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Research Article

Adaptive Parallel Simultaneous Stabilization of a Class of Nonlinear Descriptor Systems via Dissipative Matrix Method

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This paper investigates the adaptive parallel simultaneous stabilization and robust adaptive parallel simultaneous stabilization problems of a class of nonlinear descriptor systems via dissipative matrix method. Firstly, under an output feedback law, two nonlinear descriptor systems are transformed into two nonlinear differential-algebraic systems by nonsingular transformations, and a sufficient condition of impulse-free is given for two resulting closed-loop systems. Then, the two systems are combined to generate an augmented dissipative Hamiltonian differential-algebraic system by using the system-augmentation technique. Based on the dissipative system, an adaptive parallel simultaneous stabilization controller and a robust adaptive parallel simultaneous stabilization controller are designed for the two systems. Furthermore, the case of more than two nonlinear descriptor systems is investigated. Finally, an illustrative example is studied by using the results proposed in this paper, and simulations show that the adaptive parallel simultaneous stabilization controllers obtained in this paper work very well.

1. Introduction

In practical control designs, a commonly encountered problem is to design feedback controller(s) to stabilize a given family of parallel systems. It is straightforward to consider each system individually and design a stabilization controller for each system. However, a more economical approach to the problem is to design a single controller, which may take measurements/signals from all members of the family, to stabilize all the systems simultaneously [1, 2]. In this way, the controller implementation cost will be greatly reduced. This control is referred to the parallel simultaneous stabilization. It is noted that this kind of stabilization is different from the traditional simultaneous stabilization problem [3, 4]. The traditional simultaneous stabilization is concerned with designing a control law such that any individual system within the collection of systems can be stabilized by the control law. In other words, the resulting closed-loop system which consists of an individual system and its corresponding controller via its state or output feedback based on that control law is asymptotically stable. It is also noted that the traditional simultaneous stabilization problem is one of the important research topics in the area of robust control and has received a considerable attention in the past few decades [3–8].

The descriptor system is a natural representation of dynamic systems and describes a larger class of systems than the normal system model [9–16]. In the last three decades, many nice results have been obtained for the controller design of linear descriptor systems; see [9, 10, 13, 14] and references therein. In general, it is not an easy task to design a controller for nonlinear descriptor systems (NDSs) and, accordingly, there are fewer works on NDSs except several special case studies [11, 12, 15, 16]; particularly, it is more difficult to design a parallel simultaneous stabilization controller for a class of nonlinear descriptor systems; the pertinent results were proposed for this case in [1]. For nonlinear differential-algebraic systems, an H_∞ controller was designed in [15] based on the condition for the existence of H_∞ controller of nonlinear systems, while the stabilization and robust stabilization of the systems were considered by the feedback linearization approach in [11] and the Hamiltonian function method in [12], respectively. In [16], based on the linear matrix inequality method, the generalized absolute stability was studied for linear descriptor systems with feedback-connected nonlinearities. Using a nonlinear performance index to the nominal system, a robust adaptive control scheme was presented in [17] for a class of nonlinear uncertain descriptor systems. For the case in which the singular matrix $E_i = M_i \operatorname{diag}\{I_r, 0\} M_i$ with M_i being an orthogonal matrix, the parallel simultaneous stabilization and robust adaptive parallel simultaneous stabilization problems were, respectively, studied in [1, 18] for two or a family of nonlinear descriptor systems via the Hamiltonian function method. It should be pointed out that there are, to the best of the authors' knowledge, fewer works on the robust adaptive parallel simultaneous stabilization of NDSs [18].

In this paper, motivated by the Hamiltonian function method [2, 19–29], we apply the structural properties of dissipative matrices to investigate the adaptive parallel simultaneous stabilization and robust adaptive parallel simultaneous stabilization problems for a class of NDSs via output feedback law [30, 31], and propose a new approach, called the dissipative matrix method, to study NDSs. Firstly, under an output feedback law, two NDSs are transformed into two nonlinear differential-algebraic systems by nonsingular transformations, and a sufficient condition of impulse-free is given for two closed-loop systems. Then, the two systems are combined to generate an augmented dissipative Hamiltonian differential-algebraic system by using the system-augmentation technique. Based on the dissipative system, an adaptive parallel simultaneous stabilization controller and a robust adaptive parallel simultaneous stabilization controller are designed for two NDSs, in which the singular matrix $E_i \geq 0 (\leq 0)$. Furthermore, the case of more than two NDSs is investigated. Finally, an illustrative example is studied by using the results proposed in this paper, and simulations show that the adaptive parallel simultaneous stabilization controllers obtained in this paper work very well.

The paper is organized as follows. In Section 2, we study the adaptive parallel simultaneous stabilization of two NDSs based on an augmented dissipative Hamiltonian form. Section 3 presents the robust adaptive parallel simultaneous stabilization controller for two NDSs with external disturbances and investigates the case of more than two NDSs. In Section 4, an illustrative example is provided, which is followed by the conclusion in Section 5.

2. Adaptive Parallel Simultaneous Stabilization of Two NDSs

This section investigates adaptive parallel simultaneous stabilization problem for two NDSs via dissipative matrix method. Firstly, based on suitable output feedback, two NDSs are transformed into two nonlinear differential-algebraic systems by new coordinate transformations, and then the two systems are combined to generate an augmented dissipative Hamiltonian differential-algebraic system by using the system-augmentation technique, based on which an adaptive parallel simultaneous stabilization controller is designed for the two systems.

Consider the following two NDSs:

$$\begin{aligned} E_1 \dot{x} &= f_1(x, p_1) + g_1(x) u, \\ E_1 x(0) &= E_1 x_0, \\ f_1(0, p_1) &= f_{p_1}(p_1), \\ f_1(0,0) &= 0, \\ y &= g_1^T(x) x, \end{aligned} \tag{1}$$

$$\begin{aligned} E_2 \dot{\xi} &= f_2(\xi, p_2) + g_2(\xi) u, \\ E_2 \xi(0) &= E_2 \xi_0, \\ f_2(0, p_2) &= f_{p_2}(p_2), \\ f_2(0,0) &= 0, \\ \eta &= g_2^T(\xi) \xi, \end{aligned} \tag{2}$$

where $x = [\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_n]^T$, $\xi = [\tilde{\xi}_1, \tilde{\xi}_2, \cdots, \tilde{\xi}_n]^T \in \mathbb{R}^n$ and $y, \eta \in \mathbb{R}^m$ are the states and outputs of the two systems, respectively; $u \in \mathbb{R}^m$ is the control input; $p_i \in \mathbb{R}^s$ is an unknown parameter perturbation vector and is assumed to be small enough to keep the dissipative structure unchanged; i.e., if $R(x) > 0$, then $R(x, p_i) > 0$; $f_i(x, p_i) \in \mathbb{R}^n$ is sufficiently smooth vector fields, $g_1(x), g_2(\xi) \in \mathbb{R}^{n\times m}$; $E_i \in \mathbb{R}^{n\times n}$, $0 < \operatorname{rank}(E_i) = r < n$, and $E_i \geq 0$ or $E_i \leq 0$, $i = 1, 2$. Without loss of generality, we discuss $E_i \geq 0$, $i = 1, 2$.

Definition 1 (see [32]). A control law $u = u(x)$ is called an admissible control law if, for any initial condition Ex_0, the resulting closed-loop descriptor system has no impulsive solution.

Lemma 2 (see [33]). *If a vector function $h(x)$ with $h(0) = 0$ ($x \in \mathbb{R}^n$) has continuous nth-order partial derivatives, then $h(x)$ can be expressed as*

$$h(x) = a_1(x) x_1 + \cdots + a_n(x) x_n, \tag{3}$$

where $a_i(x)$, $i = 1, 2, \cdots, n$, are vector functions.

According to Lemma 2, systems (1) and (2) can be transformed into the following form:

$$\begin{aligned} E_1 \dot{x} &= A_1(x, p_1) \alpha_1(x, p_1) + g_1(x) u, \\ y &= g_1^T(x) x, \end{aligned} \tag{4}$$

$$\begin{aligned} E_2 \dot{\xi} &= A_2(\xi, p_2) \alpha_2(\xi, p_2) + g_2(\xi) u, \\ \eta &= g_2^T(\xi) \xi, \end{aligned} \tag{5}$$

where the structural matrix $A_i(x, p_i) \in \mathbb{R}^{n\times n}$, $\alpha_i(x, p_i) \in \mathbb{R}^n$ is some vector of x and p_i satisfying $\alpha_i(x, 0) = x$, $i = 1, 2$.

