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Research Article

Sampling Schemes by Variables Inspection for the First-Order Autoregressive Model between Linear Profiles

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We present four new sampling schemes by variables inspection to deal with the first-order autoregressive model between linear profiles. The first plan is based on exponentially weighted moving average (EWMA) and the rest of three plans are using the resubmitted sampling, repetitive group sampling (RGS), and multiple dependent state (MDS) sampling schemes. The nonlinear optimization problem is developed to find the number of profiles and the corresponding acceptance criteria, such that the producer's and consumer's risk are satisfied simultaneously. The efficiency of the proposed plans is compared with the conventional single sampling plan in terms of average sample number and the probability of acceptance. The result implies that all of the proposed sampling plans are superior to the single acceptance sampling plan by variables. In addition, the EWMA method appeared to be better than the others. The applications of proposed plans are shown with the help of industrial examples taken from calibration of an optical imaging system, and tire cornering stiffness test.

1. Introduction

Functional profiles express the relationship between one or more explanatory variables, the controllable inputs and a response variable which is the critical-to-quality characteristics. Thus, in statistical profile monitoring instead of individual or vector of quality characteristics we monitor the curve. Montgomery [1] defines a profile that is characterized by a functional relationship between a response variable and one or more independent variables. For example, profile monitoring has extensive application in the calibration of an optical imaging system; in tire cornering stiffness test the relationship between force and displacement can be modeled by linear profile. The fundamental concepts, methods, and issues regarding linear and nonlinear profiles monitoring methods can be found in [2]. A process yield index S_{pkA} with the lower confidence bound for simple nonlinear and linear profiles can be found in [3] and [4], respectively. In the presence of autocorrelation between profiles [5] developed a process yield index $S_{pkA;\mathrm{AR}(1)}$ with its approximate lower confidence bound. The author's simulation study confirmed the method performs well-regarding bias and coverage rate.

Acceptance sampling plans lay down conditions for acceptance or rejection of the immediate lot inspected. Process capability is of utmost importance in acceptance sampling; in no event should the requirements of a sampling procedure exceed the producer's process capability [6]. Moreover, in a highly competitive environment, sampling plans must be appropriately applied. In practice, the fraction of defectives a customer is willing to accept is getting very small, often measured in parts per million. That is, in order to reflect the actual lot quality, the sample size must be very large. In such circumstances, various authors [7–10] found variables sampling plans based on capability indices to be the most efficient methods of lot sentencing. In addition, variables sampling plans, based on the exponentially weighted moving average (EWMA) statistic, resubmitted sampling, repetitive group sampling (RGS), and multiple dependent state (MDS) sampling are proved to guarantee a reduction in sample size [11]. However, each sampling scheme should be applied cautiously to different scenarios. The scenarios are described as follows.

Scenario one: when the relationships between suppliers and customers have a long-term orientation, say ten or more previous lots are accepted, there would be an accumulated quality history. The EWMA sampling scheme takes into consideration past histories of the supplier to accept or reject the immediate lot inspected. The weights decline geometrically with the time of the observations. In quality control charts this EWMA statistic is known to be efficient to detect a small shift in the process [12–14]. Aslam et al. [15] applied the EWMA statistic in an acceptance sampling plan. For a process with linear profiles, [16] provided a single variable sampling plan based on EWMA model using the yield index S_{pkA}. In the literature Aslam et al. [15] and Wang [16] suggested that a sampling plan based on EWMA yield index for a normal process and a process with linear profiles has more flexibility and economy than the single sampling plan based on yield index while providing the same protection to both suppliers and buyers. More information about EWMA sampling plan can be found in [17].

Scenario two: if either the producer or the consumer does not agree with the sample inspection result, resubmission might be necessary for reaching an appropriate decision. That is, if a lot is not accepted, then a resubmitted lot is tested and a decision is made irrespective of the previous test. In resubmitted sampling plans for a normal process, Liu et al. [19] propose variables sampling plan for two-sided specification; [20] presented the operating procedure of the resubmitted sampling plan based on one-sided capability indices. When the data can be considered as identically, independently, and normally distributed linear profiles, [21] proposed variables sampling plan for resubmitted lots based on the yield index S_{pkA}. The author found that the resubmitted sampling plan has better-operating characteristics curve (OC) than single sampling plan. Other research related to sampling inspection for resubmitted lots can be found in the literature [22].

Scenario three: in some situations testing is costly and destructive; a small sample is taken from the lot and analyzed. In the case of noncompliance, the second small sample is taken from the lot and analyzed. This process is repeated till the lot is accepted or rejected. The procedure is similar to sequential sampling. Balamurali and Jun [23] reported that the RGS plan is more efficient in terms of average sample number (ASN), cost, and time than single and double sampling plans. When the inspection is costly and destructive an RGS by variables based on yield index C_{pk} can be found in [24]. More information on RGS can be found in the literature [25, 26].

Scenario four: when the test result is marginal there might be a feeling the result is an inaccurate representation of the quality submitted. The inspection efficiency in such instances can be improved by utilizing the accumulated quality history of previous lots information. The MDS sampling plans are not only based on the current lot, but also on preceding lots [27]. With the absence of autocorrelation between linear profiles, [16] provided MDS sampling plan based on yield index S_{pkA}. More information on MDS can be found in the literature [28].

The principal advantage of variables plans is the reduction in sample size. It must be remembered, however, that the superiority of the variables plan rests on the assumption of the underlying distribution of the measurements [6]. Whether the performance measure is single quality characteristic [7–10, 15] or profile data [16, 21] sampling schemes in the literature share two basic assumptions: the quality characteristic follows a normal distribution and data is independent. However, in some manufacturing processes, data is correlated or self-dependent. The effect of autocorrelations on the statistical properties of yield indices can be found in [5]. Accordingly, when dealing with sampling plans based on yield index the autocorrelation effect has to be taken into consideration. However, to the best of our knowledge, there is no work on sampling plan in the literature to deal with autocorrelation between profiles. In this study, we propose four new acceptance sampling plans based on the yield index for a first-order autoregressive process. The first method is based on EWMA. The remaining three are based on resubmitted sampling, RGS, and MDS sampling, respectively. The rest of the paper is organized as follows. In the next section, yield index for autocorrelation between linear profiles is reviewed. Four sampling plans by variables are presented in Section 3. In the subsequent section, the performances of the proposed plans are compared and two illustrative examples are provided in Section 5. The former is based on the line widths of three photomask reference standards; the latter is an experimental example for tire cornering stiffness test on the ice road. Finally, we offer a conclusion and suggestions for future studies.

