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Research Article

Topology Detection for Output-Coupling Weighted Complex Dynamical Networks with Coupling and Transmission Delays

Xinwei Wang, Guo-Ping Jiang, Chunxia Fan, and Xu Wu

College of Automation, Nanjing University of Posts and Telecommunications, Nanjing 210003, China

Correspondence should be addressed to Guo-Ping Jiang; jianggp@njupt.edu.cn

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Topology detection for output-coupling weighted complex dynamical networks with two types of time delays is investigated in this paper. Different from existing literatures, coupling delay and transmission delay are simultaneously taken into account in the output-coupling network. Based on the idea of the state observer, we build the drive-response system and apply LaSalle's invariance principle to the error dynamical system of the drive-response system. Several convergent criteria are deduced in the form of algebraic inequalities. Some numerical simulations for the complex dynamical network, with node dynamics being chaotic, are given to verify the effectiveness of the proposed scheme.

1. Introduction

Complex networks have various existence [1, 2] ranging from the World-Wide Web to neural networks, from cell phone webs to social networks [3], from food webs to metabolic networks [4], and so on [5, 6]. Since the small-world [7] and scale-free [8] network models were proposed, researches on the complex network have entered a new stage. Abundant research achievements have been concentrated on many perspectives like exploration on the dynamics [9, 10] and complexity [11] of the complex network, synchronization criteria [12, 13] and control strategies [14–16] in complex networks, estimation of uncertain state variables in continuous [17–19] or discrete [20] time domain under the fixed topology, and so on. In practical circumstances, the topological structure of a complex network is barely known exactly due to the weak cognition to the network complexity. If a serious malfunction occurs in a large-scale network, it is primary to quickly identify the fault location before trouble removal. Therefore, topology detection of complex dynamical networks [21–29] has become a significant topic for further studies. It is functionally powerful to employ the adaptive feedback control strategy to the structure detection of complex networks [21, 22, 24–28]. The target topology matrix which Zhou and Lu [21] considered was not needed to be symmetric, irreducible, or diffusive. If nodes in their network had different dynamics, the detection could still be accomplished. Additionally, stochastic perturbations [22] were considered in the structure identification of complex networks. In [23], based on the generalized outer synchronization, the topological structure of the complex network was recovered by constructing the auxiliary response network which had simpler node dynamics.

Recently, the coupling delay has been taken into account for the more realistic topology detection [24–27] since it is ubiquitous in the interactions of network nodes. Sometimes it leads to additional complex dynamics [24]. In [24], the topology of the complex network with coupling delays was successfully identified by an adaptive feedback controller. Their approach was applied to complex networks with not only identical nodes but also different nodes. The inner coupling matrix in [24] was chosen as a linear one. The topology detection problem for weighted time-varying dynamical networks with nonlinear inner coupling was addressed in [25]. Besides the topology detection, the estimation for uncertain system parameters [26] was conducted in general complex delayed dynamical networks [27]. It provides positive possibility for future applications. However, in [21–27], information signals were assumed to be transmitted as instant communication between the drive and response systems, which is not practical enough. Due to long communication distance and unexpected transmission congestions, it definitely takes finite time for exchanging messages through the channel and the time delay should not be ignored. In addition, the inevitable transmission delay in reality is always hard to determine. Based on the state-coupling network model, the unknown network structure was obtained by achieving the lag synchronization between drive and response systems [28]. In their scheme, all states of nodes were transmitted for the synchronization of full state-coupling complex networks.

The output-coupling network [14] whose every two nodes connect to each other via just a scalar signal saves interior communication resource, which has received more attention recently. The transmission signals between the drive and response output-coupling networks are also scalar outputs instead of state vectors. It further reduces the capacity of communication channels and becomes more available for real engineering. Fan et al. [29] detected the structures for state-coupling and output-coupling complex networks, respectively. Numerical examples verified their scheme effective applying to different kinds of networks without any time delay.

