

Research Article

Strictly Finite-Time-Convergent Missile Guidance Law Based on Adaptive-Gain Observer

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Received 8 November 2016; Revised 2 January 2017; Accepted 10 January 2017; Published 1 February 2017

Academic Editor: M. Djemai

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In the absence of the upper bound of time-varying target acceleration, the finite-time-convergent guidance (FTCG) problem for missile is addressed in this paper. Firstly, a novel adaptive finite-time disturbance observer (AFDO) is developed based on adaptive-gain super twisting (ASTW) algorithm to estimate the unknown target acceleration. Subsequently, a new FTCG law is proposed by using the output of AFDO. The newly proposed FTCG law has several advantages over existing FTCG laws. First, for time-varying target acceleration, the proposed method can strictly guarantee the trajectory of the closed-loop system is driven onto the sliding surface rather than a neighbourhood of sliding surface in the extended-state-observer-based FTCG (ESOFTCG) law. Second, the proposed method requires no upper bound information on the target acceleration. Third, the chattering problem in the conventional FTCG (CFTCG) law is completely avoided in this paper. Simulation result demonstrates the effectiveness of the proposed AFDO and the proposed FTCG law.

1. Introduction

As a classical guidance method, proportional navigation guidance law (PNGL) [1–6] has the advantage of easy implementation in engineering. However, a series of theoretical researches and engineering practices have shown that PNGL has insufficient effect in the presence of maneuvering target. To improve the robustness of guidance system in allusion to maneuvering target, many modern control theories have been applied to design guidance laws, such as nonlinear H_∞ robust guidance law [7], L2 gain guidance law [8], Lyapunov-theory-based nonlinear guidance law [9], and sliding-mode guidance law [10]. However, the missile guidance problem considered in [7–10] is solved by the asymptotic stability analysis which implies that the system trajectories converge to the equilibrium with infinite time. Actually, in many applications, the time of termination is really quite short. For example, in the space interception where a missile is intercepting a ballistic target, the whole process of terminal guidance usually lasts for only a few seconds. Thus, the finite-time control for the guidance system is necessary in many practical guidance cases.

Since the 1990s, with the development of finite-time stability theories [11–14], the study on the finite-time-convergent (FTC) control method has increasingly become a research hotspot. Some guidance laws based on FTC control have been developed. The most representative one is the FTCG law proposed in [15]. The authors in [16, 17] adopted terminal-sliding-mode method to design FTCG laws. However, in order to guarantee the stability of guidance system under the condition of maneuvering target, the FTCG laws in [15–17] contain a discontinuous control term, which brings undesirable chattering phenomenon.

It is well known that the chattering phenomenon may reduce the performance of system and cause the instability of whole system. Thus, research on chatter-free FTCG laws has the important practical and theoretical significance. In [15], to alleviate the chattering phenomenon, a saturation function was utilized to replace the symbolic function of the guidance input. However, to do this, the disturbance rejection performance is sacrificed. To alleviate the chattering phenomenon and hold the disturbance rejection performance, some FTCG laws were designed in [18], by employing a non-smooth disturbance observer (NSDOB) to estimate the

target acceleration. The FTCL law in [18] eliminates the effect of target acceleration without using the saturation function and the symbolic function. Thus, the chattering problem is eliminated in [18].

However, for the above-mentioned NSDOB-based FTCL laws in [18], the upper bound of derivative of target acceleration must be known. In reality, the maneuvering characteristic of the target is complex; thus it is difficult to know the upper bound in advance. So far, fewer FTCL laws have been developed in the absence of the upper bound. It is well known that the extended state observer (ESO) is a powerful DO. Compared with NSDOB used in [18], ESO requires no information on the target acceleration. In [19], the target acceleration was estimated by ESO, and then a FTCL law was designed without using the upper bound of derivative of target acceleration.

The ESO-based FTCL (ESOFTCL) laws in [19], however, still have limitation: ESO cannot guarantee the estimation error fully converges to zero when the target acceleration is time-varying. Thus, ESOFTCL law only can guarantee the sliding surface converges to a neighbourhood of zero. In particular, if the target acceleration is fast varying, the estimation error is very large. And then the finite-time convergent feature of ESOFTCL law in [19] may be destroyed by the large estimation error of ESO. Actually, most of the target acceleration instances are time-varying in practical engineering. Thus, the ESOFTCL law in [19] may not work well in the practical situations.

In this paper, a new adaptive finite-time disturbance observer (AFDO) based on ASTW is proposed to estimate the time-varying target acceleration. And then a novel FTCL law is designed based on the output of AFDO. The main contributions of this paper lie in the following aspects:

- (1) In the absence of the upper bound information of time-varying target acceleration, the proposed AFDO can fully estimate the target acceleration. Compared with NSDOB used in [18], AFDO requires no a priori information on the target acceleration. Unlike ESO used in [19], the advantage of AFDO is that the estimation error of AFDO can fully converge to zero in finite time when the target acceleration is time-varying.
- (2) The proposed FTCL law is strictly finite-time convergent in the presence of time-varying target acceleration. When the target acceleration is time-varying, the proposed guidance law can strictly guarantee the sliding surface converges to zero in finite time rather than a neighbourhood of zero in [19]. Moreover, with the help of AFDO, the proposed FTCL law does not require the a priori information on the target acceleration which is needed in [18].

The remaining parts of this paper are as follows. In Section 2, the guidance model, the design objective, and the design idea of this paper are expounded. The main results are presented in Section 3. In Section 3, a novel observer AFDO is developed and the stability proof of the estimation with AFDO is presented. Then, a AFDO-based FTCL law

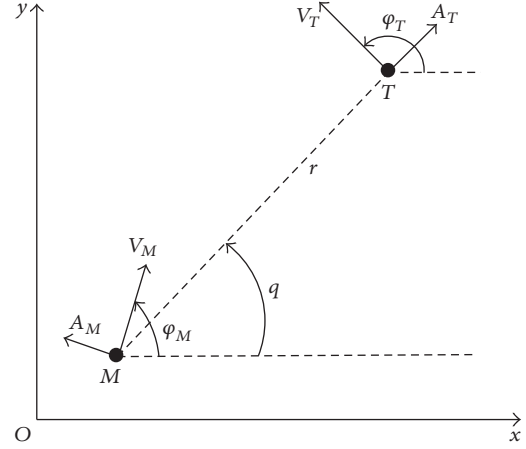


FIGURE 1: Interception geometry.

is proposed, and the finite-time stability of the law is also obtained. In Section 4, a simulation verifies the effectiveness of both AFDO and the proposed FTCL law. In Section 5, the conclusion of the whole paper is presented.

Notations. The following notations will be used in this paper: t_0 denotes the initial time. Let $\|\cdot\|$ denote the Euclidean norm of a vector and its induced norm of a matrix.

