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Research Article

Delay-Dependent Stability Analysis of TS Fuzzy Switched Time-Delay Systems

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This paper proposes a new approach to deal with the problem of stability under arbitrary switching of continuous-time switched time-delay systems represented by TS fuzzy models. The considered class of systems, initially described by delayed differential equations, is first put under a specific state space representation, called arrow form matrix. Then, by constructing a pseudo-overvaluing system, common to all fuzzy submodels and relative to a regular vector norm, we can obtain sufficient asymptotic stability conditions through the application of Borne and Gentina practical stability criterion. The stability criterion, hence obtained, is algebraic, is easy to use, and permits avoiding the problem of existence of a common Lyapunov-Krasovskii functional, considered as a difficult task even for some low-order linear switched systems. Finally, three numerical examples are given to show the effectiveness of the proposed method.

1. Introduction

Switched systems, seen as an important class of hybrid dynamical systems, are defined as a family of subsystems described by differential or difference equations and a rule that governs the switching between them [1, 2]. They have a strong engineering background in various areas and are often used to suitably model a great number of real-world systems, such as mechanical systems, chemical processes, communication networks, robotic systems, and aircraft and air traffic.

On the other hand, since the significant development that has witnessed the field of fuzzy modeling and control and more particularly model-based control, a new theory about fuzzy switched systems has emerged as an answer to more complicated real systems analysis and synthesis requirements such as multiple nonlinear systems, switched nonlinear systems, and second-order nonholonomic systems [3–10].

Originally inspired from the concept of sector nonlinearity, the main idea of Takagi-Sugeno fuzzy modeling is to partition the nonlinear system dynamics into several locally linearized models so that the overall nonlinear system could be represented by a sufficiently accurate approximation [11]. Hence, model-based control is considered as a powerful universal approximation tool and a reliable approach to deal with complex and ill-determined systems [12–15].

A fuzzy switched system is defined as a switched system which involves fuzzy models among its subsystems.

Despite the considerable efforts made for the analysis of nonlinear systems, stability study of switched systems and in particular of fuzzy switched systems is still complex. Indeed, the example of asymptotically stable subsystems which, due to a specific switching sequence, yield to an unstable behavior of the overall system is well known. Besides, the case of unstable subsystems that, via a particular stabilizing switching law, lead to a stable global system also exists.

The stability problem of such systems becomes more challenging when time delay is involved [16]. In fact, time delay often occurs in many dynamical systems, namely, in biological systems, chemical systems, metallurgical processing systems, and network systems. Its existence, whether in the state variables, control inputs, or the measurement outputs, is frequently a source of instability and poor performance.

The existing results on stability of TS fuzzy switched time-delay systems can be classified into two types: delay-independent criteria, which are applicable to delay of arbitrary size [17–20] and delay-dependent criteria, which include information on the size of the delay [21]. It is generally recognized that delay-dependent results are usually less conservative than delay-independent ones, especially when the size of the delay is small.

Stability analysis of switched systems in general has been conducted mainly on the basis of Lyapunov stability theory and concerns two major problems [22–24].

The first is related to the asymptotic stability of switched systems under arbitrary switching. An important result in this area states that a sufficient condition for the asymptotic stability of this class of systems is that all the subsystems share a common Lyapunov function [25–31]. Sufficient stability conditions are then derived through the resolution of a set of Linear Matrix Inequalities (LMIs).

In the case of switched time-delay systems, a common Lyapunov-Krasovskii functional is searched [32–34]. However, checking the existence of such a functional is a hard task even for some simple cases. Moreover, the method becomes less reliable when the number of subsystems switching between each other is important or when the number of fuzzy rules required to model each subsystem with a good accuracy is high.

A second problem concerns the stabilization of TS fuzzy switched systems by restricting the class of admissible switching signals to those in which the interval between any two consecutive switching instants is no smaller than a number τ called dwell time [35].

To overcome limitations due to the existence of Lyapunov-Krasovskii functionals, we propose, in this paper, to study the stability of TS fuzzy switched systems through the convergence of a regular vector norm, associated with a specific characteristic matrix, called arrow form matrix [36–43]. The proposed method is based on the construction of a common overvaluing/comparison system for all the fuzzy submodels and whose stability permits concluding to that of the original system. The obtained results, valuable for the case of arbitrary switching, are expressed in terms of simple algebraic conditions and explicitly involve the time delay.

The application of vector norms to switched systems has already been introduced in [44–47]. It has been extended later to switched time-delay systems in [48–53].

The remainder of the paper is organized as follows: Section 2 presents the problem formulation and some preliminaries. In Section 3, new delay-dependent stability conditions are derived for a class of TS fuzzy switched time-delay systems described by differential equations and having a single constant delay. Section 4 generalizes the main result to the case of TS fuzzy switched systems with multiple delays. Three numerical examples are provided in Section 5 to demonstrate the effectiveness of the proposed method. Finally, some concluding remarks are given in Section 6.

Notations. The notations used throughout this paper are fairly standard. $\mathfrak{R}^n$ denotes the n-dimensional Euclidean space, I_n is the identity matrix with appropriate dimensions, and $\|\cdot\|$ denotes Euclidean vector norm. For any $u = (u_i)_{1\le i\le n}$, $v = (v_i)_{1\le i\le n} \in \mathfrak{R}^n$, we define the scalar product of the vectors u and v as $\langle u, v\rangle = \sum_{i=1}^{n} u_i v_i$. $M = (m_{i,j})_{1\le i,j\le n}$, M^T and M^{-1} are its transpose and its inverse, respectively. We denote $M^* = (m_{i,j}{}^*)_{1\le i,j\le n}$ with $m_{i,j}{}^* = m_{i,j}$ if $i = j$ and $m_{i,j}{}^* = |m_{i,j}|$ if $i \ne j$ and $|M| = |m_{i,j}|$, $\forall 1 \le i, j \le n$.

2. Problem Formulation and Preliminaries

2.1. Problem Formulation. Let us consider the unforced switched nonlinear time-delay systems that are described by a differential equation of the form:

$$\begin{aligned}\dot{x}(t) &= f_{\sigma(t)}(x(t), x(t-\tau)) \\ x(t) &= \Phi(t), \quad t \in [-\tau, 0],\end{aligned} \tag{1}$$

where $x(t) \in \mathfrak{R}^n$ is the state vector, $\tau > 0$ is the time delay, $\Phi : [-\tau, 0] \to \mathfrak{R}^n$ is a differentiable vector valued initial function, $f_\sigma : \mathfrak{R}^n \to \mathfrak{R}^n$ are sufficiently regular functions that are parametrized by the index set $I = \{1, 2, \dots, N\}$, $\sigma(t) : \mathfrak{R}^n \to I$ is a piecewise constant function depending on time, called switching signal and assumed to be available in real time, and N is the number of subsystems.

The switching sequence is defined through a switching vector $\xi(t) = [\xi_1(t), \dots, \xi_N(t)]^T$ whose components $\xi_i(t)$ are given by

$$\xi_i(t) = \begin{cases} 1 & \text{if } \sigma(t) = i,\ i \in I \text{ (i.e. subsystem } i \text{ is active)} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

It is obvious that $\sum_{i=1}^{N} \xi_i(t) = 1\ \forall t \ge 0$.

Therefore, the switched system is composed of N subsystems expressed as

$$\begin{aligned}\dot{x}(t) &= f_i(x(t), x(t-\tau)), \quad i \in I \\ x(t) &= \Phi(t), \quad t \in [-\tau, 0].\end{aligned} \tag{3}$$

In addition, each subsystem $\sum_{f_i}$ is described by a set of IF-THEN rules; each rule is related to a region of the state space where the subsystem could be approximated by a local linear model.

Thus, the lth fuzzy rule associated with the ith subsystem is given by

$$\begin{aligned} R_i^l:\quad & \text{IF } z_1(t) \text{ is } M_{li}^1 \text{ and} \dots \text{and } z_p(t) \text{ is } M_{li}^p \\ & \text{THEN} \begin{cases} \dot{x}(t) = A_{li}x(t) + D_{li}x(t-\tau) \\ x(t) = \Phi(t), \quad t \in [-\tau, 0], \end{cases}\end{aligned} \tag{4}$$

where $z_1(t), \dots, z_p(t)$ are the premise variables, A_{li} and D_{li} ($l = 1, \dots, r$ and $i = 1, \dots, N$) are matrices of appropriate dimensions, M_{li}^{j} $(j = 1, \ldots, p)$ are the fuzzy sets, and r and p are the number of fuzzy rules and premise variables, respectively.

By using the product inference engine and the center of average defuzzification, the ith TS fuzzy subsystem can be inferred as

$$\dot{x}(t) = \sum_{l=1}^{r} h_{li}(z(t)) \left(A_{li}x(t) + D_{li}x(t-\tau)\right), \quad (5)$$

where $z(t) = [z_1(t), \ldots, z_p(t)]^T$ and $h_{li}(z(t)) : \Re^p \to [0,1]$ are the normalized weighting functions expressed by

$$h_{li}(z(t)) = \frac{w_{li}(z(t))}{\sum_{l=1}^{r} w_{li}(z(t))} = \frac{\prod_{j=1}^{p} M_{li}^{j}(z_j(t))}{\sum_{l=1}^{r} \prod_{j=1}^{p} M_{li}^{j}(z_j(t))}. \quad (6)$$

$M_{li}^{j}(z_j(t))$ is the firing strength of the membership function M_{li}^{j}. It is assumed that $w_{li}(z_j(t)) \geq 0$ and $\sum_{l=1}^{r} w_{li}(z_j(t)) > 0$.

Hence, $h_{li}(z(t)) \geq 0$ and $\sum_{l=1}^{r} h_{li}(z(t)) = 1$.

