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Research Article

Robust Preview Control for Uncertain Discrete Singular Systems

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In this paper, the problem of robust preview control for uncertain discrete singular systems is considered. First of all, by employing the forward difference for uncertain discrete singular systems, the singular augmented error system with the state vector, the input control vector, and the previewable reference signal is derived. Since there is a singular matrix in the system, the existing method cannot be directly applied to this problem. By considering the stability of the transposition system with Linear Matrix Inequality (LMI) method, a new stability criterion for the transposition system is introduced. Then, the robust controller for the augmented error system is obtained, which is regarded as the robust preview controller for the original singular system. At last, the numerical simulation shows the correctness and effectiveness of the results.

1. Introduction

Since the singular system model [1] is proposed by Rosenbrock in the early 1970s, the singular system has been widely studied, and many results are obtained [2, 3]. Especially after the 1980s, singular systems, as an appropriate tool to deal with large-scale complex systems with multiobjective and multidimensional, have been applied to the fields of singularly perturbed theory [4], large-scale systems theory [5], circuit system [6], and so on.

In practical, due to the modeling error, the measurement error, the linear approximation, and the change of working environment, the uncertainty of the system is objective. And they are presented as uncertain model parameters, system perturbation, measurement noise, external interference, etc. There is no doubt that any controller without considering the uncertainties may be difficult to achieve an ideal actual effect or may even cause the collapse of the system. Therefore, robust control is still regarded as one of the hotspot research directions in control theory and the applications. And the study of robust control theory and methods has formed several important branches, such as Lyapunov-Razumikhin method [7], Riccati inequality [8], Linear Matrix Inequality method [9], etc.

For singular systems, uncertain problems also exist [10]. Therefore, the modeling, analysis, and design of uncertain singular systems are particularly important. The related research of uncertain singular systems began in the 1980s. The main purpose of the study is to design a controller to make the corresponding closed-loop system stable and meet certain performance targets, when there are model uncertainty and external interference in the system.

Preview control is a control technique to improve the performance of systems by making full use of known future reference or disturbance signals in advance. Since the preview control model was first proposed in 1960s, the theoretical study and the applications of preview control have never stopped. In recent years, preview control has been extended to multisampling system [11], singular system [12], time-varying system [13], and so on. And it has been applied to many fields, such as automobile control [14], robot control [15], electromechanical valve control system [16], etc.

Based on many results on the robust preview control problem of nonsingular system, the robust preview control problem of singular system is further discussed. Because of the existence of the singular matrix, a proper Lyapunov function cannot be found by the current method to design the robust preview controller. In this paper, a singular augmented error system is constructed by using forward differences. Then, based on the method of [17], a Lyapunov function for the transposition system of the closed-loop system for the singular augmented error system is designed. And by the LMI method, a stability criterion for the transposition system is obtained. Since the two unforced systems before and after transposition have the same stability, the robust controller for the singular augmented error system can be obtained by the transposition system. As for the original singular system, this controller is the robust preview controller.

Notation. $P > 0 (P < 0)$ denotes the notion that matrix P is a positive definite (negative definite) matrix; I denotes the unit matrix; $A \in R^{m\times n}$ denotes a $m \times n$ matrix; the symbol $*$ denotes the symmetric terms in a symmetric matrix.

2. Problem Statement

Consider the following linear uncertain singular system:

$$\begin{aligned} Ex(k+1) &= (A+\Delta A)x(k) + (B+\Delta B)u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the input vector, and $y(k) \in R^p$ is the output vector. $E \in R^{n\times n}$ is the singular matrix with $\operatorname{rank}(E) = q < n$. The matrices A, B, and C are known real constant matrices with appropriate dimensions. The matrices ΔA and ΔB are uncertain matrices.

First of all, the definition about admissible is needed.

Definition 1 (see [17]). The dynamic system

$$Ex(k+1) = Ax(k) \tag{2}$$

is said to be admissible if it is regular, causal, and stable.

Then, the following assumption is needed for system (1).

Assumption 2. The matrices pair (A, B) is stabilizable and the matrix $\left[\begin{smallmatrix} A-E & B \\ C & 0 \end{smallmatrix}\right]$ is of full row rank.

By denoting the previewable reference signal $r(k)$, the following assumption is needed.

Assumption 3. The reference signal $r(k)$ is previewable. That is, at any time k, the future values $r(k+1), r(k+2), \dots, r(k+M_r)$, are available. When $\sigma > k+M_r$, the reference signal $r(\sigma)$ is assumed to be unchanged, namely,

$$r(k+j) = r(k+M_r), \quad j = M_r+1, M_r+2, \dots \tag{3}$$

where M_r is the preview length [18].

About the uncertain matrices ΔA and ΔB, the following assumption is needed.

Assumption 4. There exist real constant matrices E_i, $H_i (i = 1, 2)$ with appropriate dimensions and uncertain matrices $\Sigma_i (i = 1, 2)$ satisfying

$$\begin{aligned} \Delta A &= E_1 \Sigma_1 H_1, \\ \Delta B &= E_2 \Sigma_2 H_2, \end{aligned} \tag{4}$$

$$\Sigma_i^T \Sigma_i \le I, \quad (i = 1, 2). \tag{5}$$

Remark 5. Equation (4) shows that the uncertain matrices ΔA and ΔB in system (1) satisfy matching conditions, and (5) shows that they are norm bounded.