To study the adaptive parallel simultaneous stabilization problem of systems (4) and (5), the following assumptions are given:

(A1) $\operatorname{rank}[E_i,\ g_i(x)] = \operatorname{rank}(E_i)$, $\forall x \in \mathbb{R}^n$, $i = 1, 2$;

(A2) assume there exists $\Phi \in \mathbb{R}^{l\times m}$ such that

$$A_i(x,p_i)(\alpha_i(x,p_i)-x)=g_i(x)\Phi^T\theta, \quad \forall x\in\mathbb{R}^n,\ i=1,2, \tag{6}$$

where $\theta\in\mathbb{R}^l$ is an unknown constant vector related to p_i.

Assumption (A1) implies that fast subsystems of the descriptor systems (1) and (2) have no control u. Assumption (A2) is the so-called matched condition. In most cases, we can find Φ and θ such that (6) holds.

Under assumption (A2), systems (4) and (5) are changed as

$$\begin{aligned} E_1\dot{x}&=A_1(x,p_1)x+g_1(x)u+g_1(x)\Phi^T\theta,\\ y&=g_1^T(x)x, \end{aligned} \tag{7}$$

$$\begin{aligned} E_2\dot{\xi}&=A_2(\xi,p_2)\xi+g_2(\xi)u+g_2(\xi)\Phi^T\theta,\\ \eta&=g_2^T(\xi)\xi. \end{aligned} \tag{8}$$

Definition 3. System (4) is called (strictly) dissipative if the structural matrix $A(x)$ is (strictly) dissipative; i.e., $A(x)$ can be expressed as $A(x)=J(x)-R(x)$, where $J(x)$ is skew-symmetric and $R(x)\geq 0$ ($R(x)>0$); system (4) is called feedback (strictly) dissipative if there exists suitable state feedback $u(x)=\alpha(x)+v$ such that the resulting closed-loop descriptor system is (strictly) dissipative.

Remark 4. If $E_1\leq 0$, then systems (7) can be rewritten as

$$\begin{aligned} E_1'\dot{x}&=A_1'(x,p_1)x+g_1'(x)u+g_1'(x)\Phi^T\theta,\\ y'&=g_1'^T(x)x, \end{aligned} \tag{9}$$

where $E_1'=-E_1\geq 0$, $A_1'(x,p_1)=-A_1(x,p_1)$, and $g_1'(x)=-g_1(x)$, $y'=-y$.

We can always express $A_i(x,p_i)$ as $A_i(x,p_i)=J_i(x,p_i)-R_{i0}(x,p_i)$, where $J_i(x,p_i)=(1/2)(A_i(x,p_i)-A_i^T(x,p_i))$ is skew-symmetric and $R_{i0}(x,p_i)=-(1/2)(A_i(x,p_i)+A_i^T(x,p_i))$ is symmetric, $i=1,2$. In order to investigate adaptive parallel simultaneous stabilization of systems (4) and (5), we design an output feedback law such that the symmetric part of structural matrix of the closed-loop system can be transformed into positive definite one. Based on this, we have the following result.

Lemma 5. *Assume that there exists a symmetric matrix $K\in\mathbb{R}^{m\times m}$ such that*

$$\begin{aligned} &-\frac{1}{2}\left(A_1(x,p_1)+A_1^T(x,p_1)\right)+K_{11}(x,x)>0,\\ &-\frac{1}{2}\left(A_2(\xi,p_2)+A_2^T(\xi,p_2)\right)-K_{22}(\xi,\xi)>0, \end{aligned} \tag{10}$$

where $K_{ij}(x,\xi)=g_i(x)Kg_j^T(\xi)$, $i,j=1,2$. Then, under the following adaptive output feedback law

$$\begin{aligned} u&=-K(y-\eta)-\Phi^T\widehat{\theta}+v,\\ \dot{\widehat{\theta}}&=Q\Phi(y+\eta), \end{aligned} \tag{11}$$

systems (4) and (5) can be expressed in the following forms:

$$\begin{aligned} E_1\dot{x}&=(J_1(x,p_1)-R_1(x,p_1))x+g_1(x)Kg_2^T(\xi)\xi\\ &\quad+g_1(x)v+g_1(x)\Phi^T(\theta-\widehat{\theta}),\\ \dot{\widehat{\theta}}&=Q\Phi\left(g_1^T(x)x+g_2^T(\xi)\xi\right),\\ y&=g_1^T(x)x, \end{aligned} \tag{12}$$

$$\begin{aligned} E_2\dot{\xi}&=(J_2(\xi,p_2)-R_2(\xi,p_2))\xi-g_2(\xi)Kg_1^T(x)x\\ &\quad+g_2(\xi)v+g_2(\xi)\Phi^T(\theta-\widehat{\theta}),\\ \dot{\widehat{\theta}}&=Q\Phi\left(g_1^T(x)x+g_2^T(\xi)\xi\right),\\ \eta&=g_2^T(\xi)\xi, \end{aligned} \tag{13}$$

where $J_i(x,p_i)$ is skew-symmetric, $R_i(x,p_i)\in\mathbb{R}^{n\times n}$ is positive definite, $i=1,2$, $\widehat{\theta}$ is an estimate of θ, $Q>0$ is the adaptive gain constant matrix, and v is a new reference input.

Proof. Substituting (11) into systems (7) and (8), respectively, we can obtain systems (12) and (13), where $R_1(x,p_1)=-(1/2)(A_1(x,p_1)+A_1^T(x,p_1))+g_1(x)Kg_1^T(x)$ and $R_2(\xi,p_2)=-(1/2)(A_2(\xi,p_2)+A_2^T(\xi,p_2))-g_2(\xi)Kg_2^T(\xi)$. According to (10), we know that $R_i(x,p_i)>0$. The proof is completed. ☐

Since $E_i\geq 0$ and $0<\operatorname{rank}(E_i)=r<n$, there exists a nonsingular matrix $M_i\in\mathbb{R}^{n\times n}$ such that

$$M_i^TE_iM_i=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix},\quad i=1,2. \tag{14}$$

Denote

$$\begin{aligned} x&=M_i\overline{x},\\ \overline{x}&=\begin{bmatrix} x_1\\ x_2\end{bmatrix},\quad M_i=\begin{bmatrix} M_{i11} & M_{i12}\\ M_{i21} & M_{i22}\end{bmatrix},\\ M_i^Tg_i(x)&=\begin{bmatrix}\widetilde{g}_{i1}(x)\\ \widetilde{g}_{i2}(x)\end{bmatrix}=\begin{bmatrix}\overline{g}_{i1}(\overline{x})\\ \overline{g}_{i2}(\overline{x})\end{bmatrix},\\ M_i^TJ_i(x,p_i)M_i&=\begin{bmatrix}\widetilde{J}_{i11}(x,p_i) & \widetilde{J}_{i12}(x,p_i)\\ -\widetilde{J}_{i12}^T(x,p_i) & \widetilde{J}_{i22}(x,p_i)\end{bmatrix}\\ &=\begin{bmatrix}\overline{J}_{i11}(\overline{x},p_i) & \overline{J}_{i12}(\overline{x},p_i)\\ -\overline{J}_{i12}^T(\overline{x},p_i) & \overline{J}_{i22}(\overline{x},p_i)\end{bmatrix}, \end{aligned}$$

$$M_i^T R_i(x,p_i) M_i = \begin{bmatrix} \widetilde{R}_{i11}(x,p_i) & \widetilde{R}_{i12}(x,p_i) \\ \widetilde{R}_{i12}^T(x,p_i) & \widetilde{R}_{i22}(x,p_i) \end{bmatrix} = \begin{bmatrix} \overline{R}_{i11}(\overline{x},p_i) & \overline{R}_{i12}(\overline{x},p_i) \\ \overline{R}_{i12}^T(\overline{x},p_i) & \overline{R}_{i22}(\overline{x},p_i) \end{bmatrix},$$
$$\nabla_x H_i(x) = \frac{\partial H_i(x)}{\partial x}, \quad i = 1,2, \tag{15}$$

where $x_1 \in \mathbb{R}^r$, $x_2 \in \mathbb{R}^{n-r}$, $\widetilde{J}_{i11}(x,p_i) = \overline{J}_{i11}(\overline{x},p_i)$ and $\widetilde{J}_{i22}(x,p_i) = \overline{J}_{i22}(\overline{x},p_i)$ are skew-symmetric matrices, and $\overline{R}_{i11}(\overline{x},p_i) = \widetilde{R}_{i11}(x,p_i) > 0$, $\overline{R}_{i22}(\overline{x},p_i) = \widetilde{R}_{i22}(x,p_i) = \begin{bmatrix} M_{i12}^T & M_{i22}^T \end{bmatrix} R_i(x,p_i) \begin{bmatrix} M_{i12} \\ M_{i22} \end{bmatrix}$, which implies that $\overline{R}_{i22}(\overline{x},p_i) = \widetilde{R}_{i22}(x,p_i) > 0$, $i = 1,2$.

