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Research Article

Rebate Decisions and Leadership Strategy in Competing Supply Chain with Heterogeneous Consumers

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Rebate is a traditional type of promotion, and it can benefit manufacturers and retailers with expanded demands. However, the impact of leadership strategy in rebate competition on supply chain members and rebate decision is still somewhat unclear. Our paper focuses on a horizontal competition with respect to both rebate and leadership between two manufacturers selling substitutable products through a common retailer to consumers who are heterogeneous in their price sensitivity. Furthermore, we investigate the impacts of leadership strategy on profits and study rebate decision under different strategies. Our research indicates that Bertrand-Nash game benefits the retailer, but hurts manufacturers, while Stackelberg game benefits manufacturers but hurts the retailer, which shows no difference from previous studies. In addition, the sequential-move Stackelberg game could eliminate the classic prisoners' dilemma in rebate decision, which is also influenced by fixed cost control.

1. Introduction

Rebate is a very fashionable type of sale promotion in consumers' daily lives. Companies offer rebates as a promotion, especially in electronic and automotive industries for several reasons. In some cases, rebates are used to increase sales, expand demands, decrease inventories, and even coordinate supply chain [1, 2]. According to an industrial study, 50% of retailers and 48% of manufacturers use rebates as part of their customer loyalty and promotions mix [3]. A survey of UK shopper behavior [4] shows about one in three customers is interested in rebates on consumer packaged merchandise and three in four customers want cash back rebates on appliances and electronics. Particularly, rebate decision is commonly decided as a strategy by competing manufacturers, such as Canon and Epson, who both launch a rebate promotion. However, among Dell, HP, and Sony, Dell and HP phase out their rebate promotion, but Sony goes on to offer rebates instead [5].

This paper is closely related to the literature on rebate programs in supply chain management. Most researches in the field focus on rebate decisions with uncertain demand in a newsvendor framework. Aydin and Porteus [6] developed two rebate forms with uncertain demand, which is multiplicative, from manufacturer to retailer and from manufacturer to customer with endogenous manufacturer rebates and retail pricing. Demarag et al. [7] employed a game theoretical model to examine the impact of retailer incentive and customer rebate promotions on the manufacturer's pricing and the retailer's ordering. Furthermore, they studied a setting with two manufacturers and two retailers. They found that customer rebates can be more profitable in some cases, unlike the monopoly case where the manufacturers are always better off with retailer incentives. Demarag et al. [8, 9] considered a risk-averse retailer and formally modeled it by adopting the Conditional-Value-at-Risk (CVaR) decision criterion. Geng and Mallik [10] used a game-theoretic framework to investigate the joint decision of offering mail-in rebates (MIR) with stochastic demand, and they showed both parties offering MIR, only one party offering MIR, and neither offering MIR can be the equilibrium. Arcelus et al. [11] also examined the impact of direct rebates to the end customer from the manufacturer and/or from the retailer upon the profitability and effectiveness of the policies of both channels. They showed all three scenarios were equally profitable or the retailer-only rebate policy was dominant.

There are a number of papers in this stream that consider a one-manufacturer-one-retailer relationship. Gerstner and Hess [12] examined four types of price promotions with two consumer segments: trade deals only, manufacturer rebates only, combination of them, and retailer rebates. They showed that manufacturer rebates played a positive role not only in price discrimination but also in retail participation. Chen et al. [13] used case study comparing rebates with coupons and showed that the rebate's ability of price discriminated among consumers. Lu and Moorthy [14] studied the difference between rebate and coupon when the uncertain redemption costs were resolved and identified. They showed that rebates are more efficient in price discrimination than coupons. Chen et al. [15] investigated a two-stage game with a manufacturer and a retailer and characterized the impact of a manufacturer rebate on the expected profits of both members in the supply chain. Cho et al. [16] determined the equilibrium of a vertical competition game between the manufacturer and the retailer with three decisions, including regular price, rebate, and its value, and investigated how competition affects the rebate decisions. Khouja and Jing [17] developed a Stackelberg game, in which a manufacturer offered cash mail-in rebates as a leader with a retailer as a follower.

In the latest literature, Huang et al. [18] suggested that rebate competition among manufacturers or retailers can be studied for future research. Ha et al. [19] are the first to consider a multistage game, including two manufacturers selling substitutable products through a common a retailer with slippage effect. Our study is the most close to theirs. However, in their model, two competing manufacturers just moved simultaneously and redemption rate was not involved in consumer's utility functions, which did not conform to the reality. Thus their model setup is also quite different from ours.

