$

Step 2. Using the EMSR method [5], we can figure out the relevant bid price $b(A_n^m)$ of all strategies.

Step 3. According to the bid price and strategies submitted by airlines, the airline alliances solve the 0-1 equation that can gain the optimal solution. The deduction of the seat constraint condition is as follows.

Take two airline companies of one airline, for example, and assume that airline 1 offers two bidding strategies and airline 2 offers three bidding strategies. The front part of inequality sign in constraint equation (4) can be expressed as follows:

$$\begin{aligned}
&(A_1^1)^T \cdot \gamma_1 \cdot u_1^1 + (A_1^2)^T \cdot \gamma_1 \cdot u_1^2 + (A_2^1)^T \cdot \gamma_1 \cdot u_2^1 \\
&\quad + (A_2^2)^T \cdot \gamma_1 \cdot u_2^2 + (A_2^3)^T \cdot \gamma_1 \cdot u_2^3 \\
&= \begin{bmatrix} (A_1^1)^T \cdot \gamma_1 \\ (A_1^2)^T \cdot \gamma_1 \\ (A_2^1)^T \cdot \gamma_1 \\ (A_2^2)^T \cdot \gamma_1 \\ (A_2^3)^T \cdot \gamma_1 \end{bmatrix}^T \begin{pmatrix} u_1^1 \\ u_1^2 \\ u_2^1 \\ u_2^2 \\ u_2^3 \end{pmatrix} \\
&= \gamma_1^T \begin{bmatrix} (A_1^1)^T \\ (A_1^2)^T \\ (A_2^1)^T \\ (A_2^2)^T \\ (A_2^3)^T \end{bmatrix}^T \begin{pmatrix} u_1^1 \\ u_1^2 \\ u_2^1 \\ u_2^2 \\ u_2^3 \end{pmatrix} \\
&= \gamma_1^T \left(A_1^1 \;\; A_1^2 \;\; A_2^1 \;\; A_2^2 \;\; A_2^3 \right) \begin{pmatrix} u_1^1 \\ u_1^2 \\ u_2^1 \\ u_2^2 \\ u_2^3 \end{pmatrix}.
\end{aligned} \quad (12)$$

The strategy-like $\left(A_1^1 \;\; A_1^2 \;\; A_2^1 \;\; A_2^2 \;\; A_2^3 \right)$ can be extended as all the viable strategies.

Step 4. The optimal solution is based on a minimum reference variable unit, so the solution is a second-best solution

Table 1: Fares and demands of two airlines' ODF before joining the airline alliance.

Airline 1	ORD		Airline 2	FRA	
	Price	Demand		Price	Demand
PHX	350	(30, 7)	ORD	1000	(25, 5)
	310	(28, 6)		940	(28, 5)
	270	(40, 6)		880	(30, 6)
	230	(46, 8)		820	(38, 6)
	190	(50, 7)		760	(36, 7)
	160	(52, 8)		700	(43, 7)

The unit of price is $, the first number in the demand column represents the average demand, and the second number represents standard deviation. History data are collected from airlines.

Table 2: Fares and demands of two airlines' ODF in airline alliance.

Airline 1	ORD		FRA		Airline 2	ORD		FRA	
	Price	Demand	Price	Demand		Price	Demand	Price	Demand
PHX	350	(12, 4)	1350	(11, 4)	PHX	400	(9, 3)	1400	(15, 4)
	310	(14, 4)	1250	(16, 5)		350	(11, 5)	1290	(18, 3)
	270	(21, 6)	1150	(21, 6)		300	(19, 4)	1180	(22, 5)
	230	(23, 5)	1050	(24, 5)		250	(25, 4)	1070	(29, 6)
	190	(29, 7)	950	(33, 6)		200	(27, 4)	960	(38, 7)
	160	(35, 8)	850	(40, 5)		160	(32, 6)	850	(43, 7)
ORD			1020	(6, 2)	ORD			1000	(9, 4)
			950	(11, 3)				940	(14, 4)
			870	(19, 5)				880	(17, 5)
			800	(25, 5)				820	(23, 6)
			730	(29, 6)				760	(27, 5)
			660	(32, 6)				700	(33, 7)

Data from OneWorld website; URL: https://www.oneworld.com.

among the overall situation, and we should search for the best one around it.

4.3. The VCG Mechanism of Revenue Allocation. In this paper, we use the VCG mechanism to figure out the payment price of all the airlines, which was made by other companies' loss through the participation of the bidder. VCG is a direct mechanism with a motivation system, and its advantage can be reflected to satisfy each bidder's individual interests by omitting bidders' complicated and strategic calculation.

