

Research Article

Optimization Model for Multi-Vehicle-Type Structure in Urban Bus Systems

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Received 8 December 2016; Revised 15 March 2017; Accepted 27 March 2017; Published 9 April 2017

Academic Editor: Hakim Naceur

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Nowadays, bus lines commonly share terminals, especially in urban areas, because an independent operation of each line would cause a large waste of resources. In addition, if lines share terminal all lines can be monitored by unified management, while the vehicles of different sizes can complement each other. Although the dispatcher generally sets up fixed departure intervals for bus lines, there are differences between bus lines in terms of operating time distribution and line volume. Thus, a new optimization model for multi-vehicle-type structure in urban bus system is proposed in this paper. The optimization was conducted according to two main characteristics of bus systems, bus company expense and bus service quality. The optimization model was designed based on minimal possible expense and maximal possible quality. At the same time, we proposed a Markov method based on alternative relations to solve the model. The proposed model was verified by real urban bus system. Moreover, the simulations in MATLAB software environment have proven a high efficiency and applicability of the proposed method.

1. Introduction

A proper configuration of multi-vehicle-type structure and bus timetable in urban bus systems represents comprehensive research topic. As it is well known, public transportation represents a public benefit, and its main purpose is to satisfy the travel needs of residents. Although the public transportation has been studied for a long time, it still needs some improvements. Ferland and Michelon [1] defined the public transport problem and showed that heuristic and exact methods, developed for scheduling with time windows and single type of vehicles, can be applied on a multiple-vehicle-type structure. Moreover, Ceder [2] used empty trips to adjust the departure interval in order to arrange the lines. However, Ceder considered transport system with different vehicle types, wherein the vehicles were arranged according to their type, price, and quality. Ker et al. [3] used different sizes of vehicles on the same bus line and determined the optimal vehicle size for different time periods. In addition, in order to minimize the operating cost and passengers' travel cost, they develop the formula for calculation of a proper vehicle size. Site and Filippi [4] presented an optimal

bus service model for urban areas and optimized the lines scheduling for different operation modes, such as a multi-vehicle mode. In [5] Zhu et al. studied the bus line capacity and analyzed the factors that influence the bus line equipment. Namely, using the restrictions of bus line length, route turnover, and departure interval, they calculated the number of vehicles based on bus types and road conditions. Kliever et al. [6] discussed the multidepot multi-vehicle-type bus scheduling problem (MDVSP) considering multiple depots and different vehicle types for timetabled trips. In addition, MDVSP modeling presented in [6] was based on time-space-based networks instead of connection-based networks. In order to calculate the best size for passenger-carrying vehicles, dell'Olio et al. [7] proposed a bilevel optimization model with constraints on bus capacity, which allows buses of different sizes to be assigned to public transport routes and optimizes the headways on each route in accordance with observed demands. At the upper level, model optimizes social and operating cost of system, which represents the sum of user cost and operator cost. At the lower level, an assignment model for public transport with constraints on

vehicle capacity balances bus size and headways flow in each iteration. Sun et al. [8] used the heuristic algorithm to develop a flexible timetable optimization method based on hybrid vehicle size model to tackle the bus demand fluctuations in transit operation. Huang et al. [9] studied the public multi-vehicle transportation systems and constructed the model for balanced passenger rate based on optimization of departure intervals and different bus sizes. Moreover, the proposed model makes departure interval and maximum capacity of variation uniform. Finally, the suggested optimization model uses the heuristic algorithm and provides the case analysis. Based on the mathematical model of multi-vehicle distribution path optimization problem, Xiong and Hu [10] obtained the optimal vehicle scheduling method based on minimal fuel consumption as the objective function of genetic algorithm. Huo [11] studied the vehicle structure optimization and analyzed its influencing factor, vehicle type, and technical parameters. Furthermore, the author established an improved model based on Markov forecast method and verified it by vehicle structure optimization in Statistical Package for the Social Sciences software. In order to overcome time-consuming computations and easy fall into a local minimum, Luo [12] proposed an improved particle swarm optimization (IPSO) algorithm. In the IPSO algorithm, the Cauchy mutations were used to ensure larger mutation steps and to fall into a local minimal. Pan [13] introduced vehicle classification standards and related technical characteristics, then analyzed advantages and applicability of different vehicle types, and proposed a vehicle selection method. However, there is no research on optimization model for multi-vehicle-structure in urban bus systems.