The purpose in this paper is to design a controller for system (1) with preview compensation to make the output $y(k)$ track the reference value $r(k)$ accurately when there exist disturbances ΔA and ΔB in system (1).

3. Construction of the Augmented Error System

The error equation can be defined as follows:

$$e(k) = y(k) - r(k). \tag{6}$$

Employing the difference operator as $\Delta v(k) = v(k+1) - v(k)$ to (1) and (6), we can obtain

$$E\Delta x(k+1) = (A+\Delta A)\Delta x(k) + (B+\Delta B)\Delta u(k) \tag{7}$$

and

$$\Delta e(k) = \Delta y(k) - \Delta r(k). \tag{8}$$

Furthermore, (8) can be rewritten as

$$e(k+1) = e(k) + \Delta e(k) = e(k) + C\Delta x(k) - \Delta r(k). \tag{9}$$

Putting (7) and (9) together, we have

$$\begin{aligned} E_0 X_0(k+1) &= \left(A_0 + \Delta A_0\right) X_0(k) \\ &\quad + \left(B_0 + \Delta B_0\right)\Delta u(k) + B_r \Delta r(k), \end{aligned} \tag{10}$$

where

$$X_0(k) = \begin{bmatrix} e(k) \\ \Delta x(k) \end{bmatrix},$$

$$E_0 = \begin{bmatrix} I_m & 0 \\ 0 & E \end{bmatrix},$$

$$A_0 = \begin{bmatrix} I_m & C \\ 0 & A \end{bmatrix},$$

$$\Delta A_0 = \begin{bmatrix} 0 & 0 \\ 0 & \Delta A \end{bmatrix}, \quad (11)$$

$$B_0 = \begin{bmatrix} 0 \\ B \end{bmatrix},$$

$$\Delta B_0 = \begin{bmatrix} 0 \\ \Delta B \end{bmatrix},$$

$$B_r = \begin{bmatrix} -I_m \\ 0 \end{bmatrix}.$$

Define $X_r(k) = \begin{bmatrix} \Delta r(k) \\ \vdots \\ \Delta r(k+M_r-1) \end{bmatrix}$. Since the future M_r reference signal $r(i)$ $(i = k+1, k+2, \ldots, k+M_r)$ is known at time k, it follows from the Assumption 3 that $X_r(k)$ satisfies

$$X_r(k+1) = A_r X_r(k), \quad (12)$$

where

$$A_r = \begin{bmatrix} 0 & I_m & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & & \ddots & I_m \\ 0 & \cdots & \cdots & 0 \end{bmatrix}, \quad (13)$$

$$B_{Pr} = \begin{bmatrix} B_r & 0 & \cdots & 0 \end{bmatrix}.$$

With (10) and (12), the augmented error system can be written as follows:

$$\widehat{E}\widehat{X}(k+1) = \left(\widehat{A} + \Delta\widehat{A}\right)\widehat{X}(k) + \left(\widehat{B} + \Delta\widehat{B}\right)\Delta u(k) \quad (14)$$

where

$$\widehat{X}(k) = \begin{bmatrix} X_0(k) \\ X_r(k) \end{bmatrix},$$

$$\widehat{E} = \begin{bmatrix} E_0 & 0 \\ 0 & I_{M_r} \end{bmatrix},$$

$$\widehat{A} = \begin{bmatrix} A_0 & B_{Pr} \\ 0 & A_r \end{bmatrix},$$

$$\Delta\widehat{A} = \begin{bmatrix} \Delta A_0 & 0 \\ 0 & 0 \end{bmatrix}, \quad (15)$$

$$\widehat{B} = \begin{bmatrix} B_0 \\ 0 \end{bmatrix},$$

$$\Delta\widehat{B} = \begin{bmatrix} \Delta B_0 \\ 0 \end{bmatrix}.$$

According to the properties of ΔA and ΔB in Assumption 4, we have

$$\Delta\widehat{A} = E_{11}\Sigma_{11}H_{11}, \qquad \Delta\widehat{B} = E_{22}\Sigma_{22}H_{22} \quad (16)$$

where

$$E_{11} = \begin{bmatrix} 0 \\ E_1 \\ 0 \end{bmatrix},$$

$$\Sigma_{11} = \Sigma_1,$$

$$H_{11} = \begin{bmatrix} 0 & H_1 & 0 \end{bmatrix} \quad (17)$$

$$E_{22} = \begin{bmatrix} 0 \\ E_2 \\ 0 \end{bmatrix},$$

$$\Sigma_{22} = \Sigma_2,$$

$$H_{22} = H_2$$

The matrices $\Sigma_{ii}(i = 1, 2)$ still satisfy $\Sigma_{ii}\Sigma_{ii}^{T} \le I, (i = 1, 2)$. Therefore, $\Delta\widehat{A}$ and $\Delta\widehat{B}$ are normal bounded.

4. Design of the Robust Preview Controller

In order to obtain the robust preview controller for system (1), the stability problem of the unforced discrete singular nominal system (2) should be considered first and the following lemmas are needed.

Lemma 6 (see [17]). *The discrete singular system (2) is admissible if and only if there exists a matrix* > 0 $(P \in R^{n\times n})$, *a symmetric matrix* $\Phi \in R^{(n-q)\times(n-q)}$ *such that*

$$A^T\left(P - R^T\Phi R\right)A - E^T PE < 0, \quad (18)$$

where $RE = 0$. $R \in R^{(n-q)\times n}$ *is an annihilator basis of the range space of the matrix* E *and* $\operatorname{rank}(R) = n - q$.

