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Research Article

A New Calculation Method for the Soil Slope Safety Factor

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Based on the unified strength theory, a new method to calculate the plane soil slope safety factor was derived that considers the effect of intermediate principal stress σ_2 and at-rest lateral pressure coefficient K_0. Calculation examples from the literature were used to compare the new calculation method and the current slice method; the results showed that both provided good consistency. The new method can provide a reference for slope stability evaluation. The new method was used to calculate the soil slope safety factors for different values of intermediate principal stress parameter b, double shear stress parameters u'_τ, and static lateral pressure coefficient K_0. The results showed that the safety factor F_s increased when b was increased; F_s first increased and then decreased when u'_τ was increased; and F_s increased when K_0 was increased. These results show that the intermediate principal stress as well as the stress state and its changes cannot be ignored during soil slope stability analysis. The slope soil characteristics and stress state should be considered to determine the unified strength theoretical parameters and static lateral pressure coefficient, maximize the potential of slope soil strength, and effectively reduce the costs of soil slope engineering.

1. Introduction

In roads, bridges, and construction projects, slope stability problems are often encountered during cutting or foundation pit excavation. Slope instability is due to the destruction of the original soil stress state of equilibrium caused by external forces—such as cutting or foundation pit excavation—and the reduction of soil antishear strength by the influence of various external factors—such as rainwater intrusion and soil freeze-thaw. In practical engineering, slope stability is analyzed to test the reasonableness of the soil slope section design. If the slope is too steep, it will slump easily; if the slope is too gentle, it will increase the amount of earthwork needed.

The characteristics of the Ordinary Method of Slices [1], Bishop's Modified Method [2], Force Equilibrium Methods (e.g., Lowe and Karafiath [3]), Janbu's Genralized Procedure of Slices [4], Morgenstern and Price's Method [5], and Spencer's Method [6] were summarized in most textbooks. Fall et al. [7] have conducted study on the stability analysis of landslides by the finite element method. Cheng and Yip [8] have shown that rigorous method is necessary to give a reliable estimation of stability of landslides in 3D analyses. Zhu and Lee [9] conducted study on the factor of safety based on Bell's assumption. Bell's method was improved by Zheng and Tham [10]. Zheng and Tham' method can be regarded subsequently as the enhancement of Fellenius' method.

The slope stability safety factor F_s refers to the ratio of the soil shear strength to the shear stress of a possible sliding surface in the slope. The soil stress state and its changes are prerequisites of slope stability; the existing slope circular slipping method (Petterson 1916) and slice method (Fellenius 1927) ignore the impact of the stress state. In reality, the slope stability changes with changes in the stress state. Researchers [11–14] are currently looking for the sliding center and slip surface, supplementing and modifying the basic assumptions of the slice method, and providing a fundamental basis in engineering applications for the slice method. However, defects of slice method and statically indeterminate problem of this method [15] have produced challenges in practical engineering applications.

Based on multislip mechanism and the model of multishear element, Yu established the unified strength theory

which takes into consideration the different contribution of all stress components on the yield of failure of materials [16, 17]. The unified strength theory encompasses the twin shear strength theory [18–20] and single strength theory. The excellent agreement between the predicted results by the unified strength theory and the experiment results indicates that the unified strength theory is applicable for a wide range of stress states in many materials (Ma et al. 1985 [21]).

The coefficient of the earth pressure at rest (K_0) is defined as the ratio of the in situ horizontal effective stress to the in situ vertical effective stress. The parameter K_0 is needed in the interpretation of laboratory and in situ tests and design of retaining structures and excavation support systems. Schnaid and Yu [22] believe that K_0 is an important input parameter for the numerical analyses of geotechnical boundary value problems.

In this study, the perspective of a total stress state was considered to derive a new calculation method for the soil slope safety factor based on the unified strength theory. The slope safety factor was defined by considering the effects of the intermediate principal stress and at-rest lateral pressure coefficient. The method was compared and verified with the current slice method and can provide a reference for stability evaluation in soil slope engineering.

