

Research Article

Robust Adaptive Fuzzy Output Tracking Control for a Class of Twin-Roll Strip Casting Systems

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This paper is concerned with the adaptive fuzzy control problem for a class of twin-roll strip casting systems. By using fuzzy logic systems (FLSs) to approximate the compounded nonlinear functions, a novel robust output tracking controller with adaptation laws is designed based on the high gain observer. First, the nonlinear dynamic equations for the roll gap and the molten steel level are constructed, respectively. Then, the mean value theorem is employed to transform the nonaffine nonlinear systems to the corresponding affine nonlinear systems. Moreover, it is also proved that all the closed-loop signals are bounded and the systems output tracking errors can converge to the desired neighborhoods of the origin via the Lyapunov stability analysis. Finally, simulation results, based on semiexperimental system dynamic model and parameters, are worked out to show the effectiveness of the proposed adaptive fuzzy design method.

1. Introduction

As is well known, the strip casting combines two processes of continuous casting and hot rolling into a single production; consequently, it brings in a lot of advantages including lower investment cost, energy saving, less space requirements. More specifically, compared with the conventional continuous casting [1], the production line and the production cost of the twin-roll strip casting process are significantly shortened and reduced, respectively. Meanwhile, due to a high cooling rate for the strip casting, the mechanical properties of metallic materials can be increased [2]. Nevertheless, the strip casting process is always with nonlinear uncertainty, external disturbance, and coupled behaviors, and the roll gap and the molten steel level control problems are still important research topics to guarantee steel strip quality.

In [3], the model of the continuous casting process with various nonlinearities was proposed, and the corresponding controller was also designed. The authors in [4] developed an

adaptive algorithm for the mould level control of a continuous steel slab casting. The modeling and control problem for a class of twin-roll strip casting system was studied in [5]. Correspondingly, some successful adaptive fuzzy or neural network approaches for the molten steel level control have been studied (see, e.g., [6–8] and the references therein) in the casting process. Recently, based on the perturbation method, a decoupling control strategy in [9] was proposed to obtain a uniform sheet thickness and keep a constant roll separating force in the strip casting process. By using twin-roll casting technology (TRC), the optimized process parameters and their effects on TRC of 7050 aluminum alloys strips were studied in [10].

It should be pointed out that, in most of the results about nonlinear systems, the considered parameter uncertainties and disturbances satisfy matching condition [11, 12]. Besides, the above-mentioned control approaches require that all the states of the systems are available; thus they cannot be applied to nonaffine nonlinear system with unmeasured states. In

particular, for the roll gap and the molten steel level control of twin-roll strip casting process, it is difficult to measure the rates of change of the roll gap and the molten steel level by using proper sensors. In [13], the adaptive fuzzy tracking control problem for a class of uncertain nonaffine nonlinear systems with nonsymmetric dead-zone inputs was investigated; however, the proposed approaches can only handle the SISO nonaffine nonlinear systems rather than MIMO systems with complex coupling terms. To the best knowledge of the authors, it is the first time the adaptive fuzzy tracking control is developed for MIMO nonaffine nonlinear casting systems with immeasurable states, which are very meaningful and more practical.

The above considerations motivate our study work. Especially, inspired by [8, 13], by means of fuzzy approximation technique, the adaptive fuzzy output tracking control problem for a class of twin-roll strip casting systems is considered. Moreover, compared with the existing results, the main contributions of this paper are as follows: (1) The novel fuzzy tracking controllers with adaptation laws are designed by using fuzzy logic systems to approximate the compound nonlinear functions. (2) In order to handle the nonaffine coupling terms, the implicit function theorem and the mean value theorem are invoked, respectively. It is thus that the MIMO nonaffine nonlinear system can be transformed into the corresponding affine nonlinear system by this way. (3) By making use of adaptive mechanism driven by the estimation states obtained from the high gain observer, the influence of nonlinear parameter uncertainties and external disturbances is restrained effectively. It is also shown that the output tracking errors of the roll gap and the molten steel level can converge to the desired neighborhoods via the Lyapunov stability analysis.

The rest of the paper is organized as follows. In Section 2, the mathematical model for the strip casting process is given. The adaptive fuzzy output tracking control problem is addressed in Section 3. Simulations and experimental analysis are then provided in Section 4 to verify the effectiveness of the proposed approach. Finally, Section 5 draws the conclusions.

2. System Model for the Strip Casting Process

2.1. Molten Metal Level Equation. In this subsection, a diagram of the strip casting process is shown in Figure 1, and the corresponding mathematical model for the molten steel leveling dynamics is described as in [8]. Concretely, the following dynamic equation can be derived from (1)–(4) in [8]

$$\frac{dx_h}{dt} = F^{-1}(x_g, x_h) L^{-1} \left(Q_{in} - Q_{out} - Lx_h \frac{dx_g}{dt} \right), \quad (1)$$

where R is the roll radius, L is the length of the roll cylinders, Q_{in} and Q_{out} are the input flow and output flow of the pool between the two rolls, respectively, and $F(x_g, x_h) = x_g + 2R - 2\sqrt{R^2 - x_h^2}$.

In addition, for convenience, the input flow Q_{in} is taken as $Q_{in} = k(t)u$, where u is the control input, and the gain $k(t)$ is

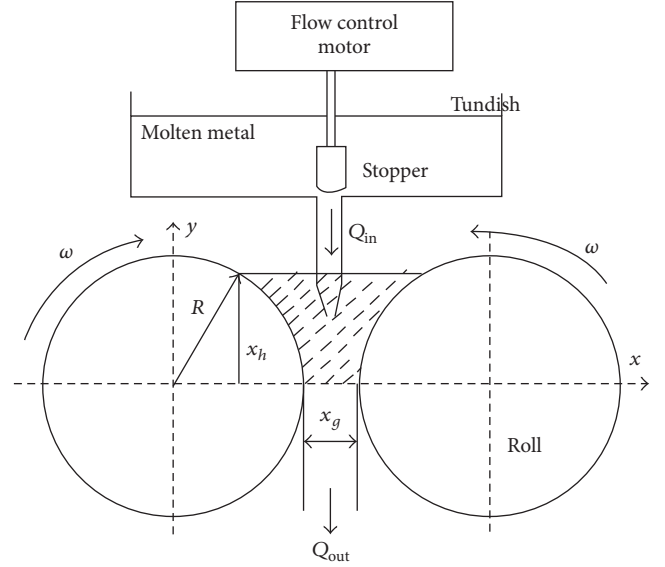


FIGURE 1: Schematic diagram of the twin-roll casting process.

determined empirically. The output flow Q_{out} can be derived from the product of roll surface tangential velocity v , roll gap x_g , and the length of the roll cylinder L ; that is, $Q_{out} = Lx_g v$. So the molten metal level equation (1) is rewritten as

$$\frac{dx_h}{dt} = F^{-1}(x_g, x_h) L^{-1} \left(ku - Lx_g v - Lx_h \frac{dx_g}{dt} \right). \quad (2)$$

By introducing the coordinate transformations $x_{11} = x_g$, $x_{12} = dx_g/dt$, $x_{21} = x_h$, and $x_{22} = dx_h/dt$, it follows from (2) that the following MIMO nonaffine nonlinear system can be obtained:

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= f_i(x, u_i) + d_i \\ y_i &= x_{i1}, \quad i = 1, 2, \end{aligned} \quad (3)$$

where $x = [x_1, x_2]^T$ and $x_i = [x_{i1}, x_{i2}]^T$ stand for the state variables, d_i is the unknown disturbance, y_i and u_i denote the system output and the control input of the i th subsystem, respectively, and $f_i(x, u_i)$ is the unknown and smooth nonaffine nonlinear function with u_i being electric servomotor control.