If the matrices E and A are replaced with E^T and A^T in system (2), we have

$$E^T x(k+1) = A^T x(k) \tag{19}$$

Since E is a singular matrix, there always exist nonsingular matrices $\overline{P}$ and $\overline{Q}$, such that $\overline{Q}E\overline{P} = \begin{bmatrix} I_q & 0 \\ 0 & N \end{bmatrix}$ and $\overline{Q}A\overline{P} = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-q} \end{bmatrix}$ according to the first restricted equivalent form in [19], where $N^h = 0$ with a constant h. Substituting $x(k) = \overline{P}\widehat{x}(k)$ into (2) gives

$$E\overline{P}\widehat{x}(k+1) = A\overline{P}\widehat{x}(k). \tag{20}$$

Premultiplying $\overline{Q}$ on both sides of (20), we have

$$\overline{Q}E\overline{P}\widehat{x}(k+1) = \overline{Q}A\overline{P}\widehat{x}(k). \tag{21}$$

(21) can be rewritten as

$$\begin{bmatrix} I_q & 0 \\ 0 & N \end{bmatrix}\begin{bmatrix} \widehat{x}_q(k+1) \\ \widehat{x}_{n-q}(k+1) \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-q} \end{bmatrix}\begin{bmatrix} \widehat{x}_q(k) \\ \widehat{x}_{n-q}(k) \end{bmatrix}. \tag{22}$$

Concretely, we have that $\widehat{x}_{n-q}(k) = 0$ is a stable solution for the second equation of (22). And the stability for the first equation of (22) is decided by the eigenvalues of A_1.

Similarly, substituting $x(k) = \overline{Q}^T\widehat{x}(k)$ into (19), we have

$$E^T\overline{Q}^T\widehat{x}(k+1) = A^T\overline{Q}^T\widehat{x}(k). \tag{23}$$

Premultiplying $\overline{P}^T$ on both sides of (20), we have

$$\overline{P}^T E^T\overline{Q}^T\widehat{x}(k+1) = \overline{P}^T A^T\overline{Q}^T\widehat{x}(k). \tag{24}$$

Corresponding to (22), (24) can be rewritten as

$$\begin{bmatrix} I_q & 0 \\ 0 & N^T \end{bmatrix}\begin{bmatrix} \widehat{x}_q(k+1) \\ \widehat{x}_{n-q}(k+1) \end{bmatrix} = \begin{bmatrix} A_1^T & 0 \\ 0 & I_{n-q} \end{bmatrix}\begin{bmatrix} \widehat{x}_q(k) \\ \widehat{x}_{n-q}(k) \end{bmatrix}. \tag{25}$$

$\widehat{x}_{n-q}(k) = 0$ is also a stable solution for the second equation of (25). And the stability for the first equation of (25) is decided by the eigenvalues of A_1^T, which are the same as A_1. Therefore, system (19) is stable if and only if system (2) is stable.

Through the definitions, it is obvious that system (19) is regular and casual if and only if system (2) is regular and casual. As a result, Lemma 7 can be obtained.

Lemma 7. *The discrete singular system (2) is admissible if there exists a matrix* $P > 0$ $(P \in R^{n\times n})$ *and a symmetric matrix* $\Phi \in R^{(n-r)\times(n-r)}$ *such that*

$$A\left(P - R\Phi R^T\right)A^T - EPE^T < 0 \tag{26}$$

where $RE = 0$. $R \in R^{n\times(n-q)}$ *is an annihilator basis of the range space of the matrix* E *and* $\operatorname{rank}(R) = n - q$.

Proof. According to Lemma 6, if (26) holds, we can get that system (19) is admissible. Because of the equality of the two systems, system (2) is also admissible and vice versa. □

Furthermore, Theorem 8 can be obtained.

Theorem 8. *The discrete singular system (2) is admissible if there exist a matrix* $P > 0$ $(P \in R^{n\times n})$, *a symmetric matrix* $\Phi \in R^{(n-q)\times(n-q)}$, *and matrices* $Y_i (i = 1, 2; Y_i \in R^{n\times n})$ *such that*

$$\Pi = \begin{bmatrix} \Pi_{11} & * \\ \Pi_{21} & \Pi_{22} \end{bmatrix} < 0 \tag{27}$$

where

$$\begin{aligned} Z &= P - T\Phi T^T \\ \Pi_{11} &= AY_1 + Y_1^T A - EPE^T \\ \Pi_{21} &= Y_2^T A^T - Y_1 \\ \Pi_{22} &= Z - Y_2 - Y_2^T, \end{aligned} \tag{28}$$

$ET = 0$. $T \in R^{n\times(n-q)}$ *is an annihilator basis of the range space of the matrix* E *and* $\operatorname{rank}(T) = n - q$.

Proof. From (27) and (28), it is easy to see that

$$\Pi = M + \Lambda\Upsilon + \Upsilon^T\Lambda^T \tag{29}$$

where

$$\begin{aligned} M &= \begin{bmatrix} -EPE^T & \\ & Z \end{bmatrix}, \\ \Lambda &= \begin{bmatrix} A \\ -I_n \end{bmatrix}, \\ \Upsilon &= \begin{bmatrix} Y_1 & Y_2 \end{bmatrix}. \end{aligned} \tag{30}$$

Define $H = [I_n \;\; A]$. It is obvious that $H\Lambda = 0$. Pre- and postmultiplying (29) by H and H^T, from (27) we can get

$$HMH^T = AZA^T - EPE^T = H\Pi H^T < 0 \tag{31}$$

According to Lemma 7, it can be concluded that system (2) is admissible. This proves the theorem. □

In the following, the robust preview controller will be designed.