Motivated by the above discussions, study on the topology detection of output-coupling complex networks with both coupling and transmission delays is increasingly closer to the reality and has very important research significance. In this paper, based on the output-coupling network model, we investigate the topology detection problem considering coupling and transmission delays at the same time. The transmission delay does not need to be known exactly, which makes our scheme applicable for a more general case. The target network serves as the drive system and a response system is built accordingly whose node dynamics is the same as the drive system. The topology of the target network can be directed and weighted. Applying LaSalle's invariance principle [24, 25, 30], the error dynamical system is stabilized to approaching zero asymptotically while the structure detection is completed and topology changes are tracked in time.

The rest parts of this paper are organized as follows. The output-coupling complex network model with the coupling delay is described in Section 2. Topology detection of a general output-coupling complex network with coupling and transmission delay is discussed in detail and some convergent criteria are given in the form of algebraic inequalities in Section 3. In Section 4, several convincing simulation results are shown to confirm the accuracy and correctness of our proposed strategy. Some conclusions are provided in Section 5.

2. Preliminaries

Consider the output-coupling complex dynamical network with coupling delays composed of N identical nodes, which is described by

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t)) + \sum_{j=1}^{N} c_{ij}(t) L y_j(t-\tau_1), \\ y_i(t) &= H x_i(t), \\ & \qquad\qquad\qquad i = 1, 2, \ldots, N, \end{aligned} \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in R^n$ is the state vector of the ith node and $y_i \in R$ is the output scalar of that. $f : R \times R^n \to R^n$ is a smooth nonlinear vector field. $H \in R^{1\times n}$ is the output matrix of each node. In this way, y_i is a linear combination of the ith node's state components, and $L \in R^{n\times 1}$ is the inner coupling matrix. τ_1 is the inner coupling delay in the complex network. For simplicity, we assume that coupling delays of all nodes are the same constant value in this paper. $C(t) = (c_{ij}(t))_{N\times N} \in R^{N\times N}$ is the uncertain configuration matrix which represents the coupling strength and topological structure of the complex network. If there exists a connection from node i to node j $(i \neq j)$, then $c_{ij}(t) \neq 0$; otherwise $c_{ij}(t) = 0$. Assume that the diagonal elements $\{c_{ii}(t) \mid i = 1, 2, \ldots, N\}$ of $C(t)$ satisfy $c_{ii}(t) = -\sum_{j=1, j\neq i}^{N} c_{ij}(t)$. Here, the configuration matrix $C(t)$ is not required to be irreducible, symmetric, or diffusive.

Remark 1. $C(t)$ could switch anytime, which is used to describe the unexpected changes or manual operations on the network topological structure. It is assumed that the switching of $C(t)$ is relatively slow. Otherwise, the identification will be difficult to achieve in time, because the topology estimators need some time to react to the swift change of the topology.

Two assumptions and one lemma are introduced in the following.

Assumption 2 (A2). Suppose that there exists a nonnegative constant α satisfying

$$\| f(x(t)) - f(y(t)) \| \leq \alpha \| x(t) - y(t) \|, \tag{2}$$

which holds for any vectors $x(t), y(t) \in R^n$, where $\|\cdot\|$ represents the Euclidean normal.

Assumption 3 (A3). Suppose that, for any given $i, j \in \{1, 2, \ldots, N\}$, $\{L y_j(t-\tau_1)\}_{j=1}^N$ is linearly independent of the orbit $\{y_i(t-\tau_1)\}_{i=1}^N$ of the synchronization manifold [31, 32].

Lemma 4 (L4). *For any vectors $x(t), y(t) \in R^n$, the inequality $2x^T y \leq x^T x + y^T y$ holds for any t.*

Remark 5. (A2) is easy to satisfy for a large class of nonlinear systems, which means the nonlinear function $f(x)$ is Lipschitz-continuous. It characterizes the property of the node dynamics. In [31–33], all the coupling terms must be linearly independent of the synchronization manifold between the drive and response systems. (A3) is a necessary condition for the purpose of detecting the network topology. Also, the coupling delay τ_1 helps to prevent the synchronization of the drive and response systems before topology detection is completed. The transmission delay τ_2 will be explained in the next section because it is generated by transmitting output signals between the drive and response systems.