2. Yield Index for Autocorrelation between Linear Profiles

The profiles addressed in this paper are of the form of autocorrelation between linear profiles given as

$$\begin{aligned} y_{ij} &= \alpha + \beta x_i + \varepsilon_{ij}, \\ \varepsilon_{ij} &= \rho \varepsilon_{i(j-1)} + a_{ij}, \\ & i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, k, \end{aligned} \tag{1}$$

where n represents the number of levels and k is the number of profiles. For each of the k profiles, the response y is measured at the same n fixed locations called levels, where x_i is the ith level of the independent variable, ρ denotes the first-order autoregressive coefficient, ε_{ij} denotes correlated random error, α and β are the intercept and slope of linear profiles, respectively, and $a_{ij} \sim N(0, \sigma^2)$.

Wang and Tamirat [5] derived the following process yield index for autocorrelation between linear profiles. This index is used to describe the relationship between manufacturing specifications and actual process yield.

$$\widehat{S}_{pkA;\mathrm{AR}(1)} = \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\left\{1 + \frac{1}{n}\sum_{i=1}^{n}\left[2\Phi\left(3\widehat{S}_{pk_i}\right) - 1\right]\right\}\right], \quad (2)$$

where

$$\begin{aligned} S_{pk_i} &= \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{\mathrm{USL}_i - \mu_i}{\sigma_i}\right) + \frac{1}{2}\Phi\left(\frac{\mu_i - \mathrm{LSL}_i}{\sigma_i}\right)\right] \\ &= \frac{1}{3}\Phi^{-1}\left[\frac{1}{2}\Phi\left(\frac{1 - C_{dr_i}}{C_{dp_i}}\right) + \frac{1}{2}\Phi\left(\frac{1 + C_{dr_i}}{C_{dp_i}}\right)\right], \end{aligned} \quad (3)$$

where μ_i and σ_i are the process mean and standard deviation of the response variable at the ith level of the independent variable, USL_i and LSL_i are the upper and lower specification limits of the response variable at the ith level of the independent variable, respectively, $C_{dr_i} = (\mu_i - m_i/d_i)$, $C_{dp_i} = \sigma_i/d_i$, $m_i = \mathrm{USL}_i + \mathrm{LSL}_i/2$, $d_i = \mathrm{USL}_i - \mathrm{LSL}_i/2$, Φ is the cumulative distribution function of a standard random normal distribution, and Φ^{-1} is the inverse function of Φ. Furthermore, [5] derived the asymptotic normal distribution of index $\widehat{S}_{pkA;\mathrm{AR}(1)}$, which is given as follows:

$$\begin{aligned} \widehat{S}_{pkA;\mathrm{AR}(1)} \sim N\Bigg(& S_{pkA;\mathrm{AR}(1)} + \sum_{i=1}^{n}\frac{1}{12n\phi\left(3S_{pkA;\mathrm{AR}(1)}\right)} \\ & \cdot\left[a_i(1-f)\right], \frac{\sum_{i=1}^{n}\left[ka_i^2F/(k-1)^2 + b_i^2 g\right]}{36n^2k\phi\left(3S_{pkA;\mathrm{AR}(1)}\right)^2}\Bigg), \end{aligned} \quad (4)$$

where

$$\begin{aligned} a_i &= \frac{d_i}{\sqrt{2}\sigma_i\left\{\left(1 - C_{dr_i}\right)\phi\left(\left(1 - C_{dr_i}\right)/C_{dp_i}\right) + \left(1 + C_{dr_i}\right)\phi\left(\left(1 + C_{dr_i}\right)/C_{dp_i}\right)\right\}}, \\ b_i &= \phi\left(\frac{1 - C_{dr_i}}{C_{dp_i}}\right) - \phi\left(\frac{1 + C_{dr_i}}{C_{dp_i}}\right), \\ f &= 1 - \frac{2}{k(k-1)}\sum_{i=1}^{k-1}(k-i)\rho i, \\ F &= k + 2\sum_{i=1}^{k-1}(k-i)\rho^{2i} + \frac{1}{k^2}\left[k + 2\sum_{i=1}^{k-1}(k-i)\rho^i\right]^2 - \frac{2}{k}\sum_{i=0}^{k-1}\sum_{j=0}^{k-i}(k-i-j)\rho^i\rho^j, \end{aligned} \quad (5)$$

where ρ_i is the ith lag autocorrelation and ϕ is the probability density function of a standard normal distribution.

3. Acceptance Sampling Plans

A sampling plan must satisfy two conditions: (1) the probability of accepting a lot at the acceptable quality level (AQL) is greater than the producer's confidence level $1 - \alpha$ and (2) the probability of accepting a lot at the lot tolerance proportion defect (LTPD) is smaller than the consumer's risk β. Thus, for a particular sampling scheme its operating characteristic (OC) curve must pass through those two designated points (AQL, $1 - \alpha$) and (LTPD, β). EWMA, resubmitted, RGS, and MDS sampling schemes based on $S_{pkA;\mathrm{AR}(1)}$ are presented and discussed in the following four subsections. The selection of a sampling scheme is based on purpose and information availability as discussed in the introduction part.

3.1. Sampling Plan Based on EWMA Model. The proposed sampling plan is given as follows.

Step 1. Choose the producer's risk and consumer's risk. Select the process yield requirements $(C_{\mathrm{AQL}}, C_{\mathrm{LTPD}})$ at two risks.

Step 2. Take a random sample of linear profiles k from the current lot and compute the process yield index $\widehat{S}^t_{pkA;\mathrm{AR}(1)}$.

Step 3. Collect process yield indices from previous lots and compute the EWMA statistic Z_t.

The EWMA model with yield index proposed by [29] is given by

$$Z_t = \lambda\widehat{S}^t_{pkA;\mathrm{AR}(1)} + (1 - \lambda)Z_{t-1}, \quad (6)$$

where $\widehat{S}^t_{pkA;\mathrm{AR}(1)}$ is process yield index from current lot, Z_{t-1} is obtained from previous lots, and λ is smoothing constant and ranges from 0 to 1. The choice of the optimal EWMA parameter is based on minimizing the error sum of squares from previous lots, $\mathrm{SSE} = \sum_{t=2}^{T}(Z_t - \widehat{S}_{pkA;\mathrm{AR}(1);t})^2$, where $Z_2 = \widehat{S}_{pkA;\mathrm{AR}(1);1}$. Wang and Tamirat [5] apply the first-order expansion of v-variate Taylor and central limit theorem to derive the asymptotic normal distribution of index $\widehat{S}_{pkA;\mathrm{AR}(1)}$; see (4).