Considering the state feedback input of system (14) described by

$$\Delta u(k) = L\widehat{X}(k), \tag{32}$$

the closed-loop system is

$$\widehat{E}\widehat{X}(k+1) = \left(\widetilde{A} + \widetilde{B}L\right)\widehat{X}(k) \tag{33}$$

where $\widetilde{A} = \widehat{A} + \Delta\widehat{A}$ and $\widetilde{B} = \widehat{B} + \Delta\widehat{B}$.

Next, the matrix L, which makes the closed-loop system (33) admissible, is to be obtained.

Theorem 9. *The closed-loop discrete singular system (33) is admissible, if there exist a matrix $\widehat{P} > 0$ ($\widehat{P} \in R^{(n+m+m\times M_r)\times(n+m+m\times M_r)}$), a symmetric matrix $\widehat{\Phi}$($\widehat{\Phi} \in R^{(n-q+1)\times(n-q+1)}$), a nonsingular matrix $\widehat{Y}$($\widehat{Y} \in R^{(n+m+m\times M_r)\times(n+m+m\times M_r)}$), a matrix $W \in R^{p\times(n+m+m\times M_r)}$, and some given scalars ρ_i $(i = 1, 2)$, such that*

$$\widehat{\Pi} = \begin{bmatrix} \widehat{\Pi}_{11} & * \\ \widehat{\Pi}_{21} & \widehat{\Pi}_{22} \end{bmatrix} < 0 \tag{34}$$

where

$$\begin{aligned}
\widehat{Z} &= \widehat{P} - \widehat{T}\widehat{\Phi}\widehat{T}^T \\
\widehat{\Pi}_{11} &= -\widehat{E}\widehat{P}\widehat{E}^T + \rho_1\widetilde{A}\widehat{Y} + \rho_1\widehat{Y}^T\widetilde{A}^T + \rho_1\widetilde{B}W + \rho_1 W^T\widetilde{B}^T \\
\widehat{\Pi}_{21} &= -\rho_1\widehat{Y} + \rho_2\widehat{Y}^T\widetilde{A}^T + \rho_2 W^T\widetilde{B}^T \\
\widehat{\Pi}_{22} &= \widehat{Z} - \rho_2\widehat{Y} - \rho_2\widehat{Y}^T,
\end{aligned} \tag{35}$$

$\widehat{E}\widehat{T} = 0$. $\widehat{T} \in R^{(n+m+m\times M_r)\times(n-q+1)}$ is an annihilator basis of the range space of the matrix $\widehat{E}$ and $\mathrm{rank}(\widehat{T}) = n - r + 1$. The state feedback controller (32) is $\Delta u(k) = W\widehat{Y}^{-1}\widehat{X}(k)$.

Proof. Based on the results of Theorem 8, replacing A with $\widetilde{A} + \widetilde{B}L$ in (27) and (28), we have (34), where $\widehat{\Pi}_{ij}$ $(i, j = 1, 2)$ are defined as

$$\begin{aligned}
\widehat{Z} &= \widehat{P} - \widehat{T}\widehat{\Phi}\widehat{T}^T \\
\widehat{\Pi}_{11} &= \widetilde{A}\widehat{Y}_1 + \widehat{Y}_1^T\widetilde{A}^T - \widehat{E}\widehat{P}\widehat{E}^T + \widetilde{B}L\widehat{Y}_1 + \widehat{Y}_1^T L^T\widetilde{B}^T \\
\widehat{\Pi}_{21} &= \widehat{Y}_2^T\widetilde{A}^T + \widehat{Y}_2^T L^T\widetilde{B}^T - \widehat{Y}_1 \\
\widehat{\Pi}_{22} &= \widehat{Z} - \widehat{Y}_2 - \widehat{Y}_2^T,
\end{aligned} \tag{36}$$

and $\widehat{Y}_i \in R^{(n+m+m\times M_r)\times(n+m+m\times M_r)}$ $(i = 1, 2)$. Since there exist some quadratic matrices variables, such as $\widehat{Y}_i^T L^T (i = 1, 2)$, (34) is not a strict LMI based on the description of (36). To tackle the problem effectively, matrices $\widehat{Y}$, W and some scalars $\rho_i (i = 1, 2)$ are introduced just like the condition in [17]. Let $\widehat{Y}_i = \rho_i\widehat{Y}$ and $W = L\widehat{Y}$. Substituting $\rho_i\widehat{Y}$ into $\widehat{Y}_i (i = 1, 2)$ and substituting W into $L\widehat{Y}$ in (36), we have (34) and (35). Because $\widehat{Y}$ is nonsingular, we have $L = W\widehat{Y}^{-1}$ and $\Delta u(k) = W\widehat{Y}^{-1}\widehat{X}(k)$. This proves the theorem. □

Since the uncertain terms $\Delta\widehat{A}$ and $\Delta\widehat{B}$ are contained in $\widetilde{A}$ and $\widetilde{B}$ in (35), the robust controller cannot be directly obtained by computing (34).

Next, the robust preview controller of system (1) is gained. First, some other lemmas are needed.