3. Topology Detection for the Complex Dynamical Network with the Transmission Delay

In order to identify unknown coupling strengths, namely, the complex network's topology, we establish a drive-response system. We consider the network model (1) as the drive system, and the response system could be constructed as

$$\begin{aligned}\dot{\hat{x}}_i(t) &= f(\hat{x}_i(t)) + \sum_{j=1}^{N}\hat{c}_{ij}(t) L\hat{y}_j(t-\tau_1) \\ &\quad - k_i(\hat{y}_i(t) - y_i(t-\tau_2)), \\ \hat{y}_i(t) &= H\hat{x}_i(t),\end{aligned} \tag{3}$$

where $i = 1, 2, \ldots, N$, $\hat{x}_i(t) = [\hat{x}_{i1}(t), \hat{x}_{i2}(t), \ldots, \hat{x}_{in}(t)]^T \in R^n$ is the state vector of the ith node, $\hat{y}_i \in R$ is the output scalar of that in the response system, $k_i \in [k_{i1}, k_{i2}, \ldots, k_{in}]^T$ is the ith node's observer gain which needs to be obtained, and τ_2 is the transmission time delay which is the time spent in the communication channel. Here, the transmission delay τ_2 is assumed to be an arbitrary constant value.

Let $e_i(t) = \hat{x}_i(t) - x_i(t-\tau_2)$, $e_{yi}(t) = \hat{y}_i(t) - y_i(t-\tau_2)$, and $\tilde{c}_{ij}(t) = \hat{c}_{ij}(t) - c_{ij}(t-\tau_2)$; then the error dynamical system (4) could be derived from the drive system (1) and response system (3):

$$\begin{aligned}\dot{e}_i(t) &= \dot{\hat{x}}_i(t) - \dot{x}_i(t-\tau_2) \\ &= f(\hat{x}_i(t)) - f(x_i(t-\tau_2)) \\ &\quad + \sum_{j=1}^{N}\hat{c}_{ij}(t) L\hat{y}_j(t-\tau_1) \\ &\quad - \sum_{j=1}^{N}c_{ij}(t-\tau_2) Ly_j(t-\tau_1-\tau_2) \\ &\quad - k_i(\hat{y}_i(t) - y_i(t-\tau_2)) \\ &= f(\hat{x}_i(t)) - f(x_i(t-\tau_2)) \\ &\quad + \sum_{j=1}^{N}\hat{c}_{ij}(t) L\hat{y}_j(t-\tau_1) \\ &\quad - \sum_{j=1}^{N}c_{ij}(t-\tau_2) L\hat{y}_j(t-\tau_1) \\ &\quad + \sum_{j=1}^{N}c_{ij}(t-\tau_2) L\hat{y}_j(t-\tau_1) \\ &\quad - \sum_{j=1}^{N}c_{ij}(t-\tau_2) Ly_j(t-\tau_1-\tau_2) - k_ie_{yi}(t) \\ &= f(\hat{x}_i(t)) - f(x_i(t-\tau_2)) \\ &\quad + \sum_{j=1}^{N}\tilde{c}_{ij}(t) L\hat{y}_j(t-\tau_1) \\ &\quad + \sum_{j=1}^{N}c_{ij}(t-\tau_2) Le_{yj}(t-\tau_1) - k_ie_{yi}(t).\end{aligned} \tag{4}$$

From (4), the output error dynamical system is obtained:

$$\begin{aligned}\dot{e}_{yi}(t) &= H\dot{e}_i(t) \\ &= H(f(\hat{x}_i(t)) - f(x_i(t-\tau_2))) \\ &\quad + H\left(\sum_{j=1}^{N}\tilde{c}_{ij}(t) L\hat{y}_j(t-\tau_1)\right) \\ &\quad + H\left(\sum_{j=1}^{N}c_{ij}(t-\tau_2) Le_{yj}(t-\tau_1)\right) \\ &\quad - Hk_ie_{yi}(t).\end{aligned} \tag{5}$$