Remark 1. Applying Newton's Second Law, it is easy to obtain that $f_1(x, u_1) = (1/M)(Ku_1 - f_M(x))$ with M being the mass of roll, $f_M(x)$ being viscous resistance, and u_1 being servomotor control. At the same time, the nonaffine nonlinear function $f_2(x, u_2)$ can be derived by taking derivative of (2).

The objective of this paper is to design an adaptive fuzzy output feedback controller u_i such that all the closed-loop error signals are uniformly ultimately bounded, and the system output y_i tracks a reference trajectory y_{di} within a desired compact set in the presence of unknown nonaffine

nonlinear coupling term. Then, to ensure the feasibility of the considered problem, the necessary assumptions are required for the nonaffine nonlinear system (3).

Assumption 2. The desired reference trajectory y_{di} is known and smooth, and its derivative is also continuous. That is, there exist unknown positive constants \bar{d}_i , $\bar{\dot{d}}_i$, and $\bar{\ddot{d}}_i$ such that $|y_{di}(t)| \leq \bar{d}_i$, $|\dot{y}_{di}(t)| \leq \bar{\dot{d}}_i$, and $|\ddot{y}_{di}(t)| \leq \bar{\ddot{d}}_i$, respectively.

Assumption 3 (see [13]). For all $x \in \mathbb{R}^4$ and $u_i \in \mathbb{R}$ in the i th subsystem (3), there always exist positive constants f_{i1} and f_{i2} such that the following inequality holds:

$$0 < f_{i1} \leq \frac{\partial f_i(x, u_i)}{\partial u_i} \leq f_{i2}, \quad i = 1, 2. \quad (4)$$

Assumption 4. The external disturbance d_i is bounded; that is, there exists an unknown positive constant d_i^* such that $|d_i(t)| \leq d_i^*$.

Remark 5. It can be seen that Assumptions 2 and 4 are quite standard in most of the references for nonlinear tracking control, which means that the external disturbances, the reference signals, and their derivatives are bounded. Assumption 3 is used to decouple the nonaffine nonlinear term for the i th subsystem, which implies that the change rate of the control input gain is bounded.

2.2. Fuzzy Logic Systems (FLSs). Generally speaking, for an FLS, it consists of four parts: the knowledge base, the singleton fuzzifier, product inference, and center average defuzzifier, respectively. First, construct the knowledge base for FLS with the following IF-THEN rules:

$$\begin{aligned} R_i: & \text{ If } X_1 \text{ is } F_1^i \text{ and } \dots \text{ and } X_n \text{ is } F_n^i. \\ & \text{ Then } Y \text{ is } B_i, i = 1, 2, \dots, N. \end{aligned}$$

Next, the FLS with the singleton fuzzifier, product inference, and center average defuzzifier can be expressed as

$$Y(X) = \frac{\sum_{i=1}^N \theta_i \prod_{j=1}^n \mu_{F_j^i}(X_j)}{\sum_{i=1}^N \left[\prod_{j=1}^n \mu_{F_j^i}(X_j) \right]}, \quad (5)$$

where $X = [X_1, X_2, \dots, X_n]^T \in \mathbb{R}^n$, $\mu_{F_j^i}$ is the membership of F_j^i , and $\theta_i = \max_{Y \in \mathbb{R}} [\mu_{B_i}(Y)]$. Let

$$\phi_i(X) = \frac{\prod_{j=1}^n \mu_{F_j^i}(X_j)}{\sum_{i=1}^N \left[\prod_{j=1}^n \mu_{F_j^i}(X_j) \right]}, \quad i = 1, 2, \dots, N, \quad (6)$$

$\phi(X) = [\phi_1(X), \phi_2(X), \dots, \phi_N(X)]^T$, and $\theta = [\theta_1, \theta_2, \dots, \theta_N]^T$. Hence, the FLS can be rewritten in the following form:

$$Y(X) = \theta^T \phi(X). \quad (7)$$

Lemma 6 (see [14]). *Let $F(X)$ be a continuous function defined on a compact set Ω_X . Then, for any given constant*

$\epsilon > 0$, *there exists a FLS $Y(X)$ in the form of (7) such that*

$$\sup_{X \in \Omega_X} |F(X) - Y(X)| = \left| F(X) - \theta^T \phi(X) \right| < \epsilon. \quad (8)$$

Similar to [13], the optimal parameter vectors θ^* of FLS are defined as

$$\theta^* = \arg \min_{\theta \in \Omega_\theta} \left[\sup_{X \in \Omega_X} |F(X) - \theta^T \phi(X)| \right], \quad (9)$$

where Ω_θ and Ω_X are compact regions for θ and X , respectively. Furthermore, from Lemma 6, the fuzzy approximation error $\delta^*(X)$ is defined as

$$F(X) = \theta^{*T} \phi(X) + \delta^*(X), \quad \forall X \in \Omega_X \subset \mathbb{R}^n. \quad (10)$$

3. Adaptive Tracking Controller Design and Stability Analysis

3.1. Adaptive Tracking Controller Design. In this subsection, we shall present an adaptive fuzzy control scheme only based on output variable. Therefore, the high gain observer is introduced to design adaptive output fuzzy tracking controller, and the corresponding lemma is given as follows.

Lemma 7 (see [15]). *Consider the following linear system:*

$$\begin{aligned} \epsilon_i \dot{z}_{i1} &= z_{i2}, \\ \epsilon_i \dot{z}_{i2} &= -\mu_{i1} z_{i2} - z_{i1} + y_i, \quad i = 1, 2, \end{aligned} \quad (11)$$

where $\epsilon_i > 0$ is a sufficiently small constant and the parameter μ_{i1} is appropriately chosen such that $s^2 + \mu_{i1}s + 1$ is an Hurwitz polynomial. If the output function y_i and its k th time derivatives $y_i^{(k)}$ are bounded, that is, there exist positive constants Y_{i0}, Y_{ik} satisfying $|y_i| \leq Y_{i0}$, $|y_i^{(k)}| \leq Y_{ik}$, then it is obtained that

$$\eta_{ik} := \frac{z_{ik}}{\epsilon_i^{k-1}} - y_i^{(k-1)} = -\epsilon_i \phi_i^{(k)}, \quad k = 1, 2, \quad (12)$$

where $\phi_i := z_{i2} + \mu_{i1} z_{i1}$ and $\phi_i^{(k)}$ represents the k th time derivative of ϕ_i . Moreover, if all the observer states satisfy that $|z_{ik}| \leq \bar{z}_{ik}$ with $\bar{z}_{ik} > 0$, then there exist $\bar{\eta}_{ik} > 0$ such that $|\eta_{ik}| \leq \epsilon_i \bar{\eta}_{ik}$.