2. Problem Formulation

Consider a standard 2D dimensional geometry of interception shown in Figure 1. The origin M is the missile and T the target. The positions of the missile are x_M and y_M . The positions of the target are x_T and y_T . q is the LOS angle, r is the range along LOS, A_M and A_T are the normal acceleration instances of the missile and target, and V_M and V_T are the velocities of missile and target. φ_m and φ_T are the flight path angles of missile and target. The relative motion between the missile and its target can be expressed by the following equations [19]:

$$\begin{aligned} \dot{r} &= V_T \cos(q - \varphi_T) - V_M \cos(q - \varphi_M), \\ r\dot{q} &= -V_T \sin(q - \varphi_T) + V_M \sin(q - \varphi_M), \\ \dot{\varphi}_M &= \frac{A_M}{V_M}, \\ \dot{\varphi}_T &= \frac{A_T}{V_T}. \end{aligned} \quad (1)$$

Let $V_r = V_T \cos(q - \varphi_T) - V_M \cos(q - \varphi_M)$, $V_\lambda = -V_T \sin(q - \varphi_T) + V_M \sin(q - \varphi_M)$, $A_{Tr} = A_T \sin(q - \varphi_T)$, and $A_{T\lambda} = A_T \cos(q - \varphi_T)$. Differentiating (1) yields [19]

$$\begin{aligned} \dot{q} &= \frac{V_\lambda}{r}, \\ \dot{V}_\lambda &= -\frac{V_r V_\lambda}{r} + A_{T\lambda} - A_M \cos(q - \varphi_M). \end{aligned} \quad (2)$$

To achieve the hit-to-kill interception, a direct interception guidance strategy is given as [19]

$$V_\lambda \longrightarrow c_0 \sqrt{r}, \quad (3)$$

where c_0 is a constant. Thus for the guidance strategy (3), the guidance error is $V_\lambda - c_0 \sqrt{r}$. In order to satisfy condition (3), the sliding surface is chosen as [19]

$$S = V_\lambda - c_0 \sqrt{r}. \quad (4)$$

If $S = 0$ can be satisfied in finite time, the objective of FTTCG will be achieved [19].

Using the method in [16], $S = 0$ can be satisfied in finite time by the following CFTCG law:

$$A_M = \frac{1}{\cos(q - \varphi_M)} \left(\tau S + \sigma |S|^\gamma \text{sign}(S) - \frac{V_r V_\lambda}{r} - \frac{(c_0/2) V_r}{\sqrt{r}} + d \text{sign}(S) \right), \quad (5)$$

where $\tau > 0$, $\sigma > 0$, $0 < \gamma < 1$. $\text{sign}(\cdot)$ represents the sign function, and its expression can be found in [16]. From [16], it can be known that $\sigma |S|^\gamma \text{sign}(S)$ play a very important role in the CFTCG law (5). The convergence time of the sliding surface S decreases as the value of σ is increased and the value of γ is decreased. From [16], it also can be known that the finite-time convergent feature of CFTCG law may be destroyed if the target acceleration $A_{T\lambda}$ is not equal to zero. Thus $d \text{sign}(S)$ in (5) is used to eliminate the effect of target acceleration. And the constant d must be selected as the upper bound of the target acceleration:

$$|A_{T\lambda}| \leq d. \quad (6)$$

However, $d \text{sign}(S)$ is discontinuous and brings chattering problem. To avoid the chattering, NSDOB is used to estimate the target acceleration in [18], a non-smooth-control-based finite-time convergent guidance (NSCFTCG) law can be designed by using the method in [18]:

$$A_M = \frac{1}{\cos(q - \varphi_M)} \left(\tau_2 S + \sigma_2 |S|^{\gamma_2} \text{sgn}(S) - \frac{V_r V_\lambda}{r} - \frac{(c_0/2) V_r}{\sqrt{r}} + p_1 \right), \quad (7)$$

where p_1 is the estimation of $A_{T\lambda}$ and given by following non-smooth disturbance observer (NSDOB):

$$\begin{aligned} \dot{p}_0 &= -\frac{V_r V_\lambda}{r} - A_M \cos(q - \varphi_M) + v_0, \\ v_0 &= -\lambda_{p0} |p_0 - V_\lambda|^{2/3} \text{sign}(p_0 - V_\lambda) + p_1, \\ \dot{p}_1 &= v_1, \\ v_1 &= -\lambda_{p1} |p_1 - v_0|^{1/2} \text{sign}(p_1 - v_0) + p_2, \\ \dot{p}_2 &= -\lambda_{p2} \text{sign}(p_2 - v_1), \end{aligned} \quad (8)$$

where $\lambda_{p0} = 2L^{1/3}$, $\lambda_{p1} = 1.5L^{1/2}$, and $\lambda_{p2} = 1.1L$. L must be selected as the upper bound of $\dot{A}_{T\lambda}$.

$$|\dot{A}_{T\lambda}| \leq L. \quad (9)$$

However, the upper bound information d and L may not be easily obtained because the maneuvering characteristic of the target is complex. If d and L are unknown, the CFTCG law (5) and NSCFTCG law (7) are no longer available. To avoid using the upper bound information and eliminate the chattering problem, ESO is used to estimate the target acceleration in [19]. The guidance laws (5) and (7) are modified as the following ESOFCTCG [19]:

$$A_M = \frac{1}{\cos(q - \varphi_M)} \left(\tau S + \sigma |S|^\gamma \text{sign}(S) - \frac{V_r V_\lambda}{r} - \frac{(c_0/2) V_r}{\sqrt{r}} + Z_2 \right), \quad (10)$$

where the estimation of target acceleration Z_2 is given by following ESO:

$$\begin{aligned} E_1 &= Z_1 - V_\lambda, \\ \dot{Z}_1 &= Z_2 - \beta_{01} E_1 - \frac{V_r V_\lambda}{r} - A_M \cos(q - \varphi_M), \\ \dot{Z}_2 &= -\beta_{02} \text{fal}(E_1, \alpha_1, \delta), \end{aligned} \quad (11)$$

$$\text{fal}(E_1, \alpha_1, \delta) = \begin{cases} |E_1|^{\alpha_1} \text{sign}(E_1), & |E_1| > \delta, \\ \frac{E_1}{\delta^{1-\alpha_1}}, & \text{otherwise.} \end{cases} \quad (12)$$

From [19], it can be known that the ESOFCTCG law (10) can drive the trajectory of the closed-loop system (2) into a neighbourhood of the sliding surface in finite time:

$$\begin{aligned} \lim_{\theta \rightarrow \theta_0} S &\in \left(S^\gamma \leq \frac{|E_2|}{(1 - \theta) \sigma} \right), \\ |E_2| &= \begin{cases} \beta_{01} \left| \frac{g(t)}{\beta_{02}} \right|^{1/\alpha_1}, & \text{if } |E_1| > \delta, \\ \frac{\beta_{01} |g(t) \delta^{1-\alpha_1}|}{\beta_{02}}, & \text{if } |E_1| \leq \delta, \end{cases} \end{aligned} \quad (13)$$

where E_2 is the estimation error of ESO (11) and satisfied $E_2 = Z_2 - A_{T\lambda}$. And $g(t)$ is the varying rate of the target acceleration; that is, $g(t) = \dot{A}_{T\lambda}$. β_{01} , β_{02} , α_1 , and δ are constant.

