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Research Article

Further Stability Analysis of Time-Delay Systems with Nonlinear Perturbations

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In this study, we focus on stability analysis for systems with time-varying delay and nonlinear perturbations. In order to cut down the conservatism of the existing stability criteria, we utilize the triple integral forms of Lyapunov-Krasovskii functional (LKF). In addition, by using single and double integral forms of Wirtinger-based inequality, we overcome some conservatism which come from Jensen's inequality. Three well-known numerical examples are given at the end. Compared with some existing results, our results have less conservatism.

1. Introduction

Time-delay has attracted a lot of interests because it is widely encountered in communication systems, neural networks, economic systems, biological systems, and networked control systems [1–6]. Since time-delay will cause serious degradation of system performance, a lot of researchers are making considerable effort for stability analysis of time-delay systems in the last few decades. The LKF approach and LMI method are the efficient instrument to get the delay-dependent stability criterion. However, only a sufficient condition we can get by utilizing these methods. Therefore, we focus on cutting down the conservatism of stability criterion. As we know, the maximum allowable delay bound (MADB) of the time-delay can measure the conservatism of stability criterion. We can get the larger MADB of time-varying delay according to the stability criterion with less conservatism.

In a lot of existing results, Jensen's inequality has played an important role. However, it will induce some conservatism hard to overcome. To cut down the conservatism, Wirtinger-based integral inequality [7], which can be used to obtain much tighter lower bound of single integral terms, was proposed. Very recently, based on Wirtinger-based integral inequality, a Wirtinger-based double integral inequality was proposed to get much tighter lower bound of double integral terms [8].

In a lot of recent literatures [9–11], researchers only utilize single and double integral forms of LKF to derive delay-dependent stability criterion. We believe that triple integral form of LKF is helpful for improving the performance of former criteria.

Motivated by the previous discussions, we are concerned about the stability analysis for the time-varying delay systems with nonlinear perturbations. We introduce the triple integral forms of LKF to cut down the conservatism. Taking the time derivative of $\int_{t-h}^{t}\int_{s}^{t}\int_{u}^{t}\dot{x}^T(v)R_i\dot{x}(v)dv\,du\,ds$, we obtain $-\int_{t-h}^{t}\int_{s}^{t}\dot{x}^T(u)R_i\dot{x}(u)du\,ds$. Instead of Jensen's inequality, Wirtinger-based double integral inequality was utilized to find much tighter upper bound of $-\int_{a}^{b}\int_{u}^{b}\dot{x}^T(u)R_i\dot{x}(u)du\,ds$. Compared with some existing results, our results have less conservatism.

The organization of this paper is as follows. The system is presented in Section 2, and some useful lemmas are given. Then the new stability criterion and proof are given in Section 3. In Section 4, the effectiveness of our results can be illustrated by three examples. Section 5 is the conclusion of our investigation.

Notations. In this paper, the superscript T means the transpose of a matrix; col$\{\cdot\}$ denotes the column vector.

2. Problem Formulation and Preliminaries

The time-varying delay systems with nonlinear perturbations are considered as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bx(t-\tau(t)) + \vartheta(t), \\ \vartheta(t) &= h(x(t),t) + p(x(t-\tau(t)),t), \\ x(t) &= \phi(t), \quad t \in [-d_M, 0] \end{aligned} \tag{1}$$

with $x(t) \in \mathbb{R}^n$ as the state variate; $A, B \in \mathbb{R}^{n*n}$ are predefined constant matrices; $d(t) \geq 0$ represent a time-varying delay satisfying

$$d(t) \in [d_m, d_M], \quad \dot{d}(t) \leq d_d. \tag{2}$$

The nonlinear perturbations $h(x(t),t) \in \mathbb{R}^n$ and $p(x(t-d(t)),t) \in \mathbb{R}^n$ can be abbreviated as h and p, assumed as follows:

$$\begin{aligned} h^T h &\leq \alpha^2 x^T(t) C^T C x(t), \\ p^T p &\leq \beta^2 x^T(t-d(t)) D^T D x(t-d(t)), \end{aligned} \tag{3}$$

where α and β are positive scalars and $C, D \in \mathbb{R}^{n*n}$ are constant matrices.

In order to derive improved stability criterion, the following lemmas will be used.

Lemma 1 (see [7]). *Z is a symmetric positive definite matrix, for differentiable function $x \in [d_1, d_2] \rightarrow \mathbb{R}^n$, and we can obtain*

$$\int_{d_1}^{d_2} \dot{x}^T(s) Z \dot{x}(s)\, ds \geq \frac{1}{d_2 - d_1} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}^T \begin{bmatrix} Z & 0 \\ * & Z \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix}, \tag{4}$$

where

$$\begin{aligned} \varepsilon_1 &= x(d_1) - x(d_2), \\ \varepsilon_2 &= \sqrt{3}x(d_1) + \sqrt{3}x(d_2) - \frac{2\sqrt{3}}{d_2 - d_1} \int_{d_1}^{d_2} x(s)\, ds. \end{aligned} \tag{5}$$