In this paper, an optimization model for multi-vehicle-type structure in urban bus systems is presented. The proposed method was tested on real urban bus system. The obtained results have shown that proposed method provides better prediction and achieves better results than commonly used methods. The paper is organized as follows. In Section 2, a problem statement is explained, and all used assumptions and constraints are provided. The proposed model is presented in Section 3. In Section 4, we propose a method to solve the optimization model, while its verification by real urban bus system is exposed in Section 5. Lastly, a brief conclusion is given in Section 6.

2. Problem Statement and Assumptions

In this section, a scheduling problem in the multi-vehicle bus systems will be explained. The optimization of multi-vehicle-type structure was based on some assumptions, which are listed in the following:

- (i) Bus lines share the same terminal, or the terminals of each bus line are very close to each other, so it can be considered that all lines share the same terminal.
- (ii) The parking function is provided.
- (iii) All buses start from and end at the same terminal.
- (iv) All buses belong to one bus company.

- (v) All bus lines have two fixed departure intervals, one for peak period and one for off-peak period, whereby the first one is smaller than the second one.
- (vi) Average daily traffic is considered constant.
- (vii) Average speed of buses is a constant value.
- (viii) All base fares are equal.
- (ix) There are K bus lines.
- (x) Work time of terminal is divided into i periods with the same length.
- (xi) There are J available bus types.

In conventional bus operating, the dispatcher generally sets up fixed departure interval. However, sometimes a dynamic departure interval is chosen, which has its own advantages and disadvantages. Nonetheless, due to restrictions on cost in real bus systems, the dispatcher mostly chooses two fixed departure intervals based on different traffic volumes, departure interval in peak period, and departure interval in off-peak period. Likewise, in this paper two fixed departure intervals were selected so that they satisfy the following conditions: $2 \leq t_{gk}, t_{pk} \leq 20$, where t_{gk} represents departure interval of bus line k in peak period and t_{pk} represents departure interval of bus line k in off-peak period.

As already mentioned the work time of terminal was divided into i intervals with the same length. The work time of the terminal is defined by $T = T_l - T_e$, where T_l is the latest operating time of bus lines and T_e is the earliest operating time of bus lines. Nevertheless, there are differences in operating distribution between bus lines. The number of passengers is also different on different lines. However, all bus lines should meet the volume demand, which is defined by

$$Q_{ik} \leq \sum_{k=1}^K \sum_{j=1}^J n_{ikj} \times C_j \times \eta_{ikj}, \quad (1)$$

where K is the number of bus lines, J is the number of available bus types, Q_{ik} represents the maximum passenger volume on bus line k in time period i , n_{ikj} denotes the number of j -type vehicles on line k in time period i , C_j is the passenger capacity of j -type vehicles, and η_{ikj} represents expected passenger volume in j -type vehicles on line k in time period i . Afterwards, the number of j -type buses, that is, j -type vehicles, on all lines in period i is defined by

$$N_{ij} = \sum_{k=1}^K n_{ikj}. \quad (2)$$

Hence, the total number of j -type vehicles is defined by

$$N_j = \max N_{ij}. \quad (3)$$

Meanwhile, the total number of vehicles is defined by

$$N = \sum_{j=1}^J N_j. \quad (4)$$

Moreover, the number of vehicles on line k should satisfy the capacity condition:

$$N_k \geq \text{EVEN} \left(\frac{60 \cdot l_k}{\bar{V}_{gk} \times t_{gk}} \right), \quad (5)$$

where N_k is the total number of vehicles on line k , l_k is the length of line k in kilometers, and \bar{V}_{gk} is the average speed of vehicles on line k in peak period. Therefore, (4) can be rewritten as

$$N = \sum_{j=1}^J N_j = \sum_{k=1}^K N_k. \quad (6)$$

3. Optimization Model

3.1. Bus Company Expense. The departure intervals were selected based on actual activities and passenger needs. If traffic volume is not considered, departure interval should meet the peak capacity demand for a single-type vehicle configuration, wherein vehicles of large or medium sizes are used. Therefore, the number of bus drivers would be strictly defined; thus, the human cost can be considered as a constant. Due to what was previously mentioned, human cost, employee benefits, and bonus were neglected in the bus company expense. On the other hand, operating and maintenance cost included repairs cost and fuel cost. In fact, the operating and maintenance cost is related to vehicle mileage. Therefore, the paper defines two cost types, production cost and vehicle mileage cost. Eventually, we analyzed three main parts of bus company expense: cost of vehicles utilization, operating and maintenance cost, and cost of carbon emission treatment.