According to Lemma 6, the estimation of unmeasurable state vector is defined as

$$\hat{x}_i = \left[x_{i1}, \frac{z_{i2}}{\epsilon_i} \right]^T = [\hat{x}_{i1}, \hat{x}_{i2}]^T, \quad i = 1, 2. \quad (13)$$

Next, to facilitate control system design from nonaffine form to affine form, the tracking error and the filtered tracking error are defined as $\hat{e}_i = \hat{x}_i - x_{id} = [\hat{e}_{i1}, \hat{e}_{i2}]^T \in \mathbb{R}^2$ and $\hat{e}_{is} = [\lambda_i, 1] \hat{e}_i$, respectively, where $x_{id} = [y_{id}, \dot{y}_{id}]^T$ is the reference state vector and λ_i is appropriately chosen coefficient such that $s + \lambda_i$ is an Hurwitz polynomial; that is, $\hat{e}_i \rightarrow 0$ as $\hat{e}_{is} \rightarrow 0$.

Then, taking the derivative of \widehat{e}_{is} gives

$$\dot{\widehat{e}}_{is} = f_i(x, u_i) - \dot{y}_{id} + \dot{\widehat{x}}_{i2} + [0, \lambda_i] \widehat{e}_i + d_i, \quad (14)$$

where $\widehat{x}_{i2} = x_{i2} + \widetilde{x}_{i2}$. By using Assumption 3 and the implicit function theorem [16], there exists a unique and continuous ideal control $u_i^* = U_i(x) \in \Omega_{u_i}$ such that $f_i(x, u_i^*) = f_i(x, U_i(x)) = 0$ for all $x \in \Omega_x$, where Ω_x and Ω_{u_i} are two compact sets. By adding and subtracting $\gamma_i \widehat{e}_{is}$ in (14) and applying it to the mean value theorem, we have

$$\begin{aligned} \dot{\widehat{e}}_{is} &= f_i(x, u_i^*) + \frac{\partial f_i(x, u_i^0)}{\partial u_i} (u_i - u_i^*) + \gamma_i \widehat{e}_{is} - \dot{y}_{id} \\ &\quad + \dot{\widehat{x}}_{i2} + [0, \lambda_i] \widehat{e}_i + d_i - \gamma_i \widehat{e}_{is} \\ &= \mu_i u_i - \frac{\partial f_i(x, u_i^0)}{\partial u_i} u_i^* + \left(\frac{\partial f_i(x, u_i^0)}{\partial u_i} - \mu_i \right) u_i \\ &\quad + \gamma_i e_{is} + [0, \lambda_i] e_i - \dot{y}_{id} + \dot{\widehat{x}}_{i2} + d_i + \gamma_i \widetilde{e}_{is} \\ &\quad + [0, \lambda_i] \widetilde{e}_i - \gamma_i \widehat{e}_{is}, \end{aligned} \quad (15)$$

where u_i^0 is some point between zero and u_i , $\gamma_i > 0$, $\mu_i > 0$, $\widehat{e}_i = e_i + \widetilde{e}_i$, and $\widehat{e}_{is} = e_{is} + \widetilde{e}_{is}$.

Denote the nonlinear functions $H_i(x, e_i) = (\partial f_i(x, u_i^0) / \partial u_i) u_i^* - (\partial f_i(x, u_i^0) / \partial u_i) u_i - \gamma_i e_{is} - [0, \lambda_i] e_i$; it follows from (10) that $H_i(x, e_i)$ can be approximated by the following form:

$$H_i(x, e_i) = \theta_i^{*T} \phi_i(x, e_i) + \delta_i^*(x, e_i) \quad (16)$$

for all $(x, e_i) \in \Omega_x \times \Omega_{e_i}$ with Ω_{e_i} being a compact set. Consequently, substituting (16) into (15) yields

$$\dot{\widehat{e}}_{is} = \mu_i u_i - \theta_i^{*T} \phi_i(x, e_i) + D_i(t) - \gamma_i \widehat{e}_{is}, \quad (17)$$

where $D_i(t) = -\delta_i^*(x, e_i) - \dot{y}_{id} + \dot{\widehat{x}}_{i2} + d_i + \gamma_i \widetilde{e}_{is} + [0, \lambda_i] \widetilde{e}_i$. From Lemma 7 and using Assumptions 2 and 4, it can be concluded that there exists an unknown upper bound D_i^* such that $|D_i(t)| \leq D_i^*$.

Remark 8. For the i th nonaffine nonlinear subsystem (3), Assumption 3 plays an important role in the controller design. The reason is that the implicit function theorem is employed to transform the nonaffine nonlinear coupling term into the corresponding affine term based on this assumption. In addition, the similar decoupling method in [13] has been developed; however, it is required that less adjustable parameters are used for the controller design in this paper.

Moreover, the adaptive fuzzy tracking controller is designed for the i th subsystem as follows:

$$\begin{aligned} u_i &= \mu_i^{-1} \left(-\alpha_i \widehat{e}_{is} + \widehat{\theta}_i^T \phi_i(\widehat{x}, \widehat{e}_i) \right. \\ &\quad \left. - \frac{\widehat{e}_{is} \widehat{D}_i^2}{\widehat{e}_{is} \widehat{D}_i \tanh(\widehat{e}_{is} / \delta_i) + \delta_i} \right) \end{aligned} \quad (18)$$

with adaptation laws

$$\dot{\widehat{\theta}}_i = -\Gamma_i \left(\phi_i(\widehat{x}, \widehat{e}_i) \widehat{e}_{is} + \sigma_i \widehat{\theta}_i \right), \quad (19)$$

$$\dot{\widehat{D}}_i = -\beta_i \sigma_i \widehat{D}_i + \beta_i |\widehat{e}_{is}|,$$

where $\widehat{\theta}_i$ and \widehat{D}_i are the estimate values of θ_i^* and D_i^* , respectively, and $\Gamma_i = \Gamma_i^T > 0$, $\alpha_i, \delta_i, \beta_i, \sigma_i, i = 1, 2$, are positive design parameters.

3.2. Stability Analysis. In this subsection, the stability of the resulting closed-loop system is given in the following theorem.

