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Research Article

Optimization Model for Reserve Fleet Sizes in Traditional Transit Systems considering the Risk of Vehicle Breakdowns

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Traditional transit systems are susceptible to unexpected costs and delays due to unforeseen events, such as vehicle breakdowns. The randomness of these events gives the appearance of an imbalance in the number of operating vehicles and of unreliable transit services. Therefore, this paper proposes the queueing theory as a means to characterize the state of any given transit system considering the risk of vehicle breakdowns. In addition, the proposed method is used to create an optimized model for reserve fleet sizes in transit systems, in order to ensure the reliability of the transit system and minimize the total cost of any transit system exposed to the risks of vehicle breakdowns. The optimization is conducted based on the two main characteristics of all bus systems, namely, operator costs and user costs, in both normal and disruptive situations. In addition, the situations in our optimization are generated in scenarios that have a certain degree of probability of experiencing delays. This paper formulates such an optimization model, presents the formulation solution method, and proves the validity of the proposed method.

1. Introduction

Obviously, the traditional transit system fleet size consists of an operating fleet, which is defined as the number of vehicles operating on a line. The fleet size is selected and determined by the desired bus frequency, in order to improve the level of service, as well as a reserve fleet, which serves as backup vehicles when operating vehicles break down or require routine and planned maintenance. In order to ensure the reliability of a transit system, it is necessary to guarantee the size of the reserve fleet under the premise and conditions of capital constraints. For instance, in London Bus, 2.4% of the company's total scheduled mileage was lost due to serious vehicle breakdowns and traffic or staff problems in 2016/2017. That figure represents 13.2 million km. While minor vehicle failures can be repaired quickly, serious failures require longer repair times. Sometimes, the disabled vehicle will even need to be towed in for lengthy repairs or long-term maintenance. In addition, the original schedule of the broken-down vehicle may deteriorate to the extent where the operating fleet vehicle schedule needs to be adjusted in real time. This has to be done by scheduling backup vehicles from the reserve fleet to cover regular routes, and this in turn depends on the availability of the reserve fleet.

From the perspective of normal public transportation systems, Lee et al. [1] used a classical analytic optimization method to minimize the total operator and user costs. These costs did not include the capital cost of vehicles in one-size operations (on a single route or multiple routes) as means by which to determine the size of bus fleets for urban operations. Yan et al. [2] showed how to manage the interrelationships between passenger trip demands and bus trip supplies, in order to produce the best timetables and bus routes based on a Lagrangian relaxation method, a subgradient method, the network simplex method, a Lagrangian heuristic and a flow decomposition algorithm. Ceder and Avishai [3] explained the concrete application of the theory, model, and algorithm of the bus line network and proposed a method of capacity allocation based on a bus chain. Ceder et al. [4] proposed minimizing the deviation of the determined headways from a desired even headway and the deviation of the observed passenger loads from a desired even-load level of the vehicles at the maximum-load point. Li et al. [5] proposed a bilevel mathematical programming problem with supply demand equilibrium constraints. Li then implemented heuristic procedures to solve small instances leading to headway values that improve the user's net utility. Castelli et al. [6] addressed a timetabling problem to minimize the weighted sum of passenger costs based on time spent in the system and operational costs based on vehicle usage. Guihaire et al. [7] addressed the transit network's timetables with an objective function based on the quantity and quality of transfers, evenness of line headways, size of feet, and length of the deadheads. Liebchen et al. [8] defined an optimization model that minimizes the expected total weighted delay for planned trips. The model considers headway bounds and fleet size constraints and develops a heuristic algorithm capable of obtaining optimal solutions to problems that can be modeled through series-parallel graphs. Raposo et al. [9, 10] presented a predictive condition monitoring maintenance approach based on engine oil analysis. This approach helps determine the size of the reserve fleet and guarantee availability. Li et al. [11] proposed an approach called new life additional benefit-cost as a means to solve the mixed bus fleet management problem. This approach maximizes the total net benefit of early replacement, where both the optimal fleet size and composition under budget constraints can be determined. Kim et al. [12] proposed a hybrid approach as a means to optimize conventional and flexible bus fleet sizes in multidimensional, nonlinear, mixed integer optimization problems. This approach considers conventional and flexible bus sizes, conventional bus route spacing, and service zone areas for flexible buses and headways.

From the perspective of disruptive public transportation systems, Li et al. [13, 14] defined the state of urban bus systems when a vehicle breaks down on a scheduled trip and one or more vehicles need to be rescheduled to serve the customers on that trip with minimum operating and delay costs. Li also considered the problem of recovery in response to breakdowns in bus passenger transportation systems, in which vehicles are reassigned to trips. Guedes et al. [15] proposed a dynamic extension of the classic multiple-depot vehicle scheduling model. In Guedes' model, a heterogeneous fleet is considered, and the model displayed in the study achieved good behavior in situations involving several simultaneous disruptions. Although bus fleet sizes have been studied for a long time, the studies always focused on the operating fleet. However, the reserve fleet, which is responsible for mitigating unexpected events (such as vehicle breakdowns), has never been seen as the major concern.

In this paper, we construct a new type of optimization model for determining the reserve fleet size in traditional transit systems. This model minimizes the costs to both operators and users in both normal and disruptive situations. The proposed method has been tested on a traditional transit system. The obtained results have shown that the proposed method provides better prediction and achieves better results than the most commonly used method. The paper is organized as follows: In Section 2, the formal description of the problems is given, and all used assumptions and constraints are provided. The queueing theory of vehicle breakdowns in traditional transit systems is also described. In Section 3, we propose the optimization model. The model's verification by a traditional transit system is exposed in Section 4. Lastly, a summary of our results and areas of future research are discussed in Section 5.