The properties of EWMA can be obtained from the formula for the sum of a geometric series. The variance of the EWMA statistic is derived by $\sigma^2_{E_t} = [\{1-(1-\lambda)^{2t}\}\lambda/(2-\lambda)]\sigma^2$. When the process is in-control, $(1-\lambda)^{2t} \rightarrow 0$, the variance of the EWMA statistic becomes $\sigma^2_{E_t} = \{\lambda/(2-\lambda)\}\sigma^2$ [12]. In a steady state process, the mean and variance of X_t are obtained as $S_{pkA;\mathrm{AR}(1)} + \sum_{i=1}^{n}([a_i(1-f)]/12n\phi(3S_{pkA;\mathrm{AR}(1)}))$ and $\mathrm{var}(X_t) = (\lambda/(2-\lambda))(\sum_{i=1}^{n}[ka_i^2F/(k-1)^2 + b_i^2 g]/36n^2k\phi(3S_{pkA;\mathrm{AR}(1)})^2)$, respectively. According to Montgomery [1], the EWMA can be viewed as a weighted average of all past and current observations; it is very insensitive to the normality assumption.

Step 4 (make a decision). Accept the lot if $Z_t \geq c_a$, where c_a is the critical value; otherwise reject it.

The lot acceptance probability, say $P_A(Z_t)$, is established as

$$P_A(Z_t \geq k_a) = 1 - \Phi\left[\frac{k_a - \left(S_{pkA;\mathrm{AR}(1)} + \sum_{i=1}^{n}\left(1/12n\phi\left(3S_{pkA;\mathrm{AR}(1)}\right)\right)[a_i(1-f)]\right)}{\sqrt{(\lambda/(2-\lambda))\left(\sum_{i=1}^{n}\left[ka_i^2F/(k-1)^2 + b_i^2 g\right]/36n^2k\phi\left(3S_{pkA;\mathrm{AR}(1)}\right)^2\right)}}\right]. \quad (7)$$

When $\lambda = 1$, this sampling plan becomes a single sampling plan based on linear profiles. The ASN for this sampling plan is k. The parameters of our proposed plan are determined through the following nonlinear optimization problem:

$$\text{Minimize} \quad k \quad (8)$$

$$\begin{aligned}\text{Subject to} \quad & 1 - \Phi\left(\frac{k_a - (C_{\mathrm{AQL}} + \sum_{i=1}^{n}(1/12n\phi(3C_{\mathrm{AQL}}))[a_i(1-f)])}{\sqrt{(\lambda/(2-\lambda))\left(\sum_{i=1}^{n}\left[ka_i^2F/(k-1)^2 + b_i^2 g\right]/36n^2k\phi(3C_{\mathrm{AQL}})^2\right)}}\right) \geq 1-\alpha,\\ & 1 - \Phi\left(\frac{k_a - (C_{\mathrm{LTPD}} + \sum_{i=1}^{n}(1/12n\phi(3C_{\mathrm{LTPD}}))[a_i(1-f)])}{\sqrt{(\lambda/(2-\lambda))\left(\sum_{i=1}^{n}\left[ka_i^2F/(k-1)^2 + b_i^2 g\right]/36n^2k\phi(3C_{\mathrm{LTPD}})^2\right)}}\right) \leq \beta,\end{aligned} \quad (9)$$

where, $C_{\mathrm{LTPD}} < C_{\mathrm{AQL}}$.

3.2. Resubmitted Sampling Plan. Resubmission is allowed $r-1$ times excluding the first inspection. The resubmitted sampling plan based on $S_{pkA;\mathrm{AR}(1)}$ is described below.

Step 1. Choose the producer's risk (α), the consumer's risk (β), two process capability indices (C_{AQL} and C_{LTPD}), and resubmission ($r-1$) times.

Step 2. Draw random linear profiles k from a submitted lot and compute the $\widehat{S}_{pkA;\mathrm{AR}(1)}$ value.

Step 3 (make a decision). Accept the lot if $\widehat{S}_{pkA;\mathrm{AR}(1)} \geq c_a$, where c_a is a critical value; otherwise resubmit a lot and go to Step 2. We can reject the lot if it is not accepted on the $r-1$ resubmission.

The lot acceptance probability, say $P_{A;\mathrm{resubmitted}}(\widehat{S}_{pkA;\mathrm{AR}(1)})$, is established as

$$P_{A;\mathrm{resubmitted}}\left(\widehat{S}_{pkA;\mathrm{AR}(1)}\right) = 1 - \left\{1 - P_A\left(\widehat{S}_{pkA;\mathrm{AR}(1)}\right)\right\}^r, \quad (10)$$

where

$$P_A\left(\widehat{S}_{pkA;\mathrm{AR}(1)}\right) = 1 - \Phi\left[\frac{k_a - \left(S_{pkA;\mathrm{AR}(1)} + \sum_{i=1}^{n}\left(1/12n\phi\left(3S_{pkA;\mathrm{AR}(1)}\right)\right)[a_i(1-f)]\right)}{\sqrt{\sum_{i=1}^{n}\left[ka_i^2F/(k-1)^2 + b_i^2 g\right]/36n^2k\phi\left(3S_{pkA;\mathrm{AR}(1)}\right)^2}}\right]. \quad (11)$$

The ASN for this resubmitted sampling plan is given as follows

$$\mathrm{ASN}_{\text{resubmitted}} = \frac{k\left(1-\left[1-P_A\left(\widehat{S}_{pkA;\mathrm{AR}(1)}\right)\right]^r\right)}{P_A\left(\widehat{S}_{pkA;\mathrm{AR}(1)}\right)}. \tag{12}$$