Remark 6. That $R_i(x,p_i) > 0$ is a sufficient not necessary condition of $\widetilde{R}_{i22}(x,p_i) > 0$. In this paper, $\widetilde{R}_{i22}(x,p_i) > 0$ can guarantee that the closed-loop descriptor systems (12) and (13) have no impulsive solution. Therefore, (10) is a sufficient condition of systems (12) and (13) to be impulse-free.

From (A1), we have

$$\begin{aligned} \operatorname{rank}[E_i,\ g_i(x)] &= \operatorname{rank} M_i^T [E_i,\ g_i(x)] \begin{bmatrix} M_i & 0 \\ 0 & I \end{bmatrix} \\ &= \operatorname{rank}\begin{bmatrix} I_r & 0 & \widetilde{g}_{i1}(x) \\ 0 & 0 & \widetilde{g}_{i2}(x) \end{bmatrix} \\ &= \operatorname{rank}\begin{bmatrix} I_r & 0 \\ 0 & \widetilde{g}_{i2}(x) \end{bmatrix} = \operatorname{rank}(E_i) \\ &= r, \end{aligned} \tag{16}$$

that is, $\overline{g}_{i2}(\overline{x}) = \widetilde{g}_{i2}(x) = 0$. Thus, according to (15) and assumption (A1), systems (12) and (13) can be transformed into the following differential-algebraic systems:

$$\begin{aligned} \dot{x}_1 &= \left(\overline{J}_{111}(\overline{x},p_1) - \overline{R}_{111}(\overline{x},p_1)\right) x_1 \\ &\quad + \left(\overline{J}_{112}(\overline{x},p_1) - \overline{R}_{112}(\overline{x},p_1)\right) x_2 \\ &\quad + \overline{g}_{11}(\overline{x}) K \overline{g}_{21}^T(\overline{\xi}) \xi_1 + \overline{g}_{11}(\overline{x}) v \\ &\quad + \overline{g}_{11}(\overline{x}) \Phi^T (\theta - \hat{\theta}), \\ 0 &= -\left(\overline{J}_{112}^T(\overline{x},p_1) + \overline{R}_{112}^T(\overline{x},p_1)\right) x_1 \\ &\quad + \left(\overline{J}_{122}(\overline{x},p_1) - \overline{R}_{122}(\overline{x},p_1)\right) x_2 \\ &=: \varphi(x_1,x_2,p_1), \\ \dot{\hat{\theta}} &= Q\Phi\left(\overline{g}_{11}^T(\overline{x}) x_1 + \overline{g}_{21}^T(\overline{\xi}) \xi_1\right), \\ y &= \overline{g}_{11}^T(\overline{x}) x_1, \end{aligned} \tag{17}$$

$$\begin{aligned} \dot{\xi}_1 &= \left(\overline{J}_{211}(\overline{\xi},p_2) - \overline{R}_{211}(\overline{\xi},p_2)\right) \xi_1 \\ &\quad + \left(\overline{J}_{212}(\overline{\xi},p_2) - \overline{R}_{212}(\overline{\xi},p_2)\right) \xi_2 \\ &\quad - \overline{g}_{21}(\overline{\xi}) K \overline{g}_{11}^T(\overline{x}) x_1 + \overline{g}_{21}(\overline{\xi}) v \\ &\quad + \overline{g}_{21}(\overline{\xi}) \Phi^T (\theta - \hat{\theta}), \\ 0 &= -\left(\overline{J}_{212}^T(\overline{\xi},p_2) + \overline{R}_{212}^T(\overline{\xi},p_2)\right) \xi_1 \\ &\quad + \left(\overline{J}_{222}(\overline{\xi},p_2) - \overline{R}_{222}(\overline{\xi},p_2)\right) \xi_2, \\ \dot{\hat{\theta}} &= Q\Phi\left(\overline{g}_{11}^T(\overline{x}) x_1 + \overline{g}_{21}^T(\overline{\xi}) \xi_1\right), \\ \eta &= \overline{g}_{21}^T(\overline{\xi}) \xi_1. \end{aligned} \tag{18}$$

Since $\overline{J}_{i22}(\overline{x},p_i) = -\overline{J}_{i22}^T(\overline{x},p_i)$ and $\overline{R}_{i22}(\overline{x},p_i) > 0$, we know that $\overline{J}_{i22}(\overline{x},p_i) - \overline{R}_{i22}(\overline{x},p_i)$ is invertible [34], $i = 1,2$. Therefore, systems (17) and (18) can be expressed in the following forms:

$$\begin{aligned} \dot{x}_1 &= \left(J_{11}(\overline{x},p_1) - R_{11}(\overline{x},p_1)\right) x_1 \\ &\quad + \overline{g}_{11}(\overline{x}) K \overline{g}_{21}^T(\overline{\xi}) \xi_1 + \overline{g}_{11}(\overline{x}) v \\ &\quad + \overline{g}_{11}(\overline{x}) \Phi^T (\theta - \hat{\theta}), \\ 0 &= -\left(\overline{J}_{112}^T(\overline{x},p_1) + \overline{R}_{112}^T(\overline{x},p_1)\right) x_1 \\ &\quad + \left(\overline{J}_{122}(\overline{x},p_1) - \overline{R}_{122}(\overline{x},p_1)\right) x_2, \\ \dot{\hat{\theta}} &= Q\Phi\left(\overline{g}_{11}^T(\overline{x}) x_1 + \overline{g}_{21}^T(\overline{\xi}) \xi_1\right), \\ y &= \overline{g}_{11}^T(\overline{x}) x_1, \end{aligned} \tag{19}$$

$$\begin{aligned} \dot{\xi}_1 &= \left(J_{21}(\overline{\xi},p_2) - R_{21}(\overline{\xi},p_2)\right) \xi_1 \\ &\quad - \overline{g}_{21}(\overline{\xi}) K \overline{g}_{11}^T(\overline{x}) x_1 + \overline{g}_{21}(\overline{\xi}) v \\ &\quad + \overline{g}_{21}(\overline{\xi}) \Phi^T (\theta - \hat{\theta}), \\ 0 &= -\left(\overline{J}_{212}^T(\overline{\xi},p_2) + \overline{R}_{212}^T(\overline{\xi},p_2)\right) \xi_1 \\ &\quad + \left(\overline{J}_{222}(\overline{\xi},p_2) - \overline{R}_{222}(\overline{\xi},p_2)\right) \xi_2, \\ \dot{\hat{\theta}} &= Q\Phi\left(\overline{g}_{11}^T(\overline{x}) x_1 + \overline{g}_{21}^T(\overline{\xi}) \xi_1\right), \\ \eta &= \overline{g}_{21}^T(\overline{\xi}) \xi_1, \end{aligned} \tag{20}$$

where $J_{i1}(\overline{x},p_i) - R_{i1}(\overline{x},p_i) = \overline{J}_{i11}(\overline{x},p_i) - \overline{R}_{i11}(\overline{x},p_i) + (\overline{J}_{i12}(\overline{x},p_i) - \overline{R}_{i12}(\overline{x},p_i))(\overline{J}_{i22}(\overline{x},p_i) - \overline{R}_{i22}(\overline{x},p_i))^{-1} \cdot (\overline{J}_{i12}^T(\overline{x},p_i) + \overline{R}_{i12}^T(\overline{x},p_i))$, $i = 1,2$. $J_{i1}(\overline{x},p_i)$ is skew-symmetric, and $R_{i1}(\overline{x},p_i)$ is positive definite, because

$$N \begin{bmatrix} \overline{J}_{i11} - \overline{R}_{i11} & \overline{J}_{i12} - \overline{R}_{i12} \\ -\left(\overline{J}_{i12}^T + \overline{R}_{i12}^T\right) & \overline{J}_{i22} - \overline{R}_{i22} \end{bmatrix} N^T$$

$$= \begin{bmatrix} J_{i1} - R_{i1} & 0 \\ * & \overline{J}_{i22} - \overline{R}_{i22} \end{bmatrix}, \tag{21}$$

where

$$N = \begin{bmatrix} I & -\left(\overline{J}_{i12} - \overline{R}_{i12}\right)\left(\overline{J}_{i22} - \overline{R}_{i22}\right)^{-1} \\ 0 & I \end{bmatrix}. \tag{22}$$

With assumptions (A1) and (A2), we have the following result.

