

Research Article

Analysis Bending Solutions of Clamped Rectangular Thick Plate

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The bending solutions of rectangular thick plate with all edges clamped and supported were investigated in this study. The basic governing equations used for analysis are based on Mindlin's higher-order shear deformation plate theory. Using a new function, the three coupled governing equations have been modified to independent partial differential equations that can be solved separately. These equations are coded in terms of deflection of the plate and the mentioned functions. By solving these decoupled equations, the analytic solutions of rectangular thick plate with all edges clamped and supported have been derived. The proposed method eliminates the complicated derivation for calculating coefficients and addresses the solution to problems directly. Moreover, numerical comparison shows the correctness and accuracy of the results.

1. Introduction

The bending problem of rectangular thick plates with various combinations of boundary conditions is sparingly common in many engineering fields, such as aerospace, concrete pavements, and mechanical and structural engineering. Moreover with the development of modern industry, relatively more accurate and practical studies on bending plate are required. Problems involving rectangular plates fall into three distinct categories [1]: (a) plates with all edges simply supported; (b) plates with a pair of opposite edges simply supported; (c) plates which do not fall into any of the above categories.

The classical plate theory (CPT) is frequently used to analyze thin plates. This theory works on the assumption to ignore the transverse shear deformation and assumes that the normal to the middle plane before deformation remains straight and normal to the middle surface after deformation. Therefore, utilizing classical plate theory to analyze thick plates leads to somehow inaccurate and even wrong results.

Following classical plate theory, a series of theories have been developed by many researchers to analyze thick plates by taking account of the shear deformation, such as Mindlin's first-order, Reddy's third-order, and Reissner's higher-order shear deformation plate theory.

The couples governing differential equations of higher order could be obtained through the analogue theory mentioned above such as Mindlin's first-order, Reddy's third-order, and Reissner's higher-order shear deformation plate theory which have two more unknowns' variables in comparison with the classical plate theory. The following three types of methods can be used to solve the governing equation which are numerical methods including finite element method [2], Ritz energy method [3], and superposition method [4] and semianalytical methods which include Levy method [5], Navier method [5, 6], and the exact analytical methods which including symplectic geometry method [7, 8], and integral transform method [9]. The imposition of boundary conditions on the governing equations increased the mathematical complexity of the solution procedure. Therefore, the analytic bending solutions of rectangular thick plates are hard to solve. Furthermore, numerical methods could be used to solve the bending problems of plate. However, only the analytical method can give the exact solution, which is used to verify the results obtained from various numerical methods.

There are two common ways to deal with the plate problem. First way is to find new plate theories [10–15] which can reduce the number of unknown equations. Houari et al. [10] use the new simple higher-order shear deformation theory

to analyze bending and free vibration of functionally graded plates. Tounsi et al. [11] perform the new 3-unknown non-polynomial shear deformation theory for the buckling and vibration analyses of functionally graded material (FGM) sandwich plates. The abovementioned theories only dealt with three unknowns as the classical plate theory. Similarly, Beldjelili et al. [12] employ a four-variable refined plate theory to discuss the hygrothermomechanical bending behavior of sigmoid functionally graded material (S-FGM) plate resting on variable two-parameter elastic foundations. The second way is to find method to simplify the coupled governing differential equations of high order. For this purpose, decoupling method is used to handle such kind of problem.

In this context, the study focused on the improvement of modified Navier method to solve bending problem of rectangular plates with all edges clamped and supported. By using decoupling, modified Navier method has been modified into a new simple approach to solve the partial differential equations for Mindlin plate. In Section 2.1, the multiple differential equations have been decoupled while adding a new variable and ascending the equations' order. Two of the obtained equations are independent which can be solved directly, and another two equations are also much simpler than the already developed equations. In Section 2.2, generalized displacement variables in governing equations are obtained by using independent equations in the modified Navier method, and other solutions of the problem, namely, bending moments, have been obtained through related expressions of variables. In the end numerical comparison studies are shown to verify the results.