2. Problem Statement

2.1. Urban Transit System Description considering the Risk of Vehicle Breakdowns. In this section, we explain the reserve fleet scheduling problem in a traditional transit system. The operating fleet in a transit system performs a service trip. This service involves picking up and dropping off passengers at a sequence of bus stops. The reserve fleet performs a backup service trip, acting as a substitute for the main operating fleet when vehicle failures and traffic accidents occur. Traditional transit systems are susceptible to such requirements due to these unforeseen events. In this article, vehicle failures and traffic accidents are generalized as "vehicle breakdowns". In addition, the scenarios are generated in such a way that they range from scenarios with a very low probability of a delay and a low delay length, to scenarios with a high probability of delay and a high delay length. The reason for generating scenarios in this way was because we wanted to cover very bad days (such as winter days with bad weather conditions or hot summer days with high vehicle failure rates), as well as days where only few disruptions occur. In this paper, uncertain demand and uncertain travel times are not major concerns, so we only focus on the influences of a traditional transit system while considering the risk of vehicle breakdowns.

The optimization of the reserve fleet size is based on some assumptions, which are as follows:

(i) Vehicles breakdowns are mutually independent, and broken-down vehicles can be repaired back to a normal condition.

(ii) Daily passenger demand is constant, and passengers normally arrive independent of vehicle arrivals.

(iii) The operating fleet and the reserve fleet belong to the same bus line.

(iv) Average daily traffic is considered constant, except in cases of vehicle breakdowns.

(v) The operator holds a constant and known preference over the planning horizon.

Assumption (i) is very critical to our model, since ignoring the independence of vehicle breakdowns could lead to significant differences in both formulations and optimal solutions. In this paper, we consider the average vehicle breakdown rate in assumption (i); we then extend the discussion to the state of an urban transit system with the queueing theory. We assume passengers arrive independently of the timetable, with no pass-ups in assumption (ii). This is because our paper translates the pass-up passengers cost into the cost of time, rather than considering the behavior of passengers as a means to calculate the fare loss. Besides, we want to determine the reserve fleet size of each line that has the lowest expected total costs in all delay scenarios. If

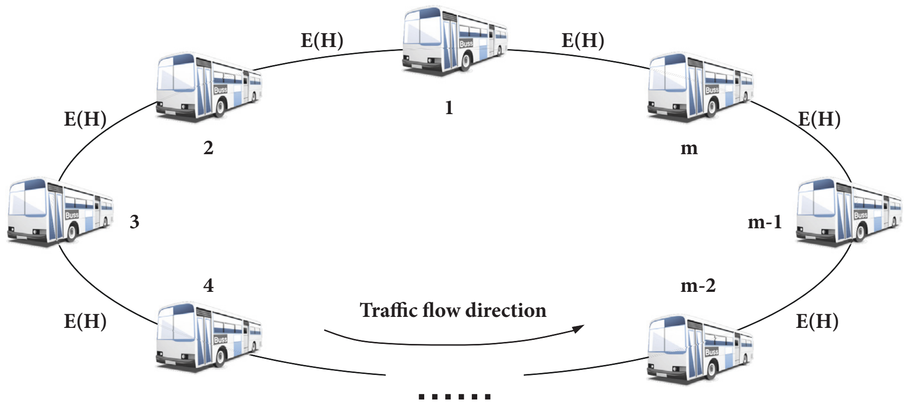

Figure 1: Transit system model.

other bus lines exist, it is possible that the backup vehicles of the other bus lines may provide the backup service trip. As a result, we assume that assumption (iii) holds in this paper. We assume the difference in scenarios is only the average vehicle breakdown rate in assumption (iv), because this paper mainly discusses the reserve fleet size considering the risk of vehicle breakdowns. Online requests and uncertain travel times are not major concerns. Additionally, the implementation of station control strategies and interstation control make it possible to keep bus headways evenly distributed. Assumption (v) also has significant importance in this paper, because the issue of uncertain risk preferences leads to another challenging topic in optimization problems. In addition, the implementation of geographical information systems and wireless communication systems in public transit systems makes it possible to handle accidents in a timely fashion when breakdowns occur.

Our idealized model treats the bus route as a dynamic system, with operating vehicles moving at a constant average velocity around a circular route of a given length, with a single depot and bus service workshops. At any point in time, each bus headway is evenly distributed on this circuit as E(H) and related to the frequency, as shown in Figure 1. In addition, all vehicles belonging to a bus company start and end each service trip at the same terminal. Additionally, each vehicle performs a feasible sequence of trips, and a trip is either served or cancelled when it is impossible to reach the starting point by a specified time. When a vehicle breakdown occurs, the driver provides the broken-down vehicle's information to the company. Based on the broken-down vehicle's information, the company will substitute the disabled vehicle with a backup vehicle from the reserve fleet at the depot. The backup vehicle will operate until the broken-down vehicle is repaired, in order to keep the bus system operating normally. The company will schedule the following operating vehicles to service the remaining passengers in the broken-down vehicle. However, if no reserve fleet is available at the depot, the operator will adjust the bus frequency, in order to keep the bus headway evenly distributed until the broken-down vehicle is repaired.