Lemma 2 (see [28]). *For positive definite $d_v : \mathbb{R}^n \rightarrow \mathbb{R}$, $d_v \in \mathcal{G} \subseteq \mathbb{R}^n$ ($v = 1 \cdots N$), reciprocally convex combination of d_v can be written as*

$$\begin{gathered} \min_{\{\rho_m | \rho_m > 0, \sum_m \rho_m = 1\}} \sum_m \frac{1}{\rho_m} d_m(t) = \sum_m d_m(t) + \max_{l_{m,w}(t)} \sum_{m \neq w} l_{m,w}(t) \\ \text{subject to} \quad \left\{ l_{m,w} : \mathbb{R}^n \longrightarrow \mathbb{R},\ l_{w,m}(t) \triangleq l_{m,w}(t),\ \begin{bmatrix} d_m(t) & l_{m,w}(t) \\ l_{m,w}(t) & d_w(t) \end{bmatrix} \geq 0 \right\}. \end{gathered} \tag{6}$$

Lemma 3 (see [8]). *R is a symmetric positive definite matrix, for differentiable function $x \in [h_a, h_b] \rightarrow \mathbb{R}^n$, and we have*

$$\begin{aligned} &\frac{(h_b - h_a)^2}{2} \int_{h_a}^{h_b} \int_s^{h_b} \dot{x}^T(u) R \dot{x}(u)\, du\, ds \\ &\quad \geq \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}^T \begin{bmatrix} R & 0 \\ * & R \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \omega_1 &= (h_b - h_a) x(h_b) - \int_{h_a}^{h_b} x(s)\, ds, \\ \omega_2 &= \frac{\sqrt{2}(h_b - h_a)}{2} x(h_b) + \sqrt{2} \int_{h_a}^{h_b} x(s)\, ds \\ &\quad - \frac{3\sqrt{2}}{h_b - h_a} \int_{h_a}^{h_b} \int_s^{h_b} x(u)\, du\, ds. \end{aligned} \tag{8}$$

3. Stability Criterion

Theorem 4. *System (1) satisfying (2)-(3) will be asymptotically stable if there exist given scalars d_m, d_M, and d_d and $\epsilon_1 > 0$ and $\epsilon_2 > 0$ and positive symmetric matrices $P \in \mathbb{R}^{5n*5n}$, $W_s \in \mathbb{R}^{n*n}$ ($s = 1, 2, 3$), and $Z_q \in \mathbb{R}^{n*n}$ ($q = 1, \ldots, 4$) and appropriate dimensions matrices S_m, satisfying the following LMIs:*

$$\begin{gathered} \begin{bmatrix} \Phi & \Delta^T N \\ * & -N \end{bmatrix} < 0, \\ \begin{bmatrix} Z_2 & S_m \\ * & Z_2 \end{bmatrix} \geq 0, \quad (m = 1, 2), \end{gathered} \tag{9}$$

where

$$\begin{aligned}
\Phi &= G_1^T P G_2 + G_2^T P G_1 + Q + M_1 + M_2 - d_m^2 \sum_{i=1}^{2} \Gamma_i^T Z_1 \Gamma_i - \sum_{i=3}^{6} \Gamma_i^T Z_2 \Gamma_i - \Gamma_3^T S_1 \Gamma_5 - \Gamma_5^T S_1^T \Gamma_3 - \Gamma_4^T S_2 \Gamma_6 - \Gamma_6^T S_2^T \Gamma_4 \\
&\quad - d_m^2 \sum_{i=7}^{8} \Gamma_i^T Z_3 \Gamma_i - d_{Mm}^2 \sum_{i=9}^{10} \Gamma_i^T Z_4 \Gamma_i, \\
G_1 &= \operatorname{col}\{e_1, e_5, e_6, e_7, e_8\}, \\
G_2 &= \operatorname{col}\{Ae_1 + Be_3 + e_{11} + e_{12}, e_1 - e_2, e_2 - e_4, d_m e_1 - e_5, d_{Mm} e_2 - e_6\}, \\
W &= \operatorname{diag}\{W_1 + W_2 + W_3 \;\; -W_2 \;\; -(1-d_d) W_1 \;\; -W_3 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\}, \\
M_1 &= \operatorname{diag}\left\{\epsilon_1 \alpha^2 C^T C \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -\epsilon_1 I \;\; 0\right\}, \\
M_2 &= \operatorname{diag}\left\{0 \;\; 0 \;\; \epsilon_2 \beta^2 D^T D \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; -\epsilon_2 I\right\}, \\
\Gamma_1 &= e_1 - e_2, \\
\Gamma_2 &= \sqrt{3} e_1 + \sqrt{3} e_2 - \frac{2\sqrt{3}}{d_m} e_5, \\
\Gamma_3 &= e_2 - e_3, \\
\Gamma_4 &= \sqrt{3} e_2 + \sqrt{3} e_3 - 2\sqrt{3} e_9, \\
\Gamma_5 &= e_3 - e_4, \\
\Gamma_6 &= \sqrt{3} e_3 + \sqrt{3} e_4 - 2\sqrt{3} e_{10}, \\
\Gamma_7 &= d_m e_1 - e_5, \\
\Gamma_8 &= \frac{\sqrt{2}}{2} d_m e_1 + \sqrt{2} e_5 - \frac{3\sqrt{2}}{d_m} e_7, \\
\Gamma_9 &= d_{Mm} e_2 - e_6, \\
\Gamma_{10} &= \frac{\sqrt{2} d_{Mm}}{2} e_2 + \sqrt{2} e_6 - \frac{3\sqrt{2}}{d_{Mm}} e_8, \\
\Delta &= Ae_1 + Be_3 + e_{11} + e_{12}, \\
d_{Mm} &= d_M - d_m, \\
e_i &= \left[0_{n\times(k-1)n} \;\; I \;\; 0_{n\times(12-k)n}\right], \quad k = 1, 2, \ldots, 12 \\
N &= N_1 + N_2 = d_m^4 Z_1 + d_{Mm}^2 Z_2 + \frac{d_m^6}{4} Z_3 + \frac{d_{Mm}^6}{4} Z_4.
\end{aligned} \tag{10}$$