2. Basic Theory and Formula Derivation

2.1. Unified Strength Theory. The Mohr–Coulomb strength theory is simple and practical. It is conducive to engineering applications but does not reflect the effect of intermediate principal stress, and the calculated results are relatively conservative. In 1991, Yu suggested the unified strength theory to compensate for the shortcomings of the Mohr–Coulomb strength theory. The unified strength theory can consider the intermediate principal stress effect of the material and can simulate nearly all materials on the partial plane to develop the material strength potentials. There are two equations with a conditional formula for both the mathematical model and the theoretical expression of the unified strength theory, which considers the different contributions of various stress components to the material yield and destruction, reduces the number of material parameters, and makes the limiting surface to reach the outer boundary of the convex criteria; these cannot be achieved by other criteria. The mathematical modeling methods from the two equations can also be used to solve issues with determining the intermediate principal shear stress. Yu derived a mathematical expression of the unified strength theory using the unified twin shear model and a new mathematical model [17]:

$$F = \sigma_1 - \frac{\alpha}{1+b}\left(b\sigma_2 + \sigma_3\right) = \sigma_t, \quad \sigma_2 \le \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}, \tag{1a}$$

$$F' = \frac{1}{1+b}\left(b\sigma_1 + \sigma_2\right)\sigma_1 - \alpha\sigma_3 = \sigma_t, \quad \sigma_2 \ge \frac{\sigma_1 + \alpha\sigma_3}{1+\alpha}, \tag{1b}$$

where F and F' are yield functions; $\alpha = \sigma_t/\sigma_c = (1 - \sin\varphi_0)/(1 + \sin\varphi_0)$ is the ratio of the tensile to compression strengths of the material; $\sigma_t = 2c_0\cos\varphi_0/(1 + \sin\varphi_0)$ is the tensile strength; c_0 and φ_0 are the cohesion and internal friction angle, respectively, of rock and soil; and b is the selected failure criterion introduced in the unified strength theory that also reflects the destructive effects of the intermediate principal shear stress and normal stress of the corresponding surface on the material.

The unified strength theory was converted into a formula similar to the Mohr–Coulomb strength theory in order to obtain the friction angle φ_{uni} and unified cohesive force c_{uni}; these are expressed by the internal friction angle φ_0 and cohesion c_0 as follows [23].

When $u'_\tau \le (1 - \sin\varphi_0)/2$,

$$\varphi_{\text{uni}} = \arcsin\left(\frac{(1+b)\sin\varphi_0}{1 + b\left(1 - u'_\tau\right) - bu'_\tau\sin\varphi_0}\right), \quad c_{\text{uni}} = \frac{(1+b)\,c_0\cos\varphi_0\cot\left(45^\circ + \varphi_{\text{uni}}/2\right)}{1 + b\left(1 - u'_\tau\right) + \left(1 + b + bu'_t\right)\sin\varphi_0}. \tag{2a}$$

When $u'_\tau \ge (1 - \sin\varphi_0)/2$,

$$\varphi_{\text{uni}} = \arcsin\left(\frac{(1+b)\sin\varphi_0}{1 + bu'_\tau + b\left(1 - u'_\tau\right)\sin\varphi_0}\right), \quad c_{\text{uni}} = \frac{(1+b)\,c_0\cos\varphi_0\cot\left(45^\circ + \varphi_{\text{uni}}/2\right)}{\left(1 + bu'_\tau\right)\left(1 - \sin\varphi_0\right)}, \tag{2b}$$

where $u'_\tau = \tau_{23}/\tau_{13} = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ is the twin shear stress parameter.

The unified internal friction angle φ_{uni} and unified cohesion c_{uni} can be used to express the Mohr–Coulomb strength theory:

$$\tau_f = c_{\text{uni}} + \sigma\tan\varphi_{\text{uni}}. \tag{3}$$

2.2. Basic Assumptions

(1) The excavated soil is simplified as plane soil slopes.

(2) Soil is homogeneous.

(3) The stress state can be represented by formula (4).

(4) The soil antishear strength satisfies formula (3).

(5) The horizontal stress of inner points along the depth causes soil slope instability.

(6) During excavation of the soil, the static lateral pressure coefficient remains unchanged.

(7) The impact of pore water and groundwater is not considered.

2.3. Formula Derivation. The extending direction of the soil slope is taken as the plane stress state, and elastic half-space plane stress analysis is performed for soil in a steady state

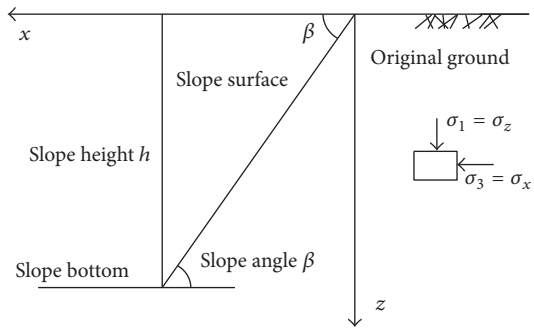

Figure 1: Simplified soil slope excavation.