Finally, the fuzzy switched system (3) can be represented by

$$\dot{x}(t) = \sum_{i=1}^{N} \sum_{l=1}^{r} \xi_i(t) h_{li}(z(t)) \left(A_{li}x(t) + D_{li}x(t-\tau)\right). \quad (7)$$

Equation (7) can also be written as

$$\dot{x}(t) = \sum_{l=1}^{r} h_{l\sigma(t)}(z(t)) \left(A_{l\sigma(t)}x(t) + D_{l\sigma(t)}x(t-\tau)\right). \quad (8)$$

By using the Newton-Leibniz formula,

$$x(t-\tau) = x(t) - \int_{t-\tau}^{t} \dot{x}(\theta)\, d\theta. \quad (9)$$

Equation (8) can be written as

$$\begin{aligned}\dot{x}(t) = \sum_{i=1}^{N} \xi_i(t) \sum_{l=1}^{r} h_{li} \Big\{ &(A_{li} + D_{li})\, x(t) \\ &- (D_{li}A_{li}) \int_{t-\tau}^{t} x(\theta)\, d\theta - (D_{li})^2 \int_{t-\tau}^{t} x(\theta - \tau)\, d\theta \Big\}.\end{aligned} \quad (10)$$

In the sequel, $h_{li}(z(t))$ will be simplified by h_{li}.

2.2. Preliminaries. This subsection recalls some of the definitions and remarks that will be useful throughout this paper.

Kotelyanski Lemma (see [54]). *The real parts of the eigenvalues of matrix A, with non-negative off-diagonal elements, are less than a real number μ if and only if all those of matrix M; $M = \mu I_n - A$ are positive, with I_n the n identity matrix.*

Definition 1 (see [43]). The matrix $A(\cdot) = (a_{i,j}(\cdot))_{1\leq i,j\leq n}$ is called an M-matrix, if the following conditions are met:

(i) $a_{ii}(\cdot) > 0$ $(i = 1, \ldots, n)$, $a_{ij}(\cdot) \leq 0$ $(i \neq j;\ i, j = 1, \ldots, n)$;

(ii) the principal minors of $A(\cdot)$ are all positive:

$$(A(\cdot)) \begin{pmatrix} 1 & 2 & \cdots & j \\ 1 & 2 & \cdots & j \end{pmatrix} > 0, \quad j = 1, \ldots, n; \quad (11)$$

(iii) for any positive real vector $\eta = [\eta_1, \ldots, \eta_n]^T$, the algebraic equations $A(\cdot)x = \eta$ have a positive solution $w = [w_1, \ldots, w_n]^T$.

Remark 2. $A(\cdot) = (a_{i,j}(\cdot))_{1\leq i,j\leq n}$ is the opposite of an M-matrix if $(-A(\cdot))$ is an M-matrix.

Remark 3. A continuous-time system characterized by $A(\cdot)$ is stable if $A(\cdot)$ is the opposite of an M-matrix. In this case, the main minors of $A(\cdot)$ are of alternating signs (the first is negative) and the Kotelyanski lemma permits concluding to the stability of the system characterized by $A(\cdot)$.

Definition 4 (see [43, 55]). If $M_c(\cdot)$ is a pseudo-overvaluing matrix of the system $\dot{x}(t) = A_i(\cdot)x(t)$ with respect to the vector norm $p(x) = [|x_1|, \ldots, |x_n|]^T$, the following inequality is satisfied:

$$D^{+} p(x) \leq M_c(\cdot)\, p(x), \quad (12)$$

where D^{+} denotes the right hand derivative operator.

Consequently, the stability of the comparison system, $\dot{z}(t) = M_c(\cdot)z(t)$ with the initial conditions such as $z_0 = p(x_0)$, implies the same property for the initial system.

3. Delay-Dependent Stability Conditions for TS Fuzzy Switched Systems with Single Delay

3.1. Main Results. In this section, new delay-dependent conditions for global asymptotic stability of system (10) under arbitrary switching are stated.

Theorem 5. *System (10) is globally asymptotically stable under an arbitrary switching rule $\sigma(t) = i$, $i \in I$, if matrix*

$$T_m = \max_{\substack{1\leq i\leq N \\ 1\leq l\leq r}} (T_{li}) \quad (13)$$

is the opposite of an M-matrix, where

$$T_{li} = (A_{li} + D_{li})^{*} + \tau\left(\left|D_{li}A_{li}\right| + \left|D_{li}{}^{2}\right|\right). \quad (14)$$

Proof. Let $w \in \Re_{+}^{*n}$ with components $(w_m > 0, \forall m = 1, \ldots, n)$ and consider the radially unbounded candidate Lyapunov functional for each fuzzy submodel $\sum_{A_{li},D_{li}}$ given by

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad (15)$$

where

$$
\begin{aligned}
V_1(t) &= \langle |x(t)|, w \rangle \\
V_2(t) &= \left\langle |D_{li}A_{li}| \int_{-\tau}^{0} \int_{t+\theta}^{t} |x(s)|\, ds\, d\theta, w \right\rangle \\
V_3(t) &= \left\langle |D_{li}{}^2| \int_{-\tau}^{0} \int_{t+\theta}^{t} |x(s-\tau)|\, ds\, d\theta, w \right\rangle \\
V_4(t) &= \tau \left\langle |D_{li}{}^2| \int_{t-\tau}^{t} |x(s)|\, ds, w \right\rangle .
\end{aligned}
\tag{16}
$$

It is clear that $V(t_0) < \infty$ and that a common Lyapunov functional for all the submodels can be given by

$$
V_c(t) = \max_{\substack{1\le i\le N\\ 1\le l\le r}} (V(t)). \tag{17}
$$

The right Dini derivative of $V(t)$ along the trajectory of system (10) gives

$$
\begin{aligned}
D^+V(t)\big|_{(10)} &= \sum_{j=1}^{4} D^+V_j(t)\big|_{(10)} \\
D^+V_1(t)\big|_{(10)} &= \left\langle \frac{d^+|x(t)|}{dt^+}, w \right\rangle \\
&= \left\langle \operatorname{sgn}(x(t)) \frac{d^+x(t)}{dt^+}, w \right\rangle
\end{aligned}
\tag{18}
$$

with

$$
\operatorname{sgn}(x(t)) = \begin{pmatrix} \operatorname{sgn} x_1(t) & & \\ & \ddots & \\ & & \operatorname{sgn} x_n(t) \end{pmatrix}. \tag{19}
$$

Then,

$$
\begin{aligned}
&D^+V_1(t)\big|_{(10)} = \Big\langle \operatorname{sgn}(x(t)) \Big\{ (A_{li}+D_{li})\, x(t) \\
&- (D_{li}A_{li}) \int_{t-\tau}^{t} x(\theta)\, d\theta \\
&- (D_{li})^2 \int_{t-\tau}^{t} x(\theta-\tau)\, d\theta \Big\}, w \Big\rangle \le \big\langle (A_{li}+D_{li})^* \\
&\cdot |x(t)|, w \big\rangle + \left\langle |D_{li}A_{li}| \int_{t-\tau}^{t} |x(\theta)|\, d\theta, w \right\rangle \\
&+ \left\langle |D_{li}{}^2| \int_{t-\tau}^{t} |x(\theta-\tau)|\, d\theta, w \right\rangle
\end{aligned}
$$

$$
\begin{aligned}
&D^+V_2(t)\big|_{(10)} = \Big\langle |D_{li}A_{li}| \Big(\tau |x(t)| - \int_{t-\tau}^{t} |x(s)|\, ds \Big), \\
&w \Big\rangle = \langle \tau |D_{li}A_{li}|\, |x(t)|, w \rangle - \Big\langle |D_{li}A_{li}| \\
&\cdot \int_{t-\tau}^{t} |x(s)|\, ds, w \Big\rangle \\
&D^+V_3(t)\big|_{(10)} = \Big\langle |D_{li}{}^2| \Big(\tau |x(t-\tau)| \\
&- \int_{t-\tau}^{t} |x(s-\tau)|\, ds \Big), w \Big\rangle = \left\langle \tau |D_{li}{}^2|\, |x(t-\tau)|, w \right\rangle \\
&- \left\langle |D_{li}{}^2| \int_{t-\tau}^{t} |x(s-\tau)|\, ds, w \right\rangle \\
&D^+V_4(t)\big|_{(10)} = \left\langle \tau |D_{li}{}^2| (|x(t)| - |x(t-\tau)|), w \right\rangle \\
&= \left\langle \tau |D_{li}{}^2|\, |x(t)|, w \right\rangle - \left\langle \tau |D_{li}{}^2|\, |x(t-\tau)|, w \right\rangle .
\end{aligned}
\tag{20}
$$

Finally, we obtain

$$
\begin{aligned}
&D^+V(t)\big|_{(10)} \\
&\le \left\langle \left\{ (A_{li}+D_{li})^* + \tau \left(|D_{li}A_{li}| + |D_{li}{}^2| \right) \right\} |x(t)|, w \right\rangle \\
&\le \langle T_m |x(t)|, w \rangle ,
\end{aligned}
\tag{21}
$$

where

$$
T_m = \max_{\substack{1\le i\le N\\ 1\le l\le r}} \left\{ (A_{li}+D_{li})^* + \tau \left(|D_{li}A_{li}| + |D_{li}{}^2| \right) \right\}. \tag{22}
$$

On the other hand, if we suppose that T_m is the opposite of an M-matrix and according to the M-matrices properties, we can find a vector $\rho \in \mathfrak{R}_+^{*n}$ ($\rho_l > 0\ \forall l = 1, \dots, n$) satisfying the relation: $T_m{}^T w = -\rho$, $\forall w \in \mathfrak{R}_+^{*n}$.