Our research is also closely related to the endogenous timing of strategy, which refers to a member's relative ability to control the decision-making process in the supply chain. For example, Apple is considered to be the Manufacturer-Stackelberg leader [20], while Wal-Mart generally is regarded as the Retailer-Stackelberg leader [21]. A great deal of studies examined the leadership strategy under specific duopoly games settings, such as pricing setting duopoly [22–33], quantity setting duopoly [34–36], and other duopolies. Gal-Or [22] first demonstrated how two identical players moved sequentially in a game when their reaction functions were upwards or downwards sloping. Amir and Stepanova [24] utilized a first-versus second-mover advantage in differentiated product Bertrand duopoly and showed that a firm with larger cost than its rival had a first-mover advantage. Chen et al. [32] examined the competition among retailers about product return strategies and leadership strategies. Moreover, most of the researches have concentrated on the impact of factors on leadership strategies. Mago and Dechenaux [37] investigated the impact of firm size asymmetry and found that large firm was subjected to be the price leader. Hirata and Matsumura [38] showed that firms with higher cost were more likely to be the Stackelberg leader. Wang et al. [39] utilized the endogenous timing of strategies to model the efficient-responsive choice for two firms, and they showed that moving simultaneously can be an equilibrium. Niu et al. [40] analyzed the price leadership of two manufacturers and showed that the market size had a major impact on equilibrium. Chen et al. [32] examined how competing retailers should choose leadership strategies with product return strategies and the impact of them. They found Money-Back Guarantee (MBG) returns policy significantly influenced the leadership strategy. Our paper contributes to this body of work by examining leadership strategy in manufacturer competition with both of wholesale prices and rebate values duopoly setting.

In our study, we examined how the leadership strategies affected the profits of supply chain members and rebate decisions from a horizontal competitive perspective referring to two manufacturers and one retailer with certain demand. To address these questions, we developed a multistage game model, in which two manufacturers sell substitutable products through a common retailer to consumers who are heterogeneous in their price sensitivity. The two competing manufacturers faced the same redemption rate, but played the game with different leadership strategies, which are Bertrand-Nash game strategy and sequential-move Stackelberg game strategy, respectively. We discussed how manufacturers may decide leadership strategies, either moving simultaneously, or moving first as the leader, or moving second as the follower. In addition, we investigated how they made the rebate decisions with different leadership strategies.

Our contributions to the literature are twofold. First, we demonstrated that leadership strategy has an opposite impact on profits for manufacturers and retailer in rebate competition, which coincides with previous studies, and in most cases, manufacturers have the second-move advantages to decide the wholesale prices and rebate values. Second, sequential-move Stackelberg strategy could eliminate the classic prisoners' dilemma scenario in rebate decision, which is also influenced by fixed cost control.

2. The Benchmark Model

2.1. The Model Setting. We consider the duopoly model involving two manufacturers (indexed by X or Y) selling substitutable products through a common retailer (he) to consumers who are heterogeneous in their price sensitivity, where the manufacturers are viewed as to be the Stackelberg leaders and the retailer is the follower. We assume that each manufacturer (she) has to make a rebate decision whether to offer a rebate project before other decisions, as it takes time to design and launch the rebate project. We also assume that the manufacturers can move simultaneously or sequentially when determining wholesale prices and rebate values. We denote the duopoly's price leadership strategy as $\mathrm{L} = \{\mathrm{B}, \mathrm{X}, \mathrm{Y}\}$, that is, three basic games: a simultaneous game (Bertrand-Nash game, denoted as a superscript B), a sequential-move Stackelberg game with manufacturer X as the leader (manufacturer X Stackelberg, denoted as a superscript X), or a sequential-move Stackelberg game with manufacturer Y as the leader (manufacturer Y Stackelberg, denoted as a superscript Y).

Moreover, we consider that the manufacturer i ($i = X, Y$) bears a fixed cost F_i, which captures the costs related to designing a rebate project, launching a rebate promotion and advertising. Without loss of generality, we assume that the unit variable rebate cost, the unit manufacturing cost, and the unit selling cost are constant and normalized to zero.