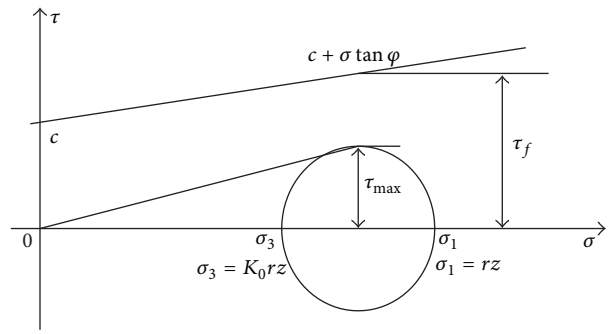

Figure 2: Schematic diagram of accumulated safety in soil.

under gravity stress; the main stress expression of any point is as follows:

$$\sigma_1 = \sigma_z = rz,$$
$$\sigma_3 = \sigma_x = K_0 rz, \tag{4}$$

where σ_1 is the maximum principal stress, σ_3 is the minimum principal stress, r is the soil gravity, K_0 is the static lateral pressure coefficient of the soil, z is the distance from the ground surface to any point, σ_x is the horizontal stress at any point, and σ_z is the vertical stress at any point.

As shown in Figure 1, assuming that the cutting or foundation soil is not excavated, when slope angle $\beta = 0^\circ$, then $\sigma_x = K_0 rz$. In vertical excavation of the soil, when slope angle $\beta = 90^\circ$, then $\sigma_x = 0$. The horizontal stress of the soil excavation slope angle satisfies the following formula:

$$\sigma_x = K_0 rz\left(1 - \sin\beta\right). \tag{5}$$

According to the conventional definition of the factor of safety for some point within a soil mass, the safety factor is the ratio between the shear strength and shear stress at that point [24, 25]:

$$F_s = \frac{\tau_f}{\tau} = \frac{\left(c + \sigma\tan\varphi\right)}{\tau}. \tag{6}$$

In line with the Mohr–Coulomb criterion for the shear strength of soils, for stresses at some point within a soil mass, differences in the magnitude of shear stress in an arbitrary direction will result in shear strength differences. In other words, the factor of safety at a point in a soil mass, defined in (6), will vary with the direction. This leads to complexities and difficulties in the methods of slope stability analysis and to a variety of assumptions in calculational theories. To ensure the uniqueness of safety factors computed at each point within a soil mass, the factor of safety was defined as described below.

$$F_s = \frac{\tau_f}{\tau_{\max}} = \frac{\left(c + \left(\left(\sigma_1 + \sigma_3\right)/2\right)\tan\varphi\right)}{\tau_{\max}}. \tag{7}$$

Given a point with a determined stress state within some soil mass, its margin of safety is the ratio between the shear strength corresponding to the maximum shear strength at that point and the total maximum shear strength as described in Figure 2.

Then, the safety margin of a slope is the ratio between the cumulative shear strength and the cumulative maximum shear stress within the slope's height; thus

$$F_s = \frac{\int_0^z \tau_f dz}{\int_0^z \tau_{\max} dz} \tag{8}$$

since

$$\tau_{\max} = \frac{1}{2}\left(\sigma_1 - \sigma_3\right) = \frac{1}{2} rz\left(1 - K_0\left(1 - \sin\beta\right)\right),$$
$$\tau_f = c_{\text{uni}} + \frac{1}{2}\left(\sigma_1 + \sigma_3\right)\tan\varphi_{\text{uni}} \tag{9}$$
$$= c_{\text{uni}} + \frac{1}{2} rz\left(1 + K_0\left(1 - \sin\beta\right)\right)\tan\varphi_{\text{uni}}.$$

Therefore, (9) are substituted into (8) to obtain

$$F_s = \frac{4c_{\text{uni}}\left(1 + K_0 - K_0\sin\beta\right)}{rz\left(1 - K_0^2\left(1 - \sin\beta\right)^2\right)} + \frac{rz\left(1 + K_0 - K_0\sin\beta\right)^2\tan\varphi_{\text{uni}}}{rz\left(1 - K_0^2\left(1 - \sin\beta\right)^2\right)}. \tag{10}$$

3. Safety Factor Calculation and Analysis

The new approach was derived from cutting or foundation pit soil excavation. Slope engineering calculation examples in textbooks [26, 27] and literature [11] were used to verify the general application of the calculation method with formula (10).