The cost of vehicles utilization is defined by

$$C_c = \sum_{j=1}^J N_j \times c_j, \quad (7)$$

where c_j is the price of j -type vehicle. Subsequently, daily cost of vehicles utilizations is defined by

$$C_{cd} = \frac{\sum_{j=1}^J N_j \times c_j}{d}. \quad (8)$$

Furthermore, the operating and maintenance cost is defined by

$$C_y = \sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times r_j, \quad (9)$$

where m_{kj} is the number of departure times of j -type vehicles on bus line k on daily base and r_j is the unit operating cost of line k .

Moreover, the cost of carbon emission treatment is defined by

$$\begin{aligned} C_E &= \mu \sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times E_j \\ &= \mu \sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times \xi \times \frac{B_j}{100}, \end{aligned} \quad (10)$$

where μ is the unit cost of carbon treatment, E_j denotes the carbon emission, ξ represents the fuel consumption per 100 km of j -type vehicles, and B is the carbon emission factor.

The number of departure times of all lines on daily base is defined by

$$M = \sum_{j=1}^J M_j = \sum_{j=1}^J \sum_{i=1}^I \sum_{k=1}^K m_{ikj} = \sum_{j=1}^J \sum_{i=1}^I \sum_{k=1}^K n_{ikj}, \quad (11)$$

where M_j is the number of departure times of j -type vehicles on daily base. Thus, the bus company expense is defined by

$$\begin{aligned} C &= C_{cd} + C_y + C_E \\ &= \frac{\sum_{j=1}^J N_j \times c_j}{d} + \sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times r_j \\ &\quad + \mu \sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times E_j, \end{aligned} \quad (12)$$

where d represents the length of bus operation life. However, the operating cost of various types of vehicles and the cost of carbon treatment are related to bus mileage. Due to differences in lines' length and number of vehicle types on each line, the minimal sum of operating and maintenance cost and cost of carbon emission treatment in time period i is defined by

$$\begin{aligned} \min(C_{yi} + C_{Ei}) &= \min \left(\sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times r_j \right. \\ &\quad \left. + \mu \sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times E_j \right). \end{aligned} \quad (13)$$

3.2. Bus Service Quality. In addition to the bus company expense, we analyzed the bus service quality. Due to fixed departure intervals and fixed operating time of lines, the bus service quality can be determined by daily passenger capacity provided by all lines. As mentioned before, the bus fare is equal for all lines; thus, the bus service quality can be expressed by total daily passenger capacity. Considering that average passenger volume is constant, greater capacity would provide higher bus service quality. Consequently, bus service quality is defined by

$$S = p \sum_{k=1}^K \sum_{j=1}^J m_{kj} \times \eta_j, \quad (14)$$

where p is the base fare and η_j represents the expected passenger volume in j -type vehicles.

3.3. Multi-Vehicle-Type Model. The aim of this paper is to determine the total bus company expense and to calculate the bus service quality and, finally, to optimize the bus system in order to reduce the cost and to increase the quality. In fact, different interest groups have different opinion on weight coefficients α and β ; for instance, bus companies profit if α is much larger than β , while the opposite solution is more convenient for bus passengers; thus, we did not specify their values here. Therefore, the multi-vehicle-type configuration model can be described by

$$G = \max(-\alpha C + \beta S) = \max \left\{ -\alpha \left[\frac{\sum_{j=1}^J N_j \times c_j}{d} + \sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times r_j + \mu \sum_{k=1}^K \sum_{j=1}^J 2 \times l_k \times m_{kj} \times E_j + \beta p \sum_{k=1}^K \sum_{j=1}^J m_{kj} \times \eta_j \right] \right\}, \quad (15)$$

where C represents the bus company expense, S denotes the bus service quality, α is the weight coefficient of bus company expense, and β is the weight coefficient of bus service quality. Obviously, different stakeholders have different opinions on α and β . In this study, Delphi method was used to determine the numerical value of α and β . According to experts on public transport, the specific values represent the average values.

4. The Markov Method

The proportion of bus vehicle types on regular bus lines is affected by various random factors. The nonaftereffect property of vehicle structure can satisfy the analysis condition of Markov method; thus, the development trend of vehicle structure can be analyzed and predicted by Markov method.