The supply chain model we study can be described as a multistage game with the following sequence of events:

(1) Each manufacturer i in the duopoly needs to decide her rebate project, either offering the rebate (R) or not offering the rebate (N) with a related fixed cost F_i. There are four combinations of the two manufactures. We denote the manufacturers' rebate decision as $\mathrm{Z} = \mathrm{Z_X Z_Y} = \{\mathrm{NN}, \mathrm{NR}, \mathrm{RN}, \mathrm{RR}\}$. The first and second letters denote manufacturer X's and manufacturer Y's rebate decisions, respectively, where $\mathrm{Z_X} = \{\mathrm{R}, \mathrm{N}\}$ and $\mathrm{Z_Y} = \{\mathrm{R}, \mathrm{N}\}$.

(2) After observing the rebate project decision, each manufacturer i determines her wholesale price and rebate value if a rebate project is offered, according to the leadership strategy between the two manufacturers. If the leadership strategy is $\mathrm{L} = \mathrm{B}$, both manufacturers decide their wholesale price (p_X^{BZ}, p_Y^{BZ}) and rebate values (r_X^{BZ}, r_Y^{BZ}), if rebate projects are offered simultaneously; if the leadership strategy is $\mathrm{L} = X$, anticipating manufacturer Y's wholesale price and rebate value if a rebate is offered, manufacturer X announces her wholesale price (p_X^{XZ}) and rebate value (r_X^{XZ}) if a rebate is offered, and then manufacturer Y announces her wholesale price (p_Y^{XZ}) and rebate value (r_Y^{XZ}) if a rebate is offered; if the leadership strategy is $\mathrm{L} = \mathrm{Y}$, similarly, manufacturer Y announces her wholesale price (p_Y^{YZ}) and rebate value (r_Y^{YZ}) if a rebate is offered, and then manufacturer X announces her wholesale price (p_X^{YZ}) and rebate value (r_X^{YZ}) if a rebate is offered.

(3) Given the wholesale prices and rebate values (if rebates are offered), the retailer determines his retailer prices p_i for both products.

(4) The manufacturers produce to meet their demands and the firms acquire their payoffs.

2.2. The Demand Functions. Lu and Moorthy [14] argued that consumers with different income might have different redemption costs because they differed in their opportunity cost of time, and consumers may incur different redemption costs. Chen et al. [15] pointed out through some studies that consumers systematically exhibited overconfidence in the personal forecast. Such optimistic bias we call "slippage," which can let customers overestimate their likelihood of redeeming a rebate offer and make an error in estimating the effort involved in the redemption. Tasoff and Letzler [41] further investigated that experimental results showed the expected redemption rates exceed actual redemption rates by 49% because of stamp and envelope costs, the cost of time, loss of the form, and so forth, which explained high redemption cost.

As mentioned by Cai [42], rebates helped discriminate consumers who were heterogeneous in their price sensitivity, as not every consumer redeems the rebates. We develop two segments with consumers who are heterogeneously rebate-sensitive and rebate-insensitive. The rebate-sensitive consumers incur a lower redemption cost C_L, and the rebate-insensitive consumers incur a higher redemption cost C_H for the complexity of the redemption steps or their higher cost of time, where $0 \le C_L < C_H$. We derive the demand functions by following the similar approach from Ha et al. [19], which is also developed from Zhang et al. [27] and Chung [30]. The utility function of a representative consumer is given by

$$\begin{aligned} &\left(q_X^{LZ} + q_Y^{LZ}\right) a - \frac{1}{2}\left[\left(q_X^{LZ}\right)^2 + \left(q_Y^{LZ}\right)^2 + 2\gamma q_X^{LZ} q_Y^{LZ}\right] \\ &\quad - \left(p_X^{LZ} - \max\left[m r_X^{LZ}, \left(r_X^{LZ} - C\right)\right]\right) q_X^{LZ} \\ &\quad - \left(p_Y^{LZ} - \max\left[m r_Y^{LZ}, \left(r_Y^{LZ} - C\right)\right]\right) q_Y^{LZ}, \end{aligned} \tag{1}$$

where a is the market size, q_i^{LZ}, p_i^{LZ}, and r_i^{LZ} are, respectively, the consumption quantity, retail price, and rebate value of product i produced by manufacturer i, and C is the consumer's estimated redemption cost. Here $\gamma \in [0, 1)$, generally interpreted as the competition intensity between the manufacturers, captures the substitutability of the two products.

Particularly, in order to take "slippery effect" into account to differentiate the consumers heterogeneous in their rebate sensitivity, we consider the redemption rate $m \in (0, 1]$ in the utility function; that is, we assume when $C = C_L = 0$, the consumers are rebate-sensitive and will incur a 100% redemption rate ($m = 1$), and when $C = C_H$, the redemption cost is very high such that the consumers are rebate-insensitive and will incur a redemption rate less than 100% ($m \in (0, 1)$), instead of being prohibited from redeeming rebate. For the purpose of tractability, let the proportion of rebate-sensitive consumers be identical to rebate-insensitive consumers in the market (if not, too many parameters will be stacked in formulas).