2.2. The State of an Urban Transit System considering the Risk of Vehicle Breakdowns. This paper considers daily vehicle breakdowns to be random, accidental events. Breakdown arrival patterns postulate for an ordinary Poisson process, whereby one event at most can occur at any given time. In addition, the broken-down vehicles require fixed repair times for towing the disabled vehicle for repairs and then receiving maintenance from the bus service workshop. The bus operation process assumes that vehicle breakdown arrival patterns postulate for an ordinary Poisson process with parameter λ_j for different scenarios j. Also, the total time, which is comprised of the bus service workshop service time for broken-down vehicles and the time it takes for the repaired vehicles to come back to the depot as backup vehicles of the reserve fleet, is assumed to be subject to exponential service rate μ, ignoring any past system history. There are m operating buses moving at a constant average velocity v around a circular route of length L, with evenly spaced headway E(H) in the operation system. The backup service system consists of N backup vehicles as the reserve fleet to be used to respond to any emergency with the operating buses. In addition, the maintenance service system consists of n bus service workshops, which carry out routine and emergency maintenance. Usually, n>N in real life. Therefore, the transit system is M/M/n/m+N/m queueing model, including the operation system, maintenance service system, and backup service system. The transit system state flow graph can be drawn as in Figure 2.

The transit system can be one of the possible states $E = \{0, 1, 2, 3 \cdots m + N\}$. When the transit system is in state 0, there are no vehicle breakdowns, and the reserve fleet and the bus service workshop are idle. When the transit system is in state k $(0 < k \leq N - 1)$, there are k vehicles broken down, and breakdown vehicles are sent to the k bus service workshops for repair in the maintenance service system. The backup service system sends k backup vehicles to substitute for the k broken-down vehicles, and the bus system operates as usual. Once the maintenance service is finished, the transit system is in state $k - 1$, and the repaired vehicle joins the reserve fleet as a new backup vehicle within the reserve fleet.

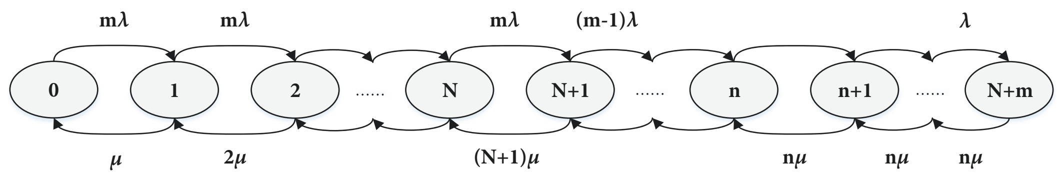

Figure 2: Transit system state flow graph.

Because the possibility of k breakdown vehicles from broken-down to repaired ones is equal, the total failure rate of the system is $m\lambda_j$ per unit time in state k, and the transition intensity of the state from k to $k-1$ is $k\mu$. When the transit system is in state k ($N \leq k \leq n-1$), due to the fact that the reserve fleets all substitute for the broken-down vehicles, the backup service system is congested, and the bus system is under inadequate operation. The total failure rate of the system is $(m-k+N)\lambda_j$ per unit of time, and the transition intensity is $k\mu$. When the transit system is in state k ($n \leq k \leq N+m$), the reserve fleets all substitute for the broken-down vehicles, and the bus service workshops are all repairing the broken-down vehicles. Therefore, the backup service system and the maintenance service system are both congested. The bus system is also under inadequate operation. The total failure rate of the system is $(m-k+N)\lambda_j$ per unit of time, and the transition intensity is $n\mu$. Through the state flow graph, we determine the state algebraic equation based on the system equilibrium conditions as follows:

For state 0, there is

$$m\lambda_j p_0 = \mu p_1, \quad \text{and } p_1 = m\rho_1 p_0 \tag{1}$$

For state 1, there is

$$m\lambda_j p_1 = 2\mu p_2, \quad \text{and } p_2 = \frac{m^2}{2!}\rho_1^2 p_0 \tag{2}$$

For state N-1, there is

$$m\lambda_j p_{N-1} = N\mu p_N, \quad \text{and } p_N = \frac{m^N}{N!}\rho_1^N p_0 \tag{3}$$

For state N, there is

$$m\lambda_j p_N = (N+1)\mu p_{N+1}, \quad \text{and } p_{N+1} = \frac{m^N m!}{(N+1)!(m-1)!}\rho_1^{N+1} p_0 \tag{4}$$

For state n-1, there is

$$(m-n+1+N)\lambda_j p_{n-1} = n\mu p_n, \quad \text{and } p_n = \frac{m^N m!}{n!(m-n+N)!}\rho_1^n p_0 \tag{5}$$

For state n, there is

$$(m-n+N)\lambda_j p_n = n\mu p_{n+1}, \quad \text{and } p_{n+1} = \frac{m^N m!}{n!(m-n-1+N)!n}\rho_1^{n+1} p_0 \tag{6}$$

For state N+m-1, there is

$$\lambda_j p_{N+m-1} = n\mu p_{N+m}, \quad \text{and } p_{N+m} = \frac{m^N m!}{n! n^{N+m-n}}\rho_1^{N+m} p_0 \tag{7}$$