The plan parameters of this scheme will be determined from the solution to the following nonlinear optimization problem:

$$\begin{aligned} &\text{Minimize} \quad \mathrm{ASN}_{\text{resubmitted}} \\ &\text{Subject to} \quad 1-\left(\Phi\left[\frac{k_a-\left(C_{\mathrm{AQL}}+\sum_{i=1}^{n}\left(1/12n\phi\left(3C_{\mathrm{AQL}}\right)\right)\left[a_i\left(1-f\right)\right]\right)}{\sqrt{\sum_{i=1}^{n}\left[ka_i^2F/\left(k-1\right)^2+b_i^2g\right]/36n^2k\phi\left(3C_{\mathrm{AQL}}\right)^2}}\right]\right)^r \geq 1-\alpha, \\ &\qquad 1-\left(\Phi\left[\frac{k_a-\left(C_{\mathrm{LTPD}}+\sum_{i=1}^{n}\left(1/12n\phi\left(3C_{\mathrm{LTPD}}\right)\right)\left[a_i\left(1-f\right)\right]\right)}{\sqrt{\sum_{i=1}^{n}\left[ka_i^2F/\left(k-1\right)^2+b_i^2g\right]/36n^2k\phi\left(3C_{\mathrm{LTPD}}\right)^2}}\right]\right)^r \leq \beta, \end{aligned} \tag{13}$$

where $C_{\mathrm{LTPD}} < C_{\mathrm{AQL}}$.

3.3. Repetitive Group Sampling Plan. The RGS plan based on $S_{pkA;\mathrm{AR}(1)}$ is described as follows.

Step 1. Choose the producer's risk, the consumer's risk, and two process capability indices (C_{AQL} and C_{LTPD}).

Step 2. Draw random linear profiles k from a submitted lot and compute the $\widehat{S}_{pkA;\mathrm{AR}(1)}$ value.

Step 3 (make a decision). Accept the lot if $\widehat{S}_{pkA;\mathrm{AR}(1)} \geq c_a$, and reject the lot if $\widehat{S}_{pkA;\mathrm{AR}(1)} < c_r$, where c_a and c_r are critical values; otherwise repeat Step 2.

The lot acceptance probability, say $P_{A;\mathrm{RGS}}(\widehat{S}_{pkA;\mathrm{AR}(1)})$, is established as

$$\begin{aligned} &P_{A;\mathrm{RGS}}\left(\widehat{S}_{pkA;\mathrm{AR}(1)}\right) \\ &\quad = \frac{P_A\left(S_{pkA;\mathrm{AR}(1)} \geq c_a\right)}{P_A\left(S_{pkA;\mathrm{AR}(1)} \geq c_a\right)+P_A\left(S_{pkA;\mathrm{AR}(1)} < c_r\right)}. \end{aligned} \tag{14}$$

The ASN for this RGS sampling plan is given as follows:

$$\begin{aligned} &\mathrm{ASN}_{\mathrm{RGS}}\left(C_{\mathrm{LTPD}}\right) \\ &\quad = \frac{k}{P_A\left(S_{pkA;\mathrm{AR}(1)} \geq c_a\right)+P_A\left(S_{pkA;\mathrm{AR}(1)} < c_r\right)}. \end{aligned} \tag{15}$$

When $c_a = c_r$, this RGS plan becomes a traditional single sampling plan.

The parameters of the repetitive sampling plan can be determined through the following nonlinear optimization problem:

$$\begin{aligned} &\text{Minimize} \quad \mathrm{ASN}_{\mathrm{RGS}} \\ &\text{Subject to} \quad \frac{P_A\left(C_{\mathrm{AQL}} \geq k_a\right)}{P_A\left(C_{\mathrm{AQL}} \geq k_a\right)+P_A\left(C_{\mathrm{AQL}} < k_r\right)} \\ &\qquad \geq 1-\alpha, \\ &\qquad \frac{P_A\left(C_{\mathrm{LTPD}} \geq k_a\right)}{P_A\left(C_{\mathrm{LTPD}} \geq k_a\right)+P_A\left(C_{\mathrm{LTPD}} < k_r\right)} \leq \beta, \end{aligned} \tag{16}$$

where $0 < c_r < c_a$ and $C_{\mathrm{LTPD}} < C_{\mathrm{AQL}}$.

3.4. Multiple Dependent State Sampling Plan. The MDS sampling plan based on $S_{pkA;\mathrm{AR}(1)}$ is described as follows.

Step 1. Choose the producer's risk, the consumer's risk, two process capability indices (C_{AQL} and C_{LTPD}), and the number of proceeding lots "l."

Step 2. Draw random linear profiles n from a submitted lot and compute the $\widehat{S}_{pkA;\mathrm{AR}(1)}$ value.

Step 3 (make a decision). (1) Accept the lot if $\widehat{S}_{pkA;\mathrm{AR}(1)} \geq c_a$ and reject the lot if $\widehat{S}_{pkA;\mathrm{AR}(1)} < c_r$, where c_a and c_r are critical values; (2) accept the current lot if the proceeding l lots were accepted under the condition of $\widehat{S}_{pkA;\mathrm{AR}(1)} \geq c_a$; otherwise repeat Step 2.

The lot acceptance probability, say $P_{A;\mathrm{MDS}}(\widehat{S}_{pkA;\mathrm{AR}(1)})$, is established as

$$P_{A;\mathrm{MDS}}\left(S_{pkA;\mathrm{AR}(1)}\right) = \frac{P_1\left(S_{pkA;\mathrm{AR}(1)}\right)}{1-P_{\text{repetitive}}\left(S_{pkA;\mathrm{AR}(1)}\right)}, \tag{17}$$

where $P_1(\widehat{S}_{pkA;\mathrm{AR}(1)}) = P(\widehat{S}_{pkA;\mathrm{AR}(1)} \geq k_a)+P(k_r \leq \widehat{S}_{pkA;\mathrm{AR}(1)} < k_a)[P(\widehat{S}_{pkA;\mathrm{AR}(1)} \geq k_a)]^l$ and $P_{\text{repetitive}}(S_{pkA;\mathrm{AR}(1)}) = P(k_r \leq \widehat{S}_{pkA;\mathrm{AR}(1)} < k_a)(1-[P(\widehat{S}_{pkA;\mathrm{AR}(1)} \geq k_a)]^l)$.

The average sample number for the MDS plan is established as

$$\begin{aligned} &\mathrm{ASN}_{\mathrm{MDS}} \\ &= \frac{1}{2}\left\{\frac{k}{1-P\left(c_r \leq \widehat{S}_{pkA;\mathrm{AR}(1)} < c_a\right)\left(1-\left[P\left(\widehat{S}_{pkA;\mathrm{AR}(1)} \geq c_a\right)\right]^l\right)}\right. \\ &\left. + \frac{k}{1-P\left(c_r \leq \widehat{S}_{pkA;\mathrm{AR}(1)} < c_a\right)\left[P\left(\widehat{S}_{pkA;\mathrm{AR}(1)} \geq c_a\right)\right]^l}\right\}. \end{aligned} \tag{18}$$

When $l = 0$, this MDS plan becomes the RGS plan.