So the utility function of a representative rebate-sensitive consumer is given by

$$\begin{aligned} &\left(q_X^{LZ} + q_Y^{LZ}\right) a - \frac{1}{2}\left[\left(q_X^{LZ}\right)^2 + \left(q_Y^{LZ}\right)^2 + 2\gamma q_X^{LZ} q_Y^{LZ}\right] \\ &\quad - \left(p_X^{LZ} - r_X^{LZ}\right) q_X^{LZ} - \left(p_Y^{LZ} - r_Y^{LZ}\right) q_Y^{LZ}. \end{aligned} \tag{2}$$

Given p_i^{LZ} and r_i^{LZ}, the optimal consumption quantities t_i^{LZ} for the rebate-sensitive consumers are given by the following demand functions:

$$t_X^{LZ} = \frac{(1-\gamma) a - \left(p_X^{LZ} - r_X^{LZ}\right) + \gamma\left(p_Y^{LZ} - r_Y^{LZ}\right)}{1-\gamma^2}, \tag{3}$$

$$t_Y^{LZ} = \frac{(1-\gamma) a - \left(p_Y^{LZ} - r_Y^{LZ}\right) + \gamma\left(p_X^{LZ} - r_X^{LZ}\right)}{1-\gamma^2}. \tag{4}$$

The utility function of a representative rebate-insensitive consumer is given by

$$\left(q_X^{LZ}+q_Y^{LZ}\right)a-\frac{1}{2}\left[\left(q_X^{LZ}\right)^2+\left(q_Y^{LZ}\right)^2+2\gamma q_X^{LZ}q_Y^{LZ}\right] \\ -\left(p_X^{LZ}-mr_X^{LZ}\right)q_X^{LZ}-\left(p_Y^{LZ}-mr_Y^{LZ}\right)q_Y^{LZ}. \quad (5)$$

Given p_i^{LZ}, the optimal consumption quantities k_i^{LZ} for the rebate-insensitive consumers are given by the following demand functions:

$$k_X^{LZ}=\frac{(1-\gamma)a-\left(p_X^{LZ}-mr_X^{LZ}\right)+\gamma\left(p_Y^{LZ}-mr_Y^{LZ}\right)}{1-\gamma^2}, \quad (6)$$

$$k_Y^{LZ}=\frac{(1-\gamma)a-\left(p_Y^{LZ}-mr_Y^{LZ}\right)+\gamma\left(p_X^{LZ}-mr_X^{LZ}\right)}{1-\gamma^2}. \quad (7)$$

Let $D_i^{LZ}=(1/2)(t_i^{LZ}+k_i^{LZ})$ as the total demand of product i.

3. Prices and Rebate Values Decision under Leadership Strategy

For given rebate decision (Z) and leadership strategy (L) from two manufacturers, we solve for the equilibrium retail prices, wholesale prices, and rebate values (if rebates are offered) by optimizing the retailer's profit function and manufacturers' profit functions and obtain the firms' profit. Particularly, the fixed cost of a rebate project is not involved in manufacturers' profit functions because it is a sunk cost and does not have any impact on manufacturers' price and rebate values or leadership strategy.

Given ω_i^{LZ} and r_i^{LZ}, the retailer maximizes his profit (π_R^{LZ})

$$\left(p_X^{LZ}-\omega_X^{LZ}\right)D_X^{LZ}+\left(p_Y^{LZ}-\omega_Y^{LZ}\right)D_Y^{LZ}, \quad (8)$$

by choosing the following best-response function:

$$\widehat{p}_i^{LZ}\left(\omega_i^{LZ},r_i^{LZ}\right)=\frac{1}{4}\left[2a+2\omega_i^{LZ}+(1+m)r_i^{LZ}\right], \quad (9)$$

where the Hessian matrix is negative due to $\partial^2\pi_R^{LZ}/\partial p_i^{LZ^2}=-2/(1-\gamma^2)<0$ and $(\partial^2\pi_R^{LZ}/\partial p_1^{LZ^2})(\partial^2\pi_R^{LZ}/\partial p_2^{LZ^2})-(\partial^2\pi_R^{LZ}/\partial p_1^{LZ}\partial p_2^{LZ})(\partial^2\pi_R^{LZ}/\partial p_2^{LZ}\partial p_1^{LZ})=4/(1-\gamma^2)>0$ for any $\gamma\in[0,1)$; thus the profit of the retailer satisfies the second-order condition for a maximum.