Proof. Construct the following LKF:

$$V(t) = \sum_{i=1}^{4} V_i(t), \tag{11}$$

where

$$V_1(t) = \nu^T(t) P \nu(t),$$

$$\begin{aligned}
V_2(t) &= \int_{t-d(t)}^{t} x^T(s) W_1 x(s)\, ds + \int_{t-d_m}^{t} x^T(s) W_2 x(s)\, ds \\
&\quad + \int_{t-d_M}^{t} x^T(s) W_3 x(s)\, ds,
\end{aligned}$$

$$
\begin{aligned}
&V_3(t) \\
&= d_m^3 \int_{-d_m}^{0} \int_{t+s}^{t} \dot{x}^T(u) Z_1 \dot{x}(u)\, du\, ds \\
&+ d_{Mm} \int_{-d_M}^{-d_m} \int_{t+s}^{t} \dot{x}^T(u) Z_2 \dot{x}(u)\, du\, ds, \\
&V_4(t) \\
&= \frac{d_m^4}{2} \int_{t-d_m}^{t} \int_{s}^{t} \int_{u}^{t} \dot{x}^T(v) Z_3 \dot{x}(v)\, dv\, du\, ds \\
&+ \frac{d_{Mm}^4}{2} \int_{t-d_M}^{t-d_m} \int_{s}^{t-d_m} \int_{u}^{t} \dot{x}^T(v) Z_4 \dot{x}(v)\, dv\, du\, ds, \\
&u_1(t) = \int_{t-d_m}^{t} x^T(s)\, ds, \\
&u_2(t) = \int_{t-d_M}^{t-d_m} x^T(s)\, ds, \\
&u_3(t) = \int_{t-d_m}^{t} \int_{s}^{t} x^T(u)\, du\, ds, \\
&u_4(t) = \int_{t-d_M}^{t-d_m} \int_{s}^{t-d_m} x^T(u)\, du\, ds, \\
&\nu^T(t) = \left[x^T(t) \;\; u_1(t) \;\; u_2(t) \;\; u_3(t) \;\; u_4(t) \right].
\end{aligned} \tag{12}
$$

The time derivative of $V(t)$ is as follows:

$$
\dot{V}(t) = \sum_{i=1}^{4} \dot{V}_i(t), \tag{13}
$$

where

$$
\begin{aligned}
&\dot{V}_1(t) = 2\nu^T(t) P \dot{\nu}(t) = \xi^T(t) \left(G_1^T P G_2 + G_2^T P G_1 \right) \xi(t), \\
&\dot{V}_2(t) = \sum_{i=1}^{3} x^T(t) W_i x(t) - \left(1 - \dot{d}(t)\right) x^T(t - d(t)) \\
&\quad * W_1 x(t - d(t)) - x^T(t - d_m) W_2 x(t - d_m) \\
&\quad - x^T(t - d_M) W_3 x(t - d_M) \le \xi^T(t) Q \xi(t), \\
&\dot{V}_3(t) = \dot{x}^T(t) \left[d_m^4 Z_1 + d_{Mm}^2 Z_2 \right] \dot{x}(t) + \sum_{i=1}^{3} \zeta_i = \xi^T(t) \\
&\quad \cdot \Delta^T N_1 \Delta \xi(t) + \sum_{i=1}^{3} \zeta_i, \\
&\dot{V}_4(t) = \frac{d_m^4}{2} \int_{t-d_m}^{t} \int_{s}^{t} \left[\dot{x}^T(t) Z_3 \dot{x}(t) - \dot{x}^T(u) Z_3 \right. \\
&\quad \left. * \dot{x}(u) \right] du\, ds + \frac{d_{Mm}^4}{2} \\
&\quad \cdot \int_{t-d_M}^{t-d_m} \int_{s}^{t-d_m} \left[\dot{x}^T(t) * Z_4 \dot{x}(t) \right. \\
&\quad \left. - \dot{x}^T(u) Z_4 \dot{x}(u) \right] du\, ds = \xi^T(t) \\
&\quad \cdot \Delta^T N_2 \Delta \xi(t) + \sum_{i=4}^{5} \zeta_i,
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
&N_1 = d_m^4 Z_1 + d_{Mm}^2 Z_2, \\
&N_2 = \frac{d_m^6}{4} Z_3 + \frac{d_{Mm}^6}{4} Z_4, \\
&\zeta_1(t) = -d_m^3 \int_{t-d_m}^{t} \dot{x}^T(s) Z_1 \dot{x}(s)\, ds, \\
&\zeta_2(t) = -d_{Mm} \int_{t-d(t)}^{t-d_m} \dot{x}^T(s) Z_2 \dot{x}(s)\, ds, \\
&\zeta_3(t) = -d_{Mm} \int_{t-d_M}^{t-d(t)} \dot{x}^T(s) Z_2 \dot{x}(s)\, ds, \\
&\zeta_4(t) = -\frac{d_m^4}{2} \int_{t-d_m}^{t} \int_{s}^{t} \dot{x}^T(u) Z_3 \dot{x}(u)\, du\, ds, \\
&\zeta_5(t) = -\frac{d_{Mm}^4}{2} \int_{t-d_M}^{t-d_m} \int_{s}^{t-d_m} \dot{x}^T(u) Z_4 \dot{x}(u)\, du\, ds, \\
&u_5(t) = \frac{1}{d(t) - d_m} \int_{t-d(t)}^{t-d_m} x^T(s)\, ds, \\
&u_6(t) = \frac{1}{d_M - d(t)} \int_{t-d_M}^{t-d(t)} x^T(s)\, ds, \\
&\xi^T(t) = \left[x^T(t) \;\; x^T(t - d_m) \;\; x^T(t - d(t)) \;\; x^T(t - d_M) \;\; u_1(t) \cdots u_6(t) \;\; h^T \;\; p^T \right].
\end{aligned} \tag{15}
$$

According to Lemma 1, we obtain

$$\zeta_1(t) \le -d_m^2 \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix}^T \begin{bmatrix} Z_1 & 0 \\ * & Z_1 \end{bmatrix} \begin{bmatrix} \varepsilon_1(t) \\ \varepsilon_2(t) \end{bmatrix}, \quad (16)$$

$$\zeta_2(t) \le -\frac{1}{\rho_1} \begin{bmatrix} \varepsilon_3(t) \\ \varepsilon_4(t) \end{bmatrix}^T \begin{bmatrix} Z_2 & 0 \\ * & Z_2 \end{bmatrix} \begin{bmatrix} \varepsilon_3(t) \\ \varepsilon_4(t) \end{bmatrix}, \quad (17)$$

$$\zeta_3(t) \le -\frac{1}{\rho_2} \begin{bmatrix} \varepsilon_5(t) \\ \varepsilon_6(t) \end{bmatrix}^T \begin{bmatrix} Z_2 & 0 \\ * & Z_2 \end{bmatrix} \begin{bmatrix} \varepsilon_5(t) \\ \varepsilon_6(t) \end{bmatrix}, \quad (18)$$

where

$$\begin{aligned}
\rho_1 &= \frac{d(t) - d_m}{d_{Mm}}, \\
\rho_2 &= \frac{d_M - d(t)}{d_{Mm}}, \\
\varepsilon_1(t) &= x(t) - x(t - d_m), \\
\varepsilon_2(t) &= \sqrt{3}x(t) + \sqrt{3}x(t - d_m) - \frac{2\sqrt{3}}{d_m}u_1(t), \\
\varepsilon_3(t) &= x(t - d_m) - x(t - d(t)), \\
\varepsilon_4(t) &= \sqrt{3}x(t - d_m) + \sqrt{3}x(t - d(t)) - 2\sqrt{3}u_5(t), \\
\varepsilon_5(t) &= x(t - d(t)) - x(t - d_M), \\
\varepsilon_6(t) &= \sqrt{3}x(t - d(t)) + \sqrt{3}x(t - d_M) - 2\sqrt{3}u_6(t)
\end{aligned} \quad (19)$$

and inequality (16) can be denoted as

$$\zeta_1(t) \le -\xi^T(t)\, d_m^2 \left[\Gamma_1^T Z_1 \Gamma_1 + \Gamma_2^T Z_1 \Gamma_2\right] \xi(t). \quad (20)$$

It is clear that the real numbers ρ_1 and ρ_2 satisfy $\rho_1 > 0$, $\rho_2 > 0$, and $\rho_1 + \rho_2 = 1$. Then introduce appropriate dimensions matrices S_1 and S_2, such that

$$\begin{bmatrix} Z_2 & S_2 \\ * & Z_2 \end{bmatrix} \ge 0, \qquad \begin{bmatrix} Z_2 & S_1 \\ * & Z_2 \end{bmatrix} \ge 0. \quad (21)$$