Motivation of This Paper. Unlike the CFTCG law (5) and NSCFTCG law (7), the ESOFCTCG law (10) does not need the upper bounds d and L . However, from (13), it is clear that ESOFCTCG law (10) cannot strictly guarantee the sliding surface S converges to zero in finite time if the varying rate of the target acceleration $g(t) \neq 0$. Moreover, the upper bound

of $|S|$ will increase progressively as $|g(t)|$ become bigger. Thus the requirement of FTCCG cannot be guaranteed by the FTCCG law (10) if the varying rate $g(t)$ is very large (in Section 4 of this paper, the simulation result also demonstrates that the performance of existing ESOFCCG law is poor when the target acceleration is fast time-varying). This motivates the research topic of this paper, that is, for the missile in the presence of time-varying target acceleration, designing a new FTCCG law to strictly guarantee S converge to zero in finite time without using the upper bounds d and L .

Like [19], the following assumption should be assumed to be valid throughout this paper.

Assumption 1. The target acceleration $A_{T\lambda}$ is unknown bounded disturbance and satisfied the following condition:

$$\begin{aligned} |A_{T\lambda}| &\leq A_{T\lambda}^{\max}, \\ |\dot{A}_{T\lambda}| &\leq \dot{A}_{T\lambda}^{\max}, \end{aligned} \quad (14)$$

where $A_{T\lambda}^{\max}$ and $\dot{A}_{T\lambda}^{\max}$ are unknown positive constant.

Assumption 1 implies that it is unnecessary to know the upper bound information d of (6) and L of (9).

3. Main Result

3.1. Observer Design and Stability Analysis. In this section, a new adaptive finite-time disturbance observer (AFDO) will be proposed to estimate the target acceleration based on ASTW algorithm. The performance of AFDO will not be affected by the varying rate of target acceleration. And AFDO requires no a priori information on the target acceleration.

Before giving the AFDO and guidance law of this paper, the following Lemmas 2 and 3, which play important role in the subsequent analysis, are recalled here for convenience.

Lemma 2 (see [14]). *Provided that $V(x, t)$ is a differentiable and nonnegative scalar function, and $V(x, t)$ satisfies the differential inequality $\dot{V}(x, t) \leq -\alpha V^\gamma(x, t)$, where $\alpha > 0$ and $0 < \gamma < 1$, α and γ are constants, then we have*

$$V(x, t) = 0, \quad \forall t > t_r, \quad (15)$$

where $t_r \leq t_0 + V^{1-\gamma}(x_0, t_0)/\alpha(1-\gamma)$, $V(x_0, t_0)$ is the initial value and t_0 is the initial time, and $V(x_0, t_0)$ is bounded.

The ASTW algorithm is given by the following lemma.

Lemma 3 (ASTW algorithm [20]). *Consider the following differential inclusion:*

$$\begin{aligned} \dot{e} &= -\lambda(t) |e|^{1/2} \text{sign}(e) + \varphi, \\ \dot{\varphi} &= -k(t) \text{sign}(e) + \dot{\varphi}, \end{aligned} \quad (16)$$

where $\lambda(t)$ and $k(t)$ are given as

$$\begin{aligned} \lambda(t) &= 2\sqrt{l(t)}, \\ k(t) &= 4l(t), \end{aligned} \quad (17)$$

where $l(t)$ is a positive time-varying scalar. The adaptive law of $l(t)$ is given by

$$\dot{l}(t) = \begin{cases} \bar{q}, & \text{if } e \neq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (18)$$

where \bar{q} is a positive constant. If the following condition can be satisfied,

$$|\dot{v}| \leq \bar{c}, \quad (19)$$

where \bar{c} is an unknown constant, then e and \dot{e} will converge to zero in finite time.

Other similar ASTW algorithms can be seen in [21–24]. Then AFDO and the stability analysis are given in Theorem 4.

Theorem 4. *Taking guidance system (2) into consideration, the AFDO (20) is constructed.*

$$\begin{aligned} H_1 &= V_\lambda - V_1, \\ \dot{V}_1 &= \lambda_1 |H_1|^{1/2} \text{sign}(H_1) + J_1 - \frac{V_r V_\lambda}{r} \\ &\quad - A_M \cos(q - \varphi_M), \\ \dot{J}_1 &= k_1 \text{sign}(H_1), \\ \lambda_1 &= 2\sqrt{l_1}, \\ k_1 &= 4l_1. \end{aligned} \quad (20)$$

The adaptive law of l_1 is given by

$$\dot{l}_1 = \begin{cases} \bar{q}_1, & \text{if } H_1 \neq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (21)$$

where $l_1(t_0)$ and \bar{q}_1 are positive constants. Assumption 1 is valid. The estimation error of AFDO is defined as $H_2 = J_1 - A_{T\lambda}$. Then the estimation error H_2 will converge to zero in finite time.

Proof. Differentiating H_1 gives

$$\dot{H}_1 = \dot{V}_\lambda - \dot{V}_1. \quad (22)$$

Substituting the expression of \dot{V}_1 in (20) and \dot{V}_λ in (2) into (22) yields

$$\begin{aligned} \dot{H}_1 &= -\frac{V_r V_\lambda}{r} + A_{T\lambda} - A_M \cos(q - \varphi_M) \\ &\quad - \left(\lambda_1 |H_1|^{1/2} \text{sign}(H_1) + J_1 - \frac{V_r V_\lambda}{r} \right. \\ &\quad \left. - A_M \cos(q - \varphi_M) \right) = -\lambda_1 |H_1|^{1/2} \text{sign}(H_1) - J_1 \\ &\quad + A_{T\lambda}. \end{aligned} \quad (23)$$

Let $\varphi_1 = -J_1 + A_{T\lambda}$. Differentiating φ_1 gives

$$\dot{\varphi}_1 = -\dot{J}_1 + \dot{A}_{T\lambda}. \quad (24)$$

Substituting the expression of \dot{J}_1 in (20) into (24) yields

$$\dot{\varphi}_1 = -k_1 \text{sign}(H_1) + \dot{A}_{T\lambda} \quad (25)$$

Combining (23) and (25), we have

$$\begin{aligned} \dot{H}_1 &= -\lambda_1 |H_1|^{1/2} \text{sign}(H_1) + \varphi_1, \\ \dot{\varphi}_1 &= -k_1 \text{sign}(H_1) + \dot{A}_{T\lambda}, \end{aligned} \quad (26)$$

where

$$\lambda_1 = 2\sqrt{l_1} \quad (27)$$

$$k_1 = 4l_1$$

$$l_1 = \begin{cases} \bar{q}_1, & \text{if } H_1 \neq 0 \\ 0, & \text{otherwise.} \end{cases} \quad (28)$$

It is not difficult to note that (26), (27), and (28) have the same structure as (16), (17), and (18) in Lemma 3. Moreover, since Assumption 1 is valid, condition (19) can be satisfied. Then, according to Lemma 3, the following equations can be satisfied:

$$\begin{aligned} H_1 &= 0, \\ \dot{H}_1 &= 0, \\ \forall t &\geq t_1, \end{aligned} \quad (29)$$

where t_1 is a finite time.