Example 1. Slope height $h = 6$ m, slope angle $\beta = 55^\circ$, soil gravity $r = 18.6\,\text{kN/m}^3$, angle of soil internal friction $\varphi_0 = 12^\circ$, and cohesion $c_0 = 16.7$ kPa were known. The Fellenius' slice method and Bishop formula were used to calculate the slope safety factor; the results were 1.18 and 1.19, respectively [9].

Using $b = 0$ and $K_0 = 1 - \sin\varphi_0 = 0.79$, which were calculated using Jaky's formula, formula (10) was used to calculate the slope safety factor, which was 0.98.

Table 1: Material properties.

Soil number	Gravity (kN/m^3)	Cohesion (kN/m^2)	Internal friction angle (°)
#1 soil	19.5	0.0	38.0
#2 soil	19.5	5.3	23.0
#3 soil	19.5	7.2	20.0

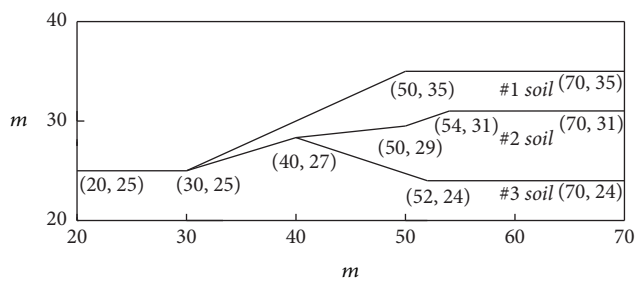

Figure 3: Contour map of ACADS assessment question EX1 (c).

Example 2. Question EX1 (c) from Australian Association of Computer Applications (ACADS) assessment in 1987: a heterogeneous soil slope of material properties was shown in Table 1, and slope shape was shown in Figure 3. The problem was simplified into one with a homogeneous soil slope with a height of $h = 10\,\text{m}$, slope gradient $\tan\beta = 1:2$, soil gravity $r = 19.5\,\text{kN/m}^3$, soil internal friction angle $\varphi_0 = 38^\circ$, and cohesion $c_0 = 5.3\,\text{kPa}$. The Bishop formula and genetic algorithm were used in the calculation, and the ten safety factors of the slide facing were in the range of 1.398~1.40 [11]. Using $b = 0$ and $K_0 = 1 - \sin\varphi_0 = 0.38$, which were calculated using Jaky's formula, the slope safety factor was calculated using formula (10) to be 1.34. The reference value was 1.39.

The examples above show that the new method is simpler than the current slice method, has a clearer theoretical basis and concept, does not require programming, and effectively reduces the computational workload. It can be used as a general basis for safety evaluation of slope stability.

In Example 1, the data substituted into formulas (2a) and (2b) were used to analyze the relationship among the unified internal friction angle φ_{uni}, cohesion c_{uni}, b, and u'_τ. The calculation results are shown in Figures 4 and 5.

Figures 4 and 5 show that the unified internal friction angle φ_{uni} and cohesion c_{uni} increased when b was increased. They firstly increased and then decreased when u'_τ was increased and reached their maximum when $u'_\tau = (1 - \sin\varphi_0)/2 = 0.4$.

φ_{uni} and c_{uni} were obtained with different b and u'_τ values and substituted into formula (10) to calculate various soil slope safety factors F_s, as shown in Figure 6. Figure 6 shows that when u'_τ was a fixed value, the safety factor F_s increased with increasing b. When b was a fixed value, u'_τ changed from 0 to 1, and the safety factor F_s first increased and then decreased. When $u'_\tau = (1 - \sin\varphi_0)/2 = 0.4$, the safety factor F_s was at its maximum; when $u'_\tau = 0$, or $u'_\tau = 1$, the safety factor F_s was at its minimum value. When $b = 0$, formula (10)

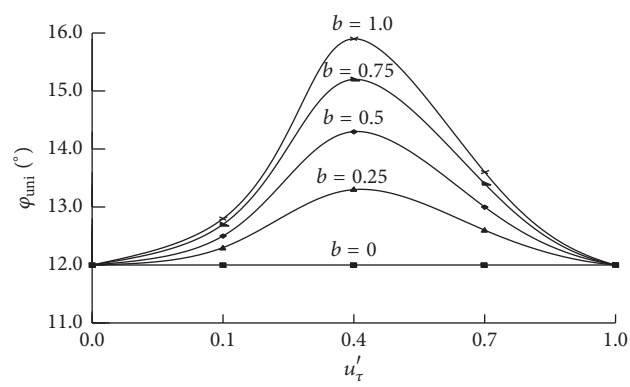

Figure 4: Relationship among φ_{uni}, u'_τ, and b.

