Hindawi
Mathematical Problems in Engineering
Volume 2017, Article ID 8270649, 5 pages
https://doi.org/10.1155/2017/8270649

Research Article

A Nonmonotone Projection Method for Constrained System of Nonlinear Equations

Yazheng Dang and Wenwen Liu

School of Management, University of Shanghai for Science and Technology, 334 Jungong Road, Shanghai 200093, China

Correspondence should be addressed to Yazheng Dang; jgdyz@163.com

Received 22 September 2016; Revised 6 December 2016; Accepted 21 December 2016; Published 18 January 2017

Academic Editor: Benjamin Ivorra

This paper deals with the nonmonotone projection algorithm for constrained nonlinear equations. For some starting points, the previous projection algorithms for the problem may encounter slow convergence which is related to the monotone behavior of the iterative sequence as well as the iterative direction. To circumvent this situation, we adopt the nonmonotone technique introduced by Dang to develop a nonmonotone projection algorithm. After constructing the nonmonotone projection algorithm, we show its convergence under some suitable condition. Preliminary numerical experiment is reported at the end of this paper, from which we can see that the algorithm we propose converges more quickly than that of the usual projection algorithm for some starting points.

1. Introduction

Recall that $G : R^n \to R^n$ is a nonlinear mapping with continuity and C is a nonempty closed set in R^n with convexity; then the constrained nonlinear equations are defined as seeking a point $x^* \in C$ so that the following equation is established:

$$G(x^*) = 0. \tag{1}$$

Many iteration methods and algorithms for solving such problem have been proposed in [1–11]. For instance, there are some variants of the Levenberg-Marquardt type methods [1–3] which have strong convergence property. In addition, Wang et al. presented a projection algorithm [5] for solving problem (1) in 2007 and a superlinearly convergent projection method [12] in 2009. From the numerical performances given in [12], we can see that the algorithm in [12] is more efficient than the method in [5] for solving such problem. Recently, a hybrid conjugate gradient projection algorithm has been established which is on the basis of the Dai-Yuan and Hestenes-Stiefel conjugate gradient method, seen in [11].

However, the projection algorithms may encounter "tunneling effect" [13] which will result in slow convergence. That is to say, during the iteration, the projection onto two or more convex sets may encounter a narrow channel, and the projection iterative sequences will become very slow. Applying the nonmonotone technique to the projection algorithm is an effective way to avoid this effect, which is based on the idea of taking a big step to interrupt the monotone behavior. The "tunneling effect" is associated with the monotone iterative sequence. Inspired by the work of Dang and Gao [14] for convex feasibility problem, we propose a nonmonotone projection algorithm, which has already been confirmed to converge faster than average in the "tunneling." From the numerical experiment, it can be verified that, comparing with the projection method in [12], this method is more effective.

The remaining part of this article is distributed as follows. In the next section, some fundamental properties will been given which is useful in the following demonstration. In Section 3, the nonmonotonic projection method will be shown and the algorithm convergence is proved theoretically. At the end of this article, an example will be given which elucidates the algorithm we propose, which converges more quickly than the existing algorithms. Based on the above understanding, we come to the conclusion.

2. Preliminaries

Let $G : R^n \to R^n$ be a nonlinear continuous mapping; then G is said to be monotone if, for $\forall x, y \in R^n$, it holds that

$$\langle G(x) - G(y), y - x \rangle \le 0. \tag{2}$$

In addition, $S := \{x \in R^n \mid G(x) = 0\}$ is convex if G is monotone.

Let C be a convex set where $C \in R^n$ and $C \notin \emptyset$. Define $P_C[*]: R^n \to C$ as a projection; it can be expressed as

$$P_C[x] = \operatorname{argmin}\{\|y - x\|\} \quad \text{where } y \in C,\ x \in R^n. \tag{3}$$

It is well known that the projection P_C has some fundamental properties. It holds that

$$\|P_C[x] - P_C[y]\| \le \|x - y\|, \tag{4}$$

$$\|P_C[x] - P_C[y]\|^2 \le \|x - y\|^2 - \|P_C[x] - x + y - P_C[y]\|^2. \tag{5}$$

According to (4) we know that $P_C[*]$ is nonexpansive. In this paper, we mainly use formula (5).