Theorem 9. Consider the MIMO nonaffine nonlinear system (3) with unmeasurable states, under the condition that Assumptions 2–4 hold and the estimation states can be obtained from the high gain observer (11). On the compact set $\Omega_x \times \Omega_{e_i}$, the adaptive fuzzy tracking controller (18) and the parameter updated laws (19) are constructed; then all the closed-loop system error signals are uniformly ultimately bounded. Moreover, the parameter estimation errors $\widetilde{D}_i, \widetilde{\theta}_i$ and tracking error e_{is} remain as the compact sets $\Omega_{\widetilde{D}_i}, \Omega_{\widetilde{\theta}_i}$, and $\Omega_{e_{is}}$ in the sense that

$$\begin{aligned} \Omega_{e_{is}} &= \left\{ e_{is} \in \mathbb{R} \mid |e_{is}| \leq \sqrt{\Omega_i} \right\} \\ \Omega_{\widetilde{D}_i} &= \left\{ \widetilde{D}_i \in \mathbb{R} \mid |\widetilde{D}_i| \leq \sqrt{\beta_i \Omega_i} \right\} \end{aligned} \quad (20)$$

$$\Omega_{\widetilde{\theta}_i} = \left\{ \widetilde{\theta}_i \in \mathbb{R}^n \mid \|\widetilde{\theta}_i\| \leq \sqrt{\frac{\Omega_i}{\lambda_{\min}(\Gamma_i^{-1})}} \right\}, \quad i = 1, 2,$$

where $\Omega_i = 2(V_i|_{t=0} + \rho_{i2} / \rho_{i1})$ with ρ_{i1} and ρ_{i2} defined as

$$\begin{aligned} \rho_{i1} &:= \min \left\{ 2 \left(\alpha_i + \gamma_i - \frac{1}{2} \right), \frac{\sigma_i - 1/\psi_i}{\lambda_{\max}(\Gamma_i^{-1})}, (\sigma_i - 2\kappa\tau_i^*) \right. \\ &\quad \left. \cdot \beta_i \right\}, \end{aligned} \quad (21)$$

$$\begin{aligned} \rho_{i2} &:= \frac{(\alpha_i + \gamma_i)^2 \tau_i^{*2}}{2} + \frac{\tau_i^{*2} \psi_i}{2} + 2 \|\theta_i^*\| + \delta_i + 2\tau_i^* D_i^* \\ &\quad + \frac{\sigma_i}{2} \|\theta_i^*\|^2 + \left(2\kappa\tau_i^* + \frac{\sigma_i}{2} \right) D_i^{*2}. \end{aligned}$$

Proof. Define the Lyapunov function $V_i(e_{is}, \widetilde{\theta}_i, \widetilde{D}_i)$ as

$$V_i(e_{is}, \widetilde{\theta}_i, \widetilde{D}_i) = \frac{1}{2} e_{is}^2 + \frac{1}{2} \beta_i^{-1} \widetilde{D}_i^2 + \frac{1}{2} \widetilde{\theta}_i^T \Gamma_i^{-1} \widetilde{\theta}_i, \quad (22)$$

where $\widetilde{D}_i = \widehat{D}_i - D_i^*$ and $\widetilde{\theta}_i = \widehat{\theta}_i - \theta_i^*, i = 1, 2$, are the parameter estimation errors. Taking the derivative of V_i yields

$$\begin{aligned} \dot{V}_i &= e_{is} \dot{e}_{is} + \beta_i^{-1} \widetilde{D}_i \dot{\widetilde{D}}_i + \widetilde{\theta}_i^T \Gamma_i^{-1} \dot{\widetilde{\theta}}_i \\ &= e_{is} \left(\mu_i u_i - \theta_i^{*T} \phi_i(x, e_i) + D_i(t) - \gamma_i \widehat{e}_{is} \right) \\ &\quad + \beta_i^{-1} \widetilde{D}_i \dot{\widetilde{D}}_i + \widetilde{\theta}_i^T \Gamma_i^{-1} \dot{\widetilde{\theta}}_i. \end{aligned} \quad (23)$$

By substituting (18) into (23), one can obtain that

$$\begin{aligned}
\dot{V}_i &= -(\alpha_i + \gamma_i) e_{is} \hat{e}_{is} + \hat{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) e_{is} \\
&\quad - \frac{e_{is} \hat{e}_{is} \widehat{D}_i^2}{\hat{e}_{is} \widehat{D}_i \tanh(\hat{e}_{is}/\delta_i) + \delta_i} - \theta_i^{*T} \phi_i(x, e_i) e_{is} \\
&\quad + e_{is} D_i + \beta_i^{-1} \widetilde{D}_i \dot{\widetilde{D}}_i + \widetilde{\theta}_i^T \Gamma_i^{-1} \dot{\widetilde{\theta}}_i \\
&\leq -(\alpha_i + \gamma_i) e_{is}^2 - (\alpha_i + \gamma_i) e_{is} \widetilde{e}_{is} \\
&\quad + (\widehat{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) - \theta_i^{*T} \phi_i(x, e_i)) e_{is} \\
&\quad - \frac{\widetilde{e}_{is}^2 \widehat{D}_i^2}{\hat{e}_{is} \widehat{D}_i \tanh(\hat{e}_{is}/\delta_i) + \delta_i} \\
&\quad + \frac{\widetilde{e}_{is} \hat{e}_{is} \widehat{D}_i^2}{\hat{e}_{is} \widehat{D}_i \tanh(\hat{e}_{is}/\delta_i) + \delta_i} + \hat{e}_{is} D_i - \widetilde{e}_{is} D_i \\
&\quad + \beta_i^{-1} \widetilde{D}_i \dot{\widetilde{D}}_i + \widetilde{\theta}_i^T \Gamma_i^{-1} \dot{\widetilde{\theta}}_i,
\end{aligned} \tag{24}$$

where $\widetilde{e}_{is} = \hat{e}_{is} - e_{is}$. Using the adaptive control laws (19), (24) becomes

$$\begin{aligned}
\dot{V}_i &\leq -(\alpha_i + \gamma_i) e_{is}^2 - (\alpha_i + \gamma_i) e_{is} \widetilde{e}_{is} \\
&\quad + (\widehat{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) - \theta_i^{*T} \phi_i(x, e_i)) e_{is} \\
&\quad - \frac{\widetilde{e}_{is}^2 \widehat{D}_i^2}{\hat{e}_{is} \widehat{D}_i \tanh(\hat{e}_{is}/\delta_i) + \delta_i} \\
&\quad + \frac{\widetilde{e}_{is} \hat{e}_{is} \widehat{D}_i^2}{\hat{e}_{is} \widehat{D}_i \tanh(\hat{e}_{is}/\delta_i) + \delta_i} + |\widetilde{e}_{is}| D_i^* + |\widetilde{e}_{is}| D_i^* \\
&\quad - \sigma_i \widetilde{D}_i \widehat{D}_i + |\widetilde{e}_{is}| \widetilde{D}_i - \widetilde{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) \hat{e}_{is} - \sigma_i \widetilde{\theta}_i^T \hat{\theta}_i.
\end{aligned} \tag{25}$$