2. Solution for Rectangular Thick Plate

2.1. Decouple Mindlin Equations. The governing equations for bending problem of rectangular thick plates are given by

$$\nabla^2 w - \left(\frac{\partial \varphi_x}{\partial x} + \frac{\partial \varphi_y}{\partial y} \right) + \frac{q}{C} = 0 \quad (1)$$

$$\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{1-\mu}{2} \frac{\partial^2 \varphi_x}{\partial y^2} + \frac{1+\mu}{2} \frac{\partial^2 \varphi_y}{\partial x \partial y} + \frac{C}{D} \left(\frac{\partial w}{\partial x} - \varphi_x \right) = 0 \quad (2)$$

$$\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{1-\mu}{2} \frac{\partial^2 \varphi_y}{\partial x^2} + \frac{1+\mu}{2} \frac{\partial^2 \varphi_x}{\partial x \partial y} + \frac{C}{D} \left(\frac{\partial w}{\partial y} - \varphi_y \right) = 0, \quad (3)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, D is the flexural rigidity of the plate and its expression is $D = Eh^3/12(1-\mu^2)$, C is the shearing stiffness of the plate and its expression is $C = 5Eh/12(1+\mu)$, and E , μ , and h are the elastic module, Poisson's ratio and the thickness of the plate, respectively. w is the transverse deflection of the middle surface. φ_x and φ_y are the rotations of a normal line due to plate bending. q is the load distribution

function. The resultant bending moments, M_x and M_y , the twisting moments M_{xy} can be obtained; namely,

$$M_x = -D \left(\frac{\partial \varphi_x}{\partial x} + \mu \frac{\partial \varphi_y}{\partial y} \right) \quad (4)$$

$$M_y = -D \left(\frac{\partial \varphi_y}{\partial y} + \mu \frac{\partial \varphi_x}{\partial x} \right) \quad (5)$$

$$M_{xy} = \frac{D(1-\mu)}{2} \left(\frac{\partial \varphi_y}{\partial y} + \frac{\partial \varphi_x}{\partial x} \right). \quad (6)$$

Another new variable M is given for decoupling the governing equations (1)–(3). According to the left expression of (1), let

$$M = \frac{\partial \varphi_x}{\partial x} + \frac{\partial \varphi_y}{\partial y} \quad (7)$$

and (1) can be expressed as

$$\nabla^2 w - M + \frac{q}{C} = 0. \quad (8)$$

Taking partial derivative of (2) and (3) with respect to x and y , respectively, then considering (7), we can obtain

$$\nabla^2 M + \frac{C}{D} (\nabla^2 w - M) = 0. \quad (9)$$

Substituting (8) into (9), the independent differential equation about M yield is obtained as follows:

$$\nabla^2 M - \frac{q}{D} = 0. \quad (10)$$

Based on (8) and (10), the independent equation about w is obtained as follows:

$$\nabla^4 w = \frac{q}{D} - \frac{\nabla^2 q}{C}. \quad (11)$$

Taking partial derivative of (1) with respect to x and y , respectively, is as follows:

$$\nabla^2 \frac{\partial w}{\partial x} - \left(\frac{\partial^2 \varphi_x}{\partial x^2} + \frac{\partial^2 \varphi_y}{\partial x \partial y} \right) + \frac{1}{C} \frac{\partial q}{\partial x} = 0 \quad (12)$$

$$\nabla^2 \frac{\partial w}{\partial y} - \left(\frac{\partial^2 \varphi_y}{\partial y^2} + \frac{\partial^2 \varphi_x}{\partial x \partial y} \right) + \frac{1}{C} \frac{\partial q}{\partial y} = 0. \quad (13)$$

Multiply (12) by coefficient $(1+\mu)/2$, and then subtract with (2) to eliminate φ_y in the equation. The expression between w and φ_x yields

$$\nabla^2 \varphi_x - Q \varphi_x = -Q \frac{\partial w}{\partial x} - \frac{1+\mu}{1-\mu} \frac{\partial}{\partial x} \left(\nabla^2 w + \frac{q}{C} \right). \quad (14)$$