3. The Algorithm and Its Convergence Analysis

The basic idea of our algorithm is as follows. Taking a well-determined big step at each of the a priori fixed moments, we try to interrupt the monotone behavior of the iteration sequence by introducing an appropriate parameter at suitable steps so as to ensure that both of the nonmonotone sequence and the iteration within the interval are monotonically decreasing. In this way, the whole sequence may converge to a point in the solution set.

Algorithm 1 (the nonmonotone projection algorithm).

Step 0. Choose $M > 2$ and $N > M$ which are positive integer numbers. Take $\varphi \in (0, 1)$. B is an arithmetic number which is as large as possible.

Step 1. Pick an initial point $x^0 \in C$; set the parameters such that $\gamma_1 > 0$, $\gamma_2 > 0$, $0 \le \kappa_0 < 1$, $0 < \alpha < 1$, and $0 < \beta < 1$.

Step 2. If $G(x^k) = 0$, stop. Or else let $\sigma_k = \min\{\kappa_0, \gamma_2\|G(x^k)\|^{1/2}\}$; solve the linear equation below:

$$\gamma_1 \left\|G\left(x^k\right)\right\|^{1/2} I\left(x - x^k\right) + G\left(x^k\right) = 0. \tag{6}$$

Find the solution $\overline{x}^k \in R^n$ to (6) so that r^k can satisfy

$$\left\|r^k\right\| \le \sigma_k \gamma_1 \left\|x^k - \overline{x}^k\right\| \cdot \left\|G\left(x^k\right)\right\|^{1/2}. \tag{7}$$

Step 3. Get y^k by $y^k = x^k - \beta^{m_k}(x^k - \overline{x}^k)$ such that

$$\left\langle G\left(y^k\right), x^k - \overline{x}^k \right\rangle \ge \alpha\gamma_1\left(1 - \sigma_k\right)\left\|G\left(x^k\right)\right\|^{1/2}\left\|x^k - \overline{x}^k\right\|^2, \tag{8}$$

where m_k is the smallest nonnegative integer that satisfies (8).

Step 4. Set $H_k = \{\langle G(y^k), x - y^k\rangle = 0 \mid x \in R^n\}$.

(1) When $k \notin \{N + pM\}_{p=0}^{+\infty}$ (p is nonnegative integer), put

$$\alpha_k^1 = \frac{\left\langle G\left(y^k\right), x^k - y^k\right\rangle}{\left\|G\left(y^k\right)\right\|^2}. \tag{9}$$

Construct x^{k+1} by

$$x^{k+1} = P_{C\cap H_K}\left[x^k - \alpha_k^1 G\left(y^k\right)\right]. \tag{10}$$

(2) When $k \in \{N + pM\}_{p=0}^{+\infty}$, determine α_k^1 as in (1). Put

$$\omega^{k+1} = P_{C\cap H_K}\left[x^k - \alpha_k^1 G\left(y^k\right)\right]. \tag{11}$$

Construct x^{k+1} by

$$x^{k+1} = \chi_{k+1} G\left(y^k\right) + \omega^{k+1}, \tag{12}$$

where

$$\chi_{k+1} = \min\left(B, \sqrt{\frac{\varphi M_{k+1}}{\left\|G\left(y^k\right)\right\|^2}}\right), \tag{13}$$

$$M_{k+1} = \frac{\left\langle G\left(y^{k+1-M}\right), x^{k+1-M} - y^{k+1-M}\right\rangle^2}{\left\|G\left(y^{k+1-M}\right)\right\|^2} + \cdots + \frac{\left\langle G\left(y^k\right), x^k - y^k\right\rangle^2}{\left\|G\left(y^k\right)\right\|^2}. \tag{14}$$

Then replace k by $k + 1$ and turn to Step 2.

Compared with the existing algorithms, we attempt to interrupt the monotone behavior of the iterative sequence $\{x^k\}_{k=0}^{\infty}$ by taking a big step at different moments and introducing χ_{k+1} at every appropriate step. Therefore, for some starting points, the nonmonotone technique may avoid the tunneling effect and improve the algorithms convergence.

Next, we analyse the convergence of our algorithm. To this end, we need the assumptions below:

(A_1) F is a monotone mapping.

(A_2) $S = \{x \in R^n \mid G(x) = 0\}$ is nonempty and convex.

(A_3) Algorithm 1 always generates an infinite sequence.