Invoking inequalities $0 \leq b \tanh(b/a) \leq |b|, \forall b \in \mathbb{R}, a > 0$ and $|b| \leq b \tanh(b/a) + \kappa a, \forall b \in \mathbb{R}, a > 0, \kappa = 0.2785$ given in [17], it follows from (25) that

$$\begin{aligned}
\dot{V}_i &\leq -(\alpha_i + \gamma_i) e_{is}^2 - (\alpha_i + \gamma_i) e_{is} \widetilde{e}_{is} \\
&\quad + (\widehat{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) - \theta_i^{*T} \phi_i(x, e_i)) e_{is} + \frac{|\widetilde{e}_{is}| \widehat{D}_i \delta_i}{|\widetilde{e}_{is}| \widehat{D}_i + \delta_i} \\
&\quad + |\widetilde{e}_{is}| \widetilde{D}_i + \kappa |\widetilde{e}_{is}| \widehat{D}_i^2 + |\widetilde{e}_{is}| D_i^* - \sigma_i \widetilde{D}_i^2 \\
&\quad - \sigma_i \widetilde{D}_i D_i^* - \widetilde{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) \hat{e}_{is} - \sigma_i \widetilde{\theta}_i^T \hat{\theta}_i.
\end{aligned} \tag{26}$$

For the FLS error term on the right side of (26), adding and subtracting $\theta_i^{*T} \phi_i(\hat{x}, \hat{e}_i)$ yield

$$\begin{aligned}
&\widehat{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) - \theta_i^{*T} \phi_i(\hat{x}, \hat{e}_i) + \theta_i^{*T} \phi_i(\hat{x}, \hat{e}_i) \\
&\quad - \theta_i^{*T} \phi_i(x, e_i) \\
&= \widetilde{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) + \theta_i^{*T} (\phi_i(\hat{x}, \hat{e}_i) - \phi_i(x, e_i)).
\end{aligned} \tag{27}$$

Using the fact that $\phi_i^T \phi_i \leq 1$ and the triangle inequality, we have

$$\begin{aligned}
&\widehat{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) - \theta_i^{*T} \phi_i(x, e_i) \leq \widetilde{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) + 2 \|\theta_i^*\| \\
&2 \widetilde{D}_i D_i^* = \widehat{D}_i^2 - D_i^{*2} - \widetilde{D}_i^2 \geq -D_i^{*2} - \widetilde{D}_i^2
\end{aligned} \tag{28}$$

$$2 \widetilde{\theta}_i^T \hat{\theta}_i = \|\widetilde{\theta}_i\|^2 + \|\hat{\theta}_i\|^2 - \|\theta_i^*\|^2 \geq \|\widetilde{\theta}_i\|^2 - \|\theta_i^*\|^2.$$

Invoking (28), (26) becomes

$$\begin{aligned}
\dot{V}_i &\leq -(\alpha_i + \gamma_i) e_{is}^2 - (\alpha_i + \gamma_i) e_{is} \widetilde{e}_{is} - \widetilde{\theta}_i^T \phi_i(\hat{x}, \hat{e}_i) \widetilde{e}_{is} \\
&\quad + 2 \|\theta_i^*\| + \frac{|\widetilde{e}_{is}| \widehat{D}_i \delta_i}{|\widetilde{e}_{is}| \widehat{D}_i + \delta_i} + |\widetilde{e}_{is}| \widetilde{D}_i + \kappa |\widetilde{e}_{is}| \widehat{D}_i^2 \\
&\quad + 2 |\widetilde{e}_{is}| D_i^* - \frac{\sigma_i}{2} \widetilde{D}_i^2 + \frac{\sigma_i}{2} D_i^{*2} - \frac{\sigma_i}{2} \|\widetilde{\theta}_i\|^2 \\
&\quad + \frac{\sigma_i}{2} \|\theta_i^*\|^2.
\end{aligned} \tag{29}$$

From the definition in (12) and the fact that $\widetilde{e}_{is} = [\lambda_i, 1] \widetilde{e}_i$, it can be obtained that \widetilde{e}_{is} is bounded; that is, there exists a positive constant τ_i^* subject to $|\widetilde{e}_{is}| \leq \tau_i^*$. Moreover, by utilizing the inequalities $2ab \leq (1/c)a^2 + cb^2, \forall c > 0, a, b \in \mathbb{R}$ and $0 \leq a/(a+b) < 1, \forall a \geq 0, b > 0$, we have

$$\begin{aligned}
\dot{V}_i &\leq -(\alpha_i + \gamma_i) e_{is}^2 + \frac{1}{2} e_{is}^2 + \frac{(\alpha_i + \gamma_i)^2 \tau_i^{*2}}{2} + \frac{1}{2\psi_i} \|\widetilde{\theta}_i\|^2 \\
&\quad + \frac{\tau_i^{*2} \psi_i}{2} + 2 \|\theta_i^*\| + \delta_i + \frac{1}{2\varphi_i} \tau_i^{*2} + \frac{\widetilde{D}_i^2 \varphi_i}{2} + 2\kappa \tau_i^* \widehat{D}_i^2 \\
&\quad + 2\kappa \tau_i^* D_i^{*2} + 2\tau_i^* D_i^* - \frac{\sigma_i}{2} \widetilde{D}_i^2 + \frac{\sigma_i}{2} D_i^{*2} - \frac{\sigma_i}{2} \|\widetilde{\theta}_i\|^2 \\
&\quad + \frac{\sigma_i}{2} \|\theta_i^*\|^2 \leq -\left(\alpha_i + \gamma_i - \frac{1}{2}\right) e_{is}^2 - \frac{1}{2} \left(\sigma_i - \frac{1}{\psi_i}\right) \\
&\quad \cdot \|\widetilde{\theta}_i\|^2 - \frac{1}{2} (\sigma_i - 2\kappa \tau_i^*) \widetilde{D}_i^2 + \left(\frac{(\alpha_i + \gamma_i)^2 \tau_i^{*2}}{2}\right. \\
&\quad \left. + \frac{\tau_i^{*2} \psi_i}{2} + 2 \|\theta_i^*\| + \delta_i + 2\tau_i^* D_i^* + \frac{\sigma_i}{2} \|\theta_i^*\|^2\right. \\
&\quad \left. + \left(2\kappa \tau_i^* + \frac{\sigma_i}{2}\right) D_i^{*2}\right) \leq -\rho_{i1} V_i + \rho_{i2},
\end{aligned} \tag{30}$$

where ρ_{i1} and ρ_{i2} are given by the following form:

$$\begin{aligned}
\rho_{i1} &:= \min \left\{ 2 \left(\alpha_i + \gamma_i - \frac{1}{2} \right), \frac{\sigma_i - 1/\psi_i}{\lambda_{\max}(\Gamma_i^{-1})}, (\sigma_i - 2\kappa \tau_i^*) \right. \\
&\quad \left. \cdot \beta_i \right\} \\
\rho_{i2} &:= \frac{(\alpha_i + \gamma_i)^2 \tau_i^{*2}}{2} + \frac{\tau_i^{*2} \psi_i}{2} + 2 \|\theta_i^*\| + \delta_i + 2\tau_i^* D_i^* \\
&\quad + \frac{\sigma_i}{2} \|\theta_i^*\|^2 + \left(2\kappa \tau_i^* + \frac{\sigma_i}{2} \right) D_i^{*2}.
\end{aligned} \tag{31}$$