Substituting (29) into (23), we have

$$A_{T\lambda} = J_1, \quad \forall t \geq t_1. \quad (30)$$

Then it is clear that the estimation error $H_2 = J_1 - A_{T\lambda}$ will converge to zero in finite time t_1 . The demonstration of Theorem 4 is completed. \square

In order to implement the AFDO (20), the following assumption is needed.

Assumption 5. $r, q, \dot{r}, \dot{q}, A_M$, and φ_M are measurable.

Remark 6. Note that the third formula J_1 in (20) is most important. Theorem 4 shows that J_1 can estimate the target acceleration $A_{T\lambda}$. Unlike Z_2 in ESO (11), which only can estimate the target acceleration with the estimation error E_2 , J_1 can fully estimate the target acceleration in finite time. And like ESO (11), AFDO (20) do not need the upper bound of $A_{T\lambda}$ and $\dot{A}_{T\lambda}$, which only need $A_{T\lambda}$ and $\dot{A}_{T\lambda}$ to be bounded.

3.2. Guidance Law Design and Stability Analysis. After estimating the target acceleration with AFDO, a novel FTCLG law is designed as follows:

$$\begin{aligned} A_M &= \frac{1}{\cos(q - \varphi_M)} \left(\tau_1 S + \sigma_1 |S|^{\gamma_1} \text{sign}(S) - \frac{V_r V_\lambda}{r} \right. \\ &\quad \left. - \frac{(c_0/2) V_r}{\sqrt{r}} + J_1 \right), \end{aligned} \quad (31)$$

where J_1 is given by the AFDO (20). $\tau_1 > 0$, $\sigma_1 > 0$, and $0 < \gamma_1 < 1$. Then Theorem 7 will prove the finite-time-convergent feature of the closed-loop system under the AFDO-based guidance law (31).

Theorem 7. Consider the guidance system (2) adopts AFDO-based guidance law (31). If Assumption 1 is valid, then the trajectory of system (2) can be driven onto the sliding surface ($S = 0$) in finite time.

Proof. From (2) and (4), we have

$$\begin{aligned} \dot{S} &= \dot{V}_\lambda - \frac{(c_0/2) V_r}{\sqrt{r}} \\ &= -\frac{V_r V_\lambda}{r} + A_{T\lambda} - A_M \cos(q - \varphi_M) - \frac{(c_0/2) V_r}{\sqrt{r}}. \end{aligned} \quad (32)$$

Substituting the proposed AFDO-based guidance law (31) into (32), we have

$$\dot{S} = -\tau_1 S - \sigma_1 |S|^{\gamma_1} \text{sign}(S) + A_{T\lambda} - J_1. \quad (33)$$

Construct Lyapunov function V_2 as

$$V_2 = \frac{1}{2} S^2. \quad (34)$$

Then calculating the time derivative of V_2 along the trajectories of (33), we get

$$\begin{aligned} \dot{V}_2 &= S\dot{S} = S(-\tau_1 S - \sigma_1 |S|^{\gamma_1} \text{sign}(S) - J_1 + A_{T\lambda}) \\ &= -\tau_1 S^2 - \sigma_1 |S|^{\gamma_1+1} - (J_1 - A_{T\lambda}) S \\ &\leq -2^{(\gamma_1+1)/2} \sigma_1 V_2^{(\gamma_1+1)/2} - (J_1 - A_{T\lambda}) S \\ &\leq -2^{(\gamma_1+1)/2} \sigma_1 V_2^{(\gamma_1+1)/2} + \sqrt{2} |J_1 - A_{T\lambda}| V_2^{1/2}. \end{aligned} \quad (35)$$

From (35), it denotes that V_2 is affected by the estimation error $H_2 = J_1 - A_{T\lambda}$. Thus, in the following, the proof of Theorem 7 consists of two steps. In the first step, it will be proved that V_2 will not escape to infinity before $(J_1 - A_{T\lambda})$ converges to zero. In the second step, it will be proved that V_2 will converge to zero in finite time after $(J_1 - A_{T\lambda})$ converges to zero. And the total convergence time of V_2 will be calculated.

Step 1. From (25) and Assumption 1, we have

$$|\dot{\varphi}_1| \leq k_1 + |\dot{A}_{T\lambda}| \leq k_1 + \dot{A}_{T\lambda}^{\max}. \quad (36)$$

From the expression of k_1 in (20), (36) can be rewritten as

$$|\dot{\varphi}_1| \leq 4l_1 + \dot{A}_{T\lambda}^{\max}. \quad (37)$$

Then considering (21) and (37), it can be deduced that $|\dot{\varphi}_1|$ is bounded by an unknown positive constant for $t \leq t_1$:

$$|\dot{\varphi}_1| \leq 4(\bar{q}_1(t_1 - t_0) + l_1(t_0)) + \dot{A}_{T\lambda}^{\max}, \quad \forall t \leq t_1. \quad (38)$$

From (38), we have

$$\begin{aligned}
|\varphi_1| &= \left| \int_{t_0}^t \dot{\varphi}_1 dt + \varphi_1(t_0) \right| \leq \left| \int_{t_0}^t \dot{\varphi}_1 dt \right| + |\varphi_1(t_0)| \\
&\leq \int_{t_0}^t |\dot{\varphi}_1| dt + |\varphi_1(t_0)| \\
&\leq \left(4(\bar{q}_1(t_1 - t_0) + l_1(t_0)) + \dot{A}_{T\lambda}^{\max} \right) (t - t_0) \\
&\quad + |\varphi_1(t_0)| \\
&\leq \left(4(\bar{q}_1(t_1 - t_0) + l_1(t_0)) + \dot{A}_{T\lambda}^{\max} \right) (t_1 - t_0) \\
&\quad + |J_1(t_0)| + |A_{T\lambda}(t_0)|, \quad \forall t \leq t_1.
\end{aligned} \tag{39}$$

From (39), it is clear that φ_1 is bounded by unknown positive constant \bar{c}_2 in finite time t_1 :

$$|\varphi_1| = |-J_1 + A_{T\lambda}| = |J_1 - A_{T\lambda}| \leq \bar{c}_2, \quad \forall t \leq t_1, \tag{40}$$

where $\bar{c}_2 = (4(\bar{q}_1(t_1 - t_0) + l_1(t_0)) + \dot{A}_{T\lambda}^{\max})(t_1 - t_0) + |J_1(t_0)| + |A_{T\lambda}(t_0)|$ and t_1 is the convergence time of AFDO (20) and given in Theorem 4.