Lemma 2 (see [15, Lemma 3.2]). *Suppose that the underlying mapping F is monotone. Then*

$$P_C\left[x^k - \alpha_k^2 F\left(y^k\right)\right] = P_{C\cap H_K}\left[x^k - \alpha_k^1 F\left(y^k\right)\right]. \tag{15}$$

Lemma 3 (see [12, Theorem 3.1]). *When $k \notin \{N + pM\}_{p=0}^{+\infty}$, take α_k^1 in (9) and use iteration (10). Then*

$$\left\|x^{k+1} - x^*\right\|^2 \le \left\|x^k - x^*\right\|^2 - \frac{\left\langle G\left(y^k\right), x^k - y^k\right\rangle^2}{\left\|G\left(y^k\right)\right\|^2}. \tag{16}$$

From the inequality above we have the conclusion that $\|x^k - x^*\|$ is monotonically decreasing and converges. Namely, $\{x^k\}$ is bounded.

Theorem 4. *Suppose $x^* \in C$, when $k \notin \{N + pM\}_{p=0}^{+\infty}$, where $M > 2$ and $N > M$, under Assumptions (A_1) and (A_2), for the sequence produced by Algorithm 1; then*

$$\lim_{k+1\to\infty, k+1\notin\{N+1+pM\}_{p=0}^{+\infty}} \left\|x^k - x^{k+1}\right\| = \lim_{k+1\to\infty, k+1\in\{N+1+pM\}_{p=0}^{+\infty}} \left\|x^k - w^{k+1}\right\| = 0. \tag{17}$$

Proof. First, we claim that $\{y^k\}_{k=0}^{+\infty}$ is bounded.

From the demonstration in [12], we have

$$\left\langle G\left(x^k\right), x^k - y^k\right\rangle \geq \alpha\gamma_1\left(1-\sigma_k\right)\left\|G\left(x^k\right)\right\|^{1/2}\left\|x^k - y^k\right\|^2. \tag{18}$$

Substituting it into the Cauchy-Schwartz inequality with σ_k, we obtain

$$\left\|G\left(x^k\right)\right\|^{1/2} \geq \alpha\left(1-\kappa_0\right)\gamma_1\left\|x^k - y^k\right\|. \tag{19}$$

Due to the boundedness of $\{x^k\}$ and the continuity of G, $\{y^k\}$ is bounded.

Second, we show that $\lim_{k\to\infty, k\notin\{N+pM\}_{p=0}^{+\infty}}\|x^k - y^k\| = 0$.

For one thing, when $k \to \infty$, $k \notin \{N + pM\}_{p=0}^{+\infty}$, according to (16), we can see that $\|x^{k+1} - x^*\|$ and $\|x^k - x^*\|$ are limited to the same number. So we can draw a conclusion that

$$\lim_{k\to\infty, k\notin\{N+pM\}_{p=0}^{+\infty}} \left\langle G\left(x^k\right), x^k - y^k\right\rangle = 0. \tag{20}$$

For another,

$$\left\langle G\left(y^k\right), x^k - y^k\right\rangle \geq \beta^{m_k}\alpha\gamma_1\left(1-\sigma_k\right)\left\|G\left(x^k\right)\right\|^{1/2}\left\|x^k - \overline{x}^k\right\|^2, \tag{21}$$

and by the substitution of α and σ_k, we get

$$\lim_{k\to\infty, k\notin\{N+pM\}_{p=0}^{+\infty}} \left\|x^k - \overline{x}^k\right\| = 0, \tag{22}$$

and, combined with the equality of y^k, it leads to

$$\lim_{k\to\infty, k\notin\{N+pM\}_{p=0}^{+\infty}} \left\|x^k - y^k\right\| = 0. \tag{23}$$

Finally, we show that $x^k \in C$, and

$$\left\|x^k - x^{k+1}\right\| = \left\|x^k - P_{C\cap H_K}\left[x^k - \alpha_k^1 G\left(y^k\right)\right]\right\| \leq \left\|\alpha_k^1 G\left(y^k\right)\right\| = \left\|x^k - y^k\right\|. \tag{24}$$

In association with Step 2, we obtain the desired conclusion. □

Theorem 5. *Suppose $x^* \in C$, when $k \in \{N + pM\}_{p=0}^{+\infty}$, where $M > 2$ and $N > M$, for the sequence produced by Algorithm 1; then*

$$\lim_{k+1\to\infty, k+1\in\{N+1+pM\}_{p=0}^{+\infty}} \left\|x^k - x^{k+1}\right\| = 0. \tag{25}$$