However, it is difficult to obtain the transfer probability P_{ab} of a -type vehicle to b -type vehicle by Markov method. Namely, the important assumption in Markov method is that the state transition matrix does not change with time, while the structure of different bus vehicle structures changes in an unbalanced time series. Therefore, in order to overcome this problem, we established a multi-vehicle-type structure prediction model based on Markov method and alternative relation. Hence, for bus line k , one has the following:

- (a) The proportion of j -type buses in time period i is sum of $f_1(i), f_2(i), \dots, f_j(i)$:

$$\sum_{j=1}^J f_j(i) = 1. \quad (16)$$

- (b) a and b are different vehicle types contained in J vehicle types, and $a = 1, 2, \dots, J$. From time period i to time period $(i + 1)$, the proportion of a -type and b -type is $\Delta f_{ab}(i)$, and

$$f_j(i) = \sum_{j=1}^J \Delta f_{ab}(i). \quad (17)$$

- (c) In time period $(i + 1)$ the proportion of a -type is translated from 1 to J types in time period i ; therefore, it can be defined by

$$f_a(i + 1) = \sum_{b=1}^J \Delta f_{ab}(i). \quad (18)$$

- (d) There is no correlation between proportion of a -type and b -type acquisition, and the proportion of a -type is only related to the proportion of all types in the last time period $f_a(i)(i + 1)$.

- (e) $\Delta f_{ab}(i)$ from time period i to time period $(i + 1)$ is proportional to $f_a(i)$ and $f_b(i)$:

$$\Delta f_{ab}(i) = \Delta G_{ab}(i) \times f_a(i) \times f_b(i), \quad (19)$$

where G_{ab} is the scale factor whose value determines the value of $\Delta f_{ab}(i)$.

- (f) According to the above assumptions, from (17) and (19) we get

$$f_b(i) = \sum_{a=1}^J G_{ab} \times f_a(i) \times f_b(i). \quad (20)$$

Similarly, from (18) and (19), we get the following:

$$f_a(i + 1) = \sum_{b=1}^J G_{ba} \times f_b(i) \times f_a(i). \quad (21)$$

Since $f_a(i) \neq 0$, the optimization model can be obtained by (20) and (21):

$$\sum_{a=1}^J G_{ab} \times f_a(i) = 1, \quad (22)$$

$$\sum_{b=1}^J G_{ba} \times f_b(i) = \frac{f_a(i + 1)}{f_a(i)}.$$

- (g) Using the linear regression analysis in SPSS software, we first estimate the parameter G_{ab} ; namely, (22) is used to estimate the initial G_0 . Then, the objective function is selected by

$$R_s = \frac{1}{I} \sum_{i=1}^I \sum_{a=1}^J \left\{ \left[1 - \sum_{b=1}^J G_{ab} \times f_b(i) \right]^2 + \left[\frac{f_a(i + 1)}{f_a(i)} - \sum_{a=1}^J G_{ba} \times f_a(i) \right]^2 \right\}. \quad (23)$$

TABLE 1: Related information on bus lines.

Line	A	B
L [km]	11	8
Time range [h]	14	12
N_k	14	10
Vehicle type	Large	Large
C	110	110
t_g [min]	5	6
t_p [min]	8	8
Average vehicle speed in peak period [km/h]	19	16
Average vehicle speed in off-peak period [km/h]	22	20
Line character	Commuter service	Shopping service

Afterwards, the target condition of planning solutions is set; therefore, we get the minimum of R_s :

$$\min R_s = \frac{1}{I} \sum_{i=1}^I \sum_{a=1}^J \left\{ \left[1 - \sum_{b=1}^J G_{ab} \times f_b(i) \right]^2 + \left[\frac{f_a(i+1)}{f_a(i)} - \sum_{a=1}^J G_{ba} \times f_a(i) \right]^2 \right\}, \quad (24)$$

where $\sum_{a=1}^J f_a(i) = 1$. Lastly, the optimal solution G_{ab} can be obtained by (23)-(24).

5. Model Verification

Model verification was conducted with two bus lines that had the same terminal in Jilin. The base fare of both lines was 1 yuan. According to the traffic system, Line A connected several schools and enterprises; thus, Line A mainly serviced the commuter volume. On the other hand, Line B connected several shopping centers and residential areas; thus, Line B mainly serviced the shopping volume. All related information on bus lines is presented in Table 1. In addition, the maximum hourly passenger volume of both lines and the sum of maximum hourly passenger volumes are presented in Figure 1.

The operating time of both lines was 14 hours, and we divided that period into 14 periods of the same length. The technical parameters of vehicles are presented in Table 2.

According to the distribution of hourly passenger volume of Line A and Line B presented in Figure 1, it can be noticed that from 7:30 to 8:30 traffic volume was the highest. If we assume that expected passenger volume on Line A in that time period is 1, according to (5), the capacity of Line A is $Q_{13} = 1260 > 925$. The analysis of results has proven that Line A satisfies traffic volume condition in 7:30 to 8:30 period. Similarly, the capacity provided by Line B is $Q_{23} = 720 > 285$.