Lemma 10 (see [20]). *Let F and G be matrices of appropriate dimensions. Let $\Xi = \mathrm{diag}(\Xi_1, \Xi_2, \ldots, \Xi_s)$, where $\Xi_1, \Xi_2, \ldots, \Xi_s$ are uncertain matrices that satisfy $\Xi_i^T\Xi_i \le I, (i = 1, 2, \ldots, s)$. Then, for arbitrary positive scalars $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_s$, we have*

$$F\Xi G + G^T\Xi^T F^T \le F\widetilde{\Lambda}F^T + G^T\widetilde{\Lambda}^{-1}G, \tag{37}$$

where $\widetilde{\Lambda} = \mathrm{diag}(\varepsilon_1 I, \varepsilon_2 I, \ldots, \varepsilon_s I)$.

Lemma 11 (see [21]). *Considering the matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$, where S_{11} and S_{22} are symmetric matrices and invertible, the following three conditions are equivalent:*

(1) $S < 0$;

(2) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Then, we can get Theorem 12.

Theorem 12. *If there exist a matrix $\widehat{P} > 0$ ($\widehat{P} \in R^{(n+m+m\times M_r)\times(n+m+m\times M_r)}$), a symmetric matrix $\widehat{\Phi}$($\widehat{\Phi} \in R^{(n-q+1)\times(n-q+1)}$), a nonsingular matrix $\widehat{Y}$($\widehat{Y} \in R^{(n+m+m\times M_r)\times(n+m+m\times M_r)}$), a matrix $W \in R^{p\times(n+m+m\times M_r)}$, and some given scalars $\rho_i, (i = 1, 2)$, $\varepsilon_1 > 0$, $\varepsilon_2 > 0$ such that*

$$\begin{bmatrix} \Lambda_1 & * & * & * \\ \Lambda_2 & \widehat{Z} - \rho_2\widehat{Y} - \rho_2\widehat{Y}^T & * & * \\ \rho_1 H_{11}\widehat{Y} & \rho_2 H_{11}\widehat{Y} & -\varepsilon_1 I & 0 \\ \rho_1 H_{22} W & \rho_2 H_{22} W & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{38}$$

where

$$\begin{aligned}
\widehat{Z} &= \widehat{P} - \widehat{T}\widehat{\Phi}\widehat{T}^T \\
\Lambda_1 &= \rho_1\widehat{A}\widehat{Y} + \rho_1\widehat{Y}^T\widehat{A}^T - \widehat{E}\widehat{P}\widehat{E}^T + \rho_1\widehat{B}W + \rho_1 W^T\widehat{B}^T \\
\Lambda_2 &= \rho_2\widehat{Y}^T\widehat{A}^T + \rho_2 W^T\widehat{B}^T - \rho_1\widehat{Y},
\end{aligned} \tag{39}$$

the controller

$$\begin{aligned}
u(k) &= K_e\sum_{j=1}^{k} e(j-1) + K_x x(k) \\
&\quad + \sum_{j=0}^{M_r-1}\sum_{i=1}^{k} K_r(j)\Delta r(i+j-1) + u(0) \\
&\quad - K_x x(0)
\end{aligned} \tag{40}$$

is the robust preview controller for the discrete singular system (1).

Proof. From (34) and (35), we can get

$$\begin{aligned}
&\begin{bmatrix} \rho_1\left(\widehat{A}+\Delta\widehat{A}\right)\widehat{Y}+\rho_1\widehat{Y}^T\left(\widehat{A}+\Delta\widehat{A}\right)^T-\widehat{E}\widehat{P}\widehat{E}^T+\rho_1\left(\widehat{B}+\Delta\widehat{B}\right)W+\rho_1W^T\left(\widehat{B}+\Delta\widehat{B}\right)^T & * \\ \rho_2\widehat{Y}^T\left(\widehat{A}+\Delta\widehat{A}\right)^T+\rho_2W^T\left(\widehat{B}+\Delta\widehat{B}\right)^T-\rho_1\widehat{Y} & \widehat{Z}-\rho_2\widehat{Y}-\rho_2\widehat{Y}^T \end{bmatrix} \\
&= \begin{bmatrix} \rho_1\widehat{A}\widehat{Y}+\rho_1\widehat{Y}^T\widehat{A}^T-\widehat{E}\widehat{P}\widehat{E}^T+\rho_1\widehat{B}W+\rho_1W^T\widehat{B}^T & * \\ \rho_2\widehat{Y}^T\widehat{A}^T+\rho_2W^T\widehat{B}^T-\rho_1\widehat{Y} & \widehat{Z}-\rho_2\widehat{Y}-\rho_2\widehat{Y}^T \end{bmatrix} \\
&+ \begin{bmatrix} \rho_1\left(E_{11}\Sigma_{11}H_{11}\right)\widehat{Y}+\rho_1\left(E_{22}\Sigma_{22}H_{22}\right)W & \rho_2\left(E_{11}\Sigma_{11}H_{11}\right)\widehat{Y}+\rho_2\left(E_{22}\Sigma_{22}H_{22}\right)W \\ 0 & 0 \end{bmatrix} \\
&+ \begin{bmatrix} \rho_1\widehat{Y}^T\left(E_{11}\Sigma_{11}H_{11}\right)^T+\rho_1W^T\left(E_{22}\Sigma_{22}H_{22}\right)^T & 0 \\ \rho_2\widehat{Y}^T\left(E_{11}\Sigma_{11}H_{11}\right)^T+\rho_2W^T\left(E_{22}\Sigma_{22}H_{22}\right)^T & 0 \end{bmatrix}
\end{aligned} \tag{41}$$