The same as the derivation for (14), the expression between w and φ_y can be obtained as

$$\nabla^2 \varphi_y - Q \varphi_y = -Q \frac{\partial w}{\partial y} - \frac{1+\mu}{1-\mu} \frac{\partial}{\partial y} \left(\nabla^2 w + \frac{q}{C} \right). \quad (15)$$

According to (8), further simplifying (14) and (15) yields

$$\begin{aligned}\nabla^2 \varphi_x - Q\varphi_x &= -Q \frac{\partial w}{\partial x} - \frac{1 + \mu}{1 - \mu} \frac{\partial M}{\partial x} \\ \nabla^2 \varphi_y - Q\varphi_y &= -Q \frac{\partial w}{\partial y} - \frac{1 + \mu}{1 - \mu} \frac{\partial M}{\partial y}.\end{aligned}\quad (16)$$

The basic governing equations are reexpressed as follows:

$$\nabla^2 M - \frac{q}{D} = 0 \quad (17a)$$

$$\nabla^4 w = \frac{q}{D} - \frac{\nabla^2 q}{C} \quad (17b)$$

$$\nabla^2 \varphi_x - Q\varphi_x = -Q \frac{\partial w}{\partial x} - \frac{1 + \mu}{1 - \mu} \frac{\partial M}{\partial x} \quad (17c)$$

$$\nabla^2 \varphi_y - Q\varphi_y = -Q \frac{\partial w}{\partial y} - \frac{1 + \mu}{1 - \mu} \frac{\partial M}{\partial y}. \quad (17d)$$

2.2. Solution Method of Decoupled Equation. Considering the example of rectangular thick plates with all edges clamp supported, the solution of basic governing equations (1)–(3) is obtained through the modified Navier method. First, the boundary condition equations for CCCC plates are given by

$$\begin{aligned}x = 0, a: \\ w &= 0; \\ \varphi_x &= 0; \\ \varphi_y &= 0 \\ y = 0, b: \\ w &= 0; \\ \varphi_x &= 0; \\ \varphi_y &= 0\end{aligned}\quad (18)$$

and (18) shows the basic form of boundary condition. The expressions of w , φ_x , and φ_y are assumed as double sine series

$$w(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{nm} \sin \alpha_n x \sin \beta_m y \quad (19)$$

$$\varphi_x(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} X_{nm} \sin \alpha_n x \sin \beta_m y \quad (20)$$

$$\varphi_y(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_{nm} \sin \alpha_n x \sin \beta_m y, \quad (21)$$

where $\alpha_n = n\pi/a$ and $\beta_m = m\pi/b$. Based on the definition of M as (7), the boundary condition for M is obtained as follows:

$$\begin{aligned}x = 0, a: M &= 0 \\ y = 0, b: M &= 0.\end{aligned}\quad (22)$$

The expression of M is also assumed as double sine series:

$$M(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} M_{nm} \sin \alpha_n x \sin \beta_m y. \quad (23)$$

Expanding q in the form of double sine series,

$$q = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Q_{nm} \sin \alpha_n x \sin \beta_m y, \quad (24)$$

where Q_{nm} is defined as

$$Q_{nm} = \frac{4}{ab} \int_0^a \int_0^b q(x, y) \sin \alpha_n x \sin \beta_m y dx dy. \quad (25)$$

Substituting (23) and (24) into (17a) gives

$$\begin{aligned}\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} M_{nm} (\alpha_n^2 + \beta_m^2) \sin \alpha_n x \sin \beta_m y \\ = -\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{Q_{nm}}{D} \sin \alpha_n x \sin \beta_m y.\end{aligned}\quad (26)$$

According to the uniqueness theorem of Fourier expansion, equating the coefficient M_{nm} in (26),

$$\begin{aligned}M_{nm} &= -\frac{Q_{nm}}{D(\alpha_n^2 + \beta_m^2)} = -\frac{4}{abD(\alpha_n^2 + \beta_m^2)} \\ &\cdot \int_0^a \int_0^b q(x, y) \sin \alpha_n x \sin \beta_m y dx dy.\end{aligned}\quad (27)$$