Theorem 7. *Consider systems (1) and (2) with their equivalent forms (4) and (5). Assume assumptions (A1) and (A2) hold; if there exist symmetric matrices $K \in \mathbb{R}^{m\times m}$ and $\Phi \in \mathbb{R}^{l\times m}$ such that (10) and (6) hold, respectively, then the admissible adaptive parallel controller (11) ($v = 0$) can simultaneously stabilize systems (1) and (2).*

Proof. If assumptions (A1) and (A2) hold, then systems (4) and (5) can be transformed into systems (19) and (20) by the adaptive feedback law (11), which are of index one at the equilibrium point 0 (system (12) is said to have index one at the equilibrium point 0 if $\partial\varphi(x_1, x_2, p_1)/\partial x_2$ in (17) is nonsingular in a neighborhood of 0); i.e., systems (19) and (20) are impulse-free. According to the implicit function theorem, there exist continuous functions $q_i(\cdot)$ such that $x_2 = q_1(x_1)$, $\xi_2 = q_2(\xi_1)$, $q_i(0) = 0$. Thus, systems (19) and (20) can be rewritten as ($v = 0$)

$$\begin{aligned} \dot{X} &= \left(J(X,p) - R(X,p)\right)\frac{\partial H(X)}{\partial X}, \\ 0 &= -\left(\overline{J}^T_{112}(\overline{x}, p_1) + \overline{R}^T_{112}(\overline{x}, p_1)\right)x_1 \\ &\quad + \left(\overline{J}_{122}(\overline{x}, p_1) - \overline{R}_{122}(\overline{x}, p_1)\right)x_2, \\ 0 &= -\left(\overline{J}^T_{212}(\overline{\xi}, p_2) + \overline{R}^T_{212}(\overline{\xi}, p_2)\right)\xi_1 \\ &\quad + \left(\overline{J}_{222}(\overline{\xi}, p_2) - \overline{R}_{222}(\overline{\xi}, p_2)\right)\xi_2, \end{aligned} \tag{23}$$

where

$$\begin{aligned} X &= \begin{bmatrix} x_1 \\ \xi_1 \\ \widehat{\theta} \end{bmatrix}, \\ p &= \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}, \\ R(X,p) &= \begin{bmatrix} R_{11}(x_1, q_1(x_1), p_1) & 0 & 0 \\ 0 & R_{21}(\xi_1, q_2(\xi_1), p_2) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ J(X,p) &= \begin{bmatrix} J_{11}(x_1, q_1(x_1), p_1) & \overline{g}_{11}(x_1, q_1(x_1))K\overline{g}^T_{21}(\xi_1, q_2(\xi_1)) & -\overline{g}_{11}(x_1, q_1(x_1))\Phi^T Q \\ -\left(\overline{g}_{11}(x_1, q_1(x_1))K\overline{g}^T_{21}(\xi_1, q_2(\xi_1))\right)^T & J_{21}(\xi_1, q_2(\xi_1), p_2) & -\overline{g}_{21}(\xi_1, q_2(\xi_1))\Phi^T Q \\ \left(\overline{g}_{11}(x_1, q_1(x_1))\Phi^T Q\right)^T & \left(\overline{g}_{21}(\xi_1, q_2(\xi_1))\Phi^T Q\right)^T & 0 \end{bmatrix}, \\ H(X) &= \frac{1}{2}\left(x_1^T x_1 + \xi_1^T \xi_1\right) + \frac{1}{2}\left(\theta - \widehat{\theta}\right)^T Q^{-1}\left(\theta - \widehat{\theta}\right). \end{aligned} \tag{24}$$

Obviously, $J(X,p) = -J^T(X,p)$, $R(X,p) \geq 0$, $H(X) \geq 0$. Therefore, system (23) is a dissipative Hamiltonian system. Choosing $V(X) = H(X)$, then $H(X)$ has a local minimum at $X_0 = (0^T, 0^T, \widehat{\theta}_0^T)^T$. Then, based on system (23) we have

$$\begin{aligned} \dot{V}(X) &= \frac{\partial^T H(X)}{\partial X}\dot{X} \\ &= \frac{\partial^T H(X)}{\partial X}\left(J(X,p) - R(X,p)\right)\frac{\partial H(X)}{\partial X} \\ &= -\frac{\partial^T H(X)}{\partial X}R(X,p)\frac{\partial H(X)}{\partial X} \\ &= -x_1^T R_{11}(x_1, q_1(x_1), p_1)x_1 \\ &\quad - \xi_1^T R_{21}(\xi_1, q_2(\xi_1), p_2)\xi_1 \leq 0. \end{aligned} \tag{25}$$

Thus, system (23) converges to the largest invariant set contained in

$$\begin{aligned} \left\{X : \dot{V}(X) = 0\right\} \subset \Big\{X : R_{11}^{1/2}(x_1, q_1(x_1), p_1)x_1 \\ = 0,\ R_{21}^{1/2}(\xi_1, q_2(\xi_1), p_2)\xi_1 = 0,\ \forall t \geq 0\Big\} := S. \end{aligned} \tag{26}$$

From systems (19) and (20), we know that both $R_{11}^{1/2}(x_1, q_1(x_1), p_1)$ and $R_{21}^{1/2}(\xi_1, q_2(\xi_1), p_2)$ are nonsingular, which implies that $R_{11}^{1/2}(x_1, q_1(x_1), p_1)x_1 = 0 \Longrightarrow x_1 = 0$ and $R_{21}^{1/2}(\xi_1, q_2(\xi_1), p_2)\xi_1 = 0 \Longrightarrow \xi_1 = 0$. That is, the largest invariant set only contains one point, i.e., $S = \{[0^T, 0^T, \widehat{\theta}_0^T]^T\}$, with which it is easy to see that $x_1 \longrightarrow 0$ and $\xi_1 \longrightarrow 0$, as $t \longrightarrow \infty$. Moreover, according to systems (19) and (20), it is clear that $x_2 \longrightarrow 0$ and $\xi_2 \longrightarrow 0$, as $t \longrightarrow \infty$. Thus, $x = M_1\overline{x} \longrightarrow 0$, $\xi = M_2\overline{\xi} \longrightarrow 0$, as $t \longrightarrow \infty$. Therefore, under the admissible adaptive parallel control law (11), systems (1) and (2) can be simultaneously stabilized. □

3. Robust Adaptive Parallel Simultaneous Stabilization of Two NDSs and More Than Two NDSs

In this section, we investigate the robust adaptive parallel simultaneous stabilization problem of two NDSs with external disturbances and parameters perturbation and discuss the case of more than two NDSs. Firstly, for a given disturbance attenuation level $\gamma > 0$, we design an adaptive parallel L_2 disturbance attenuation output feedback law such that under the law the L_2 gain (from w to z) of the closed-loop system is less than γ. Then, we show that the two systems are simultaneously asymptotically stable when $w = 0$.

To design the robust adaptive parallel simultaneous stabilization controller, the following lemma is recalled, first.

Lemma 8 (see [34]). *Consider a dissipative Hamiltonian system as follows:*

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\nabla H + g_1(x)u + g_2(x)w, \\ z &= h(x)g_1^T(x)\nabla H, \end{aligned} \tag{27}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $w \in \mathbb{R}^q$ is the disturbance, $J(x)$ is skew-symmetric, $R(x) \geqslant 0$, $H(x)$ has a strict local minimum at the system's equilibrium, z is the penalty function, and $h(x)$ is a weighting matrix. Given a disturbance attenuation level $\gamma > 0$, if

$$R(x) + \frac{1}{2\gamma^2}\left[g_1(x)g_1^T(x) - g_2(x)g_2^T(x)\right] \geq 0, \tag{28}$$

then an L_2 disturbance attenuation controller of system (27) can be given as

$$u = -\left[\frac{1}{2}h^T(x)h(x) + \frac{1}{2\gamma^2}I_m\right]g_1^T(x)\nabla H, \tag{29}$$

and the γ-dissipation inequality

$$\begin{aligned} &\dot{H} + \nabla^T H\left[R(x)\right. \\ &\left.+ \frac{1}{2\gamma^2}\left(g_1(x)g_1^T(x) - g_2(x)g_2^T(x)\right)\right]\nabla H \\ &\leq \frac{1}{2}\left\{\gamma^2\|w\|^2 - \|z\|^2\right\} \end{aligned} \tag{30}$$

holds along the trajectories of the closed-loop system consisting of (27) and (29).

Now, we consider the following NDSs (1) and (2) with external disturbances:

$$\begin{aligned} E_1\dot{x} &= f_1(x, p_1) + g_1(x)u + d_1 w, \\ E_1 x(0) &= E_1 x_0, \\ f_1(0, p_1) &= f_{p_1}(p_1), \\ f_1(0,0) &= 0, \\ y &= g_1^T(x)x, \end{aligned} \tag{31}$$

$$\begin{aligned} E_2\dot{\xi} &= f_2(\xi, p_2) + g_2(\xi)u + d_2 w, \\ E_2\xi(0) &= E_2\xi_0, \\ f_2(0, p_2) &= f_{p_2}(p_2), \\ f_2(0,0) &= 0, \\ \eta &= g_2^T(\xi)\xi, \end{aligned} \tag{32}$$

where $w \in \mathbb{R}^q$ is the disturbance, $d_i(x) \in \mathbb{R}^{n\times q}$, $i = 1, 2$, other variables are the same as those in systems (1) and (2), and

$$M_i^T d_i(x) = \begin{bmatrix} \widetilde{d}_{i1}(x) \\ \widetilde{d}_{i2}(x) \end{bmatrix} = \begin{bmatrix} \overline{d}_{i1}(\overline{x}) \\ \overline{d}_{i2}(\overline{x}) \end{bmatrix}. \tag{33}$$

Given a disturbance attenuation level $\gamma > 0$, choose

$$z = \Lambda(y + \eta) \tag{34}$$

as the penalty function, where $\Lambda \in \mathbb{R}^{s\times m}$ is a weighting matrix.