The equilibrium probability of each state can be obtained as follows:

$$\rho_1^{jk} = \frac{\lambda_j}{\mu} \tag{8}$$

$$p_{j0} = \left[\sum_{k=0}^{N-1}\frac{m^k}{k!}\rho_1^{jk} + \sum_{k=N}^{n-1}\frac{m^N m!}{k!(m-k+N)!}\rho_1^{jk} + \sum_{k=n}^{N+m}\frac{m^N m! \rho_1^{jk}}{n!(m-k+N)! n^{k-n}}\right]^{-1} \tag{9}$$

$$p_{jk} = \begin{cases} \dfrac{m^k}{k!}\rho_1^{jk} p_0, & 0 \leq k \leq N-1 \\ \dfrac{m^N m!}{k!(m-k+N)!}\rho_1^{jk} p_0, & \mathrm{N} \leq k \leq n-1 \\ \dfrac{m^N m!}{n!(m-k+N)! n^{k-n}}\rho_1^{jk} p_0, & \mathrm{n} \leq k \leq N+m \end{cases} \tag{10}$$

However, it is basically impossible for most of the vehicles to fail in the operation process of the real bus line. Therefore, the probability of the state needs to be adjusted according to the historical data of the vehicle breakdowns or the fault tolerance rate φ. That is, if the probability of the state is less than φ, it will be impossible for that state to happen. The possible number of vehicle breakdowns is m_u. Subsequently, the adjusted probability of the state is defined by

$$p_{jk}^a = \begin{cases} \dfrac{p_{jk}}{\left(1 - \sum_{m_u}^{N+m} p_{jk}\right)} & 0 \leq k \leq m_u \\ 0 & k > m_u \end{cases} \tag{11}$$

In terms of the number of operating vehicles, when the transit system is in state k ($0 \leq k \leq N$), there are available reserve fleet vehicles, and the number of the

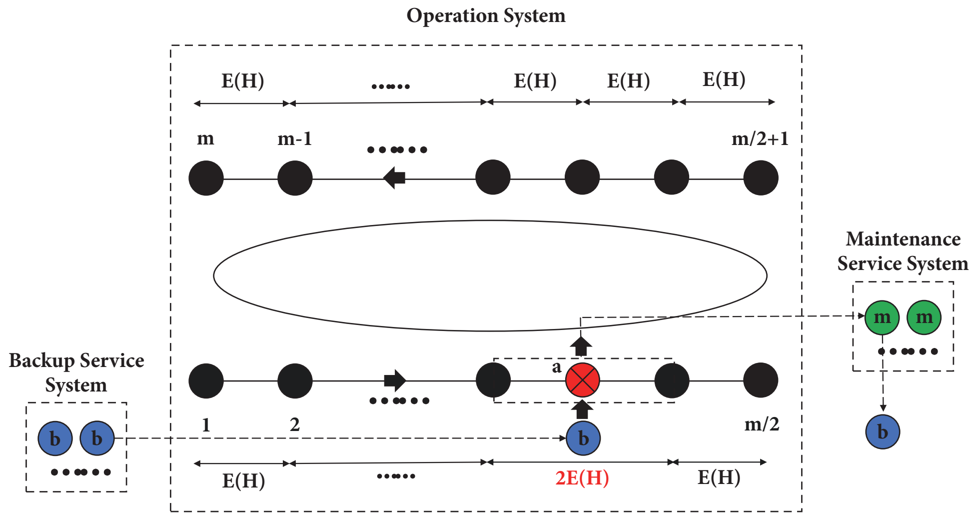

Figure 3: Transit system in disruptive situation 1.

operating vehicles in the transit system is m. When the transit system is in state k ($N+1 \leq k \leq m_u$), the reserve fleet vehicle performs a backup service trip, substituting for the broken-down vehicle, and then there are no available reserve fleet vehicles. Therefore, the number of operating vehicles in the transit system is diminishing with the following state. Subsequently, the number of operating vehicles in different states is defined by

$$m_k = \begin{cases} m & 0 \leq k \leq N \\ m+N-k & N+1 \leq k \leq m_u \end{cases} \tag{12}$$

where m_k is the total number of operating vehicles on line under the bus system state of k.

In terms of the average headway, when the transit system is in state k ($k = 0$), the average headway is related to the preliminary frequency, which is determined by the operator in the bus line's normal state. After setting the frequency, the operator usually considers the turnover time and the shortest stop time at each bus terminal station, in order to minimize the preliminary number of operating fleet vehicles. Additionally, the operator prepares N backup vehicle for the reserve fleet. When the transit system is in state k ($0 < k \leq N$), the operator does not change the frequency, and the headway E(H) basically stays the same because of the availability of the reserve fleet. For example, when the transit system is in state 1, vehicle "a" breaks down in the operation system, and the headway of adjacent buses will be twice the normal headway. In order to ensure the reliability of the system, the backup service system sends backup vehicle "b" to substitute for the broken-down vehicle. Either that or the following operating bus will fill the vacancy, and the backup vehicle will depart the depot, depending on the distance between the location of vehicle "a" and the depot. Then, broken-down vehicle "a" leaves the operation system to enter the maintenance service system for repairs and maintenance. When the broken-down vehicle is repaired, it goes back to the depot to join the reserve fleet, as shown in Figure 3.

When the transit system is in state k ($N+1 \leq k \leq m_u$), the operator will change the frequency, and the headway will be adjusted with an inadequate operating fleet. For example, when the transit system is in state N+1, the reserve fleets all substitute for the broken-down vehicles, and there is no available reserve fleet in the backup service system. The operation system adjusts the headway E(H) to E'(H) with the inadequate operating fleet. This is done by adjusting the bus speed (by either speeding up or slowing down) at a time control point and changing the arrival and departure frequency at the depot, in order to ensure adherence to the schedule at the next time control point, as shown in Figure 4.