Theorem 6. *Suppose that (A2) and (A3) hold. Use the following control law:*

$$\dot{\hat{c}}_{ij}(t) = -e_{yi}^T(t) HL\hat{y}_j(t-\tau_1). \tag{6}$$

If there exist matrices k_i ($i = 1, 2, \ldots, N$) satisfying the inequality

$$\left(\alpha + \lambda_{\max}\left(\frac{1}{2}P(t-\tau_2) P^T(t-\tau_2)\right) + \frac{1}{2} - \min(Hk_i)\right) < 0, \tag{7}$$

where $\lambda_{\max}((1/2)P(t-\tau_2)P^T(t-\tau_2))$ denotes the maximum eigenvalue of the matrix $((1/2)P(t-\tau_2)P^T(t-\tau_2))$ and $\min(Hk_i)$ denotes the minimum value among $\{Hk_i \mid i = 1, 2, \ldots, N\}$, $P(t-\tau_2) = C(t-\tau_2) \otimes HL$. As a result, the drive system (1) and the response system (3) will achieve synchronization asymptotically and the original topology matrix $C(t)$ will be detected by $\widehat{C}(t)$ eventually; that is,

$$\begin{aligned}&\lim_{t\to\infty}\left\|e_{yi}(t)\right\| = 0, \\ &\lim_{t\to\infty}\left\|\widehat{C}(t) - C(t)\right\| = 0, \\ &\qquad i, j = 1, 2, \ldots, N.\end{aligned} \tag{8}$$

Proof. Consider the scalar function V as

$$\begin{aligned}V &= \frac{1}{2}\sum_{i=1}^{N}e_{yi}{}^T(t) e_{yi}(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}^2(t) \\ &\quad + \frac{1}{2}\int_{t-\tau_1}^{t}\sum_{i=1}^{N}e_{yi}^T(\theta) e_{yi}(\theta) d\theta.\end{aligned} \tag{9}$$

Calculate the derivative of the scalar function (9) with the control law (6) and we get

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N}e_{yi}^T(t)\dot{e}_{yi}(t) + \sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}(t)\dot{\hat{c}}_{ij}(t) \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}e_{yi}^T(t) e_{yi}(t) - \frac{1}{2}\sum_{i=1}^{N}e_{yi}^T(t-\tau_1) e_{yi}(t-\tau_1)\end{aligned}$$

$$
\begin{aligned}
&= \sum_{i=1}^{N} e_{yi}^{T}(t) H\left(f\left(\widehat{x}_i(t)\right) - f\left(x_i\left(t-\tau_2\right)\right)\right) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} \widetilde{c}_{ij}(t) e_{yi}^{T}(t) HL\widehat{y}_j\left(t-\tau_1\right) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}\left(t-\tau_2\right) e_{yi}^{T}(t) HLe_{yj}\left(t-\tau_1\right) \\
&\quad - \sum_{i=1}^{N} e_{yi}^{T}(t) Hk_i e_{yi}(t) + \sum_{i=1}^{N}\sum_{j=1}^{N} \widetilde{c}_{ij}(t) \dot{\widehat{c}}_{ij}(t) \\
&\quad + \frac{1}{2}\sum_{i=1}^{N} e_{yi}^{T}(t) e_{yi}(t) - \frac{1}{2}\sum_{i=1}^{N} e_{yi}^{T}\left(t-\tau_1\right) e_{yi}\left(t-\tau_1\right) \\
&= \sum_{i=1}^{N} e_{yi}^{T}(t) H\left(f\left(\widehat{x}_i(t)\right) - f\left(x_i\left(t-\tau_2\right)\right)\right) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}\left(t-\tau_2\right) e_{yi}^{T}(t) HLe_{yj}\left(t-\tau_1\right) \\
&\quad - \sum_{i=1}^{N} e_{yi}^{T}(t) Hk_i e_{yi}(t) + \frac{1}{2}\sum_{i=1}^{N} e_{yi}^{T}(t) e_{yi}(t) \\
&\quad - \frac{1}{2}\sum_{i=1}^{N} e_{yi}^{T}\left(t-\tau_1\right) e_{yi}\left(t-\tau_1\right).
\end{aligned}
\tag{10}
$$