The parameters of the MDS sampling plan can be determined through the following nonlinear optimization problem:

$$\begin{aligned} &\text{Minimize} \quad \mathrm{ASN}_{\mathrm{MDS}} \\ &\text{Subject to} \quad \frac{P_1(C_{\mathrm{AQL}})}{1-P_{\text{repetitive}}(C_{\mathrm{AQL}})} \geq 1-\alpha, \\ &\qquad\qquad\quad \frac{P_1(C_{\mathrm{LTPD}})}{1-P_{\text{repetitive}}(C_{\mathrm{LTPD}})} \leq \beta, \end{aligned} \tag{19}$$

where $0 < k_r < k_a$, $C_{\mathrm{LTPD}} < C_{\mathrm{AQL}}$, $P_1(C_{\mathrm{AQL}})$, and $P_{\text{repetitive}}(C_{\mathrm{AQL}})$ are obtained by (19).

4. Comparison Study

In this section, the average sample numbers and probability of acceptance of the proposed plans are compared. Computations are carried out using computer programs written in the R language [30]. The programs are available from the authors. For instance, in order to determine the plan parameters for different combinations of $(C_{\mathrm{AQL}}, C_{\mathrm{LTPD}})$ using the sampling plan based on EWMA model, we used a simple algorithm to search the space of possible solutions. It consists of the following three steps.

Step 1. Assuming k ranges from 2 to 300 and c_a is generated from a uniform distribution between C_{LTPD} and C_{AQL}, 1,000 combinations of k and c_a were randomly generated.

Step 2. Evaluate 1,000 combinations of (k, c_a) that must satisfy the two constraints (9) with the objective of minimizing the number of profiles.

Step 3. Repeat Steps 1 and 2 for 100 times to determine the optimal parameters.

As shown in Table 1 we consider some different combinations of C_{AQL} and C_{LTPD} at a specified producer's risk ($\alpha = 0.05$) and customer's risk ($\beta = 0.1$). The probability of acceptance and the average number of profiles were calculated for the specified plans. From Table 1 all the proposed methods outperform the existing single variable scheme. The EWMA requires the smallest average number of profiles. Furthermore, it can be observed that the difference in ASN values between the variables single sampling plan and the proposed sampling plans increases when the gap between C_{AQL} and C_{LTPD} narrows. This implies that, with the same protection level, the proposed plans will minimize the time and cost of inspection especially for cases when the process fraction of defectives required is very low.

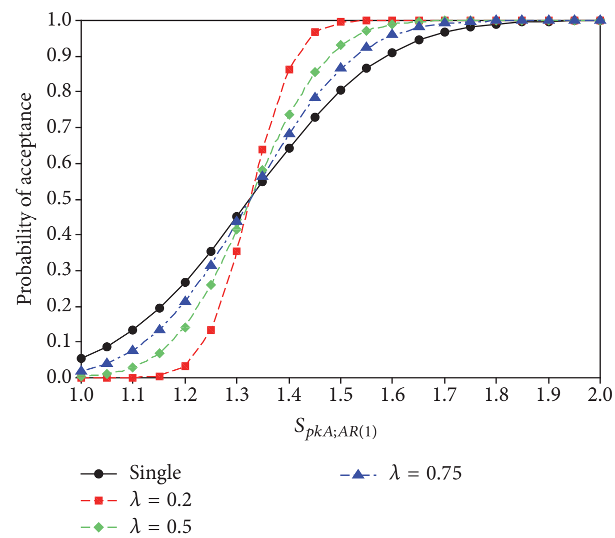

Figure 1: Comparison of EWMA OC curves under $k = 100$, $c = 1.33$, and $\rho = 0.5$.

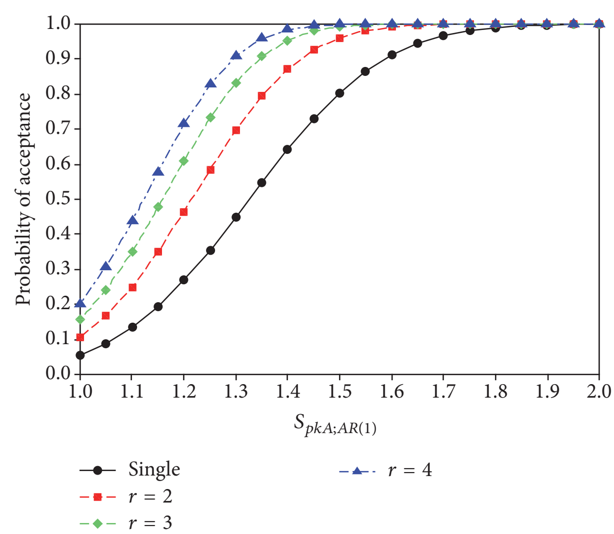

Figure 2: Comparison of resubmitted OC curves under $k = 100$, $c = 1.33$, and $\rho = 0.5$.

The slope of the OC curves in Figures 1–4, respectively, displays the discriminatory power of the sampling scheme between good and bad lots. The steeper the slope of the OC curve, the greater the discriminatory power. Figures 1–4 also illustrate how different values of an input variable impact the sensitive the OC curves of each sampling scheme. Figure 1

Table 1: Comparison of different sampling plans under $n = 4$, $\rho = 0.5$ to match at $\alpha = 0.05$, and $\beta = 0.1$.