By appropriately adjusting the design parameters μ_i , α_i , β_i , γ_i , σ_i , δ_i , and ψ_i such that $\alpha_i + \gamma_i - 1/2 > 0$, $\sigma_i - 1/\psi_i > 0$, and $\sigma_i - 2\kappa\tau_i^* > 0$, respectively, the closed-loop system stability can be guaranteed. Finally, by multiplying $e^{\rho_{i1}t}$ and integrating over $[0, t]$ on both sides of (30), we have

$$V_i \leq \left(V_i|_{t=0} - \frac{\rho_{i2}}{\rho_{i1}} \right) e^{-\rho_{i1}t} + \frac{\rho_{i2}}{\rho_{i1}} \leq V_i|_{t=0} + \frac{\rho_{i2}}{\rho_{i1}}. \quad (32)$$

It follows from (22) and (32) that

$$\begin{aligned} |e_{is}| &\leq \sqrt{2 \left(V_i|_{t=0} + \frac{\rho_{i2}}{\rho_{i1}} \right)} \\ |\bar{D}_i| &\leq \sqrt{2\beta_i \left(V_i|_{t=0} + \frac{\rho_{i2}}{\rho_{i1}} \right)} \\ \|\tilde{\theta}_i\| &\leq \sqrt{\frac{2 \left(V_i|_{t=0} + \rho_{i2}/\rho_{i1} \right)}{\lambda_{\min}(\Gamma_i^{-1})}}. \end{aligned} \quad (33)$$

Consequently, the parameter estimation errors \bar{D}_i , $\tilde{\theta}_i$ and tracking error e_s are bounded from (31) and remain as the compact sets $\Omega_{\bar{D}_i}$, $\Omega_{\tilde{\theta}_i}$, and $\Omega_{e_{is}}$ in the sense that

$$\begin{aligned} \Omega_{e_{is}} &= \left\{ e_{is} \in \mathbb{R} \mid |e_{is}| \leq \sqrt{\Omega_i} \right\} \\ \Omega_{\bar{D}_i} &= \left\{ \bar{D}_i \in \mathbb{R} \mid |\bar{D}_i| \leq \sqrt{\beta_i \Omega_i} \right\} \\ \Omega_{\tilde{\theta}_i} &= \left\{ \tilde{\theta}_i \in \mathbb{R}^n \mid \|\tilde{\theta}_i\| \leq \sqrt{\frac{\Omega_i}{\lambda_{\min}(\Gamma_i^{-1})}} \right\}, \quad i = 1, 2, \end{aligned} \quad (34)$$

where $\Omega_i = 2(V_i|_{t=0} + \rho_{i2}/\rho_{i1})$ with ρ_{i1} and ρ_{i2} defined as (31). The proof is completed. \square

Remark 10. It is worth mentioning that the authors in [13] considered the adaptive fuzzy tracking control problem for a class of SISO nonaffine nonlinear systems. However, the approach proposed in [13] cannot be applied to MIMO nonaffine nonlinear casting systems with immeasurable coupling states. In this paper, the mean value theorem and the fuzzy approximation method are employed to transform the nonaffine nonlinear systems to the corresponding affine nonlinear systems. Also, based on Lyapunov stability analysis, it is shown that the proposed adaptive fuzzy output tracking control scheme can guarantee that the roll gap x_g and the molten steel level x_h can track to the empirical reference signals.

4. Simulations and Experimental Analysis

In this section, in order to verify the effectiveness of the proposed adaptive fuzzy control method, the following numerical simulation is performed for MIMO nonaffine nonlinear system (3), and the corresponding system parameters are selected as $R = 150$ mm, $L = 200$ mm, $M = 300$ kg, and $v_r = 10$ mpm. These values are chosen from [9]. In addition, the initial roll gap and the desired roll gap are set to be 0 mm and 3 mm, and the initial molten steel level and desired molten steel level are set to be 20 mm and 70 mm, respectively.

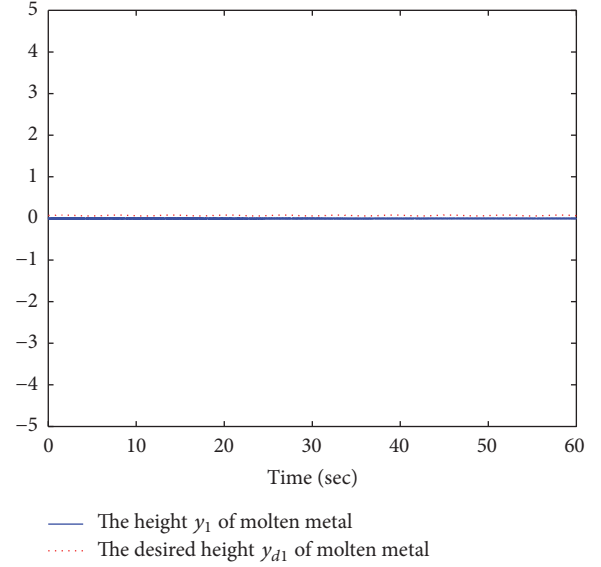


FIGURE 2: Trajectories of the system output y_1 and the desired reference signal y_{d1} .

Furthermore, the fuzzy membership functions are chosen as follows:

$$\begin{aligned} \mu_{F_j^1} &= \exp \left[-\frac{(X_j + 1.5)^2}{2} \right], \\ \mu_{F_j^2} &= \exp \left[-\frac{(X_j + 1)^2}{2} \right], \\ \mu_{F_j^3} &= \exp \left[-\frac{(X_j + 0.5)^2}{2} \right], \\ \mu_{F_j^4} &= \exp \left[-\frac{X_j^2}{2} \right], \\ \mu_{F_j^5} &= \exp \left[-\frac{(X_j - 0.5)^2}{2} \right], \\ \mu_{F_j^6} &= \exp \left[-\frac{(X_j - 1)^2}{2} \right], \\ \mu_{F_j^7} &= \exp \left[-\frac{(X_j - 1.5)^2}{2} \right], \end{aligned} \quad (35)$$

$j = 1, 2, \dots, n.$

Define fuzzy basis functions as

$$\phi_i(X) = \frac{\prod_{j=1}^n \mu_{F_j^i}(X_j)}{\sum_{i=1}^7 \left[\prod_{j=1}^n \mu_{F_j^i}(X_j) \right]}, \quad i = 1, 2, \dots, 7, \quad (36)$$

where $X = [X_1, X_2, \dots, X_n]^T$.