Knowing that

$$
\langle T_m |x(t)|, w \rangle = \left\langle T_m{}^T w, |x(t)| \right\rangle , \tag{23}
$$

we can write

$$
\begin{aligned}
D^+V(t)\big|_{(10)} &\le \left\langle T_m{}^T w, |x(t)| \right\rangle = \langle -\rho, |x(t)| \rangle \\
&= -\sum_{l=1}^{n} \rho_l |x_l(t)| < 0.
\end{aligned}
\tag{24}
$$

This completes the proof of Theorem 5. □

3.2. Extension of the Results to the Case of TS Fuzzy Switched Systems Described by Delayed Differential Equations with Single Delay. In this section, we consider the class of TS fuzzy switched time-delay systems that are governed by the following differential equation:

$$\begin{aligned} & y^{(n)}(t)+\sum_{i=1}^{N}\xi_i(t) \\ & \quad \cdot\left\{\sum_{l=1}^{r}h_{li}\left(\sum_{j=0}^{n-1}a_{li}^{j}y^{(j)}(t)+\sum_{j=0}^{n-1}d_{li}^{j}y^{(j)}(t-\tau)\right)\right\} \\ & \quad = 0, \end{aligned} \tag{25}$$

where $y(t) \in \Re^n$ is the state vector and a_{li}^j and d_{li}^j ($l = 1, \ldots, r$; $i = 1, \ldots, N$; and $j = 0, \ldots, n-1$) are constant coefficients.

A change of variable under the form $x_{j+1}(t) = y^{(j)}(t)$, $j = 0, \ldots, n-1$, allows system (25) to be represented in the state space as follows:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{N}\xi_i(t)\sum_{l=1}^{r}h_{li}\left(A_{li}x(t)+D_{li}x(t-\tau)\right) \\ x(t) &= \Phi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{26}$$

where matrices A_{li} and D_{li} are given by

$$\begin{aligned} A_{li} &= \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -a_{li}^0 & \cdots & \cdots & -a_{li}^{n-1} \end{pmatrix}, \\ D_{li} &= \begin{pmatrix} 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \\ -d_{li}^0 & \cdots & \cdots & -d_{li}^{n-1} \end{pmatrix}. \end{aligned} \tag{27}$$

A change of base of the form $z(t) = Px(t)$ of (26) under the arrow form matrix gives

$$\begin{aligned} \dot{z}(t) &= \sum_{i=1}^{N}\xi_i(t)\sum_{l=1}^{r}h_{li}\left(M_{li}z(t)+N_{li}z(t-\tau)\right) \\ z(t) &= P\Phi(t), \quad t \in [-\tau, 0], \end{aligned} \tag{28}$$

where $M_{li} = P^{-1}A_{li}P$, $N_{li} = P^{-1}D_{li}P$, and P is the corresponding passage matrix such that

$$P = \begin{pmatrix} 1 & 1 & \cdots & 1 & 0 \\ \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 0 \\ (\alpha_1)^2 & (\alpha_2)^2 & \cdots & (\alpha_{n-1})^2 & \vdots \\ \vdots & \vdots & \cdots & \vdots & 0 \\ (\alpha_1)^{n-1} & (\alpha_2)^{n-1} & \cdots & (\alpha_{n-1})^{n-1} & 1 \end{pmatrix}. \tag{29}$$

Arrow form matrices M_{li}, $l = 1, \ldots, r$ and $i \in I$, are given by

$$M_{li} = \begin{pmatrix} \alpha_1 & & & \beta_1 \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & \beta_{n-1} \\ \gamma_{li}^1 & \cdots & \gamma_{li}^{n-1} & \gamma_{li}^n \end{pmatrix} \tag{30}$$

with

$$\begin{aligned} \beta_j &= \prod_{\substack{q=1 \\ q\neq j}}^{n-1}\left(\alpha_j-\alpha_q\right)^{-1} \quad \forall j = 1, \ldots, n-1 \\ \gamma_{li}^j &= -P_{A_{li}}\left(\alpha_j\right) \quad \forall j = 1, \ldots, n-1 \\ \gamma_{li}^n &= -a_{li}^{n-1}-\sum_{j=1}^{n-1}\alpha_j, \end{aligned} \tag{31}$$

whereas matrices N_{li} are given by

$$N_{li} = \begin{pmatrix} 0_{n-1,n-1} & & & 0_{n-1,1} \\ \delta_{li}^1 & \cdots & \delta_{li}^{n-1} & \delta_{li}^n \end{pmatrix} \tag{32}$$

with

$$\begin{aligned} \delta_{li}^j &= -P_{D_{li}}\left(\alpha_j\right) \quad \forall j = 1, \ldots, n-1 \\ \delta_{li}^n &= -d_{li}^{n-1} \\ P_{D_{li}}(\lambda) &= \sum_{j=0}^{n-1}d_{li}^j\lambda^j. \end{aligned} \tag{33}$$

Note that α_j, $j = 1, \ldots, n-1$, are distinct constant parameters that can be chosen arbitrarily and $P_{A_{li}}(\lambda)$ is the instantaneous characteristic polynomial of matrix A_{li} given by

$$P_{A_{li}}(\lambda) = \lambda^n+\sum_{j=0}^{n-1}a_{li}^j\lambda^j. \tag{34}$$

Therefore, the expression of the common comparison matrix in the new base is given through the sum of the following terms:

$$\begin{aligned} & \left(M_{li}+N_{li}\right)^* \\ & \quad = \begin{pmatrix} \alpha_1 & & & \left|\beta_1\right| \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & \left|\beta_{n-1}\right| \\ \left|\gamma_{li}^1+\delta_{li}^1\right| & \cdots & \left|\gamma_{li}^{n-1}+\delta_{li}^{n-1}\right| & \gamma_{li}^n+\delta_{li}^n \end{pmatrix} \end{aligned}$$

$$|N_{li}M_{li}| = \begin{pmatrix} 0_{n-1,n-1} & & & 0_{n-1,1} \\ |\psi_{li}(\alpha_1)| & \cdots & |\psi_{li}(\alpha_{n-1})| & \left|\delta_{li}^n\gamma_{li}^n + \sum_{j=1}^{n-1}\delta_{li}^j\beta_j\right| \end{pmatrix}$$

$$|N_{li}{}^2| = \begin{pmatrix} 0_{n-1,n-1} & & 0_{n-1,1} \\ |-\delta_{li}^1 d_{li}^{n-1}| & |-\delta_{li}^{n-1} d_{li}^{n-1}| & |(d_{li}^{n-1})^2| \end{pmatrix}. \quad (35)$$

The new polynomial $\psi_{li}(\alpha_j)$ is defined by

$$\psi_{li}(\alpha_j) = -P_{D_{li}}(\alpha_j) \times \alpha_j + d_{li}^{n-1} P_{A_{li}}(\alpha_j). \quad (36)$$

Notice that $|N_{li}M_{li}|$ can be simplified. In fact, $\delta_{li}^n\gamma_{li}^n + \sum_{j=1}^{n-1}\delta_{li}^j\beta_j = \mathrm{trace}(N_{li}M_{li})$. However, $\mathrm{trace}(N_{li}M_{li}) = \mathrm{trace}(D_{li}A_{li}) = a_{li}^{n-1}d_{li}^{n-1} - d_{li}^{n-2}$. Consequently, we can construct for each individual submodel $\sum_{A_{li},D_{li}}$ the following overvaluing system:

$$T_{li} = (M_{li} + N_{li})^* + \tau\left(|N_{li}M_{li}| + |N_{li}{}^2|\right), \quad (37)$$

where

$$T_{li} = \begin{pmatrix} \alpha_1 & & & |\beta_1| \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & |\beta_{n-1}| \\ t_{li}^1 & \cdots & t_{li}^{n-1} & t_{li}^n \end{pmatrix},$$

$$t_{li}^j = |\gamma_{li}^j + \delta_{li}^j| + \tau\left(|\psi_{li}(\alpha_j)| + |-\delta_{li}^j d_{li}^{n-1}|\right)$$

$$t_{li}^n = (\gamma_{li}^n + \delta_{li}^n) + \tau\left(|a_{li}^{n-1}d_{li}^{n-1} - d_{li}^{n-2}| + (d_{li}^{n-1})^2\right). \quad (38)$$

Finally, the common overvaluing matrix is given by

$$T_m = \begin{pmatrix} \alpha_1 & & & |\beta_1| \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & |\beta_{n-1}| \\ t_1 & \cdots & t_{n-1} & t_n \end{pmatrix} \quad (39)$$

with

$$t_j = \max_{\substack{1\le i\le N \\ 1\le l\le r}} (t_{li}^j) \quad \forall j = 1,\dots,n. \quad (40)$$

At this level, we can state the following theorem.

Theorem 6. *System (26) is globally asymptotically stable under an arbitrary switching rule $\sigma(t) = i$, $i \in I$, if there exist $\alpha_j < 0$, $\forall j = 1,\dots,n-1$, $\alpha_j \ne \alpha_q$, $\forall j \ne q$, satisfying the following condition:*

$$-t_n + \sum_{j=1}^{n-1} t_j |\beta_j| \alpha_j^{-1} > 0. \quad (41)$$

Proof. It is sufficient to check that the matrix

$$T_m = \max_{\substack{1\le i\le N \\ 1\le l\le r}} \left((M_{li} + N_{li})^* + \tau\left(|N_{li}M_{li}| + |N_{li}{}^2|\right)\right) \quad (42)$$

is the opposite of an M-matrix by verifying that all the successive principal minors of T_m, denoted by Δ_j, are of alternating signs; that is,

$$(-1)^j \Delta_j > 0 \quad \forall j = 1,\dots,n-1. \quad (43)$$

It is clear that, for an arbitrary choice of $\alpha_j < 0$, relation (43) is satisfied for $j = 1,\dots,n-1$.