Combining (40) with (35), we have

$$\begin{aligned}
\dot{V}_2 &\leq -2^{(\gamma_1+1)/2} \sigma_1 V_2^{(\gamma_1+1)/2} + \sqrt{2} |J_1 - A_{T\lambda}| V_2^{1/2} \\
&\leq \left(-2^{(\gamma_1+1)/2} \sigma_1 V_2^{\gamma_1/2} + \sqrt{2} \bar{c}_2 \right) V_2^{1/2}, \quad \forall t \leq t_1.
\end{aligned} \tag{41}$$

From (41), it is clear that $\dot{V}_2 \leq 0$ if $V_2 \geq (\bar{c}_2/\sigma_1 2^{\gamma_1/2})^{2/\gamma_1}$. Thus, it can be known that V_2 is bounded in finite time t_1 :

$$V_2 \leq \max \left\{ \left(\frac{\bar{c}_2}{\sigma_1 2^{\gamma_1/2}} \right)^{2/\gamma_1}, V_2(t_0) \right\}, \quad \forall t \leq t_1. \tag{42}$$

Thus V_2 will not escape to infinity before $(A_{T\lambda} + J_1)$ converges to zero; that is, $V_2(t_1)$ is bounded.

Step 2. Since Assumption 1 is valid, (30) will be satisfied in finite time t_1 . Then combining (30) with (35), we have

$$\dot{V}_2 \leq -2^{(\gamma_1+1)/2} \sigma_1 V_2^{(\gamma_1+1)/2}, \quad \forall t \geq t_1. \tag{43}$$

$V_2(t_1)$ is bounded and has been proved in Step 1. As $V_2(t_1)$ is bounded, $\sigma_1 > 0$ and $0 < \gamma_1 < 1$, and V_2 and S will converge to zero in finite time t_2 based on Lemma 2:

$$S = V_2 = 0, \quad \forall t \geq t_2. \tag{44}$$

The convergence time t_2 satisfies the following equation:

$$t_2 \leq \frac{V_2^{(1-\gamma_1)/2}(t_1)}{2^{(1-\gamma_1)/2} \sigma_1 (1-\gamma_1)} + t_1. \tag{45}$$

The demonstration of Theorem 7 is completed. \square

Remark 8. From the result of Theorem 7, it is clear that the problem that ESOTFCG law (10) cannot strictly guarantee S converge to zero is solved by the proposed AFDO-based FTFCG law.

Remark 9. From Remark 4.3 in [19], it can be known that the boundary layer of the sliding surface in [19] is determined by the estimation error of the ESO. Thus, the parameter selection of the ESO is more important since it not only determines the performance of the ESO but also impacts the behavior of the sliding surface. However, in this paper, the estimation error of AFDO will converge to zero in finite time as soon as the parameters satisfy $l_1(t_0) > 0$ and $\bar{q}_1 > 0$. Thus, the parameter selection in this paper is much simpler.

Remark 10. From [21], if the final miss distance is less than 0.25 m, the hit-to-kill interception also can be satisfied.

Remark 11. Condition $H_1 = 0$ is difficult to be satisfied in practice due to numerical approximations and measurement noise. From [25], it can be known that the condition $H_1 = 0$ can be modified by the following dead-zone technique:

$$\dot{i}_1 = \begin{cases} \bar{q}_1, & \text{if } |H_1| \geq \eta, \\ 0, & \text{otherwise,} \end{cases} \tag{46}$$

where η is a sufficiently small positive value.

4. Simulation Results

This subsection shows the performances of the AFDO and the proposed AFDO-based guidance law. The initial positions of the missile are $x_M(t_0) = 0$ and $y_M(t_0) = 0$. The initial positions of the target are $x_T(t_0) = 2000$ m and $y_T(t_0) = 2000$ m. The initial path angles are $\varphi_M(t_0) = \pi/4$ rad and $\varphi_T(t_0) = \pi/3.8$ rad. Seeker measurement delays for 30 ms. In addition, the maximum limit of the missile acceleration command is selected as 200 m/s².

For the comparison, the ESOTFCG law (10) given in [19] are also considered in this section. The parameters of ESO (8) are chosen as $\beta_{01} = 50$, $\beta_{02} = 100$, $\alpha_1 = 0.2$, and $\delta = 0.15$. The parameters of ESOTFCG law (10) are chosen as $\tau = 10$, $\sigma = 1$, and $\gamma = 0.5$. Note that, in this paper, the parameters of ESOTFCG law and ESO are the same as those in [19] and used here to ensure the fairness of comparison.

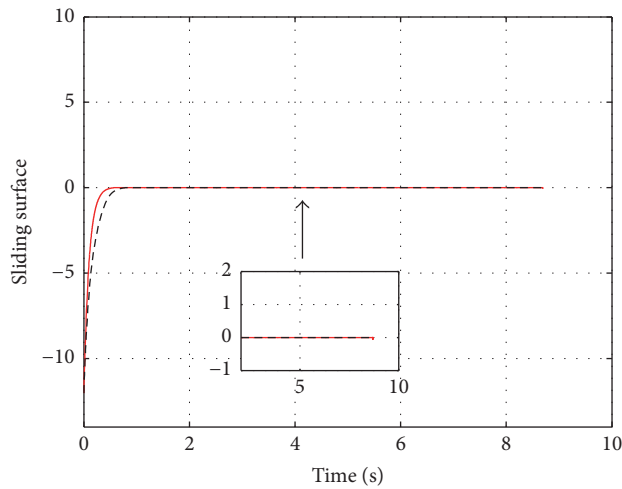
The parameters of AFDO (20) are chosen as $\eta = 0.0005$, $l_1(t_0) = 5$, and $\bar{q}_1 = 1000$. The parameters of the proposed law (31) are chosen as $\tau_1 = 10$, $\sigma_1 = 1$, and $\gamma_1 = 0.5$.

Like [19], the parameter of the sliding surface S in the ESOTFCG law and the proposed law is selected as $c_0 = 0.1$.