Denote

$$\begin{aligned}
\Psi &= \begin{bmatrix} \rho_1\left(\widehat{A}+\Delta\widehat{A}\right)\widehat{Y}+\rho_1\widehat{Y}^T\left(\widehat{A}+\Delta\widehat{A}\right)^T-\widehat{E}\widehat{P}\widehat{E}^T+\rho_1\left(\widehat{B}+\Delta\widehat{B}\right)W+\rho_1W^T\left(\widehat{B}+\Delta\widehat{B}\right)^T & * \\ \rho_2\widehat{Y}^T\left(\widehat{A}+\Delta\widehat{A}\right)^T+\rho_2W^T\left(\widehat{B}+\Delta\widehat{B}\right)^T-\rho_1\widehat{Y} & \widehat{Z}-\rho_2\widehat{Y}-\rho_2\widehat{Y}^T \end{bmatrix}, \\
\Omega &= \begin{bmatrix} \rho_1\widehat{A}\widehat{Y}+\rho_1\widehat{Y}^T\widehat{A}^T-\widehat{E}\widehat{P}\widehat{E}^T+\rho_1\widehat{B}W+\rho_1W^T\widehat{B}^T & * \\ \rho_2\widehat{Y}^T\widehat{A}^T+\rho_2W^T\widehat{B}^T-\rho_1\widehat{Y} & \widehat{Z}-\rho_2\widehat{Y}-\rho_2\widehat{Y}^T \end{bmatrix} \\
\Xi_1 &= \begin{bmatrix} E_{11} & E_{22} \\ 0 & 0 \end{bmatrix}, \\
\Xi_2 &= \begin{bmatrix} \Sigma_{11} & 0 \\ 0 & \Sigma_{22} \end{bmatrix}, \\
\Xi_3 &= \begin{bmatrix} \rho_1H_{11}\widehat{Y} & \rho_2H_{11}\widehat{Y} \\ \rho_1H_{22}W & \rho_2H_{22}W \end{bmatrix}
\end{aligned} \tag{42}$$

Then (41) can be rewritten as

$$\Psi = \Omega + \Xi_1\Xi_2\Xi_3 + {\Xi_3}^T{\Xi_2}^T{\Xi_1}^T \tag{43}$$

According to the Assumption 4, $\Xi_2{\Xi_2}^T \leq I$. Applying Lemma 10 to (43), we have

$$\Psi < \Omega + \Xi_1\overline{\Lambda}{\Xi_1}^T + {\Xi_3}^T\overline{\Lambda}^{-1}\Xi_3 \tag{44}$$

where $\overline{\Lambda} = \left[\begin{smallmatrix} \varepsilon_1 I & \\ & \varepsilon_2 I \end{smallmatrix}\right], (\varepsilon_1, \varepsilon_2 > 0)$.

Therefore, if there exist $\varepsilon_1, \varepsilon_2 > 0$ satisfying $\Omega + \Xi_1\overline{\Lambda}{\Xi_1}^T + {\Xi_3}^T\overline{\Lambda}^{-1}\Xi_3 < 0$, according to Lemma 11, namely, if

$$\begin{bmatrix} \widehat{\Lambda}_1 & * & * & * \\ \widehat{\Lambda}_2 & Z-\rho_2\widehat{Y}-\rho_2\widehat{Y}^T & * & * \\ \rho_1H_{11}\widehat{Y} & \rho_2H_{11}\widehat{Y} & -\varepsilon_1 I & 0 \\ \rho_1H_{22}W & \rho_2H_{22}W & 0 & -\varepsilon_2 I \end{bmatrix} < 0 \tag{45}$$

where

$$\begin{aligned}
\widehat{Z} &= \widehat{P} - \widehat{T}\widehat{\Phi}\widehat{T}^T \\
\widehat{\Lambda}_1 &= \rho_1\widehat{A}\widehat{Y}+\rho_1\widehat{Y}^T\widehat{A}^T-\widehat{E}\widehat{P}\widehat{E}^T+\rho_1\widehat{B}W+\rho_1W^T\widehat{B}^T \\
&\quad + \varepsilon_1E_{11}E_{11}^T + \varepsilon_2E_{22}E_{22}^T \\
\widehat{\Lambda}_2 &= \rho_2\widehat{Y}^T\widehat{A}^T+\rho_2W^T\widehat{B}^T-\rho_1\widehat{Y}.
\end{aligned} \tag{46}$$

The state feedback controller for system (14) is

$$\Delta u(k) = W\widehat{Y}^{-1}\widehat{X}(k). \tag{47}$$

Let $L = W\widehat{Y}^{-1}$ be blocked into $L = [K_e \ \ K_x \ \ K_r(1) \ \cdots \ K_r(M_r)]$, then (47) can be expressed as

$$\Delta u(k) = K_e e(k) + K_x\Delta x(k) + \sum_{i=0}^{M_r-1} K_r(i)\Delta r(k+i). \tag{48}$$

With regard to the discrete singular system (1), since $\Delta u(k) = u(k+1) - u(k)$, we have

$$\begin{aligned} u(1) - u(0) &= K_e e(0) + K_x \Delta x(0) \\ &\quad + \sum_{j=0}^{M_r-1} K_r(j)\,\Delta r(j), \\ u(2) - u(1) &= K_e e(1) + K_x \Delta x(1) \\ &\quad + \sum_{j=0}^{M_r-1} K_r(j)\,\Delta r(j+1), \\ &\vdots \\ u(k) - u(k-1) &= K_e e(k-1) + K_x \Delta x(k-1) \\ &\quad + \sum_{j=0}^{M_r-1} K_r(j)\,\Delta r(j+k-1). \end{aligned} \tag{49}$$