Case	EWMA $\lambda = 0.25$	Resubmitted	RGS	MDS	Single variable $\lambda = 1.00$
$C_{\text{AQL}} = 1.33$ $C_{\text{LTPD}} = 1.10$	$k = 8$ $c_a = 1.2975$ $P_{a1} = 0.9515$ $P_{a2} = 0.0990$ ASN = 8	$k = 30$ $c_a = 1.3239$ $r = 2$ $P_{a1} = 0.9527$ $P_{a2} = 0.0906$ ASN = 36.5	$k = 33$ $l = 2$ $c_a = 1.2342$ $c_r = 1.0957$ $P_{a1} = 0.9514$ $P_{a2} = 0.0987$ ASN = 39.9	$k = 32$ $l = 2$ $c_a = 1.2214$ $c_r = 0.8577$ $P_{a1} = 0.9510$ $P_{a2} = 0.0999$ ASN = 34.9	$k = 48$ $c_a = 1.2988$ $P_{a1} = 0.9518$ $P_{a2} = 0.0848$ ASN = 48
$C_{\text{AQL}} = 1.33$ $C_{\text{LTPD}} = 1.00$	$k = 5$ $c_a = 1.1526$ $P_{a1} = 0.96781$ $P_{a2} = 0.0962$ ASN = 5	$k = 14$ $c_a = 1.2884$ $r = 2$ $P_{a1} = 0.9558$ $P_{a2} = 0.0969$ ASN = 16.9	$k = 16$ c_a =1.1840 $c_r = 0.9918$ $P_{a1} = 0.9523$ $P_{a2} = 0.0937$ ASN = 19.1	$k = 15$ $l = 2$ $c_a = 1.1669$ $c_r = 0.8156$ $P_{a1} = 0.9506$ $P_{a2} = 0.0992$ ASN = 16.4	$k = 23$ $c_a = 1.2290$ $P_{a1} = 0.9500$ $P_{a2} = 0.0882$ ASN = 23
$C_{\text{AQL}} = 1.50$ $C_{\text{LTPD}} = 1.35$	$k = 24$ $c_a = 1.4707$ $P_{a1} = 0.9539$ $P_{a2} = 0.0989$ ASN = 24	$k = 104$ $c_a = 1.4940$ $r = 2$ $P_{a1} = 0.9532$ $P_{a2} = 0.0938$ ASN = 126.5	$k = 102$ $c_a = 1.4417$ $c_r = 1.3488$ $P_{a1} = 0.9502$ $P_{a2} = 0.0985$ ASN = 124.9	$k = 101$ $l = 2$ $c_a = 1.4321$ $c_r = 1.2756$ $P_{a1} = 0.9514$ $P_{a2} = 0.0993$ ASN = 110	$k = 155$ $c_a = 1.4713$ $P_{a1} = 0.9518$ $P_{a2} = 0.0985$ ASN = 155
$C_{\text{AQL}} = 1.50$ $C_{\text{LTPD}} = 1.25$	$k = 9$ $c_a = 1.4237$ $P_{a1} = 0.9511$ $P_{a2} = 0.0904$ ASN = 9	$k = 33$ $c_a = 1.4993$ $r = 2$ $P_{a1} = 0.9513$ $P_{a2} = 0.09234$ ASN = 40.3	$k = 36$ $c_a = 1.3938$ $c_r = 1.2537$ $P_{a1} = 0.9521$ $P_{a2} = 0.0998$ ASN = 43.1	$k = 35$ $l = 2$ $c_a = 1.3817$ $c_r = 1.1818$ $P_{a1} = 0.9528$ $P_{a2} = 0.0998$ ASN = 38.1	$k = 53$ $c_a = 1.4363$ $P_{a1} = 0.9503$ $P_{a2} = 0.0999$ ASN = 53
$C_{\text{AQL}} = 1.67$ $C_{\text{LTPD}} = 1.50$	$k = 23$ $c_a = 1.6463$ $P_{a1} = 0.9500$ $P_{a2} = 0.0911$ ASN = 23	$k = 97$ $c_a = 1.6587$ $r = 2$ $P_{a1} = 0.9503$ $P_{a2} = 0.0982$ ASN = 118.6	$k = 99$ $c_a = 1.6058$ $c_r = 1.4871$ $P_{a1} = 0.9513$ $P_{a2} = 0.0966$ ASN = 122.9	$k = 98$ $l = 2$ $c_a = 1.5927$ $c_r = 1.3230$ $P_{a1} = 0.9527$ $P_{a2} = 0.0999$ ASN = 106.7	$k = 143$ $c_a = 1.6683$ $P_{a1} = 0.9510$ $P_{a2} = 0.0984$ ASN = 143
$C_{\text{AQL}} = 1.67$ $C_{\text{LTPD}} = 1.40$	$k = 9$ $c_a = 1.6180$ $P_{a1} = 0.95081$ $P_{a2} = 0.0977$ ASN = 9	$k = 36$ c_a =1.6553 $r = 2$ $P_{a1} = 0.9557$ $P_{a2} = 0.0984$ ASN = 43.6	$k = 39$ $c_a = 1.5531$ $c_r = 1.4175$ $P_{a1} = 0.9513$ $P_{a2} = 0.0984$ ASN = 46.1	$k = 37$ $l = 2$ $c_a = 1.5438$ $c_r = 1.3817$ $P_{a1} = 0.9509$ $P_{a2} = 0.0988$ ASN = 40.3	$k = 56$ $c_a = 1.6159$ $P_{a1} = 0.9562$ $P_{a2} = 0.0973$ ASN =56

P_{a1} is probability of accepting at C_{AQL} and P_{a2} is probability of accepting at C_{LTPD}.

indicates that the smaller the value of λ, the better the power. For example, if the quality level desired by the consumer is $S_{pk} = 1.50$, the probability of acceptance at λ values of 0.25, 0.5, 0.75, and 1.0 became 1.0, 0.99, 0.96, and 0.91, respectively. That is, the associated producer's risk turned out to be 0, 0.01, 0.04, and 0.09, respectively. Figures 2 and 3 indicate that increasing the number of resubmissions and repetitions improves the power. For instance, at a given $C_{\text{AQL}} = 1.33$ and $C_{\text{LTPD}} = 1.00$, increasing the allowable resubmissions from two to four improves the probability of acceptance of a good lot from 0.96 to 0.99. For the MDS the effect of a number of preceding lots on the power is very marginal. From these comparisons, we may conclude that all of the proposed sampling plans are superior to the variable single acceptance sampling plan in terms of power. Figure 5 compared the efficiency of the proposed sampling plans with an existing single sampling plan ($\lambda = 1$) at $C_{\text{AQL}} = 1.50$, $\alpha = 0.05$, and $\beta = 0.10$ for several values of $C_{\text{LTPD}} = 1.0(0.05)1.4$. The result indicates that the EWMA sampling plan is more efficient in terms of average sample number than other four schemes, MDS is second best, and resubmitted and RGS are very close to each other.