Under the VCG mechanism, if all the airlines adopt the dominant strategy (i.e., reaching the dominant strategy equilibrium), the utility of the space value can reach the maximum. Therefore, the bidders gain the profits produced by the VCG mechanism and increase their independent profits.

5. The Empirical Study

There are two airlines in an airline alliance, and airline 1 operates Phoenix (PHX)–Chicago (ORD); meanwhile, airline 2 operates Chicago (ORD)–Frankfurt (FRA). Airline 1 provides 300 seats and airline 2 provides 250 seats. According to the airline alliance agreement, the increased revenue sharing proportion between airlines 1 and 2 is 30% versus 70%. Moreover, airlines offer several fare classes for each itinerary, an airline product defines an OD pair with a specific fare class and it is referred to as an "ODF." Forecasts about fares and demands of two airlines' ODF before and after joining the airline alliances are seen as in Tables 1 and 2.

Two airlines' respective fares and demands before they joined the airline alliance are shown in Table 1. Table 2 reflects each airline's fares and demands for different pairs of O-Ds after joining the airline alliance. According to the model calculation, Table 3 lists many relevant contents in a way of combinatorial auction, such as the airlines' seat capacity allocation results, the bidding price used by airlines, the price needed to be paid, and the allotment of the increased revenue in the traditional mechanism.

Under the VCG mechanism, we can get $\sum_{n=1}^{N} h_n'^m - \sum_{n=1}^{N} r_n' < 0$ through a calculation, and the revenue sharing proportion of the increased revenue is close to the contribution proportion of the ticket selling (as shown in Figure 1), which explains that this distribution method pays more attention on airlines' sales volume. Using the model can motivate the airlines and promote the development of the whole airline alliance.

Table 3: Result date of example.

		Airline 1	Airline 2
Alone	Revenue ($)	47775.0	124210.0
In the airline alliance	Seat capacity allocation	(75 67 44)	(72 86 53)
	Bid price/tickets selling fares ($)	125960.0	158570.0
	Payment price ($)	67030.0	72665.0
	Revenue sharing under VCG ($)	49241.5	63303.5
	Revenue sharing under traditional proportion ($)	33763.5	78781.5
	Total revenue ($)	284530.0	

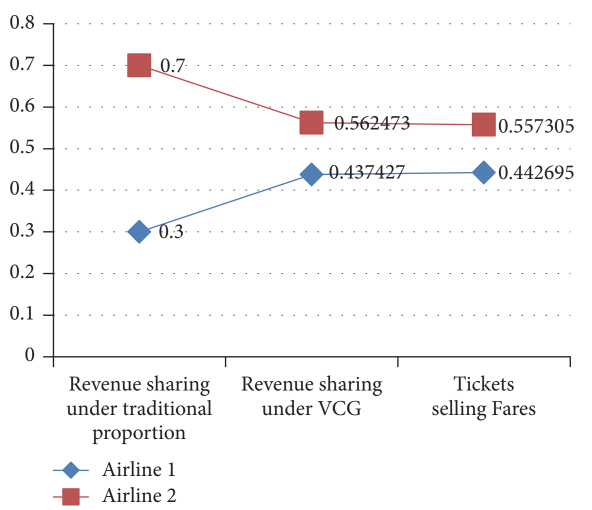

Figure 1: The comparison of the revenue sharing proportion between two airlines.

6. Conclusion

In this paper, we establish the combinatorial auction model to allocate the seat capacity to the different O-Ds of the different airlines. On the condition of not breaking the antitrust laws, this method can maximize the revenue of the airline alliances and ensure that every airline can get the seat capacity allocation on the different O-Ds. Compared to the traditional revenue distribution method, this method based on the combinatorial auction not only can gain revenue from their alliances but also can obtain profit independently from the seat capacity that they bid for; that is to say, the higher ticket price they sell, the more they earn. It stimulates the airlines to increase their own revenue by bidding more seat capacity and selling the higher price. The VCG- mechanism can calculate the final payment price and allocation amount of the increased revenue for airlines. Experimental results show that the established mathematic model in this paper is in line with the actual needs of the airline alliances, and the arithmetic can satisfy the needs of seat capacity allocation in the airline alliances.

There are also some limitations in this paper; in the common calculation software, the computation time can be long due to the large amount of calculation and the complexity of the airline alliances. Besides commerce confidence, we cannot get the real date for an airline alliance. How to figure out the space allocation results for reality application more quickly and more accurately is the direction in the further research.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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