To design the adaptive parallel L_2 disturbance attenuation output feedback control law for systems (31) and (32), the following assumption is given:

(A3) rank $[E_i,\ d_i(x)] = \text{rank}(E_i)$, $\forall x \in \mathbb{R}^n$, $i = 1, 2$.

Assumption (A3) implies that fast subsystems of the descriptor systems (31) and (32) have not been disturbed. Similar to (A1), from (A3) we can obtain that $\widetilde{d}_{i2}(x) = \overline{d}_{i2}(\overline{x}) = 0$.

Based on Section 2, systems (31) and (32) can be transformed into the following forms:

$$\begin{aligned} E_1\dot{x} &= A_1(x, p_1)\alpha_1(x, p_1) + g_1(x)u + d_1(x)w, \\ y &= g_1^T(x)x, \end{aligned} \tag{35}$$

$$\begin{aligned} E_2\dot{\xi} &= A_2(\xi, p_2)\alpha_2(\xi, p_2) + g_2(\xi)u + d_2(x)w, \\ \eta &= g_2^T(\xi)\xi. \end{aligned} \tag{36}$$

Next, we design an adaptive parallel L_2 disturbance attenuation controller for systems (31) and (32).

Theorem 9. *Consider systems (31) and (32) with their equivalent forms (35) and (36), the penalty function (34), and the disturbance attenuation level $\gamma > 0$. Assume that assumptions (A1)~ (A3) hold for systems (35) and (36). If*

(1) *there exists a symmetric matrix $K \in \mathbb{R}^{m\times m}$ such that (10) holds,*

(2) $g_i = d_i,\ i = 1, 2,$

then, the following admissible adaptive parallel feedback law

$$u = -K(y-\eta) - \left[\frac{1}{2}\Lambda^T\Lambda + \frac{1}{2\gamma^2}I_m\right](y+\eta) - \Phi^T\hat{\theta},$$
$$\dot{\hat{\theta}} = Q\Phi(y+\eta) \tag{37}$$

can simultaneously stabilize systems (31) and (32).

Proof. Rewrite (37) as follows

$$\begin{aligned} u &= -K(y-\eta) - \Phi^T\hat{\theta} + v, \\ \dot{\hat{\theta}} &= Q\Phi(y+\eta), \\ v &= -\left[\frac{1}{2}\Lambda^T\Lambda + \frac{1}{2\gamma^2}I_m\right](y+\eta). \end{aligned} \tag{38}$$

Substituting the first part of (38) into systems (35) and (36), according to the proof of Theorem 7 and assumption (A2), we know that systems (35) and (36) are impulse controllable and can be expressed as the following dissipative Hamiltonian form:

$$\begin{aligned} \dot{X} &= [J(X,p) - R(X,p)]\frac{\partial H(X)}{\partial X} + G(X)v \\ &\quad + D(X)w, \\ 0 &= -\left(\overline{J}_{112}^T(\overline{x},p_1) + \overline{R}_{112}^T(\overline{x},p_1)\right)x_1 \\ &\quad + \left(\overline{J}_{122}(\overline{x},p_1) - \overline{R}_{122}(\overline{x},p_1)\right)x_2, \\ 0 &= -\left(\overline{J}_{212}^T(\overline{\xi},p_2) + \overline{R}_{212}^T(\overline{\xi},p_2)\right)\xi_1 \\ &\quad + \left(\overline{J}_{222}(\overline{\xi},p_2) - \overline{R}_{222}(\overline{\xi},p_2)\right)\xi_2, \end{aligned} \tag{39}$$

and

$$z = \Lambda G^T(X)\frac{\partial H(X)}{\partial X}, \tag{40}$$

where X, $J(X,p)$, $R(X,p)$, and $H(X)$ are given in (23), $G(X) = \left[\overline{g}_{11}^T(x_1,q_1(x_1))\ \ \overline{g}_{21}^T(\xi_1,q_2(\xi_1))\ \ 0\right]^T$ and $D(X) = \left[\overline{d}_{11}^T(x_1,q_1(x_1))\ \ \overline{d}_{21}^T(\xi_1,q_2(\xi_1))\ \ 0\right]^T$.

Because $g_i = d_i,\ i = 1, 2$, it is easy to show

$$\begin{aligned} &R(X,p) + \frac{1}{2\gamma^2}\left[G(X)G^T(X) - D(X)D^T(X)\right] \\ &= R(X,p) \geq 0. \end{aligned} \tag{41}$$

Thus, system (39) with the penalty function (40) satisfies all the conditions of Lemma 8. From Lemma 8, an L_2 disturbance attenuation controller of system (39) can be designed as

$$v = -\left[\frac{1}{2}\Lambda^T\Lambda + \frac{1}{2\gamma^2}I_m\right](y+\eta), \tag{42}$$

which is the second part of (38), and, furthermore, the γ-dissipation inequality

$$\dot{H} + \frac{\partial^T H}{\partial X}R(X,p)\frac{\partial H}{\partial X} \leq \frac{1}{2}\left\{\gamma^2\|w\|^2 - \|z\|^2\right\} \tag{43}$$

holds along the trajectories of the closed-loop system consisting of (39) and (42).

Therefore, the feedback law (37) is an L_2 disturbance attenuation controller of systems (31) and (32). According to [34], the L_2 gain from w to z is less than γ. On the other hand, because $(\partial^T H/\partial X)R(X,p)(\partial H/\partial X) = x_1^T R_{11}(x_1,q_1(x_1),p_1)x_1 + \xi_1^T R_{21}(\xi_1,q_2(\xi_1),p_2)\xi_1 > 0$, from (43), we know that system (39) is asymptotically stable when $w = 0$; that is, $x_1 \longrightarrow 0$ and $\xi_1 \longrightarrow 0$ (as $t \longrightarrow \infty$). Moreover, it is clear that $x_2 = q_1(x_1) \longrightarrow 0$, $\xi_2 = q_2(\xi_1) \longrightarrow 0$ (as $t \longrightarrow \infty$). Therefore, $x = M_1\overline{x} \longrightarrow 0$ and $\xi = M_2\overline{\xi} \longrightarrow 0$ (as $t \longrightarrow \infty$). Thus, the admissible adaptive parallel control law (37) can simultaneously stabilize systems (31) and (32). □

Theorem 10. *Consider systems (31) and (32) with their equivalent forms (35) and (36), the penalty function (34), and the disturbance attenuation level $\gamma > 0$. Assume that assumptions (A1) ~ (A3) hold for systems (35) and (36). If*

(1) *there exists a symmetric matrix $K \in \mathbb{R}^{m\times m}$ such that (10) holds, and*

$$\begin{aligned} &-\frac{1}{2}\left(A_1(x,p_1) + A_1(x,p_1)^T\right) + K_{11}(x,x) \\ &\quad + \frac{1}{2\gamma^2}\left[g_1(x)g_1^T(x) - d_1(x)d_1^T(x)\right] > 0, \\ &-\frac{1}{2}\left(A_2(\xi,p_2) + A_2(\xi,p_2)^T\right) - K_{22}(\xi,\xi) \\ &\quad + \frac{1}{2\gamma^2}\left[g_2(\xi)g_2^T(\xi) - d_2(\xi)d_2^T(\xi)\right] > 0, \end{aligned} \tag{44}$$

where $K_{ij}(x,\xi) = g_i(x)Kg_j^T(\xi),\ i,j = 1,2;$

(2) $g_1g_2^T = 0$ *and* $d_1d_2^T = 0$,

then, the admissible adaptive parallel L_2 disturbance attenuation controller (37) can simultaneously stabilize systems (31) and (32).