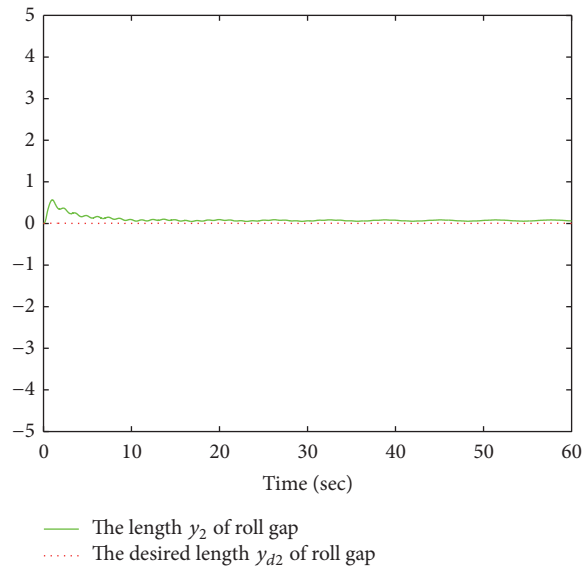


FIGURE 3: Trajectories of the system output y_2 and the desired reference signal y_{d2} .

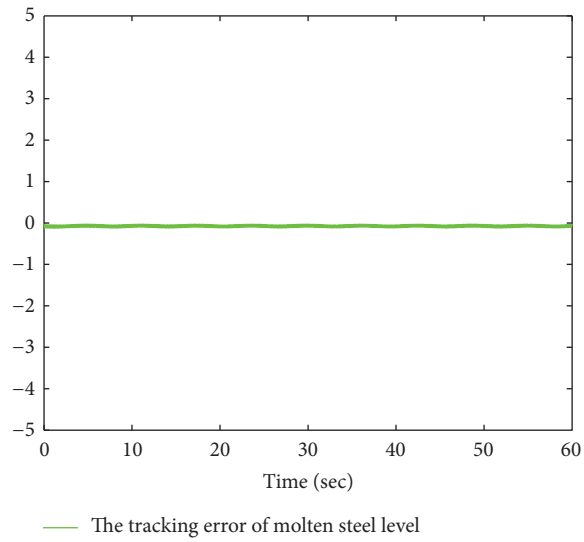


FIGURE 4: Trajectory of the system tracking error of molten steel level.

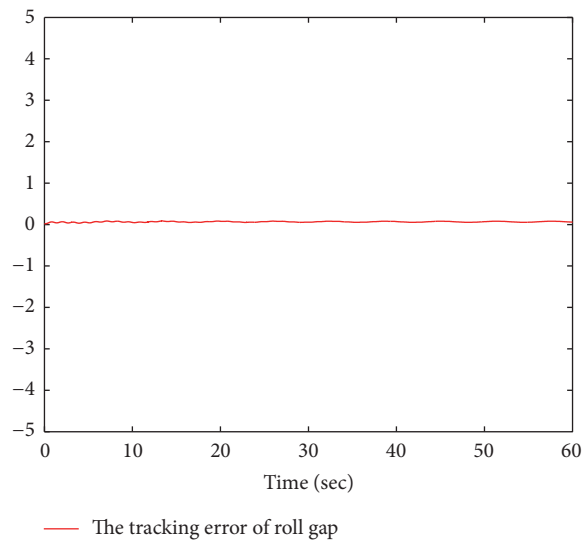


FIGURE 5: Trajectory of the system tracking error of roll gap.

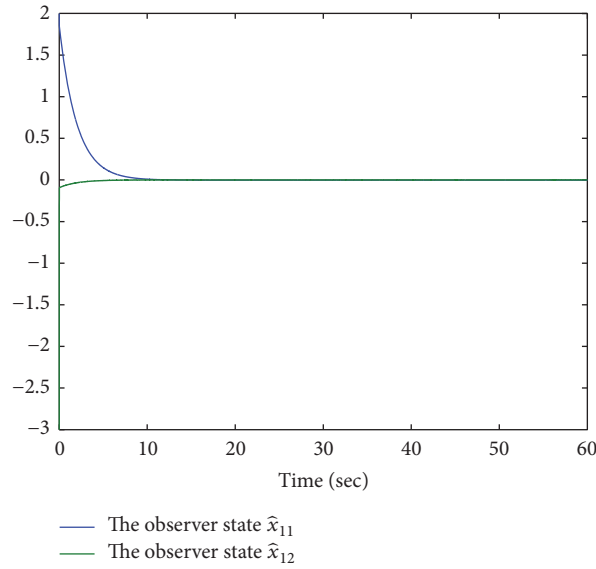


FIGURE 6: The observer states \hat{x}_{11} and \hat{x}_{12} .

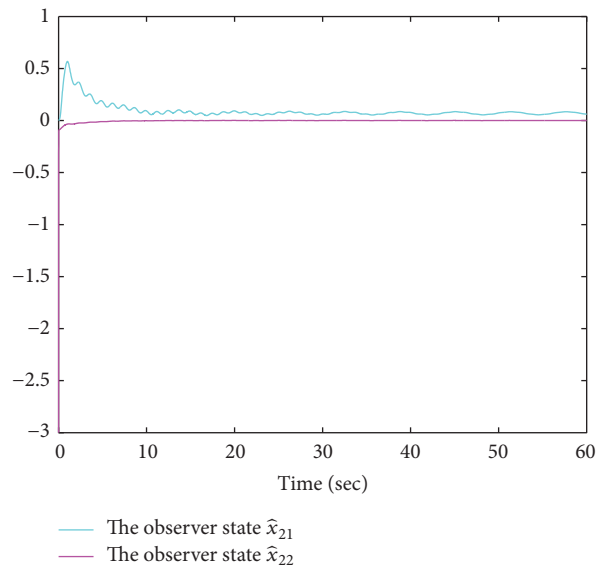


FIGURE 7: The observer states \hat{x}_{21} and \hat{x}_{22} .