Proof. First, according to (12), the following equality exists, for any $x^* \in C$:

$$\left\|x^{k+1} - x^*\right\|^2 = \left\|P_{C\cap H_K}\left[x^k - \alpha_k^2 G\left(y^k\right)\right] + \chi_{k+1}G\left(y^k\right) - x^*\right\|^2. \tag{26}$$

Combining this with (4), we obtain

$$\begin{aligned}
\left\|x^{k+1} - x^*\right\|^2 &\leq \left\|x^k - x^* - \alpha_k^2 G\left(y^k\right) + \chi_{k+1}G\left(y^k\right)\right\|^2 \\
&\quad - \left\|x^{k+1} - x^k + \alpha_k^2 G\left(y^k\right) - \chi_{k+1}G\left(y^k\right)\right\|^2 \\
&\leq \left\|x^k - x^*\right\|^2 \\
&\quad - 2\left\langle \alpha_k^1 G\left(y^k\right) - \chi_{k+1}G\left(y^k\right), x^k - x^*\right\rangle \\
&\quad + \left\|\alpha_k^1 G\left(y^k\right) - \chi_{k+1}G\left(y^k\right)\right\|^2 = \left\|x^k - x^*\right\|^2 \\
&\quad - \frac{\left\langle G\left(y^k\right), x^k - y^k\right\rangle^2}{\left\|G\left(y^k\right)\right\|^2} + \left(\chi_{k+1}\right)^2\left\|G\left(y^k\right)\right\|^2 \\
&\leq \left\|x^{k-1} - x^*\right\|^2 - \frac{\left\langle G\left(y^{k-1}\right), x^{k-1} - y^{k-1}\right\rangle^2}{\left\|G\left(y^{k-1}\right)\right\|^2} \\
&\quad - \frac{\left\langle G\left(y^k\right), x^k - y^k\right\rangle^2}{\left\|G\left(y^k\right)\right\|^2} + \left(\chi_{k+1}\right)^2\left\|G\left(y^k\right)\right\|^2 \\
&\quad \vdots \\
&\leq \left\|x^{k+1-M} - x^*\right\|^2 - \left(1-\varphi\right)M_{k+1}.
\end{aligned} \tag{27}$$

So

$$\left\|x^{k+1} - x^*\right\|^2 \leq \left\|x^{k+1-M} - x^*\right\|^2. \tag{28}$$

Second, when x^{k+1} was chosen as in (12), hence

$$\left\|x^{k+1} - x^k\right\| = \left\|\omega^{k+1} - x^k + \sqrt[2]{\eta M_{k+1}}\right\| \leq \left\|x^k - \omega^{k+1}\right\| + \left\|\sqrt[2]{\eta M_{k+1}}\right\|. \tag{29}$$

From (28), $\|x^{k+1-M} - x^*\|$ is not incremental; thus $\|x^{k+1-M} - x^*\|$ and $\|x^{k+1} - x^*\|$ tend to the same number. It leads to

$$\lim_{k+1\to\infty, k+1\in\{N+1+pM\}_{p=0}^{+\infty}} M_{k+1} = 0. \tag{30}$$

Thus, $\lim_{k+1\to\infty, k+1\in\{N+1+pM\}_{p=0}^{+\infty}}\sqrt[2]{\eta M_{k+1}}$ exists and is equal to zero. Combined with Theorem 4, the conclusion above is proved. □

Remark 6. We know that the value of M_{k+1} generated by algorithm in [12] may be very small if the algorithm encounters "tunneling effect" during the process of iteration from the

$(k+1-M)$th step to the $(k+1)$th step. In order to make the current iterate point x^k as close as possible to the optimum point, M_{k+1} needs to be more maximized. In Algorithm 1, we use the nonmonotone technique so that M_{k+1} may be very large which is the superior place of the Algorithm 1.

Theorem 7. *Under Assumptions* (A_1)–(A_3), *combined with Theorems 4 and 5, we have the conclusion that* $\{x^k\}_{k=0}^{+\infty}$ *constructed by our method globally converges to S.*

Proof. In our algorithm, $\{x^k\}_{k=0}^{+\infty}$ contains the subsequence $\{x^{N+pM}\}_{p=0}^{+\infty}$ which was generated by the nonmonotone technique. For brevity, we denote $\{x^{N+pM}\}_{p=0}^{+\infty}$ as $\{x^{k_j}\}_{j=0}^{+\infty}$. □

Next, we accomplish the demonstration in the following three steps.