According to (A2), (A3), and (L4), we have

$$
\begin{aligned}
\dot{V} &\le \sum_{i=1}^{N} e_{yi}^{T}(t) \alpha e_{yi}(t) \\
&\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}\left(t-\tau_2\right) e_{yi}^{T}(t) HLe_{yj}\left(t-\tau_1\right) \\
&\quad - \sum_{i=1}^{N} e_{yi}^{T}(t) Hk_i e_{yi}(t) + \frac{1}{2}\sum_{i=1}^{N} e_{yi}^{T}(t) e_{yi}(t) - \frac{1}{2} \\
&\quad \cdot \sum_{i=1}^{N} e_{yi}^{T}\left(t-\tau_1\right) e_{yi}\left(t-\tau_1\right) \le \alpha e_{y}^{T}(t) e_{y}(t) + e_{y}^{T}(t) \\
&\quad \cdot P\left(t-\tau_2\right) e_{y}\left(t-\tau_1\right) - \min\left(Hk_i\right) e_{y}^{T}(t) e_{y}(t) + \frac{1}{2} \\
&\quad \cdot e_{y}^{T}(t) e_{y}(t) - \frac{1}{2} e_{y}^{T}\left(t-\tau_1\right) e_{y}\left(t-\tau_1\right) \le \alpha e_{y}^{T}(t) \\
&\quad \cdot e_{y}(t) + \frac{1}{2} e_{y}^{T}(t) P\left(t-\tau_2\right) P^{T}\left(t-\tau_2\right) e_{y}(t) + \frac{1}{2} \\
&\quad \cdot e_{y}^{T}\left(t-\tau_1\right) e_{y}\left(t-\tau_1\right) - \min\left(Hk_i\right) e_{y}^{T}(t) e_{y}(t) \\
&\quad + \frac{1}{2} e_{y}^{T}(t) e_{y}(t) - \frac{1}{2} e_{y}^{T}\left(t-\tau_1\right) e_{y}\left(t-\tau_1\right) \le \Big(\alpha \\
&\quad + \lambda_{\max}\left(\frac{1}{2} P\left(t-\tau_2\right) P^{T}\left(t-\tau_2\right)\right) + \frac{1}{2} \\
&\quad - \min\left(Hk_i\right)\Big) e_{y}^{T}(t) e_{y}(t),
\end{aligned}
\tag{11}
$$

where $e_y(t) = [e_{y1}^T(t), e_{y2}^T(t), \ldots, e_{yN}^T(t)]^T \in R^N$, and hence we get

$$
\begin{aligned}
\dot{V} \le \Big(&\alpha + \lambda_{\max}\left(\frac{1}{2} P\left(t-\tau_2\right) P^{T}\left(t-\tau_2\right)\right) + \frac{1}{2} \\
&- \min\left(Hk_i\right)\Big) e_{y}^{T}(t) e_{y}(t).
\end{aligned}
\tag{12}
$$

Taking $(\alpha + \lambda_{\max}((1/2)P(t-\tau_2)P^T(t-\tau_2)) + 1/2 - \min(Hk_i)) < 0$ from (7), we have $\dot{V} \le 0$. Evidently, $\dot{V} = 0$ if and only if $e_y(t) = 0$; then $E = \{\dot{V} = 0\} = \{e_y(t) = 0\}$. Along with the output error dynamical system (5), if $\widehat{C}(t) - C(t) \ne 0$, then $\dot{e}_{yi}(t) \ne 0$, $e_{yi}(t)$ would not be fixed at the zero point. That means the largest invariant set M contained in E is $M = \{e_y(t) = 0, \widehat{C}(t) - C(t) = 0\}$. According to LaSalle's invariance principle [29], with arbitrary initial values of the output error dynamical system (5), the trajectories converge asymptotically to the set M, which ensures us the detection quality of the complex network's topology. The proof is completed. □