Case 1 (constant target acceleration). The target acceleration $A_{T\lambda}$ is given as

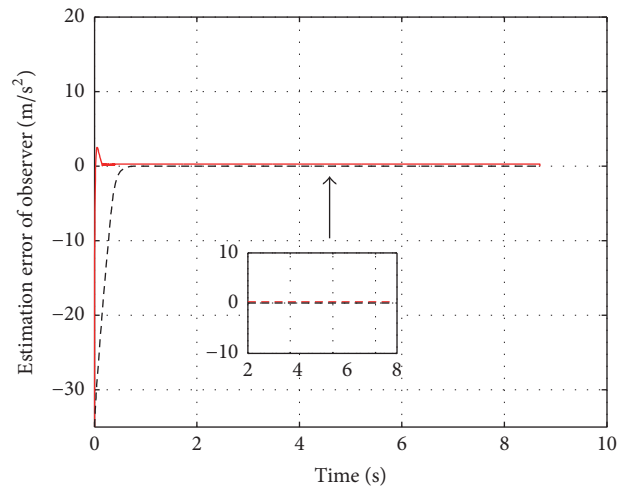
$$A_{T\lambda} = -35 \text{ m/s}^2. \tag{47}$$

From (47), it can be known that the target acceleration is constant in Case 1. Figures 2(a)–2(d) and Table 1 show the simulation results for Case 1. From Figure 2(a), it is clear that the proposed law and the ESOTFCG law can guarantee the sliding surface converges to zero in finite time. Figure 2(b) shows that AFDO and ESO ensure the estimation error converges to zero. From Figure 2(c) and Table 1, it can be



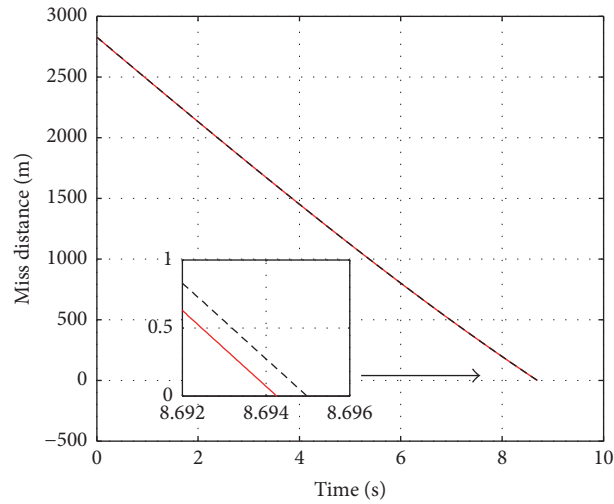
— Proposed law
 - - - ESOFTCG law

(a) Sliding surface



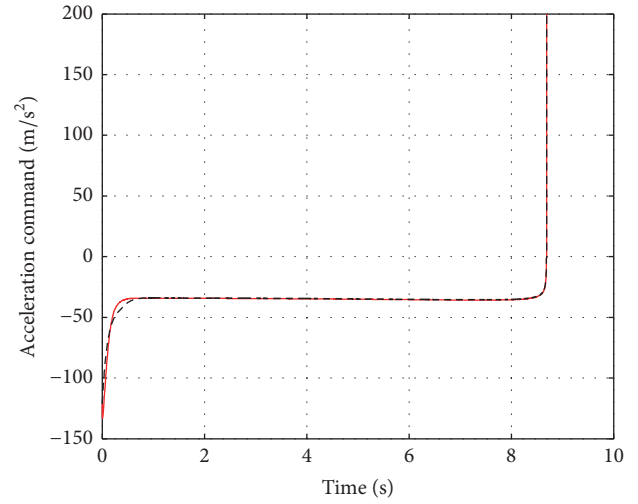
— AFDO
 - - - ESO

(b) Estimation error of observer



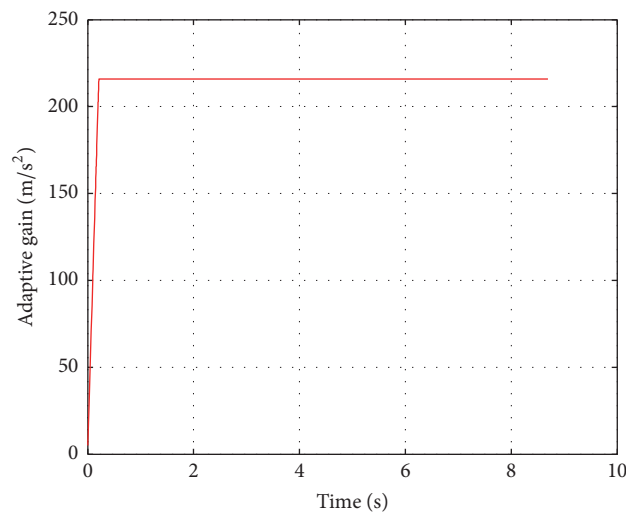
— Proposed law
 - - - ESOFTCG law

(c) Time history of miss distance



— Proposed law
 - - - ESOFTCG law

(d) Acceleration command



(e) Adaptive gain l_1

FIGURE 2: Simulation result for Case 1 (constant target acceleration).

TABLE 1: Performance of guidance laws in Section 4.

Case	Guidance law	Final miss distance (m)
1	Proposed law	0.033
	ESOFTECG law	0.054
2	Proposed law	0.079
	ESOFTECG law	0.237
3	Proposed law	0.087
	ESOFTECG law	3.23

known that the proposed law and ESOFTECG law guarantee the final miss distances are less than 0.1m, which means that the proposed law and ESOFTECG law can guarantee the missile accurately attacks the target (see Remark 10). From Figure 2(d), it is clear that the acceleration commands of the proposed law and ESOFTECG law are chatter-free. Thus the proposed method and method in [19] exhibit good performance in the presence of constant target acceleration.

Case 2 (slowly varying target acceleration). The target acceleration $A_{T\lambda}$ with a frequency $1/(6\pi)$ HZ is given as

$$A_{T\lambda} = 175 \cos\left(\frac{t}{3}\right) \text{ m/s}^2 \quad (48)$$

From (48), it can be known that the target acceleration is time-varying in Case 2. But the varying rate in Case 2 is small. Figures 3(a)–3(d) and Table 1 show the simulation results for Case 2. From Figure 3(a), it is clear that the proposed law guarantees the sliding surface converges to zero, but ESOFTECG law only can guarantee the sliding surface converges to a neighbourhood of zero. The reason for Figure 3(a) is that ESO cannot guarantee the estimation error converges to zero when the target acceleration is time-varying, while AFDO can guarantee the estimation error of AFDO converges to zero. The reason can be observed from Figure 3(b). Figure 3(c) and Table 1 show that that the proposed law and ESOFTECG law guarantee the final miss distances are less than 0.25 m, which means that the proposed law and ESOFTECG law can accomplish hit-to-kill interception (see Remark 10). And, from Table 1, it also can be known that the proposed guidance law can achieve a smaller final miss distance.