Proof. From the proof of Theorem 9, we know that under the controller (37), systems (35) and (36) are impulse controllable and can be expressed as (39). From condition (2), it can be seen that

$$\begin{aligned} M_1^T g_1 g_2^T M_2 &= \begin{bmatrix} \overline{g}_{11}(\overline{x}) \\ 0 \end{bmatrix}\left[\overline{g}_{21}^T(\overline{x})\ \ 0\right] \\ &= \begin{bmatrix} \overline{g}_{11}(\overline{x})\overline{g}_{21}^T(\overline{x}) & 0 \\ 0 & 0 \end{bmatrix} = 0, \end{aligned} \tag{45}$$

that is, $\overline{g}_{11}(\overline{x})\overline{g}_{21}^T(\overline{x}) = 0$, and in a similar way, we can obtain $\overline{d}_{11}(\overline{x})\overline{d}_{21}^T(\overline{x}) = 0$. Moreover, according to condition (1), we have

$$\begin{aligned} &M_1^T\left(-\frac{1}{2}\left(A_1(x,p_1)+A_1^T(x,p_1)\right)\right.\\ &\quad\left.+g_1(x)Kg_1^T(x)\right)M_1+\frac{1}{2\gamma^2}M_1^T\left[g_1(x)g_1^T(x)\right.\\ &\quad\left.-d_1(x)d_1^T(x)\right]M_1\\ &\quad=\begin{bmatrix}\overline{R}_{111}(\overline{x},p_1) & \overline{R}_{112}(\overline{x},p_1)\\ \overline{R}_{112}^T(\overline{x},p_1) & \overline{R}_{122}(\overline{x},p_1)\end{bmatrix}+\frac{1}{2\gamma^2}\\ &\quad\cdot\begin{bmatrix}\overline{g}_{11}(\overline{x})\overline{g}_{11}^T(\overline{x})-\overline{d}_{11}(\overline{x})\overline{d}_{11}^T(\overline{x}) & 0\\ 0 & 0\end{bmatrix}>0. \end{aligned} \tag{46}$$

Thus,

$$\begin{aligned} &\overline{R}_{111}(\overline{x},p_1)+\frac{1}{2\gamma^2}\left[\overline{g}_{11}(\overline{x})\overline{g}_{11}^T(\overline{x})-\overline{d}_{11}(\overline{x})\overline{d}_{11}^T(\overline{x})\right]\\ &\quad\coloneqq\overline{R}_{111}(\overline{x},p_1)+C(\overline{x})>0. \end{aligned} \tag{47}$$

Since

$$\begin{aligned} &N\begin{bmatrix}\overline{J}_{111}-\overline{R}_{111}-C & \overline{J}_{112}-\overline{R}_{112}\\ -\left(\overline{J}_{112}^T+\overline{R}_{112}^T\right) & \overline{J}_{122}-\overline{R}_{122}\end{bmatrix}N^T\\ &\quad=\begin{bmatrix}J_{11}-R_{11}-C & 0\\ * & \overline{J}_{22}-\overline{R}_{22}\end{bmatrix}, \end{aligned} \tag{48}$$

where $\overline{J}_{111}$ is skew-symmetric and N is the same as that in (22), we have

$$\begin{aligned} &\widehat{R}_1(\overline{x},p_1)\\ &\quad\coloneqq R_{11}(\overline{x},p_1)\\ &\qquad+\frac{1}{2\gamma^2}\left[\overline{g}_{11}(\overline{x})\overline{g}_{11}^T(\overline{x})-\overline{d}_{11}(\overline{x})\overline{d}_{11}^T(\overline{x})\right]>0. \end{aligned} \tag{49}$$

In a similar way,

$$\begin{aligned} &\widehat{R}_2(\overline{\xi},p_2)\\ &\quad\coloneqq R_{21}(\overline{\xi},p_2)\\ &\qquad+\frac{1}{2\gamma^2}\left[\overline{g}_{21}(\overline{\xi})\overline{g}_{21}^T(\overline{\xi})-\overline{d}_{21}(\overline{\xi})\overline{d}_{21}^T(\overline{\xi})\right]>0. \end{aligned} \tag{50}$$

Therefore,

$$\begin{aligned} &R(X,p)+\frac{1}{2\gamma^2}\left[G(X)G^T(X)-D(X)D^T(X)\right]\\ &\quad=\begin{bmatrix}\widehat{R}_1(x_1,q_1(x_1),p_1) & 0 & 0\\ 0 & \widehat{R}_2(\xi_1,q_2(\xi_1),p_2) & 0\\ 0 & 0 & 0\end{bmatrix}\\ &\quad\geq 0. \end{aligned} \tag{51}$$

Thus, system (39) with the penalty function (40) satisfies all the conditions of Lemma 8. From Lemma 8, an adaptive parallel L_2 disturbance attenuation controller of system (39) can be designed as (42), and, furthermore, the γ-dissipation inequality

$$\begin{aligned} &\dot{H}+\frac{\partial^T H}{\partial X}\left\{R(X,p)\right.\\ &\quad\left.+\frac{1}{2\gamma^2}\left[G(X)G^T(X)-D(X)D^T(X)\right]\right\}\frac{\partial H}{\partial X}\\ &\quad\leq\frac{1}{2}\left\{\gamma^2\|w\|^2-\|z\|^2\right\} \end{aligned} \tag{52}$$

holds along the trajectories of the closed-loop system consisting of (39) and (42). Therefore, according to the proof of Theorem 9, the admissible controller (37) can simultaneously stabilize systems (31) and (32). □

Remark 11. We can utilize the results obtained on adaptive parallel simultaneous stabilization and robust adaptive parallel simultaneous stabilization problems for two NDSs to investigate the same problems of more than two NDSs.

Consider the following N NDSs:

$$\begin{aligned} E_i\dot{x}^i&=f_i(x^i,p_i)+g_i(x^i)u+d_i(x^i)w,\\ E_ix^i(0)&=E_ix_0^i,\\ f_i(0,p_i)&=f_{p_i}(p_i),\\ f_i(0,0)&=0,\\ y_i&=g_i^T(x^i)x^i,\\ &\qquad i=1,2,\cdots,N, \end{aligned} \tag{53}$$

where $x^i\in\mathbb{R}^{n_i}$, $u\in\mathbb{R}^m$, $w\in\mathbb{R}^q$, and $y_i\in\mathbb{R}^m$ are the states, control input, external disturbances, and outputs of the N systems, respectively; p_i is an unknown parameter perturbation vector and is assumed to be small enough to keep the dissipative structure unchanged; $g_i(x^i)\in\mathbb{R}^{n_i\times m}$, $0\leq E_i\in\mathbb{R}^{n_i\times n_i}$, and $0<\mathrm{rank}(E_i)=r_i<n_i$, $i=1,2,\cdots,N$.

Given a disturbance attenuation level $\gamma>0$, choose

$$z=\Lambda\sum_{i=1}^N y_i,\quad i=1,2,\cdots,N \tag{54}$$

as the penalty function, where $\Lambda\in\mathbb{R}^{s\times m}$ is a weighting matrix.

Similar to Section 2, we obtain the following forms:

$$\begin{aligned} E_i\dot{x}^i&=A_i(x^i,p_i)\alpha_i(x^i,p_i)+g_i(x^i)u+d_i(x^i)w,\\ y_i&=g_i^T(x^i)x^i, \end{aligned} \tag{55}$$

where $\alpha_i(x^i,p_i)\in\mathbb{R}^{n_i}$ is some vector of x^i and p_i satisfying $\alpha_i(x^i,0)=x^i$, $i=1,2,\cdots,N$.

Assume that $(i_1, i_2, \cdots, i_N)$ is an arbitrary permutation of $\{1, 2, \cdots, N\}$ and that L is a positive integer satisfying $1 \leqslant L \leqslant N-1$. Let $T_1 = n_{i_1} + \cdots + n_{i_L}$ and $T_2 = n_{i_{L+1}} + \cdots + n_{i_N}$.

Now, we divide the N systems into two sets as follows:

$$\begin{aligned} E_a \dot{X}^a &= A_a(X^a, p_a)\Gamma_a(X^a, p_a) + G_a(X^a)u \\ &\quad + D_a(X^a)w, \\ Y_a &= G_a^T(X^a)X^a, \end{aligned} \tag{56}$$

$$\begin{aligned} E_b \dot{X}^b &= A_b(X^b, p_b)\Gamma_b(X^b, p_b) + G_b(X^b)u \\ &\quad + D_b(X^b)w, \\ Y_b &= G_b^T(X^b)X^b, \end{aligned} \tag{57}$$

where $X^a = [(x^{i_1})^T, \cdots, (x^{i_L})^T]^T \in \mathbb{R}^{T_1}$, $X^b = [(x^{i_{L+1}})^T, \cdots, (x^{i_N})^T]^T \in \mathbb{R}^{T_2}$, $p_a = [p_{i_1}^T, \cdots, p_{i_L}^T]^T$, $p_b = [p_{i_{L+1}}^T, \cdots, p_{i_N}^T]^T$,