4. Numerical Simulations

In this section, a representative example is shown to verify the effectiveness of the topology detection scheme proposed in Section 3. Here, the node dynamics is characterized by the chaotic system. The Lorenz system is one of the most typical chaotic systems, which could be described by

$$
\begin{aligned}
\dot{x}_{i1}(t) &= a\left(x_{i2}(t) - x_{i1}(t)\right), \\
\dot{x}_{i2}(t) &= cx_{i1}(t) - x_{i2}(t) - x_{i1}(t) x_{i3}(t), \\
\dot{x}_{i3}(t) &= x_{i1}(t) x_{i2}(t) - bx_{i3}(t),
\end{aligned}
\tag{13}
$$

when $a = 10$, $b = 8/3$, and $c = 28$, and the Lorenz system will show the chaotic characteristics. For any two state vectors x_i and x_j of the Lorenz system, there exists a constant θ satisfying $\|x_{ik}\| \le \theta$ and $\|x_{jk}\| \le \theta$ since chaotic attractor is bounded in a certain region; thus we obtain

$$
\begin{aligned}
&\left\| f\left(x_i\right) - f\left(x_j\right) \right\| \\
&\quad = \sqrt{\left(x_{j1}x_{j3} - x_{i1}x_{i3}\right)^2 + \left(x_{i1}x_{i2} - x_{j1}x_{j2}\right)^2} \\
&\quad \le 2\theta \left\| x_i - x_j \right\| = \alpha \left\| x_i - x_j \right\|.
\end{aligned}
\tag{14}
$$

Obviously, (A2) can be easily satisfied in the Lorenz system.

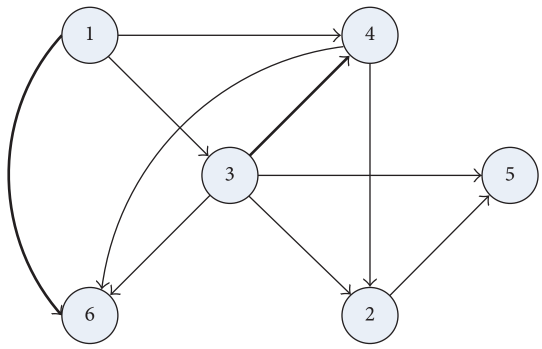

Figure 1: Drive network topology ($t < 25$).

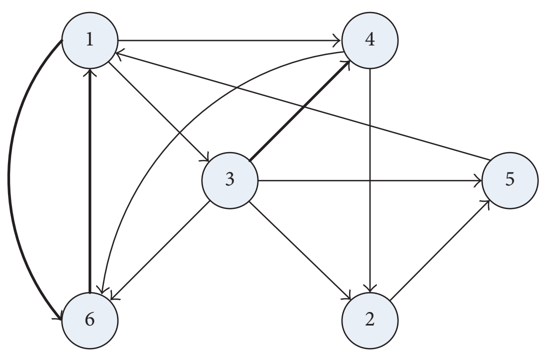

Figure 2: Drive network topology ($t > 25$).

A general complex dynamical network consisting of 6 identical nodes is chosen for the simulation. When $t < 25$, the topological structure is described by Figure 1:

$$C = \left(c_{ij}\right)_{6\times 6} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 2 & -3 & 0 & 0 \\ 0 & 1 & 1 & 0 & -2 & 0 \\ 2 & 0 & 1 & 1 & 0 & -4 \end{bmatrix}. \tag{15}$$

We suppose that when $t > 25$, the topology matrix of the drive system changes from Figures 1 and 2:

$$C = \left(c_{ij}\right)_{6\times 6} = \begin{bmatrix} -3 & 0 & 0 & 0 & 1 & 2 \\ 0 & -2 & 1 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 2 & -3 & 0 & 0 \\ 0 & 1 & 1 & 0 & -2 & 0 \\ 2 & 0 & 1 & 1 & 0 & -4 \end{bmatrix}. \tag{16}$$

Applying the network with 6 identical nodes mentioned above to the network model (1), we get

$$\begin{aligned} \dot{x}_i(t) &= f\left(x_i(t)\right) + \sum_{j=1}^{N} c_{ij} L y_j(t-1), \\ y_i(t) &= H x_i(t), \\ & \qquad\qquad i = 1, 2, \ldots, 6. \end{aligned} \tag{17}$$