As stated in the introduction each scheme is designed for specific purpose and information availability. For example,

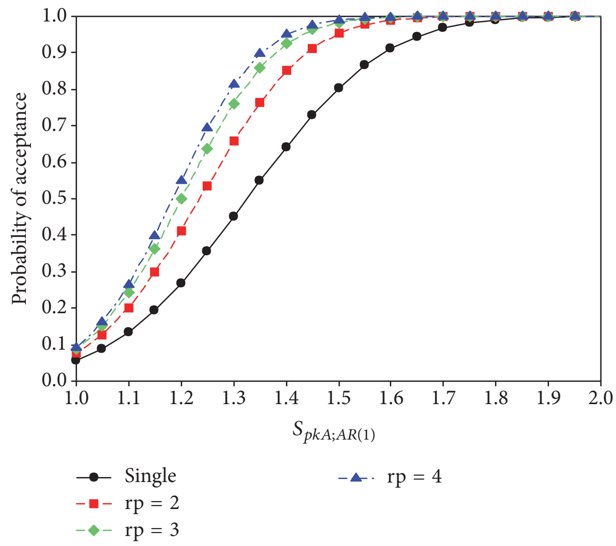

Figure 3: Comparison of RGS OC curves under $k = 100$, $c = 1.33$, and $\rho = 0.5$. rp is replication.

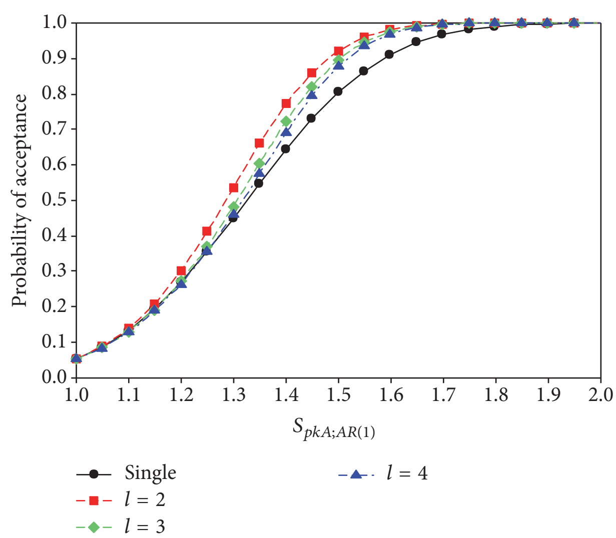

Figure 4: Comparison of MDS OC curves under $k = 100$, $c = 1.33$, and $\rho = 0.5$.

EWMA is useful when ten or more previous lots are accepted. However, in some cases, data might be available for two or more methods, for such instances based on Table 1 and Figure 5 the EWMA technique appeared to be more efficient in terms of cost than the other methods. That is, it provides the smallest ASN.

5. Illustrative Examples

Example 1. We considered the calibration of an optical imaging system in [18]. The process is in statistical control, the functional profiles from phase I analysis resulted in an AR1 model, and the relationship between the response variable

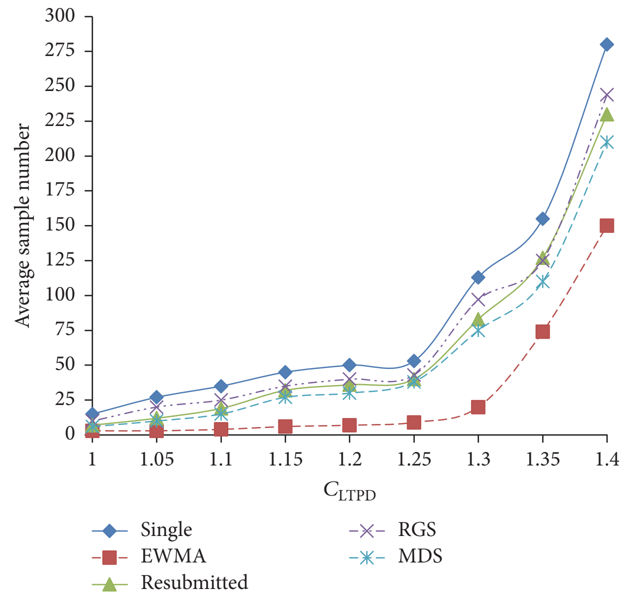

Figure 5: The average sample numbers under $C_{AQL} = 1.50$, $\alpha = 0.05$, and $\beta = 0.10$.

Table 2: Specification limits at each level for the line-width measurements modified from [18].

i	x_i	LSL_i	USL_i	$Target_i$
1	0.76	0.70	1.30	1.024
2	3.29	3.20	3.80	3.495
3	8.89	8.70	9.30	8.965

and the explanatory variable is modeled as $y_{ij} = 0.2357 + 0.9870x_i + \varepsilon_{ij}$, $\varepsilon_{ij} = 0.5\varepsilon_{i(j-1)} + a_{ij}$ with a residual standard deviation of 0.07. Table 2 shows specification limits of the response variable. Suppose that the values of C_{AQL} and C_{LTPD} are set to 1.50 and 1.25 and the two levels of risk (α, β) are set (0.05, 0.1). Table 3 depicts sample plans that are equivalent, since they all provide the same protection to both suppliers and customers. The EWMA technique provides the smallest ASN. The plan parameters are found to be $c_a = 1.4057$ and $k = 26$. Furthermore, the yield index $S_{pkA;AR(1)}$ of 10 previously accepted lots are assumed to be (1.4862, 1.5112, 1.5134, 1.5034, 1.4991, 1.5002, 1.4715, 1.4908, 1.4729, and 1.4862) the optimal value of λ is obtained as 0.1273. We collected 26 profiles and the immediate lot inspected resulted in $S_{pkA;AR(1)} = 1.2613$. The corresponding Z_t value became 1.4637 since Z_t is greater than 1.4057 according to the EWMA scheme; the current lot can be deemed as accepted.