$$\begin{aligned} E_a &= \operatorname{diag}\{E_{i_1}, \cdots, E_{i_L}\}, \\ E_b &= \operatorname{diag}\{E_{i_{L+1}}, \cdots, E_{i_N}\}, \\ A_a&(X^a, p_a) \\ &= \operatorname{diag}\{A_{i_1}(x^{i_1}, p_{i_1}), \cdots, A_{i_L}(x^{i_L}, p_{i_L})\}, \\ A_b&(X^b, p_b) \\ &= \operatorname{diag}\{A_{i_{L+1}}(x^{i_{L+1}}, p_{i_{L+1}}), \cdots, A_{i_N}(x^{i_N}, p_{i_N})\}, \\ \Gamma_a(X^a, p_a) &= \operatorname{diag}\{\alpha_{i_1}(x^{i_1}, p_{i_1}), \cdots, \alpha_{i_L}(x^{i_L}, p_{i_L})\}, \\ \Gamma_b&(X^b, p_b) \\ &= \operatorname{diag}\{\alpha_{i_{L+1}}(x^{i_{L+1}}, p_{i_{L+1}}), \cdots, \alpha_{i_N}(x^{i_N}, p_{i_N})\}, \\ Y_a &= y_{i_1} + \cdots + y_{i_L}, \\ Y_b &= y_{i_{L+1}} + \cdots + y_{i_N}, \\ G_a(X^a) &= [g_{i_1}^T(x^{i_1}), \cdots, g_{i_L}^T(x^{i_L})]^T, \\ G_b(X^b) &= [g_{i_{L+1}}^T(x^{i_{L+1}}), \cdots, g_{i_N}^T(x^{i_N})]^T, \\ D_a(X^a) &= [d_{i_1}^T(x^{i_1}), \cdots, d_{i_L}^T(x^{i_L})]^T, \\ D_b(X^b) &= [d_{i_{L+1}}^T(x^{i_{L+1}}), \cdots, d_{i_N}^T(x^{i_N})]^T. \end{aligned} \tag{58}$$

According to Section 2, (56), (57), and Theorems 9 and 10, we can easily obtain an adaptive parallel simultaneous stabilization controller (w=0) and a robust adaptive parallel simultaneous stabilization controller of systems (53).

Theorem 12. *Consider systems (53) (w=0) with their equivalent forms (55) (w=0), and assume that assumptions (A1) and (A2) hold ($i = 1, 2, \cdots, N$). If there exist a symmetric matrix $K \in \mathbb{R}^{m\times m}$, a permutation $(i_1, i_2, \cdots, i_N)$ of $\{1, 2, \cdots, N\}$, and a positive integer L ($1 \leqslant L \leqslant N-1$) such that*

$$\begin{aligned} R_a(X^a, p_a) &:= -\frac{1}{2}\left(A_a(X^a, p_a) + A_a(X^a, p_a)^T\right) \\ &\quad + K_{aa}(X^a, X^a) > 0, \\ R_b(X^b, p_b) &:= -\frac{1}{2}\left(A_b(X^b, p_b) + A_b(X^b, p_b)^T\right) \\ &\quad - K_{bb}(X^b, X^b) > 0, \end{aligned} \tag{59}$$

where

$$K_{ij}(X^i, X^j) = G_i(X^i)KG_j^T(X^j), \quad i, j = a, b, \tag{60}$$

then, the adaptive control law

$$\begin{aligned} u &= -K\left(y_{i_1} + \cdots + y_{i_L} - y_{i_{L+1}} - \cdots - y_{i_N}\right) - \Phi^T\widehat{\theta} + v, \\ \dot{\widehat{\theta}} &= Q\Phi\sum_{i=1}^{N} y_i \end{aligned} \tag{61}$$

can simultaneously stabilize the N systems given by (53) (w=0), where v is a new reference input and $\widehat{\theta}$ and Q are the same as those in (11).

Theorem 13. *Consider systems (53), the penalty function (54), and the disturbance attenuation level $\gamma > 0$. Assume that assumptions (A1) ~ (A3) ($i = 1, 2, \cdots, N$) hold. If*

(1) *there exist a symmetric matrix $K \in \mathbb{R}^{m\times m}$, a permutation $(i_1, i_2, \cdots, i_N)$ of $\{1, 2, \cdots, \mathrm{N}\}$, and a positive integer L ($1 \leqslant L \leqslant N-1$) such that (59) holds,*

(2) $g_i = d_i, i = 1, 2, \cdots, N,$

then, the following robust adaptive parallel controller

$$\begin{aligned} u &= -K\left(y_{i_1} + \cdots + y_{i_L} - y_{i_{L+1}} - \cdots - y_{i_N}\right) \\ &\quad - \left[\frac{1}{2}\Lambda^T\Lambda + \frac{1}{2\gamma^2}I_m\right]\sum_{i=1}^{N} y_i - \Phi^T\widehat{\theta}, \\ \dot{\widehat{\theta}} &= Q\Phi\sum_{i=1}^{N} y_i \end{aligned} \tag{62}$$

can simultaneously stabilize the N systems given by (53).

4. An Illustrative Example

In the following, we give an illustrative example to show how to apply Theorem 9 to investigate robust adaptive parallel simultaneous stabilization for two NDSs.

Example 14. Consider the following two NDSs:

$$\begin{aligned} E_1\dot{x} &= f_1(x,p) + g_1(x)u + d_1 w, \\ E_1 x(0) &= E_1 x_0, \\ f_1(0,p) &= f_{1,p}(p), \\ f_1(0,0) &= 0, \\ y &= g_1^T(x)x, \end{aligned} \quad (63)$$

$$\begin{aligned} E_2\dot{\xi} &= f_2(\xi,p) + g_2(\xi)u + d_2 w, \\ E_2 \xi(0) &= E_2 \xi_0, \\ f_2(0,p) &= f_{2,p}(p), \\ f_2(0,0) &= 0, \\ \eta &= g_2^T(\xi)\xi, \end{aligned} \quad (64)$$

where $x = [\tilde{x}_1, \tilde{x}_2, \tilde{x}_3]^T \in \mathbb{R}^3$, $\xi = [\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3]^T \in \mathbb{R}^3$, $u \in \mathbb{R}^2$, $w \in \mathbb{R}^2$,

$$\begin{aligned} E_1 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ E_2 &= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ f_1(x,p) &= \begin{bmatrix} -\tilde{x}_1 + 2\tilde{x}_2 \\ -2\tilde{x}_1 - \tilde{x}_2^3 - \tilde{x}_2 - 2\tilde{x}_3 - 2p \\ 2\tilde{x}_3 + 2p \end{bmatrix}, \\ f_2(\xi,p) &= \begin{bmatrix} \tilde{\xi}_1^3 - 2\tilde{\xi}_1 - \tilde{\xi}_2 - p - 2\tilde{\xi}_3 \\ -\tilde{\xi}_1 - \tilde{\xi}_2 - p \\ 2\tilde{\xi}_1 - 2\tilde{\xi}_3 \end{bmatrix}, \\ g_1(x) = d_1(x) &= \begin{bmatrix} 0 & 0 \\ 2 & \tilde{x}_2 \\ -2 & 0 \end{bmatrix}, \\ g_2(\xi) = d_2(\xi) &= \begin{bmatrix} 1 & \tilde{\xi}_1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \quad (65)$$

Choose the penalty function $z = \Lambda(y + \eta)$, where Λ is a weighting matrix.

Noticing that $f_1(0,0) = f_2(0,0) = 0$, we obtain $\alpha_1(x,p) = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 + p)^T$, $\alpha_2(\xi,p) = (\tilde{\xi}_1, \tilde{\xi}_2 + p, \tilde{\xi}_3)^T$, and

$$\begin{aligned} A_1(x,p) &= \begin{bmatrix} -1 & 2 & 0 \\ -2 & -1-\tilde{x}_2^2 & -2 \\ 0 & 0 & 2 \end{bmatrix}, \\ A_2(\xi,p) &= \begin{bmatrix} \tilde{\xi}_1^2 - 2 & -1 & -2 \\ -1 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix}. \end{aligned} \quad (66)$$

It is easy to check that assumption (A2) is satisfied, where $\Phi = [-1, 0]$ and $\theta = p$. According to Theorem 9, we obtain the following forms of systems (63) and (64) by the output feedback $u = -K(y - \eta) + v$:

$$\begin{aligned} E_1\dot{x} &= (J_1(x,p) - R_1(x,p))x + g_1(x)Kg_2^T(\xi)\xi \\ &\quad + g_1(x)v + d_1(x)w + g_1(x)\Phi^T(\theta - \hat{\theta}), \end{aligned} \quad (67)$$

$$\begin{aligned} y &= g_1^T(x)x, \\ E_2\dot{\xi} &= (J_2(\xi,p) - R_2(\xi,p))\xi - g_2(\xi)Kg_1^T(x)x \\ &\quad + g_2(\xi)v + d_2(\xi)w + g_2(\xi)\Phi^T(\theta - \hat{\theta}), \end{aligned} \quad (68)$$

$$\eta = g_2^T(\xi)\xi,$$

where

$$\begin{aligned} K &= \begin{bmatrix} 0.8 & 0 \\ 0 & -1 \end{bmatrix}, \\ J_1(x,p) &= \begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \\ R_1(x,p) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4.2 & -2.2 \\ 0 & -2.2 & 1.2 \end{bmatrix}, \\ J_2(\xi,p) &= \begin{bmatrix} 0 & 0 & -2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}, \\ R_2(\xi,p) &= \begin{bmatrix} 1.2 & 0.2 & 0 \\ 0.2 & 0.2 & 0 \\ 0 & 0 & 2 \end{bmatrix}. \end{aligned} \quad (69)$$