Condition (43) becomes for $j = n$

$$\begin{aligned} (-1)^n \Delta_n &= (-1)^n \det(T_m) \\ &= (-1)^n \left[t_n - \sum_{j=1}^{n-1} t_j |\beta_j| \alpha_j^{-1}\right] \prod_{j=1}^{n-1}\alpha_j. \end{aligned} \quad (44)$$

By dividing (44) by $\kappa = (-1)^{n-1}\prod_{j=1}^{n-1}\alpha_j$, we obtain

$$-t_n + \sum_{j=1}^{n-1} t_j |\beta_j| \alpha_j^{-1} > 0. \quad (45)$$

This achieves the proof of Theorem 6. □

Remark 7. The maximum value of τ ensuring the asymptotic stability of each individual model $\sum_{A_{li},D_{li}}$ ($\tau < \tau_{\max}^{(li)}$) is computed by

$$\tau_{\max}^{(li)} = \frac{-(\gamma_{li}^n + \delta_{li}^n) + \sum_{j=1}^{n-1}\alpha_j^{-1}\left|-\left(P_{A_{li}}(\alpha_j) + P_{D_{li}}(\alpha_j)\right)\right||\beta_j|}{(d_{li}^{n-1})^2 + |a_{li}^{n-1}d_{li}^{n-1} - d_{li}^{n-2}| - \sum_{j=1}^{n-1}\alpha_j^{-1}\left(|\psi_{li}(\alpha_j)| + |P_{D_{li}}(\alpha_j)\, d_{li}^{n-1}|\right)|\beta_j|}. \quad (46)$$

Remark 8. Expression (46) can be widely simplified. We can give, for instance, two of the possible combinations.

Case 1. One has

$$\begin{aligned}\left(P_{D_{li}}\left(\alpha_j\right)+P_{A_{li}}\left(\alpha_j\right)\right)\beta_j&<0\\ \psi_{li}\left(\alpha_j\right)\beta_j&>0\\ P_{D_{li}}\left(\alpha_j\right)d_{li}^{n-1}\beta_j&>0.\end{aligned}\tag{47}$$

Then, stability conditions will depend on the sign of $(a_{li}^{n-1}d_{li}^{n-1}-d_{li}^{n-2})$ as follows:

$$\begin{aligned}&\tau_{\max}^{(li)}\\&=\frac{P_{A_{li}}(0)+P_{D_{li}}(0)}{2\kappa\left(d_{li}^{n-2}-a_{li}^{n-1}d_{li}^{n-1}\right)+d_{li}^{n-1}\left(P_{A_{li}}(0)+P_{D_{li}}(0)\right)}\\&\qquad\text{if } a_{li}^{n-1}d_{li}^{n-1}<d_{li}^{n-2}\end{aligned}\tag{48}$$

$$\tau_{\max}^{(li)}=\frac{1}{d_{li}^{n-1}}\quad\text{if } a_{li}^{n-1}d_{li}^{n-1}>d_{li}^{n-2},\ d_{li}^{n-1}>0.$$

Case 2. One has

$$\begin{aligned}&\left(P_{D_{li}}\left(\alpha_j\right)+P_{A_{li}}\left(\alpha_j\right)\right)\beta_j<0\\&\psi_{li}\left(\alpha_j\right)\beta_j<0\\&P_{D_{li}}\left(\alpha_j\right)d_{li}^{n-1}\beta_j<0\end{aligned}\tag{49}$$

$$\begin{aligned}&\tau_{\max}^{(li)}\\&=\frac{P_{A_{li}}(0)+P_{D_{li}}(0)}{2\kappa\left(\left(a_{li}^{n-1}d_{li}^{n-1}-d_{li}^{n-2}\right)+\left(d_{li}^{n-1}\right)^2\right)-d_{li}^{n-1}\left(P_{A_{li}}(0)+P_{D_{li}}(0)\right)}\\&\qquad\text{if } a_{li}^{n-1}d_{li}^{n-1}>d_{li}^{n-2}\end{aligned}\tag{50}$$

$$\begin{aligned}&\tau_{\max}^{(li)}=\frac{P_{A_{li}}(0)+P_{D_{li}}(0)}{2\kappa\left(d_{li}^{n-1}\right)^2-d_{li}^{n-1}\left(P_{A_{li}}(0)+P_{D_{li}}(0)\right)}\\&\qquad\text{if } a_{li}^{n-1}d_{li}^{n-1}<d_{li}^{n-2}.\end{aligned}\tag{51}$$

Proof. See the Appendix. □

4. Delay-Dependent Stability Conditions for TS Fuzzy Switched Systems with Multiple Delays

Stability criteria in Theorem 5 can be generalized to systems with multiple delays.

4.1. Main Result. Consider a switched system composed of N subsystems; each subsystem is a TS fuzzy time-delay system as shown in the following differential equation:

$$\begin{aligned}\dot{x}(t)&=\sum_{i=1}^{N}\xi_i(t)\sum_{l=1}^{r}h_{li}\left(A_{li}x(t)+\sum_{k=1}^{m}D_{li,k}x\left(t-\tau_k\right)\right)\\ x(t)&=\Phi(t),\quad t\in\left[-\max_{1\le k\le m}\tau_k,0\right],\end{aligned}\tag{52}$$

where A_{li}, $D_{li,k}$ ($l=1,\dots,r$; $i=1,\dots,N$; and $k=1,\dots,m$) are constant matrices of appropriate dimensions, h_{li} are fuzzy weighting factors, previously defined in (6), and $\tau_k>0$ $\forall k=1,\dots,m$.

System (52) can also be put in the following form:

$$\begin{aligned}\dot{x}(t)&=A_{\sigma(t)}x(t)+\sum_{k=1}^{m}D_{\sigma(t),k}x\left(t-\tau_k\right)\\ x(t)&=\Phi(t),\quad t\in\left[-\max_{1\le k\le m}\tau_k,0\right],\end{aligned}\tag{53}$$

where

$$\begin{aligned}A_{\sigma(t)}&=\sum_{i=1}^{N}\xi_i(t)\sum_{l=1}^{r}h_{li}A_{li},\\ D_{\sigma(t),k}&=\sum_{i=1}^{N}\xi_i(t)\sum_{l=1}^{r}h_{li}D_{li,k}.\end{aligned}\tag{54}$$

Theorem 9. *System (52) is globally asymptotically stable under arbitrary switching rule $\sigma(t)=i\in I$, if matrix*

$$\begin{aligned}T_{m,M}=\max_{\substack{1\le i\le N\\1\le l\le r}}&\left\{\left(A_{li}+\sum_{k=1}^{m}D_{li,k}\right)^{*}\right.\\&\left.+\sum_{k=1}^{m}\tau_k\left(\left|D_{li,k}A_{li}\right|+\left|D_{li,k}{}^{2}\right|\right)\right\}\end{aligned}\tag{55}$$

is the opposite of an M-matrix.

Proof. Following the same steps as those given in the proof of Theorem 5 and by choosing the common radially unbounded Lyapunov functional as follows,

$$V_{c,M}(t)=\max_{\substack{1\le i\le N\\1\le l\le r}}\left(V_M(t)\right)=\max_{\substack{1\le i\le N\\1\le l\le r}}\left(\sum_{p=1}^{4}V_{p,M}(t)\right),\tag{56}$$

where

$$\begin{aligned}V_{1,M}(t)&=\langle|x(t),w|\rangle\\ V_{2,M}(t)&=\sum_{k=1}^{m}\left\langle\left|D_{li,k}A_{li}\right|\int_{-\tau_k}^{0}\int_{t+\theta}^{t}|x(s)|\,ds\,d\theta,w\right\rangle\\ V_{3,M}(t)&\\ &=\sum_{k=1}^{m}\left\langle\left|D_{li,k}{}^{2}\right|\int_{-\tau_k}^{0}\int_{t+\theta}^{t}\left|x\left(s-\tau_k\right)\right|ds\,d\theta,w\right\rangle\\ V_{4,M}(t)&=\sum_{k=1}^{m}\tau_k\left\langle\left|D_{li,k}{}^{2}\right|\int_{t-\tau_k}^{t}|x(s)|\,ds,w\right\rangle,\end{aligned}\tag{57}$$

we deduce that it suffices that $T_{m,M}$ is the opposite of an M-matrix to conclude to the asymptotic stability of system (52). □

4.2. Extension of the Results to the Case of TS Fuzzy Switched Systems Described by Differential Equations with Multiple Delays. The same method is applied to determine delay-dependent stability criteria for systems described by the following multiple time-delayed differential equation:

$$\begin{gathered} y^{(n)}(t)+\sum_{i=1}^{N}\xi_i(t)\sum_{l=1}^{r}h_{li} \\ \cdot\left(\sum_{j=0}^{n-1}a_{li}^{j}y^{(j)}(t)+\sum_{k=1}^{m}\sum_{j=0}^{n-1}d_{li,k}^{j}y^{(j)}(t-\tau_k)\right)=0 \\ y^{(j)}(t)=\Phi_j(t) \\ \forall t\in\left[-\max_{1\le k\le m}\tau_k,0\right],\quad j=0,\dots,n-1. \end{gathered} \tag{58}$$

The same change of variable $x_{j+1}(t)=y^{(j)}(t)$, $j=0,\dots,n-1$, as in Section 3.2 yields to the new state space representation:

$$\begin{gathered} \dot{x}(t)=\sum_{i=1}^{N}\xi_i(t)\sum_{l=1}^{r}h_{li}\left(A_{li}x(t)+\sum_{k=1}^{m}D_{li,k}x(t-\tau_k)\right) \\ x(t)=\Phi(t),\quad t\in\left[-\max_{1\le k\le m}\tau_k,0\right], \end{gathered} \tag{59}$$

where $x(t)=[x_1(t),\dots,x_n(t)]^T$ is the state vector and for each $i\in I$, $k\in[1,\dots,m]$, A_{li} and $D_{li,k}$ are given by

$$\begin{gathered} A_{li}=\begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 \\ -a_{li}^{0} & \cdots & \cdots & -a_{li}^{n-1} \end{pmatrix} \\ D_{li,k}=\begin{pmatrix} 0_{n-1,n-1} & & 0_{n-1,1} \\ -d_{li,k}^{0} & \cdots & -d_{li,k}^{n-1} \end{pmatrix}. \end{gathered} \tag{60}$$