Subsequently, the analysis has shown that Line A also meets the traffic volume condition in mentioned period, but with a huge waste of resources. According to the results obtained by MATLAB software, it is reasonable to arrange 20 buses for two lines. If we assume that numbers of large vehicles, medium vehicles, and small vehicles are $N_1, N_2,$ and $N_3,$ respectively, then the best model configuration obtained

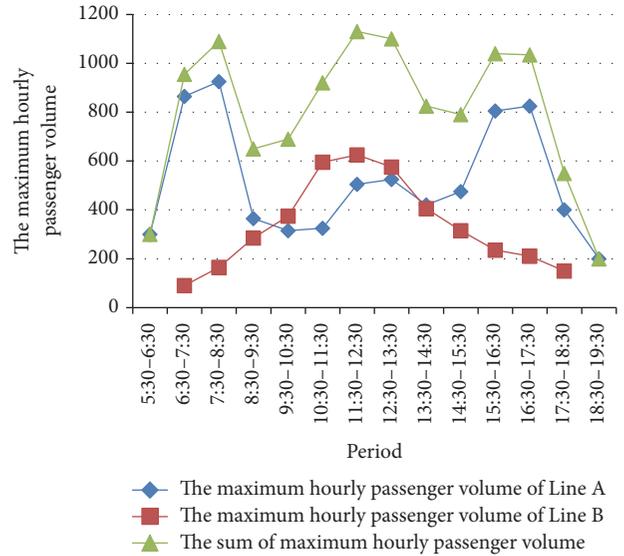


FIGURE 1: The maximum hourly passenger volumes on Lines A and B and their sum.

TABLE 2: Technical parameters of vehicles.

Index	Vehicle types	
	Large	Medium
C_j	80	50
C_c [yuan]	750000	550000
d [day]	2920	2920
C_y [yuan-km]	2.86	2.07
E_j [g/km]	62	57
r_j [yuan-g/km]	2.85	2.43
μ [yuan/km]	0.00005	0.00005
ξ [kg CO ₂ /L]	2.77	2.77
B [L]	33	26
Length of vehicle [m]	10.5	8.5
Fuel type	Diesel	Diesel

by SPSS software is $(N_1, N_2, N_3) = (8, 12, 0)$, while the optimal solutions for α and β obtained by Delphi method are $\alpha = 0.9$ and $\beta = 0.1$. The comparison of original vehicle

TABLE 3: Comparison of original vehicle configuration and optimal vehicle configuration obtained by the proposed model.

Parameter	Original vehicle configuration			Proposed model	Value added
	Line A	Line B	Sum		
N	14	10	24	20	-4
C_{cd} [yuan]	3595.890411	2568.49315	6164.38356	4315.068493	-1849.3151
C_y [yuan]	8557.12	2782.08	11339.2	12357.6	1018.4
C_E [yuan]	136.74936	61.42752	198.17688	172.93958	-25.2373
S [yuan]	10880	6720	17600	13010	-4590
G	-33111.16736	-10462.061	-43573.228	-13863.09122	29710.137

configuration and optimal vehicle configuration obtained by the proposed model is presented in Table 3.

According to the results presented in Table 3, it can be concluded that the proposed model can effectively reduce the bus company expense and improve the bus service quality.

6. Conclusion

An optimization model intended for multi-vehicle-type structure in urban bus systems is presented. The model is developed based on set of premises and assumptions. The multi-vehicle-type structure with two bus lines that have the same terminal was considered. Moreover, all buses had unified schedule in order to reduce the bus company expense and to improve the bus service quality. Furthermore, the impact of carbon treatment on the bus company expense was also analyzed. In addition, we analyzed the bus service quality and it was concluded that it can be measured by passenger capacity provided by all lines. The comparison of proposed model and original model was conducted according to the bus company expense and bus service quality, and it was concluded that proposed model can effectively reduce the bus company expense and improve the bus service quality.

Conflicts of Interest

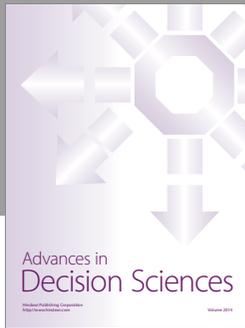
The authors declare that they have no conflicts of interest.

Acknowledgments

The authors thank Jilin Tongtai Traffic Science and Technology Consulting Co., Ltd., for bus lines data. The work was supported by the National Natural Science Foundation of China (Grant no. 51378237) and the National Natural Science Foundation of China (Grant no. 51608224).

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