Case 3 (fast varying target acceleration). The target acceleration $A_{T\lambda}$ with a frequency $2/\pi$ HZ is given as

$$A_{T\lambda} = 175 \cos(4t) \text{ m/s}^2. \quad (49)$$

From (49), it can be known that the varying rate of target acceleration in Case 3 is much larger than that in Case 2. It is noted that the method in [19] does not consider the target acceleration of this type. Figures 4(a)–4(d) and Table 1 show the simulation results for Case 3. From Figure 4(a), it is clear that the proposed law can guarantee the sliding surface converges to zero in finite time. But the ESOFTECG law cannot guarantee the sliding surface converges to zero. From Figure 4(b), it is clear that the AFDO can fully estimate

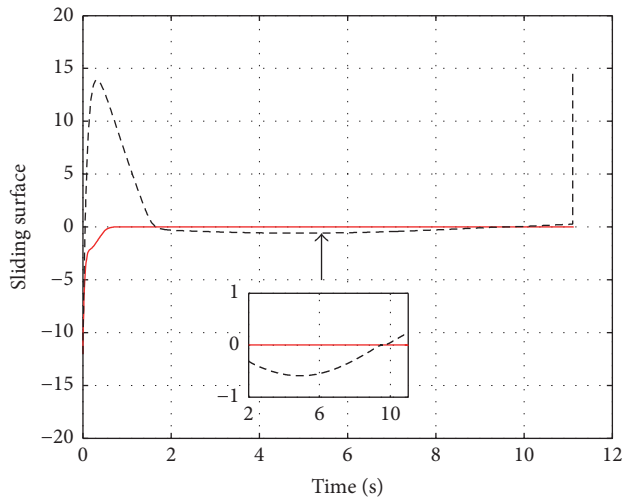
the target acceleration, but the estimation error of ESO is very large. From Figure 4(c) and Table 1, it is clear that the proposed guidance law still can guarantee the final miss distance is less than 0.1 m. Thus, the missile with the proposed law can accomplish hit-to-kill interception, while the final miss distance of ESOFTECG law is 3.2 m, which means that the missile with ESOFTECG law cannot accomplish the hit-to-kill interception (see Remark 10). The reason for Figures 4(a), 4(b), and 4(c) is that the estimation error of ESO is large when target acceleration is fast varying. And then the finite-time convergent feature of ESOFTECG law is destroyed by the large estimation error (the relationship between sliding surface, estimation error of ESO, and varying rate of target acceleration is shown in (13)). And the hit-to-kill guidance strategy cannot be accomplished by ESOFTECG law. Since AFDO can fully estimate target acceleration, the proposed law can still accomplish hit-to-kill interception in the present of the fast varying target acceleration.

According to the simulation results, the following can be concluded:

- (1) AFDO can achieve a good estimation effect on the condition of the target acceleration instances with either low or high varying rate (Figures 2–4). But ESO can only have a good estimation effect on the condition of constant acceleration (Figure 2). If the target acceleration is varying, the estimation error of ESO will increase with the increase of the varying rate of target acceleration (Figures 3 and 4).
- (2) The proposed guidance law can strictly guarantee the sliding surface converges to zero in finite time when the target acceleration is constant or time-varying (Figures 2–4). But ESOFTECG law only can guarantee the sliding surface converges to a neighbourhood of zero if the target acceleration is time-varying (Figures 3 and 4). In particular, if the target acceleration is fast varying, the estimation error of ESO is very large (Figure 4). Then the finite-time convergent feature of ESOFTECG law will be destroyed by the large estimation error of ESO (Figure 4).
- (3) Unlike the CFTECG law, the proposed law does not need the upper bound of target acceleration.
- (4) Since the target acceleration has been fully estimated by AFDO, the proposed guidance law has no discontinuous control term. Thus the chattering problem in CFTECG law is solved (Figures 2(d), 3(d), and 4(d)).

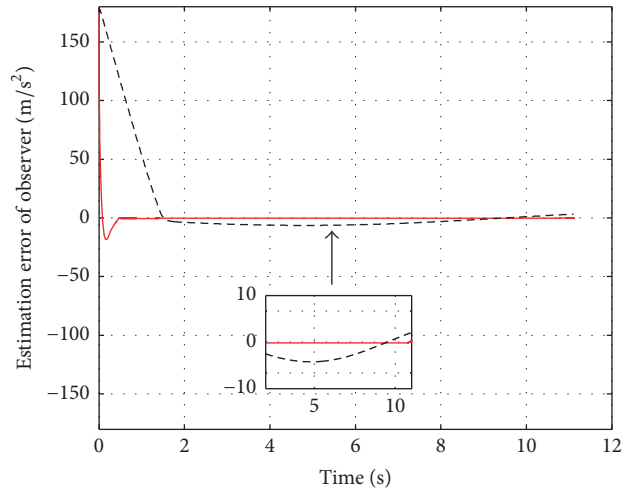
5. Conclusion

- (1) In this paper, a novel adaptive finite-time disturbance observer (AFDO) based on ASTW algorithm was proposed, which does not need to know the upper bound of the target acceleration in advance. Moreover, the estimation error of AFDO strictly converges to zero in finite time even if the target acceleration is time-varying.