Firstly, there is a subsequence in $\{x^{k_j}\}_{j=0}^{+\infty}$ converging to a point x^*. For one thing, from Theorem 5, $\{x^{k_j}\}_{j=0}^{+\infty}$ is convergent; thus there is a subsequence converging to a point $x^* \in R^n$. For another, we denote the last subsequence as $\{x^{k_z}\}$. It also converges to x^*. The following iterative yields

$$x^{k_z+1} = P_{C\cap H_K}\left[x^{k_z} - \alpha_k^1 G\left(y^{k_z}\right)\right]. \quad (31)$$

From (11), we have

$$\left\|x^{k_z+1} - x^*\right\|^2 \le \left\|x^{k_z} - x^*\right\|^2 - \frac{\left\langle G\left(y^{k_z}\right), x^{k_z} - y^{k_z}\right\rangle^2}{\left\|G\left(y^{k_z}\right)\right\|^2}. \quad (32)$$

By $\{x^{k_z}\}$ converging to x^*, $\{x^{k_z+1}\}$ also converges to x^*.

Secondly, each convergent sequence in $\{x^{k_j}\}_{j=0}^{+\infty}$ converges to the same point x^*. From Theorem 5, we know that $\|x^{k+1} - x^*\|^2 \le \|x^{k+1-1} - x^*\|^2 \le \cdots \le \|x^{k+1-M} - x^*\|^2 = \|x^{k_z} - x^*\|^2$; from the above, $\|x^{k_z} - x^*\| \to 0$ when $k \to \infty$. Hence, we get the above proposition.

Last but not least, $\{x^k\}_{k=0}^{+\infty}$ converges to x^*. From the above analysis, we see that the sequence $\{x^{N+pM}\}_{p=0}^{+\infty}$ converges to x^*. Let i be an arbitrary index. Then there are successive indices k_j and k_t of $\{x^{N+pM}\}_{p=0}^{+\infty}$, where $k_j = N+1+(p-1)M$ and $k_t = N+1+pM$. When $k_j < i < k_t$, $\|x^i - x^*\| \le \|x^{i-1} - x^*\| \le \cdots \le \|x^{k_j+1} - x^*\| \le \|x^{k_j} - x^*\|$, and $\|x^{k_j} - x^*\| \to 0$ when $k_j \to \infty$. Thus $x^i \to x^*$ when $k_i \to \infty$. When $i = k_t$, the results were significant.

4. Numerical Examples

Here we utilize our algorithm to solve a constrained system of nonlinear equations. To test the algorithm, we compare the results with the ones of the projection algorithm in [12]. For convenience, we denote our algorithm as NMPA and the projection algorithm in [12] as PA. We take the example in [12]. Set the parameters used in this example as κ (*i.e.*, $\kappa_0 \approx 0$), $\alpha = 0.95$, $\beta = 0.6$, and $\gamma_1 = \gamma_2 = 1$. We put $M = 6$, $N = 12$, and $\varphi = 0.9$. The stop criterion is $\|F(x^k)\| \le 10^{-6}$.

Table 1: Results of Alg-PA and Alg-NMPA for Example 1.

ρ	Case	Alg-PA	Alg-NMPA
$\rho = 100$	Case 1	260	43
	Case 2	424	65
$\rho = 200$	Case 1	890	101
	Case 2	1497	156

Example 1. Let the domain set C be taken as $C = C_1 \cap C_2$, where $C_1 = \{x \in R^{120} \mid \sum_{i=1}^{120} x_i \le 10\}$ and $C_2 = \{x \in R^{120} \mid \sum_{i=1}^{120} x_i^2 \le 10\}$. Let the nonlinear equations $G(x)$ be taken as $G(x) = \rho D(x) + M(x)x + q + q_0$, among which ρ is a constant; $D_i(x) = \arctan(x_i - 2)$, where i is an integer from 1 to 120; M is a $120 * 120$ asymmetrical positive definite matrix; q is the vector and q_0 is a constant vector. In addition, elements of M are produced in $(-5, 5)$ randomly and q is produced by an interval range from -10 to 10.

There are two cases below to consider:

Case 1: $x^0 = (-0.01, -0.01, \ldots, -0.01) \in R^{120}$

Case 2: $x^0 = (0.03, 0.03, \ldots, 0.03) \in R^{120}$

Table 1 gives the numbers of iterations that are required, in order to get the approximate solutions for the above two cases with $\rho = 100$ and $\rho = 200$ of Example 1 by Alg-PA and Alg-NMPA, respectively.