Besides, the simulation parameters are chosen as $\mu_i = 5$, $\alpha_i = 5$, $\beta_i = 0.2$, $\Gamma_i = 10$, $\sigma_i = 10$, and $\delta_i = 0.1$, $i = 1, 2$, and the initial values are selected as $x_1(0) = [0.02, 0]^T$, $x_2(0) = [0, 0.1]^T$, $\hat{x}_1(0) = [2, -3]^T$, $\hat{x}_2(0) = [0, -1]^T$, $\hat{\theta}_1(0) = \hat{\theta}_2(0) = [0, 0.1, 0, 0.1, 0, 0, 0, 0.1]^T$, and $\widehat{D}_1(0) = \widehat{D}_2(0) = 5$. The simulation results are obtained in Figures 2–11. From Figures 2 and 3, the system output signals of y_i and the reference signals y_{di} , respectively, can be seen. Figures 4 and 5 show that the tracking performance is satisfactory and the output tracking errors of the roll gap and the molten steel level can converge to the desired neighborhoods of the origin. The curves of the estimations states are given in Figures 6 and 7. The boundedness of parameter estimations $\hat{\theta}_i$ and \widehat{D}_i , as well

as the designed control u_i , is demonstrated in Figures 8–11. Besides, the casting 7075 aluminum alloy strip is shown in Figure 12. It can also be seen that the casting strip is flat and there are no obvious cracks on the surface.

5. Conclusion

This paper studies the robust adaptive fuzzy tracking control problem for a class of twin-roll strip casting systems. Based on fuzzy logic systems (FLSs) to approximate the compounded nonlinear functions, a novel adaptive fuzzy output tracking control scheme is developed by using the high gain observer. The mean value theorem is employed to decouple the nonaffine nonlinear systems, and thus it is proved that all

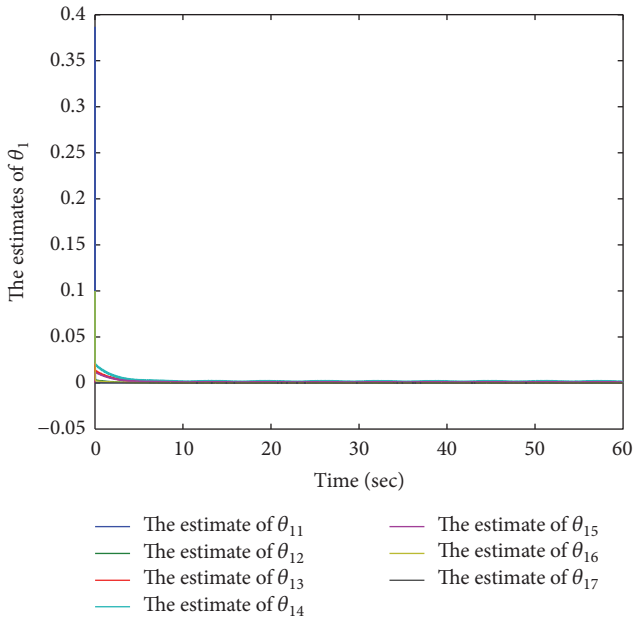


FIGURE 8: The response curves of $\hat{\theta}_1$.

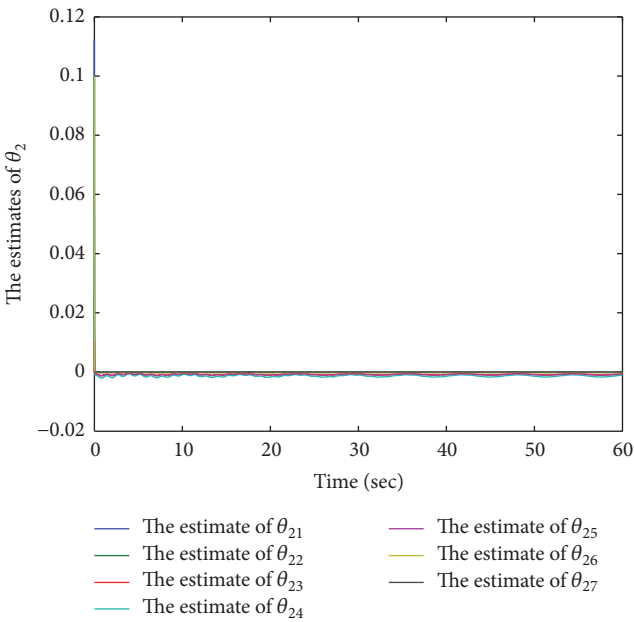


FIGURE 9: The response curves of $\hat{\theta}_2$.

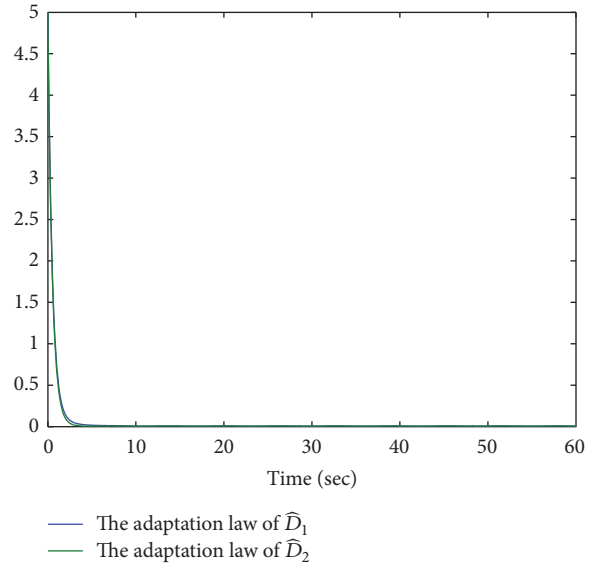


FIGURE 10: The response curves of the adaptive laws.

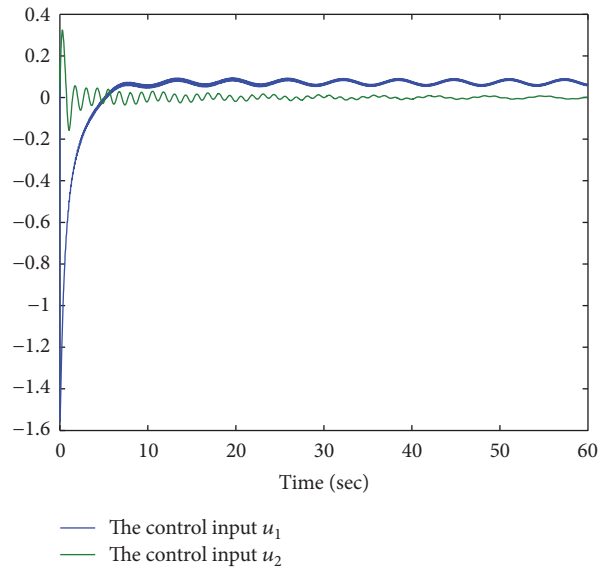


FIGURE 11: The respond curves of the control signals.

the closed-loop signals are bounded and the systems output tracking errors of the roll gap and the molten steel level can converge to the desired neighborhoods of the origin via the Lyapunov stability analysis. Simulation results show the effectiveness of the proposed adaptive control approach.

Conflicts of Interest

The authors declare that they have no conflicts of interest.



FIGURE 12: The casting 7075 aluminum alloy strip image showing the smooth surface in twin-roll strip process.

Acknowledgments

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