Treating the network model (17) as the drive system, we build the corresponding response system as follows:

$$\begin{aligned} \dot{\hat{x}}_i(t) &= f\left(\hat{x}_i(t)\right) + \sum_{j=1}^{N} \hat{c}_{ij} L \hat{y}_j(t-1) \\ &\quad - k_i\left(\hat{y}_i(t) - y_i(t-2)\right), \\ \hat{y}_i(t) &= H\hat{x}_i(t), \end{aligned} \tag{18}$$

where $i = 1, 2, \ldots, 6$, $x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T$,

$$\begin{aligned} f\left(x_i(t)\right) &= \begin{bmatrix} a\left(x_{i2}(t) - x_{i1}(t)\right) \\ c x_{i1}(t) - x_{i2}(t) - x_{i1}(t) x_{i3}(t) \\ x_{i1}(t) x_{i2}(t) - b x_{i3}(t) \end{bmatrix}, \\ \hat{x}_i(t) &= \left[\hat{x}_{i1}(t), \hat{x}_{i2}(t), \hat{x}_{i3}(t)\right]^T, \\ L &= \left[l_1, l_2, l_3\right]^T, \\ H &= \left[h_1, h_2, h_3\right]. \end{aligned} \tag{19}$$

For simplicity, the coupling time delay is set as $\tau_1 = 1$, and the transmission time delay is set as $\tau_2 = 2$. Other parameters of the complex network are chosen as

$$\begin{aligned} k_i &= \begin{bmatrix} 100 & 100 & 100 \end{bmatrix}^T, \\ L &= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T, \\ H &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{20}$$

The initial values of the drive and response system' state variables and all elements of the estimated topology matrix are set randomly in the interval $[0, 1]$.

Since (A2) and (A3) hold, the response system (18) can detect the topology and track its changes of the drive system (17) with the control law (6). Meanwhile, the outputs of error dynamical system converge to zero as $t \to \infty$. Figure 3 shows the process of the response system detecting and tracking the target topology. The trajectories of outputs of the error dynamical system are shown in Figure 4.

From Figure 4, we can see that detecting the target topological structure and tracking its changes in real time are successful. The present research exposes the feasibility of achieving the topology identification in terms of inevitable coupling and transmission time delays when unexpected changes occur in the target network topology.

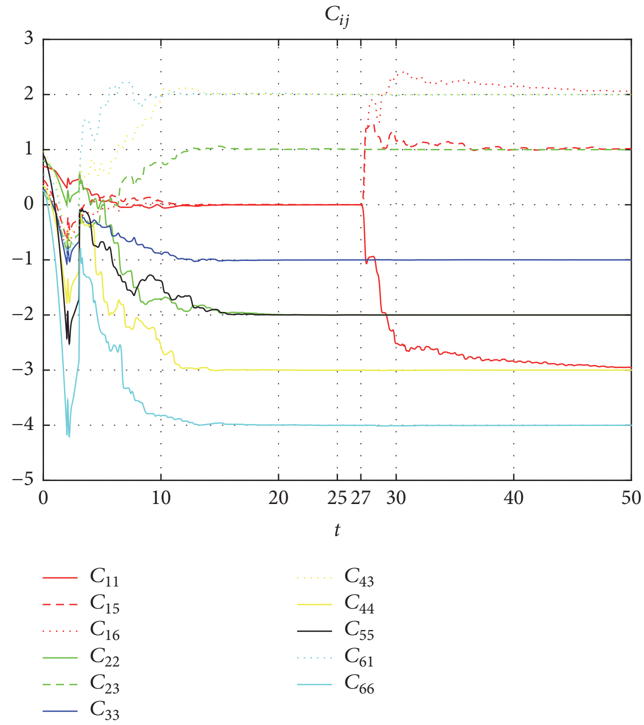

Figure 3: Detecting the topology of the drive system (17).