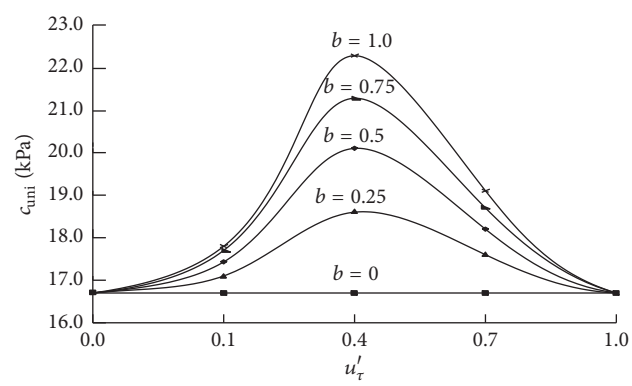

Figure 5: Relationship among c_{uni}, u'_τ, and b.

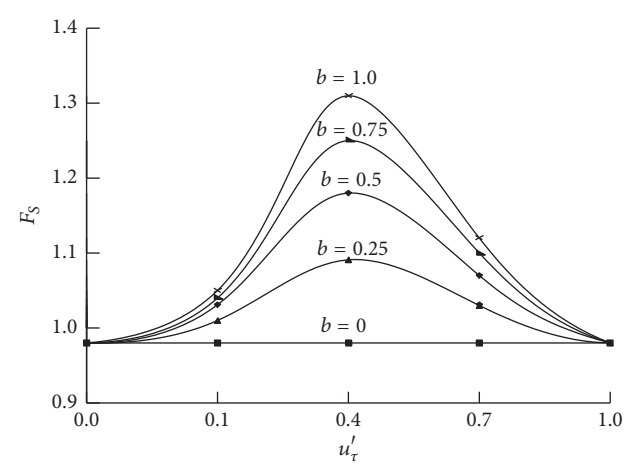

Figure 6: Relationship among F_s, u'_τ, and b.

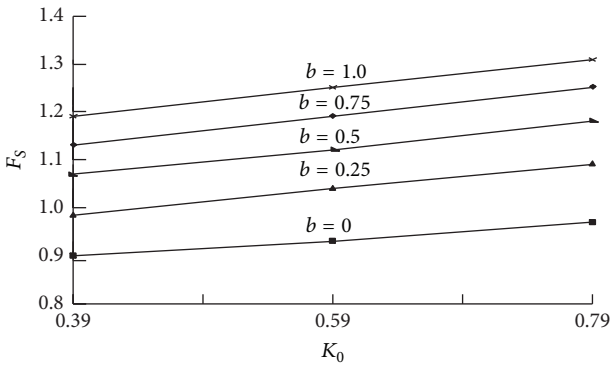

Figure 7: Relationship among F_s, K_0, and b.

degrades to a formula under the Mohr–Coulomb strength theory, and the obtained safety factor F_s will be the smallest fixed value. This indicates that the potential of the soil is far from being developed and may result in significant waste.

Using $u'_\tau = 0.4$, φ_{uni} and c_{uni} obtained with different b and K_0 were substituted into formula (10), and various soil slope safety factors F_s were obtained as shown in Figure 7. Figure 7 shows that when K_0 was a fixed value, F_s increased with b, which is consistent with the above conclusions. When b was a fixed value, K_0 varied from 0.39 to 0.79, and the safety factor F_s increased gradually. This indicates that b and K_0 are combined factors that determine slope stability. Formula (10) considers the effects of both and can provide a reference for safety evaluation of a soil slope.

4. Conclusions

The new method was verified by comparing results with the existing slice method using examples in the literature. The new method can be combined with slope engineering to obtain parameters of the unified strength theory and the static lateral pressure coefficient. The slope stability and safety were evaluated with the new method.

The effects of different factors on soil slope safety and stability were analyzed, including the intermediate principal stress parameter, twin shear stress state parameters, and static lateral pressure coefficient. These results indicate that the intermediate principal stress and static lateral pressure coefficient cannot be ignored in slope stability analysis.

This study only examined the effects of unified strength theory parameters and the static lateral pressure coefficient on the slope safety factor. In order to determine the parameters and practical applications of the new method, further research and verification are needed.

The theoretical formula was derived, calculated, and analyzed from the perspective of the total stress state. The effects on pore water pressure and groundwater should be further examined from the perspective of effective stress.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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