Subsequently, the average headway in different states is defined by

$$T = \frac{60L}{v} + t \tag{13}$$

$$E(H_k) = \begin{cases} E(H) & 0 \leq k \leq \mathrm{N} \\ \dfrac{T}{m_k} & \mathrm{N}+1 \leq k \leq m_u \end{cases} \tag{14}$$

where L is the length of the bus line in kilometers, v is the average speed of vehicles on the line during the peak period, t is the shortest stop time at a bus terminal station, and T represents the turnover time. Subsequently, the bus departure interval under the transit system state of k is defined by $E(H_k)$.

3. Optimization Model

3.1. Cost Analysis. The optimization model for determining the size of the reserve fleet has the objective of minimizing the total daily cost of the transit system, which includes the costs to both the operator and users in both normal and disruptive situations.

Regarding the operator cost, we analyze three main parts of a bus company's expenses, including the acquisition cost

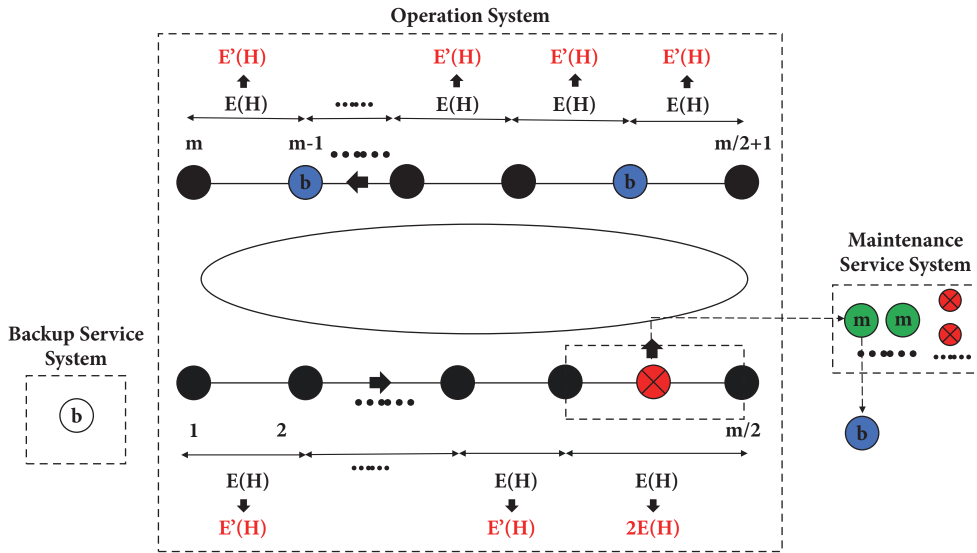

Figure 4: Transit system in disruptive situation N+1.

of vehicles, operating and maintenance costs, and the cost of carbon emissions treatment. The vehicle acquisition cost is equivalent to the daily cost of vehicle utilization within the length of the vehicles' operation life and the transit fleet size. The human cost can be considered as a constant, including employee benefits and bonuses, which are generally neglected in the bus company's expenses. Operating and maintenance costs also include repair costs and fuel costs related to total vehicle mileage. The cost of carbon emissions treatment is also directly related to vehicle mileage. Vehicle mileage is determined using the frequency under the different states of the transit system defined above.

Subsequently, the daily cost of vehicle utilizations is defined by

$$C_C = \frac{c_1 (m+N)}{D} \tag{15}$$

where c_1 represents the vehicle price, and D represents the length of vehicle operation life.

Furthermore, the operating and maintenance costs are calculated as

$$C_O = c_2 \times \sum_{j=0}^{J} \sum_{k=0}^{m_u} \frac{p_j \times p_{jk}^a \times L \times H}{E(H_k)} \tag{16}$$

where c_2 represents the lines' unit operating cost, p_j represents daily probability of scenarios, and H represents the line's operating time.

Moreover, the cost of carbon emissions treatment is defined by

$$C_E = c_3 \times \sum_{j=0}^{J} \sum_{k=0}^{m_u} \frac{p_j \times p_{jk}^a \times L \times H \times E}{100 \times E(H_k)} \tag{17}$$

where c_3 the unit cost of carbon treatment, and E denotes the vehicle carbon emissions per 100 km.

The above models can be used to calculate the total operating costs of a transit system, and the function is shown as follows:

$$\begin{aligned} C &= C_C + C_O + C_E \\ &= \frac{c_1 (m+N)}{D} + c_2 \times \sum_{j=0}^{J} \sum_{k=0}^{m_u} \frac{p_j \times p_{jk}^a \times L \times H}{E(H_k)} + c_3 \\ &\quad \times \sum_{j=0}^{J} \sum_{k=0}^{m_u} \frac{p_j \times p_{jk}^a \times L \times H \times E}{E(H_k)} \end{aligned} \tag{18}$$

Regarding the user costs, we analyze both the waiting time cost and in-vehicle time cost. With regard to the waiting time cost to passengers, since it is assumed that passengers arrive at the stop randomly and uniformly over time, the waiting time may be estimated as half of the headway. As regards the in-vehicle travel time cost to passengers, we translate the pass-up passengers' cost into the excess of in-vehicle time cost. The unit in-vehicle cost will increase with the decrease in operating fleet size in disruptive situations. To simplify our model, we assume the increase unit of in-vehicle cost is directly proportional to the increase in the in-carriage congestion level.