Example 2. This example is a real case study from the automobile parts industry. The goal was to determine whether the experimental setup is acceptable or not. The procedure is used to simulate tire cornering stiffness on the ice road. The lab measurements were done on the plunger tester equipped with the servo-hydraulic system. Type of tire tested is regular production, commercially available tire (255/80R17.5) with snow tread. Tests were conducted by pull force developed by

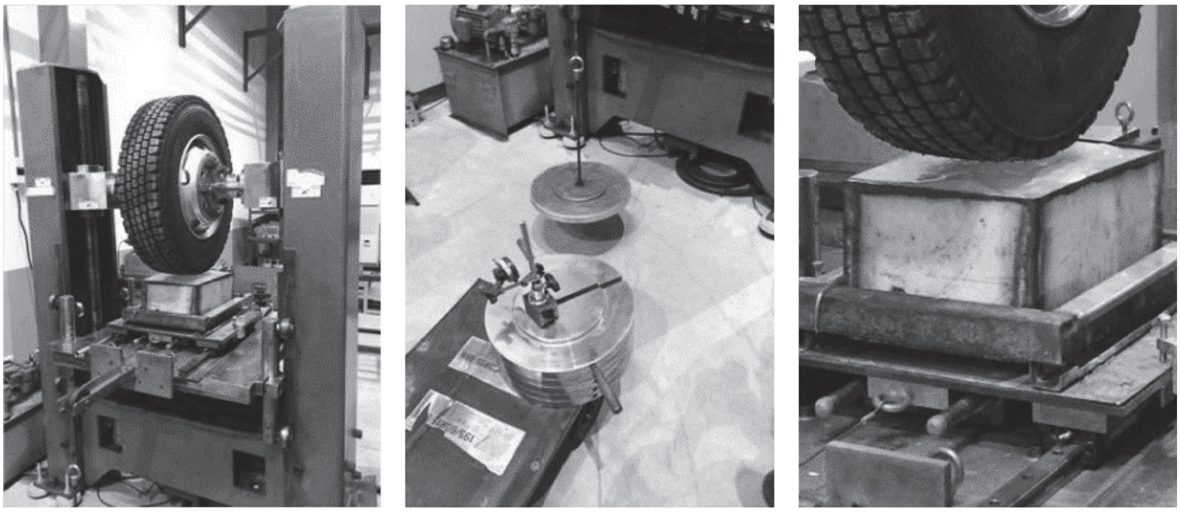

Figure 6: Experimental setup.

Table 3: Equivalent alternative sampling plans for line-width measurement.

EWMA	Resubmitted ($r = 2$)	RGS	MDS ($l = 2$)	Single variable $\lambda = 1.00$
$\lambda = 0.1273$ $k = 26$ $c_a = 1.4057$ $P_{a1} = 0.9519$ $P_{a2} = 0.0961$ ASN = 26	$k = 46$ $c_a = 1.4595$ $r = 2$ $P_{a1} = 0.9518$ $P_{a2} = 0.0913$ ASN = 56.1	$k = 50$ $c_a = 1.3887$ $c_r = 1.2806$ $P_{a1} = 0.9502$ $P_{a2} = 0.0940$ ASN = 58	$k = 46$ $l = 2$ $c_a = 1.3827$ $c_r = 1.1491$ $P_{a1} = 0.9504$ $P_{a2} = 0.0968$ ASN = 50	$k = 53$ $c_a = 1.4363$ $P_{a1} = 0.9503$ $P_{a2} = 0.0999$ ASN = 53

Table 4: Experiment output and specification limits.

Level	Weight	Target$_i$	LSL	LSL$_i$
1	20	0.12	0.04	0.2
2	40	0.65	0.57	0.73
3	60	1.37	1.29	1.45
4	80	2.01	1.93	2.09
5	100	2.85	2.77	2.93

20 kg counterweights; each time add a 20 kg counterweight and record the corresponding tire displacement. The maximum load is 100 kg; that is, the measurement is taken at 20, 40, 60, 80, and 100 kg, respectively. Table 4 shows the upper and lower specification limits at each level of the independent variable. The setup is shown in Figure 6, from left to right total experiment setup, counterweights, and close-up view of contact between tire and snow in a container.

Regression analysis was used to fit the relationship between force and displacement; we found that the relationship between the response variable and the explanatory variable can be modeled as $y_{ij} = 0.034x_i - 0.646 + \varepsilon_{ij}$, $\varepsilon_{ij} = 0.27\varepsilon_{i(j-1)} + a_{ij}$, with a residual standard deviation of 0.06, where x_i is the ith level of the independent variable ($i = 1, 2, \ldots, 5$). Suppose that the values of C_{AQL} and C_{LTPD} are set to 1.50 and 1.33 and the two levels of risk (α, β) are set $(0.05, 0.1)$. Five equivalent sampling plans are provided in Table 5. However, since we do not have prior test information it is not possible to apply EWMA and MDS. The resubmitted scheme is more appropriate than repetitive; that is, if a test is not accepted, make an improvement on the setup and the next setup is tested and a decision is made irrespective of the previous test. After the second test we found $S_{pkA;\mathrm{AR}(1)} = 1.4613$ which is greater than the critical value, $c_a = 1.4540$; it is possible to conclude the setup is acceptable.

6. Conclusion

In this paper, we developed four alternative acceptance sampling plans based on the process yield index $S_{pkA;\mathrm{AR}(1)}$ to deal with lot sentencing for autocorrelation between linear profiles. The proposed sampling plans reduce significantly the average sample number as compared with the traditional single sampling plan. Under specific conditions, the proposed sampling plans become equal and reduced to a single sampling plan by variables. Since the EWMA method considers the quality history of the previous lots the number of profiles required is smaller than the other three sampling schemes. RGS and MDS schemes are equivalent; the advantage of MDS is it takes into consideration past history, which results in a reduction of cost and time of retesting. Resubmitted

Table 5: Equivalent alternative sampling plans for tire test.

EWMA	Resubmitted ($r = 2$)	RGS	MDS ($l = 2$)	Single variable $\lambda = 1.00$
$\lambda = 0.150$ $k = 32$ $c_a = 1.4499$ $P_{a1} = 0.9529$ $P_{a2} = 0.0965$ ASN = 32	$k = 114$ $c_a = 1.4940$ $r = 2$ $P_{a1} = 0.9532$ $P_{a2} = 0.0938$ ASN = 126.5	$k = 112$ $c_a = 1.4417$ $c_r = 1.3488$ $P_{a1} = 0.9502$ $P_{a2} = 0.0985$ ASN = 124.9	$k = 111$ $l = 2$ $c_a = 1.4321$ $c_r = 1.2756$ $P_{a1} = 0.9514$ $P_{a2} = 0.0993$ ASN = 110	$k = 167$ $c_a = 1.4713$ $P_{a1} = 0.9518$ $P_{a2} = 0.0985$ ASN = 155

and RGS schemes can be used to solve disputed inspection procedures.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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