Add the above k equations together on both sides, and move $u(0)$ to the right side of the equation. When $k \geq 1$, we have

$$\begin{aligned} u(k) &= K_e \sum_{j=1}^{k} e(j-1) + K_x x(k) \\ &\quad + \sum_{j=0}^{M_r-1} \sum_{i=1}^{k} K_r(j)\,\Delta r(i+j-1) + u(0) \\ &\quad - K_x x(0). \end{aligned} \tag{50}$$

(50) is the robust preview controller for system (1). This proves the theorem. □

Remark 13. Since there exists the term $\sum_{j=0}^{M_r-1}\sum_{i=1}^{k} K_r(j)\Delta r(i+j-1)$ in (50), the future know information about the reference signal $r(t)$ is considered in the robust controller.

Remark 14. Unlike the free weighting matrix method for normal system in [22], the difficulty of designing proper Lyapunov function for singular systems is overcome by introducing the method of [17].

5. Numerical Simulation

Consider the following system:

$$\begin{aligned} Ex(k+1) &= (A + \Delta A)\,x(k) + (B + \Delta B)\,u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{51}$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.2 \\ 1 \end{bmatrix}, \\ C &= \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}, \\ \rho_1 &= 0.5, \\ \rho_2 &= 1, \\ E &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\ E_1 &= \begin{bmatrix} 0.15 & 0.3 \\ 0.3 & 0.15 \end{bmatrix}, \\ E_2 &= \begin{bmatrix} 0.3 & 0.15 \\ 0.15 & 0.15 \end{bmatrix}, \\ H_1 &= \begin{bmatrix} 0.15 & 0.15 \\ 0.15 & 0.15 \end{bmatrix}, \\ H_2 &= \begin{bmatrix} 0.15 \\ 0.3 \end{bmatrix}, \\ \Sigma_1 &= \begin{bmatrix} 1 & \\ & 0.5 \end{bmatrix}, \\ \Sigma_2 &= \begin{bmatrix} 0.5 & \\ & 0.9 \end{bmatrix}. \end{aligned} \tag{52}$$

It is proved that $\left[\begin{smallmatrix} A-E & B \\ C & 0 \end{smallmatrix}\right]$ is invertible and Σ_1, Σ_2 satisfy Assumption 4 for any k. So the system satisfies the fundamental assumptions.

The reference signal is taken as

$$r(k) = \begin{cases} 1, & k \geq 30 \\ 0, & k < 30 \end{cases} \tag{53}$$

The preview length is $M_r = 0$, $M_r = 1$, and $M_r = 7$, respectively. When $M_r = 0$, $K_e = 0.8155$, $K_x = [1.8665 \;\; 3.4830]$. When $M_r = 1$, $K_e = 0.8583$, $K_x = [1.8890 \;\; 3.5801]$, $K_r = [-0.7993 \;\; -0.5449]$. When $M_r = 7$, $K_e = 0.9344$, $K_x = [1.9397 \;\; 3.4174]$, $K_r = [-0.9115 \;\; -0.7297 \;\; -0.3429 \;\; -0.2242 \;\; -0.1183 \;\; -0.0682 \;\; -0.0352 \;\; -0.0175]$. And we have Figures 1 and 2.

Figure 1 shows the output response for system (1). Figure 2 shows the tracking error. From the figures we can see that the output response with preview can track the reference signal much faster. And along with the preview length increasing, the overshoots reduced. Meanwhile, the maximum tracking errors decreased.

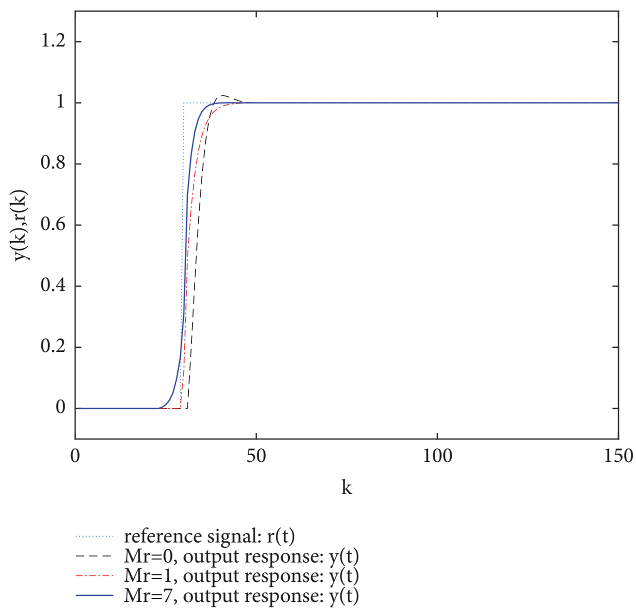

Figure 1: The output response of the closed-loop system.