The waiting time cost to passengers is calculated as

$$E(W_k) = 0.5 E(H_k) \times \left(1 + CV^2\right) \tag{19}$$

$$S_W = c_4 \times \sum_{j=0}^{J} \sum_{k=0}^{m_u} p_j \times p_{jk}^a \times E(W_k) \times Q \tag{20}$$

where CV represents the standard deviation coefficient of the headway, Q represents the daily passenger volume of the bus line, and c_4 is the per unit waiting time cost.

The total in-vehicle time cost to passengers can be expressed by

$$S_C = \frac{c_5 m}{m_k} \times \sum_{j=0}^{J} \sum_{k=0}^{m_u} p_j \times p_{jk}^{a} \times \frac{Qd}{v} \tag{21}$$

where c_5 is the unit in-vehicle cost, and d is the average travel distance of boarding passengers.

The above models can be used to calculate the total user costs of the transit system, and the function is shown as follows:

$$\begin{aligned} S &= S_W + S_C \\ &= c_4 \times \sum_{j=0}^{J} \sum_{k=0}^{m_u} p_j \times p_{jk}^{a} \times E\left(W_k\right) \times Q + \frac{c_5 m}{m_k} \\ &\quad \times \sum_{j=0}^{J} \sum_{k=0}^{m_u} p_j \times p_{jk}^{a} \times \frac{Qd}{v} \end{aligned} \tag{22}$$

3.2. Mathematical Formulation. The above models can be used to calculate the total costs of the transit system in different states. In order to reduce costs and to increase the bus system's quality, the aim of this paper is to minimize the objective of both the operator cost and the user cost in both normal and disruptive situations, in order to determine the optimum reserve fleet size. The objective function is shown as follows:

$$G = \min\left(\alpha C + \beta S\right) \tag{23}$$

subject to

$$\frac{N}{m} \geq \theta \tag{24}$$

$$N \leq W \tag{25}$$

$$N = \text{int} \tag{26}$$

$$\sum_{i=0}^{m_u} p_{jk}^{a} = 1 \tag{27}$$

$$\sum_{j=0}^{J} p_j = 1 \tag{28}$$

where C represents the operator cost, S denotes the users cost, α is the weight coefficient of the operator cost, and β is the weight coefficient of the user cost. Obviously, different stakeholders have different opinions regarding α and β. In fact, different values assigned to weight coefficients α and β may influence the result of the optimum reserve fleet size. For instance, if the operator pays more attention to profit, α is much larger than β, while in the opposite solution the operator provides more convenience for passengers, considering the bus operation to be primarily a service industry. However, the weight coefficient is not the primary scope of this paper. We wish to provide decision makers with tools that will help them to decide upon and formulate the proper service policy, once the proper weights for the chosen area have been selected, depending on the circumstances.

Equations (24)–(28) are constraints of the objective function, which is used to improve the solution method process. Constraint (24) ensures that the proportion of the reserve fleet size in the transit system fleet should be greater than the minimum reserve fleet rate θ, which is in turn dependent on the experience of the operator and the operating policy. Constraint (25) guarantees that the size of the reserve fleet at the depot cannot exceed a given number W, which is in turn dependent on the available space at the depot and the operating policy. Constraint (26) guarantees that the reserve fleet size is integer. The remaining constraints (27)-(28) ensure that the sum of probabilities is equal to one.

3.3. Solution Methods. The optimization model in the previous section is nonlinear. A solution algorithm for the optimization model is required, in order to solve the problem and obtain the optimum solution. We used a genetic algorithm to determine the optimum reserve fleet size, and values of reserve fleet size are constantly reset to find the optimal scheme. Figure 5 illustrates the solution algorithm for the above objective function.

In the initial population generation logical block, we use the binary code of reserve fleet size to reduce the efficiency of the searching procedure. In the fitness computation logical block, the total cost of the transit system in different states is used as the fitness. We convert binary code to decimal integer code that can uniformly generate a random number of reserve fleet size from the feasible integer set. This process can not only satisfy constraints (24)–(26), but also ensure the diversification of initial population. In addition, if the constraints (27)-(28) cannot be met under the corresponding individual bus service, the fitness should be equal to a large number M. Because the minimal fitness of the individual bus service is the objective, this large number can be a penalty for the inappropriate individual bus service. If the individual bus service can meet all the constraints, the optimal reserve fleet size for each individual bus service should be conducted to determine the corresponding fitness value. In the genetic manipulation logical block, the roulette wheel selection method, the two-point crossover method, and the Gaussian mutation method are used for selection, crossover, and mutation separately. In addition, the termination criterion is that the best fitness does not change during 10 successive generations.