A change of base of the form $z(t)=Px(t)$ allows system (59) to be represented by the arrow form matrix:

$$\begin{gathered} \dot{z}(t)=\sum_{i=1}^{N}\xi_i(t)\sum_{l=1}^{r}h_{li}\left(M_{li}z(t)+\sum_{k=1}^{m}N_{li,k}z(t-\tau_k)\right) \\ z(t)=P\Phi(t),\quad t\in\left[-\max_{1\le k\le m}\tau_k,0\right], \end{gathered} \tag{61}$$

where M_{li} is given by (30) and $N_{li,k}$ becomes

$$N_{li,k}=\begin{pmatrix} 0_{n-1,n-1} & & & 0_{n-1,1} \\ \delta_{li,k}^{1} & \cdots & \delta_{li,k}^{n-1} & \delta_{li,k}^{n} \end{pmatrix} \tag{62}$$

with

$$\begin{gathered} \delta_{li,k}^{j}=-P_{D_{li,k}}(\alpha_j) \\ \delta_{li,k}^{n}=-d_{li,k}^{n-1}. \end{gathered} \tag{63}$$

Define the new polynomials:

$$\begin{gathered} P_{D_{li,k}}(\lambda)=\sum_{j=0}^{n-1}d_{li,k}^{j}\lambda^{j}, \\ \psi_{li,k}(\lambda)=-P_{D_{li,k}}(\lambda)\times\lambda+d_{li,k}^{n-1}P_{A_{li}}(\lambda). \end{gathered} \tag{64}$$

Finally, we can compute the common overvaluing matrix as follows:

$$T_{m,M}=\begin{pmatrix} \alpha_1 & & & |\beta_1| \\ & \ddots & & \vdots \\ & & \alpha_{n-1} & |\beta_{n-1}| \\ t_{1,M} & \cdots & t_{n-1,M} & t_{n,M} \end{pmatrix} \tag{65}$$

with

$$\begin{gathered} t_{j,M}=\max_{\substack{1\le i\le N\\1\le l\le r}}\left(\left|\gamma_{li}^{j}+\sum_{k=1}^{m}\delta_{li,k}^{j}\right|\right. \\ \left.+\sum_{k=1}^{m}\tau_k\left(\left|\psi_{li,k}(\alpha_j)\right|+\left|-d_{li,k}^{n-1}\delta_{li,k}^{j}\right|\right)\right), \\ \forall j=1,\dots,n-1, \end{gathered} \tag{66}$$

$$\begin{gathered} t_{n,M}=\max_{\substack{1\le i\le N\\1\le l\le r}}\left(\left(\gamma_{li}^{n}+\sum_{k=1}^{m}\delta_{li,k}^{n}\right)\right. \\ \left.+\sum_{k=1}^{m}\tau_k\left(\left|a_{li}^{n-1}d_{li,k}^{n-1}-d_{li,k}^{n-2}\right|+\left(d_{li,k}^{n-1}\right)^2\right)\right). \end{gathered}$$

Hence, we can state Theorem 10.

Theorem 10. *System (59) is globally asymptotically stable under an arbitrary switching rule $\sigma(t)=i\in I$, if there exist $\alpha_j<0$, $j=1,\dots,n-1$, $\alpha_j\ne\alpha_q$, $\forall j\ne q$ such that*

$$-t_{n,M}+\sum_{j=1}^{n-1}t_{j,M}\left|\beta_j\right|\alpha_j^{-1}>0. \tag{67}$$

Proof. The same proof is as in Theorem 6; just replace matrix T_m by $T_{m,M}$. □

5. Illustrative Examples

Example 1. Consider a switched system composed of three second-order subsystems $\sum_{A_i,D_i}$, $i\in\{1,2,3\}$; each subsystem is represented by a TS fuzzy time-delay model as follows:

Subsystem (1)

$$A_{11} = \begin{pmatrix} 0 & 1 \\ -16 & -15 \end{pmatrix}, \quad D_{11} = \begin{pmatrix} 0 & 0 \\ -1.5 & -0.02 \end{pmatrix} \quad A_{21} = \begin{pmatrix} 0 & 1 \\ -16 & -13.5 \end{pmatrix}, \quad D_{21} = \begin{pmatrix} 0 & 0 \\ -1.5 & 0 \end{pmatrix}. \tag{68}$$

Subsystem (2)

$$A_{12} = \begin{pmatrix} 0 & 1 \\ -20 & -18 \end{pmatrix}, \quad A_{22} = \begin{pmatrix} 0 & 1 \\ -16 & -15 \end{pmatrix} \quad D_{12} = D_{22} = \begin{pmatrix} 0 & 0 \\ -0.5 & 0 \end{pmatrix}. \tag{69}$$

Subsystem (3)

$$A_{13} = \begin{pmatrix} 0 & 1 \\ -14 & -12 \end{pmatrix}, \quad D_{13} = \begin{pmatrix} 0 & 0 \\ -1.75 & 0 \end{pmatrix} \quad A_{23} = \begin{pmatrix} 0 & 1 \\ -14.5 & -10 \end{pmatrix}, \quad D_{23} = \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}. \tag{70}$$

A transformation of state matrices under the arrow form $F_{li} = P^{-1}A_{li}P$ and $N_{li} = P^{-1}D_{li}P$ ($l \in \{1,2\}$ and $i \in \{1,2,3\}$) with $P = \left(\begin{smallmatrix} 1 & 0 \\ \alpha & 1 \end{smallmatrix}\right)$ gives

$$F_{li} = \begin{pmatrix} \alpha & 1 \\ \gamma_{li}^1 & \gamma_{li}^2 \end{pmatrix}, \quad N_{li} = \begin{pmatrix} 0 & 0 \\ \delta_{li}^1 & \delta_{li}^2 \end{pmatrix}. \tag{71}$$

For $i \in \{1,2,3\}$ and $l \in \{1,2\}$, parameters γ_{li} and δ_{li} are computed as follows:

$$\begin{aligned}
\gamma_{11}^1 &= -P_{A_{11}}(\alpha) = -\left(\alpha^2 + 15\alpha + 16\right) \\
\gamma_{11}^2 &= -15 - \alpha; \\
\gamma_{21}^1 &= -P_{A_{21}}(\alpha) = -\left(\alpha^2 + 13.5\alpha + 16\right) \\
\gamma_{21}^2 &= -13.5 - \alpha; \\
\gamma_{12}^1 &= -P_{A_{12}}(\alpha) = -\left(\alpha^2 + 18\alpha + 20\right) \\
\gamma_{12}^2 &= -18 - \alpha; \\
\gamma_{22}^1 &= -P_{A_{22}}(\alpha) = -\left(\alpha^2 + 15\alpha + 16\right) \\
\gamma_{22}^2 &= -15 - \alpha; \\
\gamma_{13}^1 &= -P_{A_{13}}(\alpha) = -\left(\alpha^2 + 12\alpha + 14\right) \\
\gamma_{13}^2 &= -12 - \alpha; \\
\gamma_{23}^1 &= -P_{A_{23}}(\alpha) = -\left(\alpha^2 + 10\alpha + 14.5\right) \\
\gamma_{23}^2 &= -10 - \alpha; \\
\delta_{11}^1 &= -P_{D_{11}}(\alpha) = -(0.02\alpha + 1.5) \\
\delta_{11}^2 &= -0.02; \\
\delta_{21}^1 &= -P_{D_{21}}(\alpha) = -1.5 \\
\delta_{21}^2 &= 0.
\end{aligned} \tag{72}$$

For an arbitrary choice of $\alpha = -1$, we obtain, for example, for subsystem (1)

$$F_{11} = \begin{pmatrix} -1 & 1 \\ -2 & -14 \end{pmatrix}, \quad N_{11} = \begin{pmatrix} 0 & 0 \\ -1.48 & -0.02 \end{pmatrix}; \quad F_{21} = \begin{pmatrix} -1 & 1 \\ -3.5 & -12.5 \end{pmatrix}, \quad N_{21} = \begin{pmatrix} 0 & 0 \\ -1.5 & 0 \end{pmatrix}. \tag{73}$$

For $h_2 = 1 - h_1$ and by using the center of average defuzzification, state matrix $F_i = h_1F_{1i} + h_2F_{2i}$ and delayed-state matrix $N_i = h_1N_{1i} + h_2N_{2i}$ ($i \in \{1,2,3\}$) are given, respectively, by

$$\begin{aligned}
F_1 &= \begin{pmatrix} -1 & 1 \\ 1.5h_1 - 3.5 & -\left(1.5h_1 + 12.5\right) \end{pmatrix}, \\
N_1 &= \begin{pmatrix} 0 & 0 \\ 0.02h_1 - 1.5 & -0.02h_1 \end{pmatrix}, \\
F_2 &= \begin{pmatrix} -1 & 1 \\ -\left(h_1 + 2\right) & -\left(3h_1 + 14\right) \end{pmatrix}, \\
N_2 &= \begin{pmatrix} 0 & 0 \\ -0.5 & 0 \end{pmatrix}, \\
F_3 &= \begin{pmatrix} -1 & 1 \\ -\left(5.5 - 2.5h_1\right) & -\left(2h_1 + 9\right) \end{pmatrix}, \\
N_3 &= \begin{pmatrix} 0 & 0 \\ -\left(0.75h_1 + 1\right) & 0 \end{pmatrix}.
\end{aligned} \tag{74}$$

TABLE 1: Maximum allowable time delay for each submodel ($\tau_{\max}^{(li)}$).