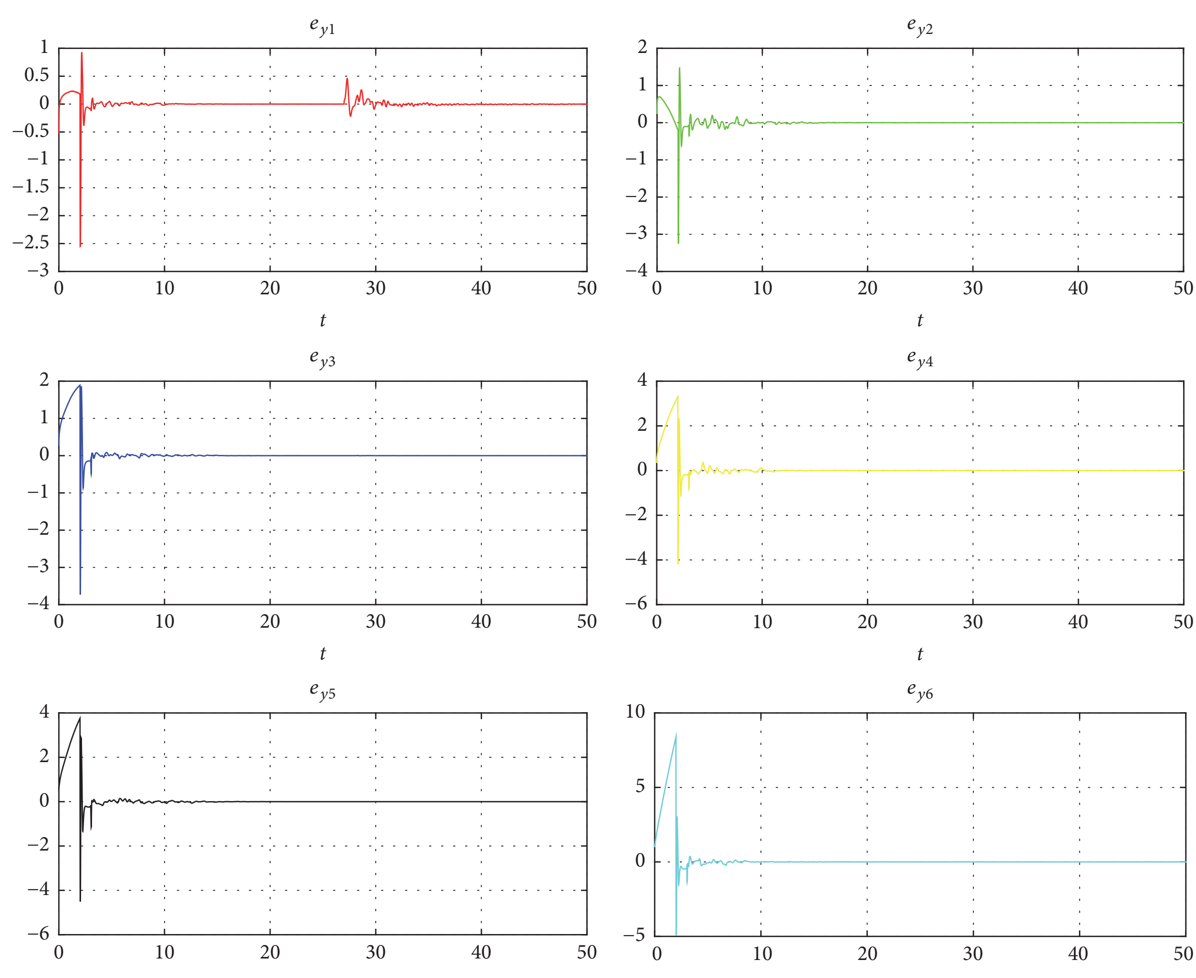

Figure 4: Trajectories of outputs of error dynamical system. The red line is the output e_{y1} of node x_1, the green line is the output e_{y2} of node x_2, the blue line is the output e_{y3} of node x_3, the yellow line is the output e_{y4} of node x_4, the black line is the output e_{y5} of node x_5, and the last line is the output e_{y6} of node x_6.

5. Conclusions

In summary, a topology detection approach based on the state observer has been proposed in this paper. We transmit the outputs of the general complex dynamical network as the control signals. The coupling delay and transmission delay are both taken into account in the output-coupling complex network model at the same time. The transmission delay is not needed to be determined exactly, which is proved to have no influence on the topology-detected process. By means of LaSalle's invariance principle, we guarantee the asymptotic stability of the output error dynamical system and complete the derivation process. Numerical simulations have indicated that the target topology matrix can be completely estimated in terms of inevitable coupling and transmission time delays. Also, we can achieve the real-time monitoring of changes of the target network topology.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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