4. Model Verification

In this section, we present several numerical examples to illustrate the performance of our models and algorithm. We begin by testing the models on a common bus line, and we compare the optimum reserve fleet size with the minimum reserve fleet size. All related information pertaining to the bus line is presented in Table 1. The bus line's hourly passenger volumes are uniform (such as the shopping volume). The two scenarios applied to the bus line's average accident rates λ, λ'

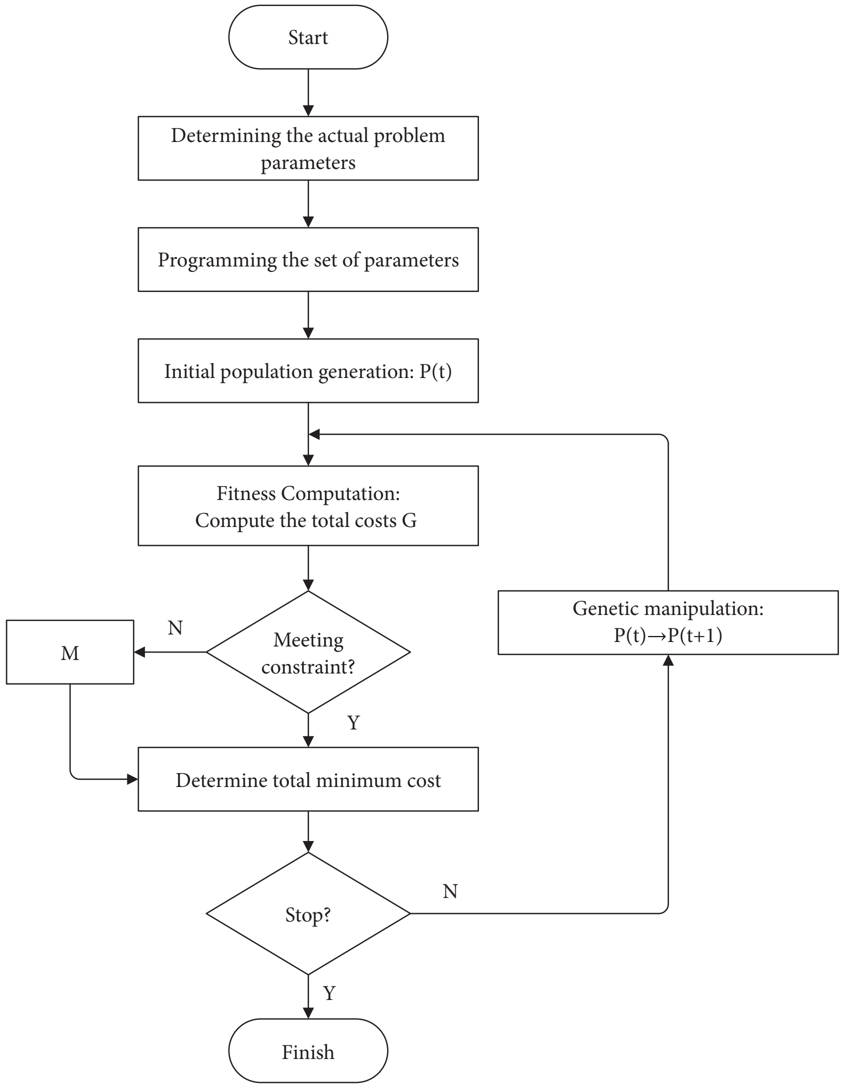

Figure 5: Solution algorithm.

are assumed to be 0.8 and 0.5 per day. The daily probability of the two scenarios p_1, p_2 are assumed to be 0.6 and 0.4. Also, on an annual basis, two or fewer accidents per hour occur simultaneously in the analyzed bus line section. The weight coefficient of the operator cost α and the weight coefficient of the user cost β are equal to 0.5.

We applied the mathematical software MATLAB to determine the total cost of the transit system and the optimal reserve fleet size, as obtained by the genetic algorithm. The comparison of the minimum reserve fleet size obtained via the experiential method (whereby the reserve fleet size is determined by the minimum reserve fleet rate) and the optimal reserve fleet size as obtained by the proposed model is presented in Table 2.

Based on the results presented in Table 2, we find that the total cost generated by the proposed model decreases in contrast to the experiential method. We can therefore conclude that the proposed model can effectively reduce the total cost while still improving the quality of the bus service. To further see the performance of the proposed model, we tested two parameters on the bus line. To begin with, we tested the models on different discrete values of $\alpha = 0.1$ to 0.9, while the values of the other parameters remained unchanged. According to the results obtained by MATLAB software, it is reasonable to compare the transit system daily total cost, as shown in Figure 6.

Figure 6 reveals how the trade-off between the weight coefficient of the operator cost and the weight coefficient of the user cost made by the stakeholder produces different effects on the reserve fleet size. Given different weight coefficients of operator cost α, when α increases, the optimal reserve fleet size becomes smaller and smaller, which is to be expected. This is because the weight coefficient of the user cost is increasing, and the larger reserve fleet can lead to a larger decrease in the user costs. What is more, when the weight coefficient of the operator costs decreases, the bus company's expense influence caused by the reserve fleet is less and less obvious. Given a weight coefficient of the

Table 1: Related bus line information.

Index	Bus Line
L [km]	20
m [vehicle]	20
n	10
H [hours/day]	12
θ	5%
t [minute]	2
Q	10000
d [km]	2
CV^2	0.15
Average vehicle speed [km/h]	20
E [g/km]	57
c_1 [¥]	500000
c_2 [¥·km]	2
c_3 [¥·g/km]	0.00005
c_4 [¥/h]	15
c_5 [¥/h]	10