Applying Lemma 2 to (17) and (18)

$$\begin{aligned}
\zeta_2(t) + \zeta_3(t) \le &-\left(\frac{1}{\rho_1}\varepsilon_3^T(t) Z_2 \varepsilon_3(t) + \frac{1}{\rho_2}\varepsilon_5^T(t) Z_2 \varepsilon_5(t)\right) - \left(\frac{1}{\rho_1}\varepsilon_4^T(t) Z_2 \varepsilon_4(t) + \frac{1}{\rho_2}\varepsilon_6^T(t) Z_2 \varepsilon_6(t)\right) \le -\begin{bmatrix} \varepsilon_3(t) \\ \varepsilon_5(t) \end{bmatrix}^T \\
&\cdot \begin{bmatrix} Z_2 & S_1 \\ * & Z_2 \end{bmatrix} \begin{bmatrix} \varepsilon_3(t) \\ \varepsilon_5(t) \end{bmatrix} - \begin{bmatrix} \varepsilon_4(t) \\ \varepsilon_6(t) \end{bmatrix}^T \begin{bmatrix} Z_2 & S_2 \\ * & Z_2 \end{bmatrix} \begin{bmatrix} \varepsilon_4(t) \\ \varepsilon_6(t) \end{bmatrix} \\
= &-\xi^T(t) \left[\sum_{i=3}^{6} \Gamma_i^T Z_2 \Gamma_i + \Gamma_3^T S_1 \Gamma_5 + \Gamma_5^T S_1^T \Gamma_3 + \Gamma_4^T S_2 \Gamma_6 + \Gamma_6^T S_2^T \Gamma_4\right] \xi(t).
\end{aligned} \quad (22)$$

Using Lemma 3 can lead to

$$\begin{aligned}
\zeta_4(t) &\le -d_m^2 \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \end{bmatrix}^T \begin{bmatrix} Z_3 & 0 \\ * & Z_3 \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \end{bmatrix} = -\xi^T(t)\, d_m^2 \sum_{i=7}^{8} \Gamma_i^T Z_3 \Gamma_i \xi(t), \\
\zeta_5(t) &\le -d_{Mm}^2 \begin{bmatrix} \omega_3(t) \\ \omega_4(t) \end{bmatrix}^T \begin{bmatrix} Z_4 & 0 \\ * & Z_4 \end{bmatrix} \begin{bmatrix} \omega_3(t) \\ \omega_4(t) \end{bmatrix} = -\xi^T(t)\, d_{Mm}^2 \sum_{i=9}^{10} \Gamma_i^T Z_4 \Gamma_i \xi(t),
\end{aligned} \quad (23)$$

where

$$\begin{aligned}
\omega_1(t) &= d_m x(t) - u_1(t), \\
\omega_2(t) &= \frac{\sqrt{2}d_m}{2}x(t) + \sqrt{2}u_1(t) - \frac{3\sqrt{2}}{d_m}u_3(t), \\
\omega_3(t) &= d_{Mm} x(t - d_m) - u_2(t), \\
\omega_4(t) &= \frac{\sqrt{2}d_{Mm}}{2}x(t - d_m) + \sqrt{2}u_2(t) - \frac{3\sqrt{2}}{d_{Mm}}u_4(t).
\end{aligned} \quad (24)$$

Form (3), we have

$$\begin{aligned}
\epsilon_1\left(h^T h - \alpha^2 x^T(t) C^T C x(t)\right) &\le 0, \\
\epsilon_2\left(p^T p - \beta^2 x^T(t - d(t)) D^T D x(t - d(t))\right) &\le 0.
\end{aligned} \quad (25)$$

Combining (13), (14), (20), (22), (23), and (25), we obtain

$$\begin{aligned}
\dot{V}(t) \le \xi^T(t) \Big[& G_1^T P G_2 + G_2^T P G_1 + W + M_1 + M_2 + \Delta^T N \Delta - d_m^2 \sum_{i=1}^{2} \Gamma_i^T Z_1 \Gamma_i - \sum_{i=3}^{6} \Gamma_i^T Z_2 \Gamma_i - \Gamma_3^T S_1 \Gamma_5 \\
& - \Gamma_5^T S_1^T \Gamma_3 - \Gamma_4^T S_2 \Gamma_6 - \Gamma_6^T S_2^T \Gamma_4 - d_m^2 \sum_{i=7}^{8} \Gamma_i^T Z_3 \Gamma_i - d_{Mm}^2 \sum_{i=9}^{10} \Gamma_i^T Z_4 \Gamma_i \Big] \xi(t).
\end{aligned} \quad (26)$$

Using Schur complement, (26) can be transform to the first LMI of (9), which completed the proof of Theorem 4. □

When $h = p = 0$, that means system without perturbation. We can consider the following system:

$$\dot{x}(t) = Ax(t) + Bx(t - d(t)), \quad x(t) = \phi(t), \quad t \in [-d_M, 0]. \tag{27}$$

Stability analysis of system (26) is studied in a lot of literatures. From Theorem 4, the stability criterion of system (26) can be obtained easily.