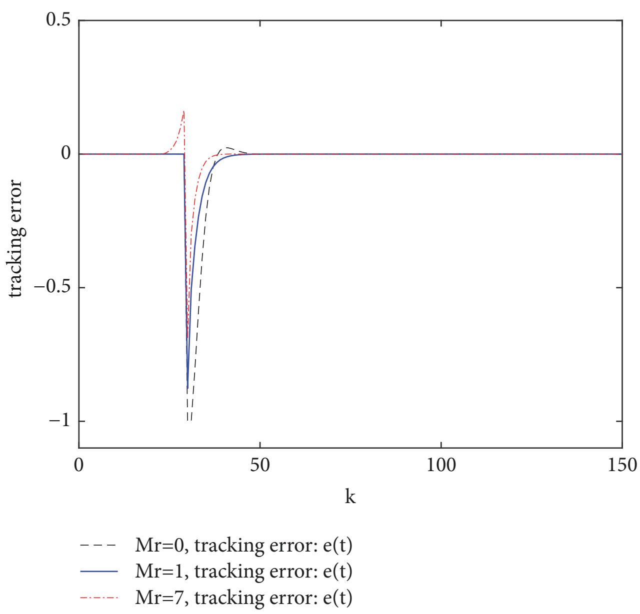

Figure 2: The tracking error of the closed-loop system.

As a comparison, the "nonrobust" controller is applied to the uncertain system (51). Similarly, the reference signal is also taken as

$$r(k)=\begin{cases}1, & k \geq 30\\ 0, & k < 30\end{cases} \tag{54}$$

and the preview length are $M_r = 0$ and $M_r = 7$, respectively. When $M_r = 0$, $K_e = 2.8488$, $K_x = [1.6456 \quad 10.8839]$. When $M_r = 7$, $K_e = 1.2201$, $K_x = [1.8265 \quad 3.7134]$, $K_r = [-1.1122 \quad -0.8906 \quad -0.3268 \quad -0.1548 \quad -0.0588 \quad -0.0297 \quad -0.0111 \quad -0.0047]$. Thus, we have Figures 3 and 4.

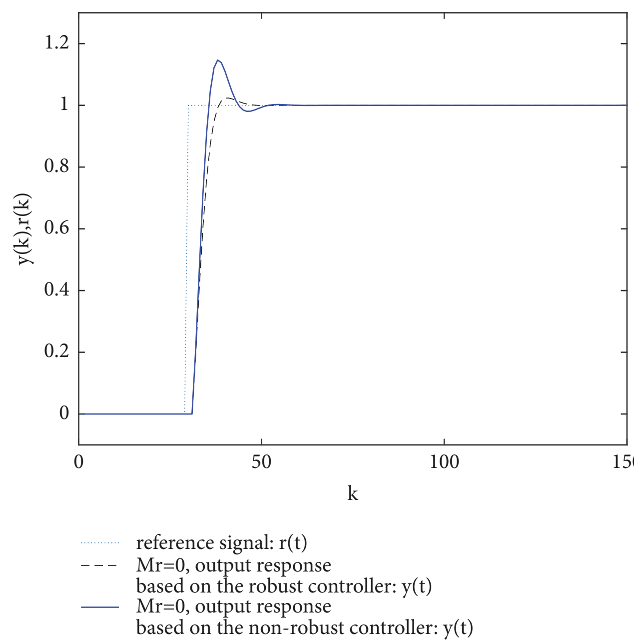

Figure 3: The output response of the closed-loop system based on the robust controller and the "nonrobust" controller when $M_r = 0$.

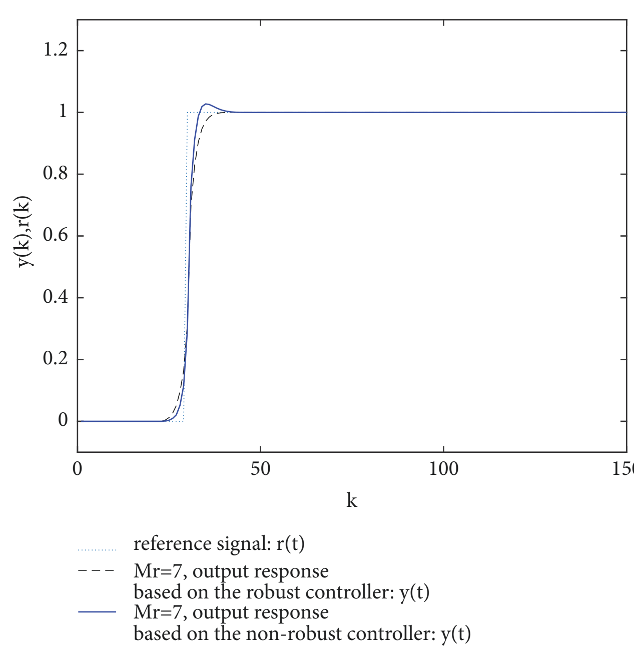

Figure 4: The output response of the closed-loop system based on the robust preview controller and the "nonrobust" preview controller when $M_r = 7$.

Figures 3 and 4 are the output response of the closed-loop system based on the robust controller and the "nonrobust" preview controller when $M_r = 0$ and $M_r = 7$, respectively. From these two figures, it can be easily seen that the overshoot of the output response based on the robust controller is smaller than that based on the nonrobust controller, which shows the effectiveness of the controller designed in this paper.

6. Conclusion

In this paper, the robust preview control problem for uncertain discrete singular systems is studied. At first, the singular augmented error system is constructed. Then, by considering the stability of the transposition system, some criterions are gained by using LMI method. At last, the robust controller for the singular augmented error system is obtained, which is the robust preview controller for the original uncertain discrete singular system. The numerical example shows the effectiveness of this paper.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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