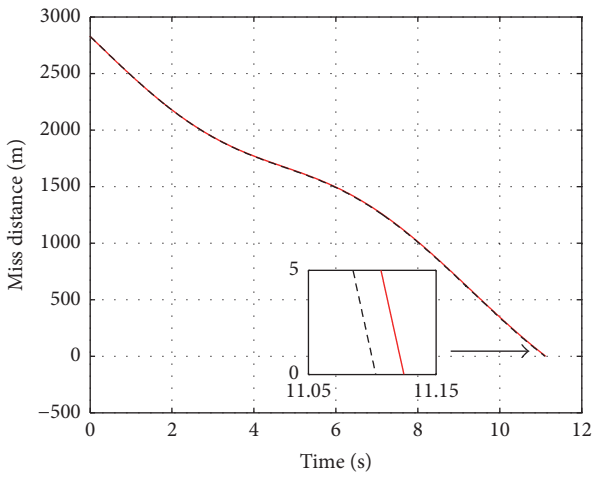
— Proposed law
 - - - ESOFTCG law

(a) Sliding surface



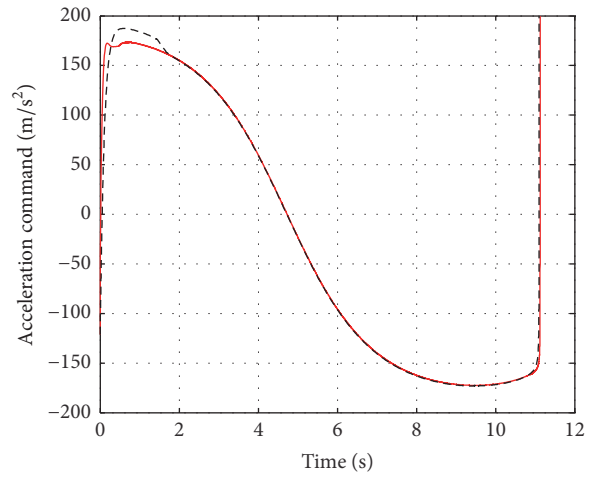
— AFDO
 - - - ESO

(b) Estimation error of observer



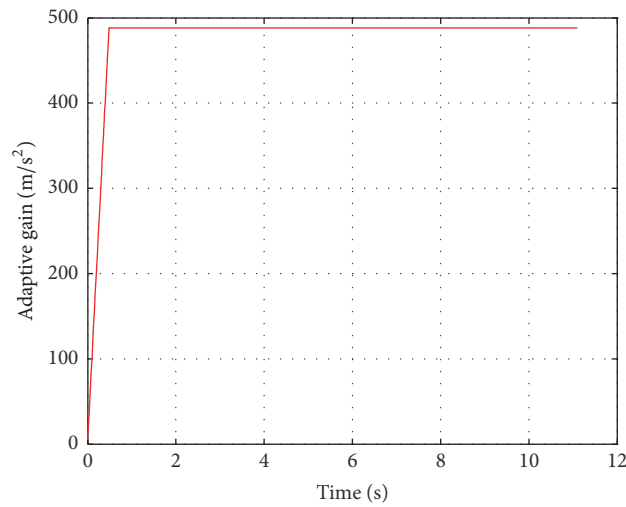
— Proposed law
 - - - ESOFTCG law

(c) Time history of miss distance



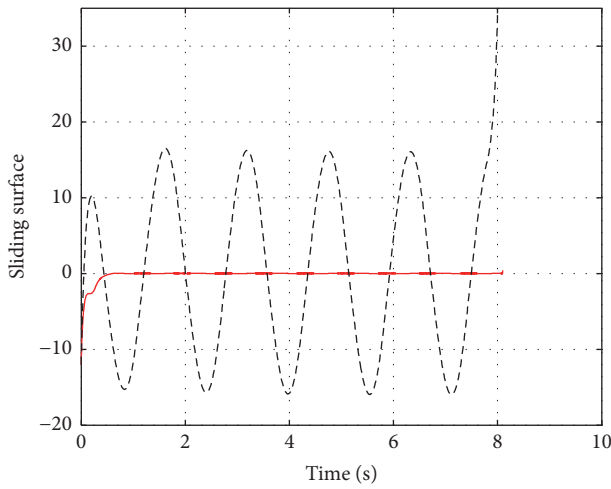
— Proposed law
 - - - ESOFTCG law

(d) Acceleration command



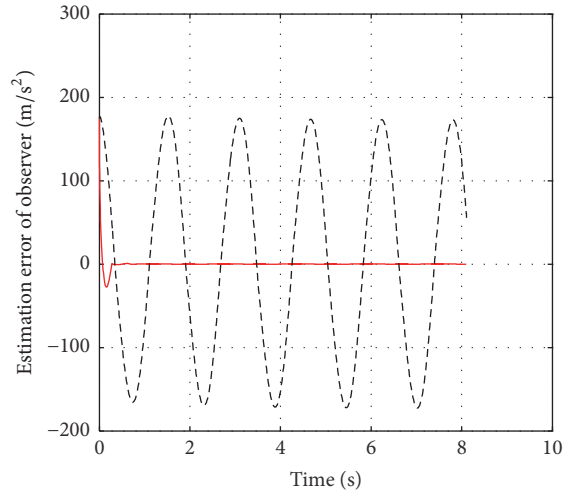
(e) Adaptive gain I_1

FIGURE 3: Simulation result for Case 2 (slowly varying target acceleration).



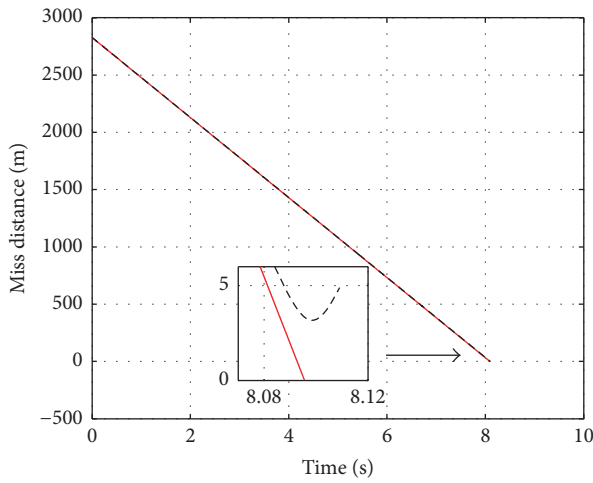
— Proposed law
 - - - ESOFTCG law

(a) Sliding surface



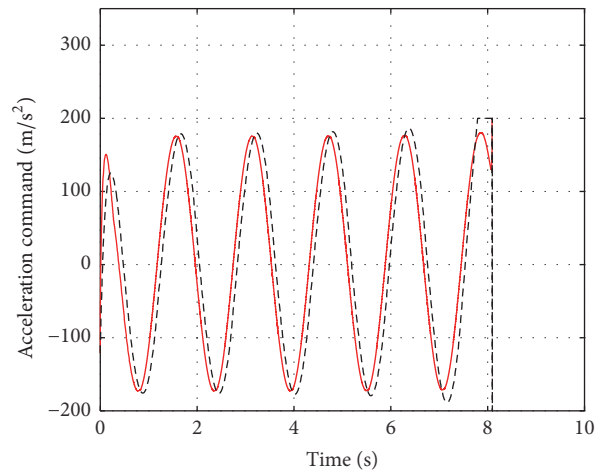
— AFDO
 - - - ESO

(b) Estimation error of observer



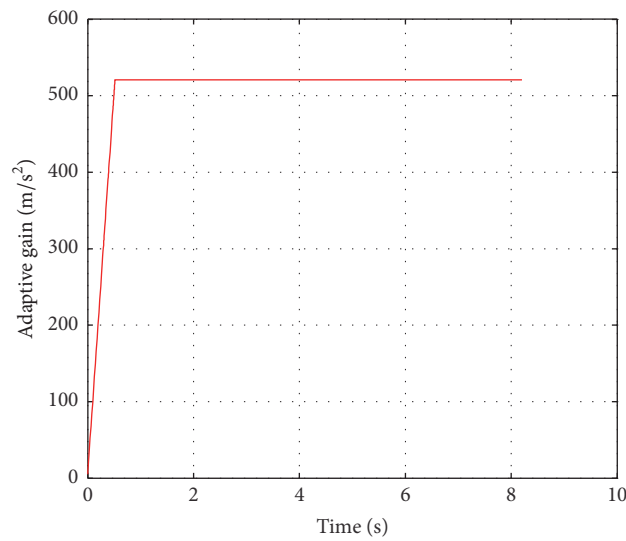
— Proposed law
 - - - ESOFTCG law

(c) Time history of miss distance



— Proposed law
 - - - ESOFTCG law

(d) Acceleration command



(e) Adaptive gain I_1

FIGURE 4: Simulation result for Case 3 (fast varying target acceleration).

- (2) Subsequently, a novel FTCL law based on AFDO was proposed. The newly proposed FTCL law has several advantages over existing FTCL laws. First, for the time-varying target acceleration, the proposed guidance law can strictly drive the trajectory of the closed-loop system onto the sliding-mode surface rather than a neighbourhood of sliding-mode surface in the ESFTCL law. Second, unlike the CFTCL law, the proposed method requires no information on the target acceleration. Third, the chattering problem in the CFTCL law is completely avoided in this paper.
- (3) Finally, mathematical simulation result demonstrated that the performances of the AFDO and the proposed guidance law are excellent.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (61304238), the National High-Tech R&D Program of China (2012AA120602, 2015AAxxx7033, and 2015AAxxx1008), and the Chinese Aerospace Supported Fund (2015-HT-XGD).

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