Substituting (27) into (23), the expression for M is obtained as follows:

$$\begin{aligned}M(x, y) &= -\frac{4}{abD} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[\frac{1}{(\alpha_n^2 + \beta_m^2)} \right. \\ &\cdot \int_0^a \int_0^b q(x, y) \sin \alpha_n x \sin \beta_m y dx dy \left. \right] \sin \alpha_n x \\ &\cdot \sin \beta_m y.\end{aligned}\quad (28)$$

Substituting (19) and (24) into (17b) yields the following result:

$$\begin{aligned}\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} W_{nm} (\alpha_n^4 + \beta_m^4 + 2\alpha_n^2 \beta_m^2) \sin \alpha_n x \sin \beta_m y \\ = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left(\frac{1}{D} + \frac{\alpha_n^2 + \beta_m^2}{C} \right) Q_{nm} \sin \alpha_n x \sin \beta_m y.\end{aligned}\quad (29)$$

According to the uniqueness theorem of Fourier expansion, W_{mn} is obtained as

$$W_{mn} = \left(\frac{1}{D(\alpha_n^2 + \beta_m^2)} + \frac{1}{C} \right) \cdot \left(\frac{4}{ab} \int_0^a \int_0^b q(x, y) \sin \alpha_n x \sin \beta_m y \, dx \, dy \right) \cdot (\alpha_n^2 + \beta_m^2)^{-1}. \quad (30)$$

Substituting (30) into (19), the expression of w is obtained as follows:

$$w(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\alpha_n^2 + \beta_m^2} \left(\frac{1}{D(\alpha_n^2 + \beta_m^2)} + \frac{1}{C} \right) \cdot \left(\frac{4}{ab} \int_0^a \int_0^b q(x, y) \sin \alpha_n x \sin \beta_m y \, dx \, dy \right) \cdot \sin \alpha_n x \sin \beta_m y. \quad (31)$$

And substituting (29) and (31) into (17c) and (17d) yielded

$$\begin{aligned} & \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} X_{nm} (\alpha_n^2 + \beta_m^2 + Q) \sin \alpha_n x \sin \beta_m y \\ &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[M_{nm} \frac{1+\mu}{1-\mu} + W_{nm} Q \right] \alpha_n \cos \alpha_n x \sin \beta_m y \\ & \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} Y_{nm} (\alpha_n^2 + \beta_m^2 + Q) \sin \alpha_n x \sin \beta_m y \\ &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left[M_{nm} \frac{1+\mu}{1-\mu} + W_{nm} Q \right] \beta_m \sin \alpha_n x \cos \beta_m y. \end{aligned} \quad (32)$$

Thus M_{nm} and W_{nm} obtained are shown as (27) and (30). Equating the unknown coefficients X_{nm} and Y_{nm} in (32), the expressions of φ_x and φ_y will be yielded as well. First unify the series core in a manner as follows:

$$\begin{aligned} \cos \alpha_n x &= \sum_{r=1}^{\infty} h_{rn} \sin \alpha_r x; \quad 0 < x < a \\ \cos \beta_m y &= \sum_{s=1}^{\infty} h_{sm} \sin \beta_s y; \quad 0 < y < b, \end{aligned} \quad (33)$$

where

$$h_{rn} = \begin{cases} \frac{4r}{\pi(r^2 - n^2)} & n+r = \text{odd}, n-r = \text{odd} \\ 0 & n+r = \text{even}, n-r = \text{even} \end{cases} \quad (34)$$

$$h_{sm} = \begin{cases} \frac{4s}{\pi(s^2 - m^2)} & s+m = \text{odd}, m-s = \text{odd} \\ 0 & m-s = \text{even}, m-s = \text{even} \end{cases}$$

based on (33), expand $\cos \alpha_n x$ and $\cos \beta_m y$ in the form of sin Fourier series, and reset the dummy variables:

$$\begin{aligned} & \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sin \alpha_n x \sin \beta_m y X_{nm} (\alpha_n^2 + \beta_m^2 + Q) \\ &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \alpha_n x \sin \beta_m y \sum_{r=1}^{\infty} \left[M_{rm} \frac{1+\mu}{1-\mu} + W_{rm} Q \right] \\ & \cdot h_{nr} \alpha_r \\ & \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sin \alpha_n x \sin \beta_m y Y_{nm} (\alpha_n^2 + \beta_m^2 + Q) \\ &= \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \sin \alpha_n x \sin \beta_m y \\ & \cdot \sum_{s=1}^{\infty} \left[M_{ns} \frac{1+\mu}{1-\mu} + W_{ns} Q \right] h_{ms} \beta_s. \end{aligned} \quad (35)$$

Finally, equating the coefficients in (35) directly according to the uniqueness theorem of Fourier expansion,

$$X_{nm} = -(\alpha_n^2 + \beta_m^2 + Q)^{-1} \sum_{r=1}^{\infty} \left[M_{rm} \frac{1+\mu}{1-\mu} + W_{rm} Q \right] \cdot h_{nr} \alpha_r \quad (36)$$

$$Y_{nm} = -(\alpha_n^2 + \beta_m^2 + Q)^{-1} \sum_{s=1}^{\infty} \left[M_{ns} \frac{1+\mu}{1-\mu} + W_{ns} Q \right] \cdot h_{ms} \beta_s. \quad (37)$$

Substituting (36) and (37) into (20) and (21), respectively, the expressions of φ_x and φ_y are obtained as follows:

$$\varphi_x(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\alpha_n^2 + \beta_m^2 + Q} \sum_{r=1}^{\infty} \frac{Q_{rm}}{\alpha_r^2 + \beta_m^2} \left\{ \frac{1+\mu}{D(1-\mu)} + \left[\frac{1}{D(\alpha_r^2 + \beta_m^2)} + \frac{1}{C} \right] Q \right\} h_{nr} \alpha_r \sin \alpha_n x \sin \beta_m y \quad (38)$$

$$\varphi_y(x, y) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{1}{\alpha_n^2 + \beta_m^2 + Q} \sum_{s=1}^{\infty} \frac{Q_{ns}}{\alpha_n^2 + \beta_s^2} \left\{ \frac{1+\mu}{D(1-\mu)} + \left[\frac{1}{D(\alpha_n^2 + \beta_s^2)} + \frac{1}{C} \right] Q \right\} h_{ms} \beta_s \sin \alpha_n x \sin \beta_m y. \quad (39)$$

TABLE 1: Nondimensional deflection and moment of a square plate, under uniform pressure and clamped (CCCC) boundary conditions.

a/b	m, n	$w _{x=a/2, y=b/2} (qa^4 \cdot D^{-1})$	$M_x _{x=0, y=b/2} (qa^2)$
3	$m, n = 10$	-0.003615	-0.04225
	$m, n = 15$	-0.003610	-0.04295
	$m, n = 20$	-0.003611	-0.04353
	FEM results	-0.00361	-0.0435
5	$m, n = 10$	-0.002116	-0.04008
	$m, n = 15$	-0.002114	-0.04321
	$m, n = 20$	-0.002114	-0.04589
	FEM results	-0.00211	-0.0458
10	$m, n = 10$	-0.001483	-0.0432
	$m, n = 15$	-0.001483	-0.04739
	$m, n = 20$	-0.001483	-0.05140
	FEM results	-0.00148	-0.0510

Similarly substituting (38) and (39) into (4) and (5), the expressions of M_x and M_y can be obtained as follows:

$$\begin{aligned}
M_x = & -D \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left\{ \alpha_n \cos \alpha_n x \sin \beta_m y (\alpha_n^2 + \beta_m^2 + Q)^{-1} \right. \\
& \cdot \sum_{r=1}^{\infty} \left[-\frac{Q_{rm}}{D(\alpha_r^2 + \beta_m^2)} \frac{1+\mu}{1-\mu} + Q \left(\frac{Q_{rm}}{D} + \frac{\alpha_r^2 + \beta_m^2}{C} \right) (\alpha_r^4 + \beta_m^4 + 2\alpha_r^2 \beta_m^2)^{-1} \right] h_{nr} \alpha_r + \mu \beta_m \sin \alpha_n x \cos \beta_m y \\
& \cdot (\alpha_n^2 + \beta_m^2 + Q)^{-1} \sum_{s=1}^{\infty} \left[-\frac{Q_{ns}}{D(\alpha_n^2 + \beta_s^2)} \frac{1+\mu}{1-\mu} + Q \left(\frac{Q_{ns}}{D} + \frac{\alpha_n^2 + \beta_s^2}{C} \right) (\alpha_n^4 + \beta_s^4 + 2\alpha_n^2 \beta_s^2)^{-1} \right] h_{ms} \beta_s \left. \right\} \quad (40) \\
M_y = & -D \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \left\{ \beta_m \sin \alpha_n x \cos \beta_m y (\alpha_n^2 + \beta_m^2 + Q)^{-1} \right. \\
& \cdot \sum_{s=1}^{\infty} \left[-\frac{Q_{ns}}{D(\alpha_n^2 + \beta_s^2)} \frac{1+\mu}{1-\mu} + Q \left(\frac{Q_{ns}}{D} + \frac{\alpha_n^2 + \beta_s^2}{C} \right) (\alpha_n^4 + \beta_s^4 + 2\alpha_n^2 \beta_s^2)^{-1} \right] h_{ms} \beta_s + \mu \alpha_n \cos \alpha_n x \sin \beta_m y \\
& \cdot (\alpha_n^2 + \beta_m^2 + Q)^{-1} \sum_{r=1}^{\infty} \left[-\frac{Q_{rm}}{D(\alpha_r^2 + \beta_m^2)} \frac{1+\mu}{1-\mu} + Q \left(\frac{Q_{rm}}{D} + \frac{\alpha_r^2 + \beta_m^2}{C} \right) (\alpha_r^4 + \beta_m^4 + 2\alpha_r^2 \beta_m^2)^{-1} \right] h_{nr} \alpha_r \left. \right\}.
\end{aligned}$$

3. Numerical Example

A thick plate with all edges clamped (CCCC) has been taken as a numerical example to justify the correctness of the above solution. The length and width of the plate are $a = b$, with the Poisson ratio of $\mu = 0.3$. Figure 1 shows the change in deflection of the plate. Table 1 highlights the comparison of nondimensional deflection results with solutions given by FEM, which shows that the results obtained are in accordance with the ones given before, and proves the correctness of the above method and the derivations.

4. Conclusion

The decoupling method and the modified Navier's solution have been used together in this study for a simple analysis of rectangular thick plates with all edges clamped and supported. Unlike the original modified Navier method, the proposed approach does not need complicated matrix derivations for calculating the coefficients. The procedure for solving the bending rectangular thick plates with all edges clamped is made simpler than before. Moreover the proposed method can be further extended to address the problem

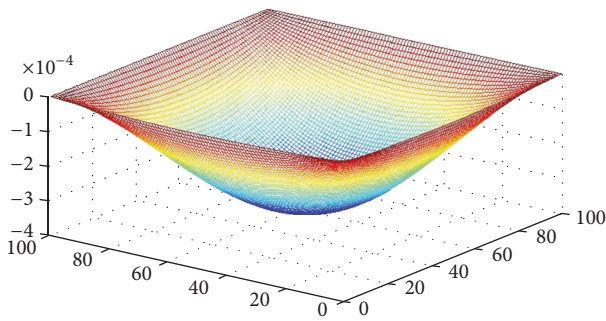


FIGURE 1: The deflections of a square plate.

of rectangular thick plates with other combinations of free and simply supported boundary conditions. The proposed method has many practical applications and can be used in foundation design of high-rise building and rigid pavements of highway and airport. Additionally, the plate support problems such as point supports and spring supports can be solved well analytically by utilizing similar approach, which would expectantly develop inspiring extensions in the field. Moreover, the results obtained from numerical example validate the precision and correctness of method and derivations.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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