Submodel (li)	(11)	(21)	(12)	(22)	(13)	(23)
$\tau_{\max}^{(li)}$(s)	3.83	2.5	13.5	11.5	1.78	1.25

Denote by T_{li}, $l \in \{1,2\}$ and $i \in \{1,2,3\}$, the minimal pseudo-overvaluing matrix of the fuzzy submodel $\sum_{A_{li},D_{li}}$ such that

$$T_{li} = \begin{pmatrix} -1 & 1 \\ t_{li}^1 & t_{li}^2 \end{pmatrix}, \tag{75}$$

where

$$\begin{aligned} T_{11} &= \begin{pmatrix} -1 & 1 \\ 3.48 + 1.5496\tau & -14.02 + 1.2\tau \end{pmatrix} \\ T_{21} &= \begin{pmatrix} -1 & 1 \\ 5 + 1.5\tau & -12.5 + 1.5\tau \end{pmatrix} \\ T_{12} &= \begin{pmatrix} -1 & 1 \\ 3.5 + 0.5\tau & -17 + 0.5\tau \end{pmatrix} \\ T_{22} &= \begin{pmatrix} -1 & 1 \\ 2.5 + 0.5\tau & -14 + 0.5\tau \end{pmatrix} \\ T_{13} &= \begin{pmatrix} -1 & 1 \\ 4.75 + 1.75\tau & -11 + 1.75\tau \end{pmatrix} \\ T_{23} &= \begin{pmatrix} -1 & 1 \\ 6.5 + \tau & -9 + \tau \end{pmatrix}. \end{aligned} \tag{76}$$

At this level, we can derive sufficient conditions for the asymptotic stability of each submodel individually as shown in Table 1.

From Table 1, we can already see that it is sufficient to have a time delay τ inferior to 1.25 s to ensure the asymptotic stability of all the submodels individually. Taking into account this value, we can notice that $\sum_{T_{23}}$ can be chosen as the common comparison system for the six submodels $\sum_{A_{li},D_{li}}$ ($l \in \{1,2\}$ and $i \in \{1,2,3\}$). Then, the TS fuzzy switched time-delay system is asymptotically stable under an arbitrary switching law if

$$\tau < 1.25\text{ s}. \tag{77}$$

The switching signal is plotted on Figure 1. For a time delay equal to $\tau = 1.2$ s, a final simulation time $t_f = 4$ s, weighting factors $h_1 = h_2 = 0.5$, and an initial state vector $x(t) = [1\ \ -2]^T$ $\forall t \in [-1.2, 0]$, the evolution of state vectors, the system's trajectory, and the state's norm are illustrated on Figures 2, 3, and 4, respectively.

This example shows that the obtained stability conditions are sufficient and very close to be necessary. Moreover, the proposed method makes it possible to avoid searching a common Lyapunov-Krasovskii functional, which is very difficult in this case.

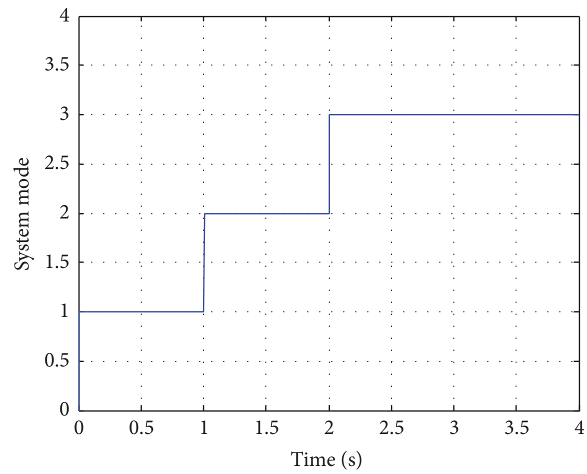

FIGURE 1: Switching signal (Example 1).

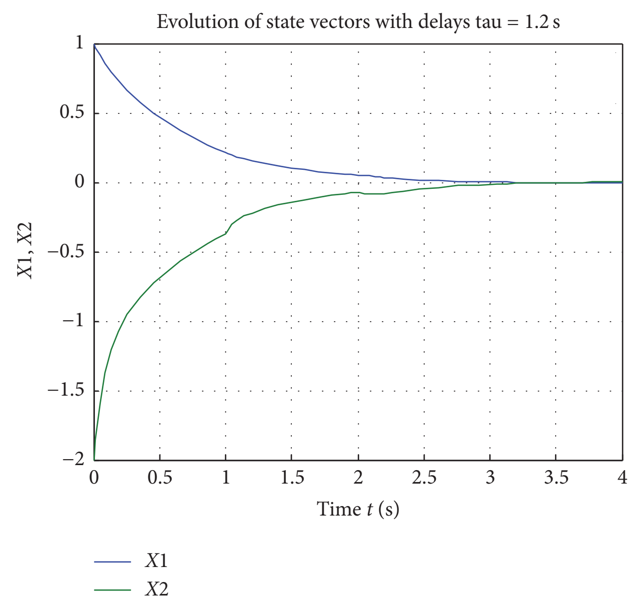

FIGURE 2: Evolution of the state vectors (Example 1).

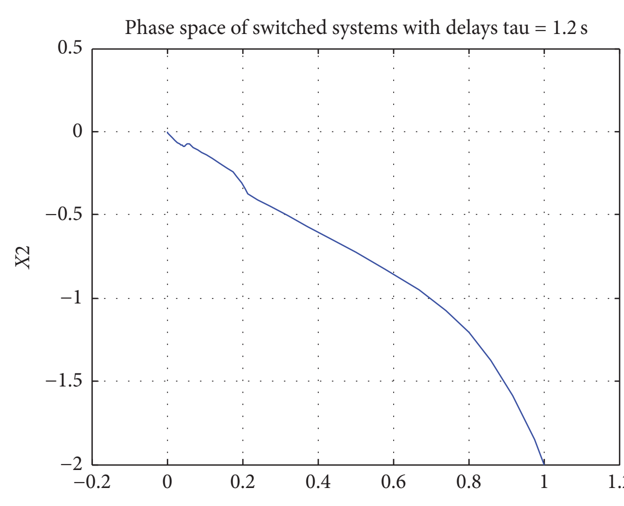

FIGURE 3: System's trajectory (Example 1).

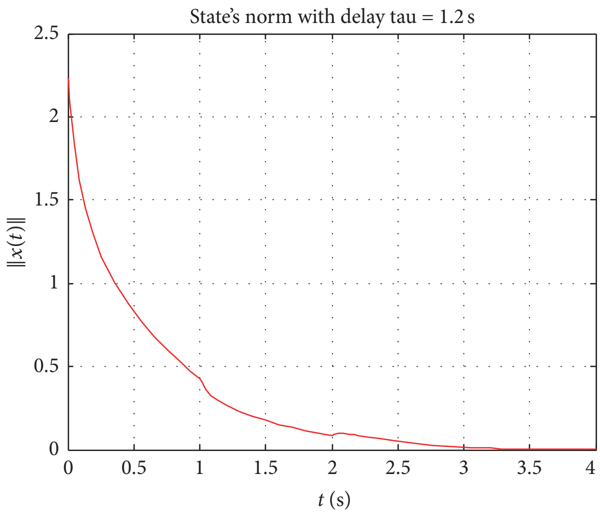

FIGURE 4: State's norm (Example 1).

Example 2. Consider the TS fuzzy switched system with two delays τ_1 and τ_2 as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} \xi_i(t) \cdot \sum_{l=1}^{2} h_{li}\left(A_{li}x(t) + D_{li,1}x(t-\tau_1) + D_{li,2}x(t-\tau_2)\right) \quad (78)$$
$$x(t) = \Phi(t), \quad t \in \left[-\max_{1\le k\le 2}\tau_k, 0\right].$$

Subsystem (1)

$$A_{11} = \begin{pmatrix} 0 & 1 \\ -4 & -3 \end{pmatrix}, \quad D_{11,1} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \quad D_{11,2} = \begin{pmatrix} 0 & 0 \\ -0.5 & 0 \end{pmatrix},$$
$$A_{21} = \begin{pmatrix} 0 & 1 \\ -3 & -3 \end{pmatrix}, \quad D_{21,1} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}, \quad D_{21,2} = \begin{pmatrix} 0 & 0 \\ -0.2 & 0 \end{pmatrix}. \quad (79)$$

Subsystem (2)

$$A_{12} = \begin{pmatrix} 0 & 1 \\ -5 & -10 \end{pmatrix}, \quad D_{12,1} = \begin{pmatrix} 0 & 0 \\ -2 & -1 \end{pmatrix}, \quad D_{12,2} = \begin{pmatrix} 0 & 0 \\ -0.1 & 0 \end{pmatrix},$$
$$A_{22} = \begin{pmatrix} 0 & 1 \\ -7 & -12 \end{pmatrix}, \quad D_{22,1} = \begin{pmatrix} 0 & 0 \\ -3 & -1 \end{pmatrix}, \quad D_{22,2} = \begin{pmatrix} 0 & 0 \\ -0.1 & 0 \end{pmatrix}. \quad (80)$$

For $\alpha = -1$, a transformation under the arrow form matrices gives

$$F_{11} = \begin{pmatrix} -1 & 1 \\ -2 & -2 \end{pmatrix}, \quad N_{11,1} = \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}, \quad N_{11,2} = \begin{pmatrix} 0 & 0 \\ -0.5 & 0 \end{pmatrix},$$
$$F_{21} = \begin{pmatrix} -1 & 1 \\ -1 & -2 \end{pmatrix}, \quad N_{21,1} = \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}, \quad N_{21,2} = \begin{pmatrix} 0 & 0 \\ -0.2 & 0 \end{pmatrix},$$
$$F_{12} = \begin{pmatrix} -1 & 1 \\ 4 & -9 \end{pmatrix}, \quad N_{12,1} = \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix}, \quad N_{12,2} = \begin{pmatrix} 0 & 0 \\ -0.1 & 0 \end{pmatrix},$$
$$F_{22} = \begin{pmatrix} -1 & 1 \\ 4 & -11 \end{pmatrix},$$

Table 2: Maximum allowable time delay for each submodel ($\tau_{2,\max}^{(li)}$).