Corollary 5. *System (26) under condition (2) will be asymptotically stable if there exist given scalars d_m, d_M, and d_d and positive symmetric matrices $P \in \mathbb{R}^{5n*5n}$, $W_s \in \mathbb{R}^{n*n}$ ($s = 1, 2, 3$), and $Z_q \in \mathbb{R}^{n*n}$ ($q = 1, \ldots, 4$) and appropriate dimensions matrices S_m, satisfying the following LMIs:*

$$\begin{bmatrix} \Phi & \Delta^T N \\ * & -N \end{bmatrix} < 0, \quad \begin{bmatrix} Z_2 & S_m \\ * & Z_2 \end{bmatrix} \geq 0, \quad (m = 1, 2), \tag{28}$$

where

$$\Phi = G_1^T P G_2 + G_2^T P G_1 + W - d_m^2 \sum_{i=1}^{2} \Gamma_i^T Z_1 \Gamma_i - \sum_{i=3}^{6} \Gamma_i^T Z_2 \Gamma_i - \Gamma_3^T S_1 \Gamma_5 - \Gamma_5^T S_1^T \Gamma_3 - \Gamma_4^T S_2 \Gamma_6 - \Gamma_6^T S_2^T \Gamma_4 - d_m^2 \sum_{i=7}^{8} \Gamma_i^T Z_3 \Gamma_i - d_{Mm}^2 \sum_{i=9}^{10} \Gamma_i^T Z_4 \Gamma_i,$$
$$G_1 = \mathrm{col}\{e_1, e_5, e_6, e_7, e_8\},$$
$$G_2 = \mathrm{col}\{Ae_1 + Be_3, e_1 - e_2, e_2 - e_4, d_m e_1 - e_5, d_{Mm} e_2 - e_6\},$$
$$W = \mathrm{diag}\{W_1 + W_2 + W_3 \;\; -W_2 \;\; -(1 - d_d) W_1 \;\; -W_3 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0\},$$
$$\Delta = Ae_1 + Be_3,$$
$$e_i = [0_{n\times(k-1)n} \;\; I \;\; 0_{n\times(10-k)n}], \quad k = 1, 2, \ldots, 10, \tag{29}$$

$\Gamma_1 \cdots \Gamma_{10}$ and N are defined in Theorem 4. If the information of delay is unavailable, by setting $Q_1 = 0$, we obtain

Corollary 6. *If there exist given scalars d_m, d_M, and d_d and positive symmetric matrices $P \in \mathbb{R}^{5n*5n}$, $W_s \in \mathbb{R}^{n*n}$ ($s = 1, 2, 3$), and $Z_q \in \mathbb{R}^{n*n}$ ($q = 1, \ldots, 4$) and appropriate dimensions matrices S_m, such that (27) with $W_1 = 0$ are feasible, system (26) under condition (2) will be asymptotically stable.*

Remark 7. From the viewpoint of control theory, the changed trend of system in reality is not only decided by the current states but also related to its past states. This is why the study of time-delay system is significant. This paper is interested in the stability analysis of the system with time-delay which has strong background in reality. The stability criterion we proposed can be used to design effective control strategy in a specific engineering file, such as synchronization of coronary artery. In order to compare with exiting results, a lot of papers related to stability analysis are concerned about the MADB which can be calculated by stability criterion. We can see the same particular examples in [10–30] as Examples 1–3 in our manuscript. In these examples structure of the matrices is prescribed. The purpose of these examples is to measure the conservatism of proposed criterion. The larger MADB, the less conservatism.

4. Numerical Examples

The advantages of our results can be illustrated by the following examples.

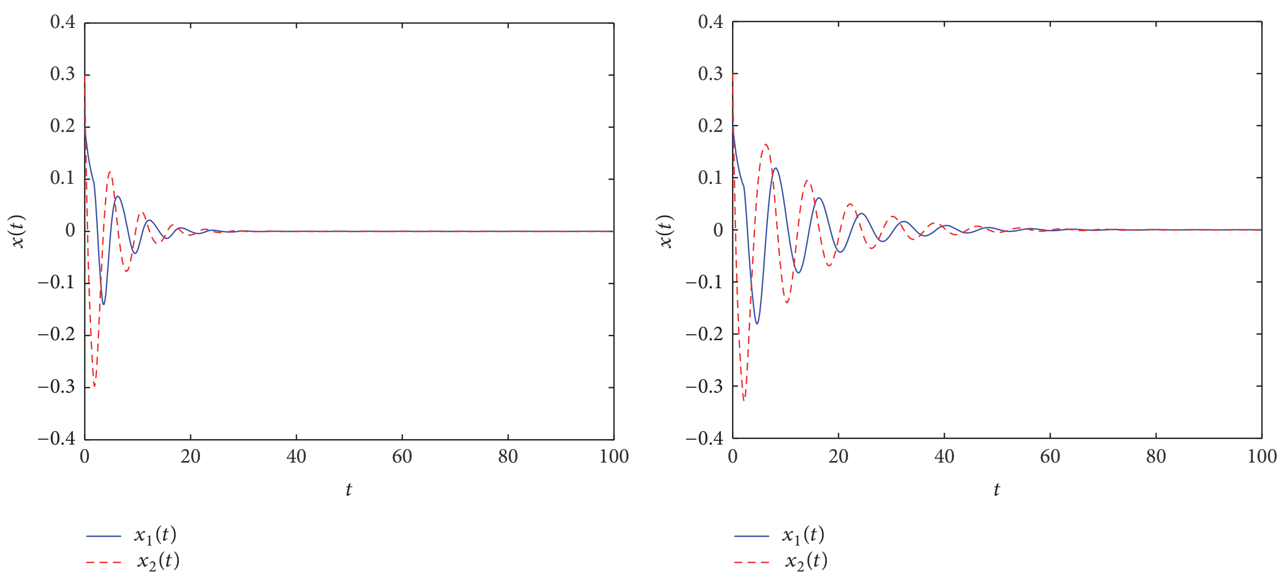
Example 1. Nonlinear system (1) is subject to (2) and (3) with