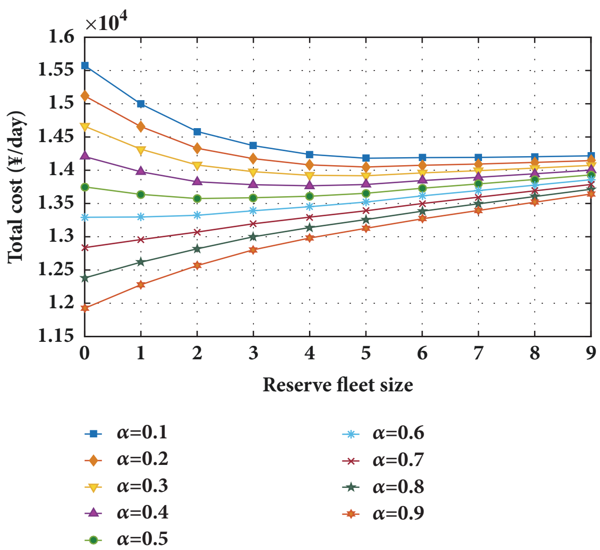

Figure 6: Transit system daily total cost with different α.

operator cost α ($0.5 \le \alpha < 0.9$), we find that when the reserve fleet size increases, the total cost gradually decreases to the lowest point and then slowly rises. This result is to be expected because the reserve fleet size is gradually increasing to the optimal reserve fleet size. Conversely, given a weight coefficient of bus company α ($0.1 \le \alpha < 0.5$), we find that the optimal reserve fleet size is zero, because the extra expense of the reserve fleet becomes increasingly obvious.

In addition, we tested the models on different discrete values of average accident rate $\lambda = 0$ to 1 incident. The total daily transit system costs under different discrete values are presented in Figure 7.

As shown in Figure 7, given different average accident rates, we find that when the accident rate increases, the total cost increases, as well as when the reserve size is less than 5. This is because the broken-down vehicles can be substituted

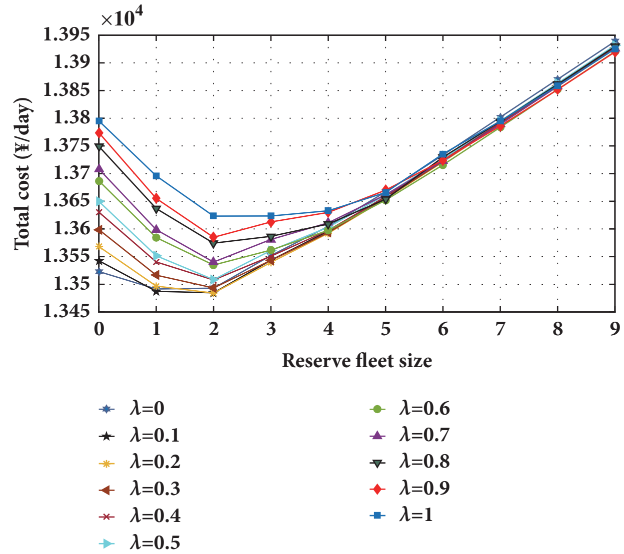

Figure 7: Transit system daily total cost under different λ.

by the increasing reserve fleet size, in order to decrease the user costs. What is more, we observe that whatever the reserve fleet size is, the total cost with different average accident rates changes only slightly when the reserve fleet size is more than 5. This is because the reserve size is sufficient to cover the broken-down vehicles. Given an average accident rate, when the reserve fleet size increases, the total cost gradually decreases to the lowest point and then gradually rises. This is to be expected, because the reserve fleet can effectively decrease the user cost until the reserve fleet size increases to the optimal reserve fleet size.

5. Conclusions

In this paper, we implement an optimization model for finding reserve fleet sizes in traditional transit systems that are exposed to the risks of vehicle breakdowns. The scenarios are generated in ways where they range from scenarios with a very low probability of a delay and a low delay length, to scenarios with a high probability and a high delay length. Using the M/M/n/m+N/m queueing model, we characterize the state of the transit system considering the risk of vehicle breakdowns, and we analyze the influence on both the operator cost and the user cost in disruptive situations. Furthermore, we minimize the objective of the operator cost and the user cost in both normal and disruptive situations, in order to determine the optimum reserve fleet size. From our tests, several conclusions can be found, as follows.

The comparison of our proposed model and the experiential method was conducted according to both the total operator cost and user cost. The proposed model can effectively reduce the total user costs of the transit system and improve the quality of the bus service, but it will also increase the total operating costs of the transit system.

Through an examination to determine the optimum reserve fleet size on the different weight coefficients of the

Table 2: Comparison of optimal and minimum reserve fleet size as obtained by the proposed model.

Parameter	Experiential method	Proposed model	Value added
N	1	2	1
C_C[¥]	2876.71	3013.70	136.99
C_O[¥]	9048.48	9287.16	238.68
C_E[¥]	12.89	13.23	0.34
S_W[¥]	4572.44	4441.02	-131.42
S_C[¥]	10763.27	10393.57	-369.7
G [¥]	13636.90	13574.34	-62.56

operator cost, we can see that a change of the weight coefficient will have an influence on the objective formulation. If the weight coefficient of the operator cost is too small, the objective of balancing the operator cost and the user cost cannot be achieved.

Compared with different average accident rates on our proposed model, we observe that, whatever the reserve fleet size is, the total cost with different average accident rates changes only slightly when the reserve fleet size is more than 5. Therefore, if the accident rate of the traditional transit systems is uncertain within limits, there will be a reserve fleet size that is sufficient to cover the broken-down vehicles.

Rules above would be beneficial for the determination of reserve fleet sizes. We will incorporate capacity constraints, uncertain demand, and uncertain correlated disruptions in our future work. In addition, we will consider multiple bus lines belong to the bus company, and we will optimize the total reserve fleet size serving different bus lines.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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