Submodel (li)	(11)	(21)	(12)	(22)
$\tau_{2,\max}^{(li)}$	$1.5-6\tau_1$	$7-12.5\tau_1$	$35.5-65\tau_1$	$50.5-70\tau_1$

$$N_{22,1}=\begin{pmatrix}0 & 0\\ -2 & -1\end{pmatrix},$$
$$N_{22,2}=\begin{pmatrix}0 & 0\\ -0.1 & 0\end{pmatrix}. \tag{81}$$

Denote by $T_{li,M}$, $l,i\in\{1,2\}$, the minimal overvaluing matrix of subsystem of index (li) and relative to the vector norm $p(x)=[|x_1|,\ldots,|x_n|]^T$ such that

$$T_{li,M}=\begin{pmatrix}-1 & 1\\ t_{li,M}^1 & t_{li,M}^2\end{pmatrix}, \tag{82}$$

where

$$\begin{gathered}T_{11,M}=\begin{pmatrix}-1 & 1\\ 1.5+2\tau_1+0.5\tau_2 & -3+4\tau_1+0.5\tau_2\end{pmatrix}\\ T_{21,M}=\begin{pmatrix}-1 & 1\\ 0.2+\tau_1+0.2\tau_2 & -3+4\tau_1+0.2\tau_2\end{pmatrix}\\ T_{12,M}=\begin{pmatrix}-1 & 1\\ 2.9+4\tau_1+0.1\tau_2 & -10+9\tau_1+0.1\tau_2\end{pmatrix}\\ T_{22,M}=\begin{pmatrix}-1 & 1\\ 1.9+4\tau_1+0.1\tau_2 & -12+10\tau_1+0.1\tau_2\end{pmatrix}.\end{gathered} \tag{83}$$

Applying Theorem 10, we can draw Table 2 which gives a sufficient stability condition $0<\tau_2<\tau_{2,\max}^{(li)}$ for each submodel.

Finally, we can construct the following common comparison system $\sum_{T_{m,M}}$ such that

$$T_{m,M}=\begin{pmatrix}-1 & 1\\ 2.9+4\tau_1+0.1\tau_2 & -3+4\tau_1+0.5\tau_2\end{pmatrix}. \tag{84}$$

Consequently, the TS fuzzy switched time-delay system is asymptotically stable under arbitrary switching if

$$0<\tau_2<\frac{0.1-8\tau_1}{0.6}. \tag{85}$$

In this example, we can notice that maximum bounds found for the delays τ_1 and τ_2 are small. Indeed, this is justified by the multiplicity of time delay, by the choice of delay matrices $N_{li,k}$, $l,i,k\in\{1,2\}$, and by the fact that arbitrary switching strategy brings conservativeness because of the "strict" condition that requires finding a comparison system which is common to all fuzzy local models.

Nevertheless, the simplicity of vector norms based approach makes the study almost feasible in most cases in the opposite of Lyapunov-based method which proves to be difficult when dealing with the problem of stability under arbitrary switching.

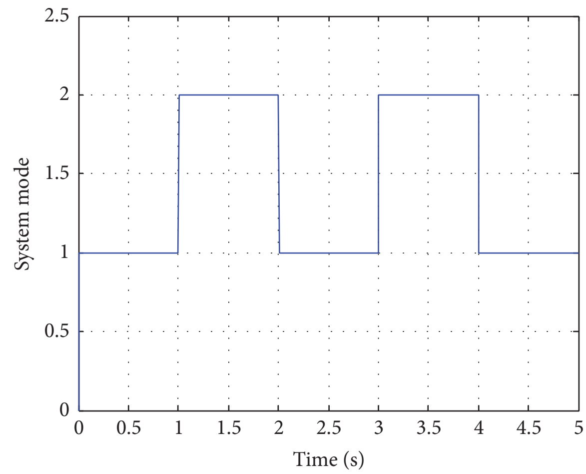

Figure 5: Switching signal (Example 2).

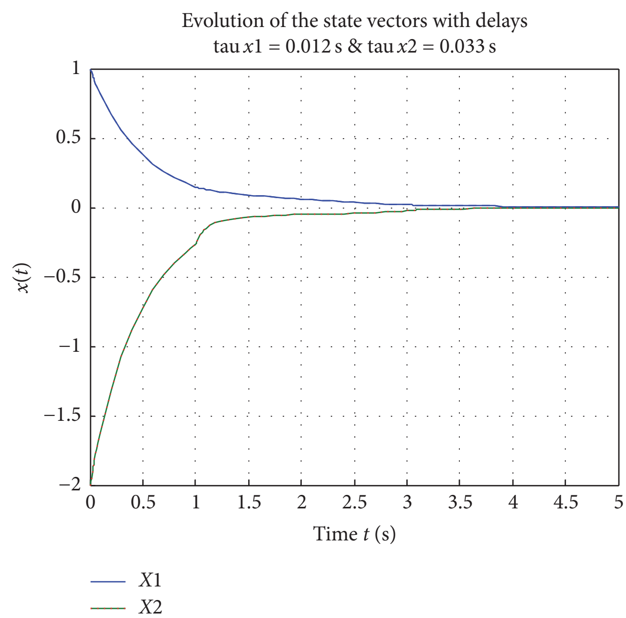

Figure 6: Evolution of state vectors (Example 2).

For a switching signal as shown on Figure 5, a choice of time delays $\tau_1=0.01$ s and $\tau_2=0.033$ s, particular values of weighting factors $h_1=0.2$ and $h_2=0.8$, and an initial state vector $x(t)=\begin{bmatrix}1 & -2\end{bmatrix}^T$ $\forall t\in[-0.033,0]$, the evolution of state responses is illustrated on Figure 6 whereas Figures 7 and 8 show the system's trajectory and the norm's state, respectively.

Example 3 (see [38, 45]). Consider a DC motor with separate excitation (Figure 9). The system acts under variable mechanical load and is subject to a retarded state feedback control input. Our aim is to synthesize a state feedback control for each local model so that the overall system is asymptotically stable under arbitrary switching despite the time delay.

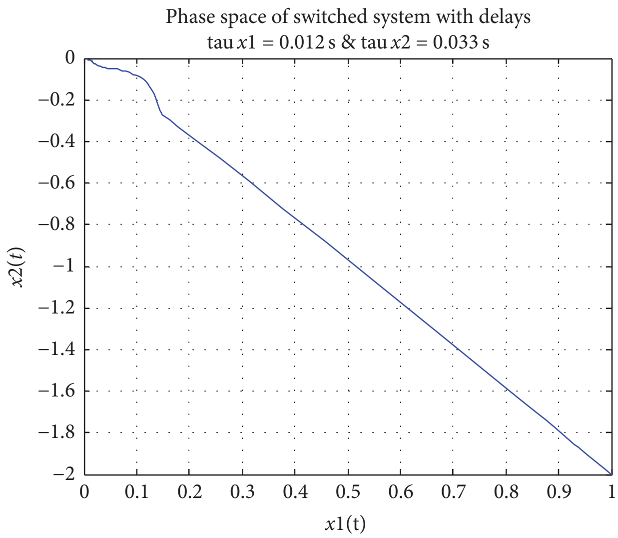

FIGURE 7: System's trajectory (Example 2).

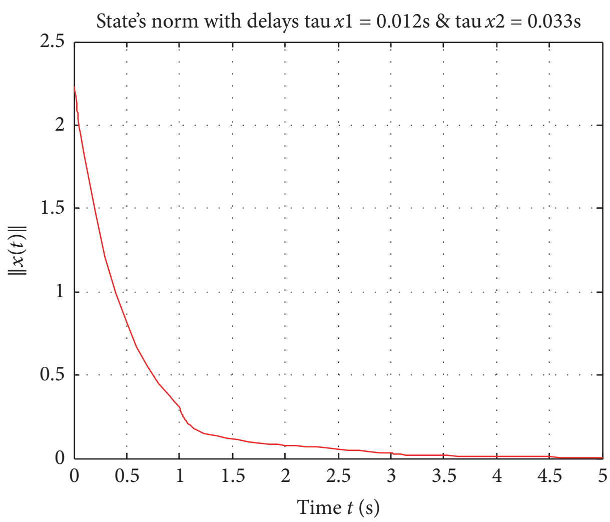

FIGURE 8: State's norm (Example 2).

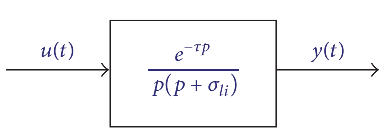

FIGURE 9: DC motor with separate excitation.

A PDC controller scheme is employed such that the lth controller fuzzy rule of the ith subsystem is given by

$$\begin{aligned} R_i^l:\quad & \text{IF } z_1(t) \text{ is } M_{li}^1 \text{ and } z_2(t) \text{ is } M_{li}^2 \\ & \text{THEN } u(t) = -K_{li}x(t), \end{aligned} \tag{86}$$

where $K_{li} = [k_{li}^0 \;\; k_{li}^1]$, $l, i \in [1, 2]$, is the local feedback gain vector.