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, \quad C = D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{30}$$

For given d_m, α, β, and d_d, utilizing Theorem 4, we calculate the MADB d_M which can guarantee the stability of system (1). The result is listed in Table 1. We can see that the stability criterion we proposed has less conservatism than others. It is worth mentioning that when $\alpha = 0$, Theorem 4

Table 1: Maximal d_M for given $d_m = 0.5$.

Methods	α, β					
	$\alpha = 0, \beta = 0.1$			$\alpha = 0.1, \beta = 0.1$		
	$d_d = 0.5$	$d_d = 0.9$	$d_d = 1.1$	$d_d = 0.5$	$d_d = 0.9$	$d_d = 1.1$
[12]	1.4420	1.3380	1.3380	1.2840	1.2450	1.2450
[10] ($N = 2$)	1.5500	1.5500	1.5500	1.3690	1.3690	1.3690
[13]	1.5580	1.5580	1.5580	1.3840	1.3840	1.3840
[14]	1.5636	1.5636	1.5636	1.3858	1.3858	1.3858
[10] ($N = 4$)	1.8240	1.8240	1.8240	1.5240	1.5240	1.5240
[15]	1.8599	1.8599	1.8599	1.6622	1.6622	1.6622
Theorem 4	2.2561	2.2561	2.2561	1.6844	1.6844	1.6844



(a) $d(t) = 1.0922 + 0.5922 * \sin(1.5t)$

(b) $d(t) = 2.0497$

Figure 1: Trajectories of $x(t)$.

Table 2: Maximal d_M with given $d_m = 0$ for $d_d = 0$.

Methods	α, β	
	$\alpha = 0, \beta = 0.1$	$\alpha = 0.1, \beta = 0.1$
[16]	0.6811	0.6129
[17]	1.3279	1.2503
[18]	2.7422	1.8753
[19]	2.7423	1.8753
[20]	2.7757	1.8959
[21]	2.7758	1.8959
[22]	2.9816	1.9805
Theorem 4	3.0715	2.0497

can get much better results. Then we consider system (1) with constant time-delay. When $d_d = 0$, the MADB d_M under different α and β is listed in Table 2. We can see that Theorem 4 provides much less conservative results than others.

Let $f(x(t)) = 0.1x(t) * \sin(x(t))$, $g(x(t-d(t))) = 0.1x(t-d(t)) * \cos(x(t-d(t)))$, and $x(0) = (0.2, 0.3)$. Figure 1(a) shows the trajectories of variable $x(t)$ with $\alpha = 0.1$, $\beta = 0.1$, $d_d = 0.5$, $d_m = 0.5$, and $d(t) = 1.0922 + 0.5922 * \sin(1.5t)$. Figure 1(b) shows the trajectories of variable $x(t)$ with $\alpha = 0.1$, $\beta = 0.1$, $d_d = 0$, $d_m = 0$, and $d(t) = 2.0497$.

Example 2. Consider system (26) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \tag{31}$$

For given $d_m = 3, 4, 5$ and different $d_d = 0.1, 0.3, 0.5, 0.9$, we calculate the MADB d_M which is listed in Table 3. Table 3 illustrates the methods presented in Corollary 5 providing less conservative results than others. It is worth noting that the effect is very obvious when $d_d \leq 0.5$.

Let $d(t) = 5.1964 + 0.1964 * \sin(4.58 * t)$, and trajectories of variable $x(t)$ can be showed in Figure 2.