From Table 1, by choosing the proper initial point, we show that the sequences are generated by the nonmonotone convergent projection algorithm. Comparing with the PA, the most prominent advantage is that our algorithm can avoid the "tunnel effect."

5. Conclusion

This paper presented a nonmonotone projection method for constrained nonlinear equations. With the introduction of monotone technology, the monotone behavior of the iterative sequence has been disorganized. Based on some assumption, algorithm global convergence is guaranteed. In comparison with the extant projection methods, the most prominent characteristics in this paper are that, for some starting points, the nonmonotone projection algorithm can circumvent the "tunneling effect," which leads to slow convergence.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported in part by Natural Science Foundation of Shanghai (no. 14ZR1429200) and Innovation Program of Shanghai Municipal Education Commission (no. 15ZZ073). The authors would like to express their gratitude to all those who have helped them during the writing of this paper.

References

[1] N. Yamashita and M. Fukushima, "On the rate of convergence of the Levenberg-Marquardt method," *Computing*, vol. 15, pp. 237–249, 2001.

[2] C. Kanzow, N. Yamashita, and M. Fukushima, "Levenberg-Marquardt methods with strong local convergence properties for solving nonlinear equations with convex constraints," *Journal of Computational and Applied Mathematics*, vol. 172, no. 2, pp. 375–397, 2004.

[3] C. Kanzow, N. Yamashita, and M. Fukushima, "Levenberg-Marquardt methods with strong local convergence properties for solving nonlinear equations with convex constraints," *Journal of Computational and Applied Mathematics*, vol. 173, no. 2, pp. 321–343, 2005.

[4] C. D. Maranas and C. A. Floudas, "Finding all solutions of nonlinearly constrained systems of equations," *Journal of Global Optimization*, vol. 7, no. 2, pp. 143–182, 1995.

[5] C. Wang, Y. Wang, and C. Xu, "A projection method for a system of nonlinear monotone equations with convex constraints," *Mathematical Methods of Operations Research*, vol. 66, no. 1, pp. 33–46, 2007.

[6] X. J. Tong and L. Qi, "On the convergence of a trust-region method for solving constrained nonlinear equations with degenerate solutions," *Journal of Optimization Theory and Applications*, vol. 123, no. 1, pp. 187–211, 2004.

[7] F. S. Bai, M. Mammadov, Z. Y. Wu, and Y. J. Yang, "A filled function method for constrained nonlinear equations," *Pacific Journal of Optimization*, vol. 4, no. 1, pp. 9–18, 2008.

[8] P. N. Anh and H. A. Le Thi, "An Armijo-type method for pseudomonotone equilibrium problems and its applications," *Journal of Global Optimization*, vol. 57, no. 3, pp. 803–820, 2013.

[9] Z. Wu, F. Bai, and M. Mammadov, "An auxiliary function method for constrained systems of nonlinear equations," in *Proceedings of the 20th International Conference EURO Mini Conference: Continuous Optimization and Knowledge-Based Technologies (EurOPT '08)*, pp. 259–265, Neringa, Lithuania, May 2008.

[10] L. Zheng, "A modified prp projection method for nonlinear equations with convex constraints," *International Journal of Pure and Applied Mathematics*, vol. 79, no. 1, pp. 87–96, 2012.

[11] M. Sun and J. Liu, "New hybrid conjugate gradient projection method for the convex constrained equations," *Calcolo*, vol. 53, no. 3, pp. 399–411, 2016.

[12] C. Wang and Y. Wang, "A superlinearly convergent projection method for constrained systems of nonlinear equations," *Journal of Global Optimization*, vol. 44, no. 2, pp. 283–296, 2009.

[13] A. Levi and H. Stark, "Restoration from phase and magnitude by generalized projections," in *Image Recovery: Theory and Application*, pp. 227–320, Academic Press, New York, NY, USA, 1987.

[14] Y. Dang and Y. Gao, "Non-monotonous accelerated parallel subgradient projection algorithm for convex feasibility problem," *Optimization. A Journal of Mathematical Programming and Operations Research*, vol. 63, no. 4, pp. 571–584, 2014.

[15] Y. J. Wang, N. H. Xiu, and C. Y. Wang, "Unified framework of extragradient-type methods for pseudomonotone variational inequalities," *Journal of Optimization Theory and Applications*, vol. 111, no. 3, pp. 641–656, 2001.