The TS fuzzy switched system is then represented in the state space by

$$\dot{x}(t) = \sum_{i=1}^{2} \xi_i(t) \sum_{l=1}^{2} h_{li} \left(A_{li}x(t) - \sum_{q=1}^{2} h_{qi} BK_{qi} x(t-\tau) \right), \tag{87}$$

where $A_{li} = \begin{pmatrix} 0 & 1 \\ 0 & -\sigma_{li} \end{pmatrix}$ and $B = (0 \;\; 1)^T$.

We can also write

$$\dot{x}(t) = \sum_{i=1}^{2} \xi_i(t) \cdot \left\{ \sum_{l=1}^{2} h_{li} A_{li} x(t) - \sum_{q=1}^{2} h_{qi} \sum_{l=1}^{2} h_{li} BK_{qi} x(t-\tau) \right\}. \tag{88}$$

Hence,

$$\dot{x}(t) = \sum_{i=1}^{2} \xi_i(t) \left(\sum_{l=1}^{2} h_{li} \left(A_{li}x(t) - BK_{li}x(t-\tau) \right) \right). \tag{89}$$

Denote $D_{li} = -BK_{li}$, and the problem is then reduced to determining admissible values of parameters (k_{li}^0, k_{li}^1), $l, i \in [1, 2]$, for a given value of time delay τ.

A transformation under the arrow form matrix with $\alpha = -1$ yields

$$\begin{aligned} F_{li} &= \begin{pmatrix} -1 & 1 \\ -(1-\sigma_{li}) & -\sigma_{li}+1 \end{pmatrix} \\ N_{li} &= \begin{pmatrix} 0 & 0 \\ k_{li}^1 - k_{li}^0 & -k_{li}^1 \end{pmatrix}. \end{aligned} \tag{90}$$

To tune parameters (k_{li}^0, k_{li}^1), suppose that for $k_{li}^1 > 0$, $\sigma_{li} < 1$ we have

$$k_{li}^0 < \sigma_{li} k_{li}^1. \tag{91}$$

Thus, we can verify that

$$\begin{aligned} -\delta_{li}^1 d_{li}^1 &= -\left(k_{li}^1 - k_{li}^0\right) \times k_{li}^1 < 0 \\ \psi_{li}(-1) &= k_{li}^0 - \sigma_{li} k_{li}^1 < 0. \end{aligned} \tag{92}$$

Since all the assumptions of Remark 8 (Case 2) are met, this allows us, according to inequality (50), to write

$$\tau < \frac{k_{li}^0}{2\left[\left(\sigma_{li}k_{li}^1 - k_{li}^0\right) + \left(k_{li}^1\right)^2\right] - k_{li}^1 k_{li}^0}. \tag{93}$$

Therefore, each fuzzy model is asymptotically stable individually if

$$k_{li}^0 > \frac{2\tau k_{li}^1 \left(\sigma_{li} + k_{li}^1\right)}{1 + \tau\left(2 + k_{li}^1\right)}. \tag{94}$$

The pseudo-overvaluing matrix for each submodel is constructed as follows:

$$T_{li} = \begin{pmatrix} -1 & 1 \\ t_{li}^{1} & t_{li}^{2} \end{pmatrix} \tag{95}$$

with

$$\begin{aligned} t_{li}^{1} &= \left(k_{li}^{1} - k_{li}^{0} + \sigma_{li} - 1\right) + \tau\left(\psi_{li}(-1) + k_{li}^{1}\left(k_{li}^{1} - k_{li}^{0}\right)\right) \\ t_{li}^{2} &= 1 - \sigma_{li} - k_{li}^{1} + \tau\left(\sigma_{li}k_{li}^{1} - k_{li}^{0} + \left(k_{li}^{1}\right)^{2}\right). \end{aligned} \tag{96}$$

For instance, if the time delay τ is equal to 0.5 s, $\sigma_{11} = 0.86$, $\sigma_{21} = 0.97$, $\sigma_{12} = 0.76$, and $\sigma_{22} = 0.89$, and local feedback gains are chosen as $K_{11} = [0.7 \;\; 0.9]$, $K_{21} = [1.15 \;\; 1.2]$, $K_{12} = [0.92 \;\; 1.22]$, and $K_{22} = [0.8 \;\; 1]$, we obtain

$$\begin{aligned} T_{11} &= \begin{pmatrix} -1 & 1 \\ 0.187 & -0.318 \end{pmatrix} \\ T_{21} &= \begin{pmatrix} -1 & 1 \\ 0.057 & -0.443 \end{pmatrix} \\ T_{12} &= \begin{pmatrix} -1 & 1 \\ 0.233 & -0.2352 \end{pmatrix} \\ T_{22} &= \begin{pmatrix} -1 & 1 \\ 0.2350 & -0.345 \end{pmatrix}. \end{aligned} \tag{97}$$

Finally, the common comparison matrix that verifies the properties of the opposite of an M-matrix is given by

$$T_{m} = \begin{pmatrix} -1 & 1 \\ 0.2350 & -0.2352 \end{pmatrix}. \tag{98}$$

This example has shown that the proposed approach is appropriate for the synthesis of a stabilizing state feedback retarded control for an initially unstable nonlinear system described by a set of linear TS fuzzy local linear models.

6. Conclusion

In this paper, new delay-dependent stability conditions by using the vector norms approach have been presented. In fact, most of the research works that have been carried out on stability analysis of TS fuzzy switched time-delay systems under arbitrary switching are based on the research of a common Lyapunov-Krasovskii functional for all the fuzzy models, which is considered as a hard task. The idea of the proposed method consists in putting the switched system under a special form of state space representation using arrow form matrices and then finding a common pseudo-overvaluing system for all the constituent submodels. The stability analysis of this comparison system, based on the aggregation techniques and the M-matrices properties, permits concluding to that of the original TS fuzzy switched system.

Vector norms approach, applied to TS fuzzy switched time-delay systems, whether with single or multiple delay, is suitable for the case of arbitrary switching and provides delay-dependent algebraic criteria. The applicability of the obtained conditions is shown through three numerical examples. It would be interesting to generalize the study to the case of systems with control input.

Appendix

Demonstrations of (48), (50), and (51) are based on the following expressions:

$$\begin{aligned} &-\gamma_{li}^{n} + \sum_{j=1}^{n-1} \alpha_j^{-1} \gamma_{li}^{j} \beta_j = \frac{P_{A_{li}}(0)}{\prod_{j=1}^{n-1}\left(-\alpha_j\right)} = \frac{P_{A_{li}}(0)}{\kappa} \\ &-\delta_{li}^{n} + \sum_{j=1}^{n-1} \alpha_j^{-1} \delta_{li}^{j} \beta_j = \frac{P_{D_{li}}(0)}{\prod_{j=1}^{n-1}\left(-\alpha_j\right)} = \frac{P_{D_{li}}(0)}{\kappa} \\ &d_{li}^{n-1} + \sum_{j=1}^{n-1} \alpha_j^{-1} \delta_{li}^{j} \beta_j = \frac{P_{D_{li}}(0)}{\kappa} \\ &-\left(a_{li}^{n-1} d_{li}^{n-1} - d_{li}^{n-2}\right) + \sum_{j=1}^{n-1} \alpha_j^{-1} \psi_{li}\left(\alpha_j\right) \beta_j \\ &\quad = \frac{-d_{li}^{n-1} P_{A_{li}}(0)}{\kappa}. \end{aligned} \tag{A.1}$$

Considering assumptions of Case 1 and the condition $a_{li}^{n-1} d_{li}^{n-1} > d_{li}^{n-2}$, these relations permit us to write

$$\begin{aligned} \tau_{\max}^{(li)} &= \frac{P_{A_{li}}(0) + P_{D_{li}}(0)}{\kappa\left[\left(a_{li}^{n-1} d_{li}^{n-1} - d_{li}^{n-2}\right) - \sum_{j=1}^{n-1} \alpha_j^{-1} \psi_{li}\left(\alpha_j\right) \beta_j + d_{li}^{n-1}\left(d_{li}^{n-1} + \sum_{j=1}^{n-1} \alpha_j^{-1} \delta_{li}^{j} \beta_j\right)\right]} \\ &= \frac{P_{A_{li}}(0) + P_{D_{li}}(0)}{\kappa\left(d_{li}^{n-1}\left(\left(P_{A_{li}}(0) + P_{D_{li}}(0)\right)/\kappa\right)\right)} = \frac{1}{d_{li}^{n-1}}. \end{aligned} \tag{A.2}$$

Considering assumptions of Case 2 and the same condition $a_{li}^{n-1} d_{li}^{n-1} > d_{li}^{n-2}$, it becomes

$$\tau_{\max}^{(li)} = \frac{P_{A_{li}}(0) + P_{D_{li}}(0)}{\kappa\left[2\left(a_{li}^{n-1}d_{li}^{n-1} - d_{li}^{n-2}\right) - \left(a_{li}^{n-1}d_{li}^{n-1} - d_{li}^{n-2}\right) + \sum_{j=1}^{n-1}\alpha_j{}^{-1}\psi_{li}\left(\alpha_j\right)\beta_j + 2\left(d_{li}^{n-1}\right)^2 - d_{li}^{n-1}\left(d_{li}^{n-1} + \sum_{j=1}^{n-1}\alpha_j{}^{-1}\delta_{li}^{j}\beta_j\right)\right]}$$
$$= \frac{P_{A_{li}}(0) + P_{D_{li}}(0)}{2\kappa\left(\left(a_{li}^{n-1}d_{li}^{n-1} - d_{li}^{n-2}\right) + \left(d_{li}^{n-1}\right)^2\right) - d_{li}^{n-1}\left(P_{A_{li}}(0) + P_{D_{li}}(0)\right)}. \quad \text{(A.3)}$$

The same steps will be followed for the other cases.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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