Table 3: Maximal d_M with given d_m for different d_d.

d_m	Methods	$d_d = 0.1$	$d_d = 0.3$	$d_d = 0.5$	$d_d = 0.9$
3	[23]	4.3979	3.3408	3.3408	3.3408
	[24]	4.4506	3.4186	3.4186	3.4186
	[11]	4.8247	3.6616	3.6616	3.6616
	Corollary 5	5.1213	4.2496	4.2208	3.5974
4	[23]	4.1978	4.1690	4.1690	4.1690
	[24]	4.2367	4.2097	4.2097	4.2097
	[11]	4.5762	4.3788	4.3788	4.3788
	Corollary 5	5.1916	4.9391	4.9338	4.4987
5	[23]	5.0275	5.0275	5.0275	5.0275
	[24]	5.0440	5.0440	5.0440	5.0440
	[11]	5.1453	5.1453	5.1453	5.1453
	Corollary 5	5.7065	5.7043	5.6965	5.3928

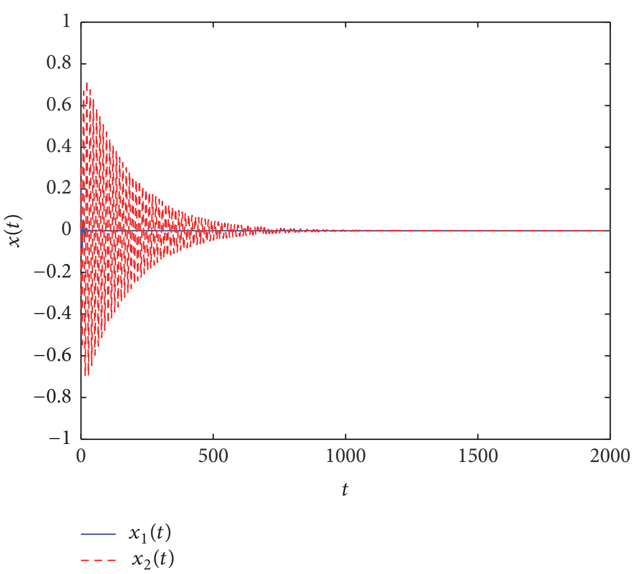


Figure 2: Trajectories of $x(t)$ with $d(t) = 5.1964 + 0.1964 * \sin(4.58 * t)$.

Table 4: Maximal d_M with given d_m for unknown d_d.

Methods	$d_m = 0.3$	$d_m = 0.5$	$d_m = 0.8$
[25]	1.0715	1.2191	1.4539
[23] ($N = 2$)	1.0716	1.2196	1.4552
[26]	1.2043	1.3429	1.5663
[27]	1.2246	1.3619	1.5838
[28]	1.2400	1.3800	1.6000
[29]	1.2700	1.3900	1.6100
[30]	1.3500	1.4700	1.6800
Corollary 6	1.5607	1.6426	1.7101

Table 5: Maximal d_M with given $d_m = 0$ for $d_d = 0.1, 0.3$.

Methods	$d_d = 0.1$	$d_d = 0.3$
[25]	5.4630	2.2160
[23] ($N = 2$)	5.4764	2.2160
[26]	5.4780	2.2850
[27]	5.4940	2.3070
Corollary 5	7.6301	3.0937

Example 3. Consider another system (26) with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}. \quad (32)$$

Sometimes the information of $d(t)$ is unavailable or $\dot{d}(t)$ is undifferentiable. We calculate the MADB d_M under different d_m for unknown d_d. From Table 4, it is observed that the results obtained by Corollary 6 are much less conservative than others. Finally, for given $d_m = 0$ and different $d_d = 0.1, 0.3$, using Corollary 5 we get the MADB d_M listed in Table 5. It is clear to see that our results have much less conservatism when the lower bound is $d_m = 0$.

Let $d(t) = 1.54685 + 1.54685 * \sin(0.194 * t)$, and trajectories of variable $x(t)$ can be showed in Figure 3.

5. Conclusion

We have researched the stability analysis problem for the time-varying delay systems. We utilize the new augmented LKF which is constructed based on single and double integral forms of Wirtinger-based inequality. Combining reciprocally convex method, we get an improved delay-dependent stability criterion. Finally, examples illustrate that the stability criterion we obtained is less conservative than some existing results.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

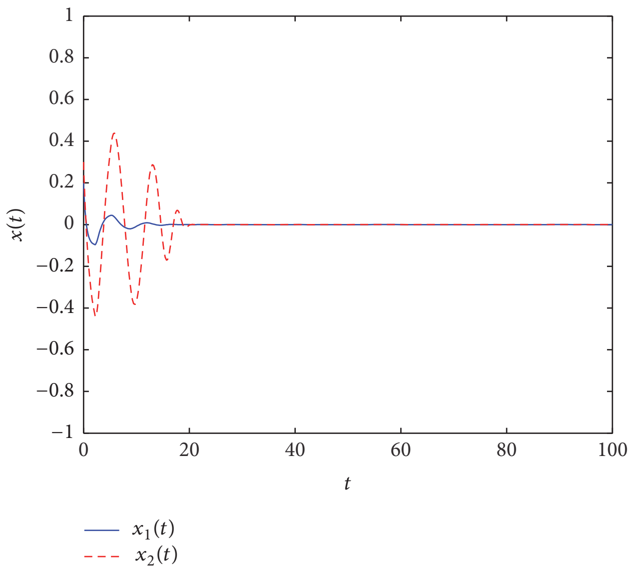


Figure 3: Trajectories of $x(t)$ with $d(t) = 1.54685 + 1.54685 * \sin(0.194 * t)$.

Acknowledgments

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