Since $E_1 \geq 0$ and $E_2 \geq 0$, we can give nonsingular matrices

$$M_1 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & \dfrac{\sqrt{5}}{5} & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

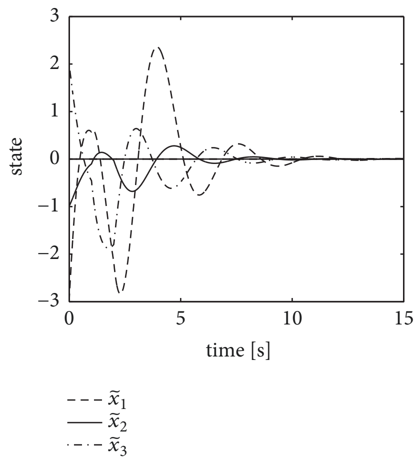

Figure 1: Response of the state x.

$$M_2 = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{70}$$

Moreover, it is clear that (A1) and (A3) are also satisfied. Thus, all the conditions of Theorem 9 hold. Therefore, an admissible adaptive parallel simultaneous stabilization controller of systems (63) and (64) can be designed as

$$u = -K(y-\eta) - \left[\frac{1}{2}\Lambda^T\Lambda + \frac{1}{2\gamma^2}I_m\right](y+\eta) - \Phi^T\widehat{\theta}, \\ \dot{\widehat{\theta}} = Q\Phi(y+\eta). \tag{71}$$

In order to test the effectiveness of the controller (71), we carry out some numerical simulations with the following choices: initial condition: $E_1x(0) = [0,-5,2]^T$, $E_2\xi(0) = [2,-4,0]^T$, $\widehat{\theta}^0 = -0.5$; parameter: $\gamma = 1$, $p = 0.5$, $Q = 1$, and weighting matrix $\Lambda = I_2$. To test the robustness of the controller with respect to external disturbances, we add a square-wave disturbance of amplitude $[2,-4]^T$ to the systems in the time duration $[1s \sim 2s]$. The responses of the states, control signal, and $\widehat{\theta}$ are shown in Figures 1–3, respectively.

It can be observed from Figures 1–3 that the states quickly converge to the origin after the disturbance is removed. The simulation results show that the controller (71) is very effective in simultaneously stabilizing the two systems and has strong robustness against external disturbances and parameters perturbation.

5. Conclusion

This paper has investigated the (robust) adaptive parallel simultaneous stabilization problems of a class of nonlinear descriptor systems via dissipative matrix method. Firstly, under a suitable output feedback law, two nonlinear descriptor systems have been changed as two equivalent nonlinear differential-algebraic systems by nonsingular transforms, and a sufficient condition of impulse-free has been given for two closed-loop systems. Then, the two systems are combined to generate an augmented dissipative Hamiltonian differential-algebraic system, with which an adaptive parallel simultaneous stabilization controller has been designed for the two systems via the Hamiltonian function method. When there are external disturbances in the two systems, a robust adaptive parallel simultaneous stabilization controller has been presented. Finally, the case of more than two nonlinear descriptor systems has also been investigated in this paper.

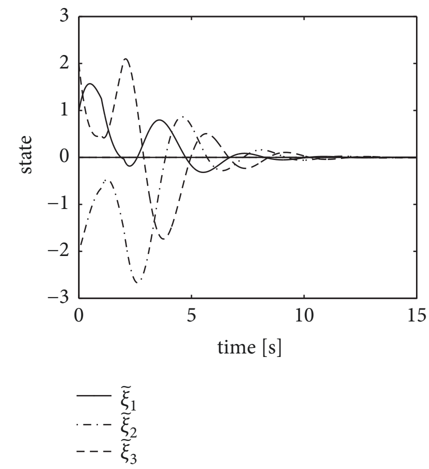

Figure 2: Response of the state ξ.

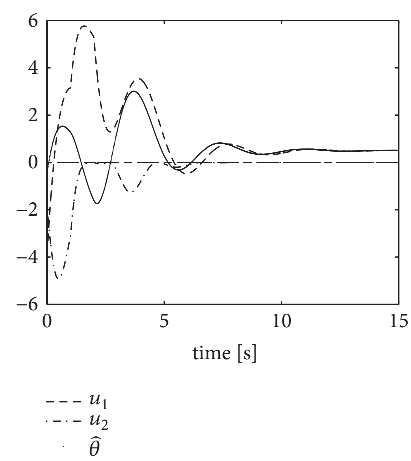

Figure 3: The control u and estimate $\widehat{\theta}$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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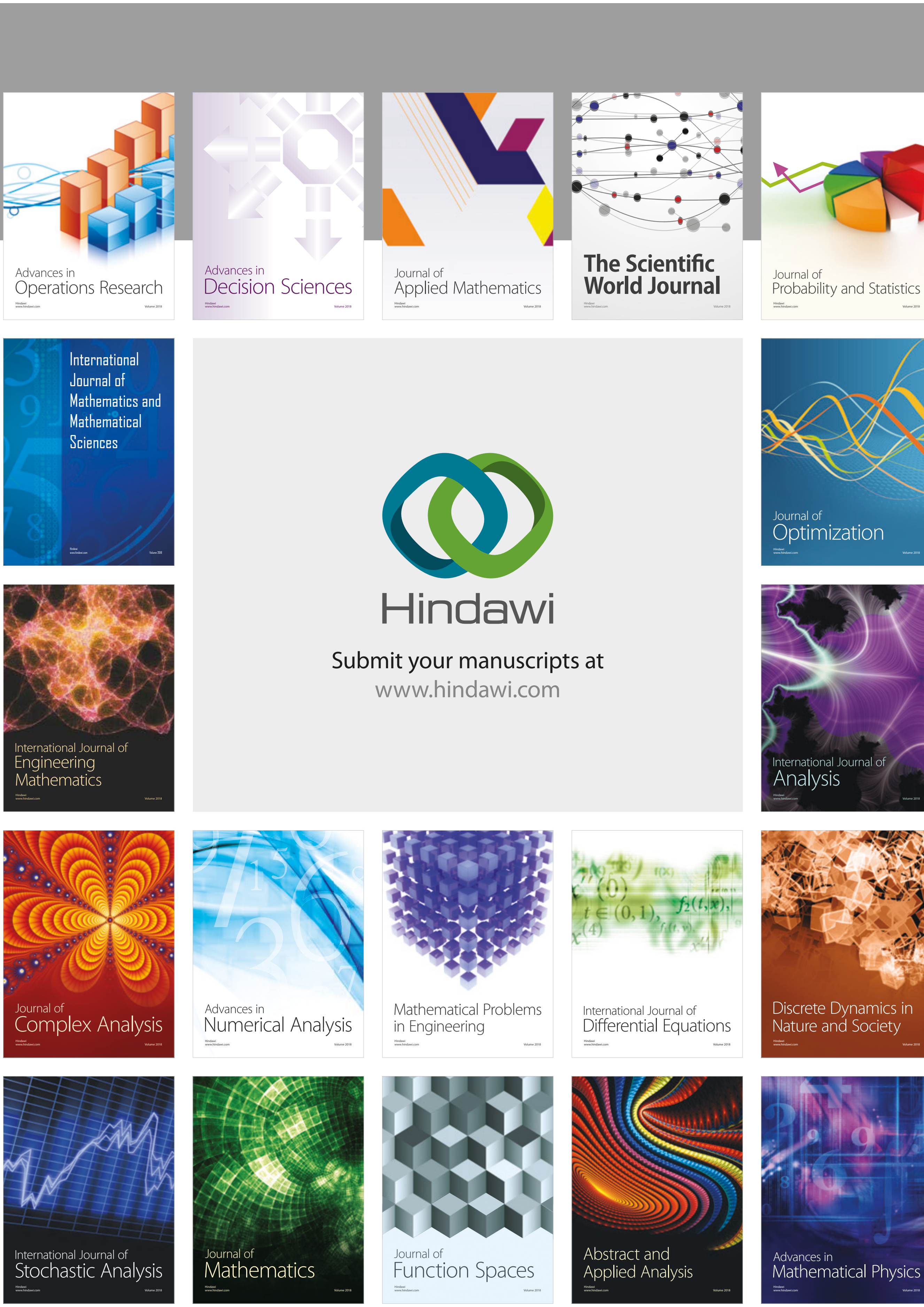
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