Given rebate decision (Z) and leadership strategy (L) from two manufacturers, by optimizing the manufacturers' profit functions over ω_i^{LZ} and r_i^{LZ} (if rebates are offered), we derive a unique optimal equilibrium of them; then the corresponding p_i^{LZ}, D_i^{LZ}, π_i^{LZ}, and π_R^{LZ} can also be acquired. All the results under different hypothesis leadership strategies are summarized in Table 1, and detailed manufacturers' profit functions and related derivations are shown in Proof A in the Appendix.

where $g(\gamma)$, $s(m)$, and $v_e(m,\gamma)$, $e=1,\ldots,22$, are given in Table 3 ($g(\gamma)>0$, $s(m)>0$, and $v_e(m,\gamma)>0$).

3.1. Bertrand-Nash Game Scenario. As benchmark case, we first consider the Bertrand-Nash game between the two competing manufacturers; that is, two manufacturers move simultaneously to decide wholesale prices and rebate values. The unique optimal equilibrium results, presented in Table 1(a), indicate the following rank orders in wholesale prices and profits for manufacturers shown in Lemma 1, rebate values, retail prices, and demands for consumers shown in Lemma 2, and profits for retailer shown in Lemma 3 (proofs of all the lemmas are given in Proof B in Appendix, $i=X,Y$).

Lemma 1. *For $L=B$,*

(1) $\omega_X^{BNR}(\omega_Y^{BRN})<\omega_i^{BNN}<\omega_i^{BRR}$, $\pi_X^{BNR}(\pi_Y^{BRN})<\pi_i^{BNN}<\pi_i^{BRR}$;

(2) $\omega_X^{BNR}(\omega_Y^{BRN})<\omega_i^{BNN}<\omega_X^{BRN}(\omega_Y^{BNR})$, $\pi_X^{BNR}(\pi_Y^{BRN})<\pi_i^{BNN}<\pi_X^{BRN}(\pi_Y^{BNR})$.

Lemma 1 implies that the rank orders of profits for manufacturers accord with the ones of wholesale prices, which indicates that manufacturers' profits greatly depend on wholesale prices. And all the rank orders are intuitive, except that the comparison of wholesale prices or profits for manufacturers under $Z=\{RR\}$ and $Z=\{RN\}$ for manufacturer X ($Z=\{NR\}$ for manufacturer Y) is related to γ as seen in Figure 1, which is different from the previous research. When the competition between manufacturers is less intense, the wholesale prices or profits for manufacturers under $Z=\{RR\}$ are higher than $Z=\{RN\}$ for manufacturer X ($Z=\{NR\}$ for manufacturer Y) shown in Figures 1(a) and 1(b). And when the competition is more intense, the result is just on the contrary shown in Figures 1(c) and 1(d), which illustrates that one of the two manufacturers will taper off offering a rebate when the competition becomes more intense.

Lemma 2. *For $L=B$,*

(1) $r_X^{BRN}(r_Y^{BNR})<r_i^{BRR}$;

(2) $p_X^{BNR}(p_Y^{BRN})<p_i^{BNN}<p_X^{BRN}(p_Y^{BNR})<p_i^{BRR}$;

(3) $D_X^{BNR}(D_Y^{BRN})<D_i^{BNN}<D_i^{BRR}<D_X^{BRN}(D_Y^{BNR})$.

Lemma 2 shows that the rank orders of rebate values, retail prices, and demands for consumers under different scenarios of $Z=\{RR,RN,NR,NN\}$ are not affected by the intensity of competitive rivalry. The intuition is that when the competition is more intense, manufacturers cannot lower the rebate values with decreasing wholesale prices, because the higher rebate values would raise the redemption rate intrinsically to help manufacturers maintain a healthy profit margin ($\omega-mr$) for the rebate-sensitive consumers. Consequently, higher wholesale prices and rebate values definitely induce higher retail prices, but lower demands for consumers.

Lemma 3. *For $L=B$, $\pi_R^{BNN}<\pi_R^{BRN}(\pi_R^{BNR})<\pi_R^{BRR}$.*

As indicated in Lemma 3, the rank order of profits for retailer is also not related to γ. But it should be noted that

Table 1: Equilibrium results under different leadership strategies.

(a) Bertrand-Nash game ($L = B$)

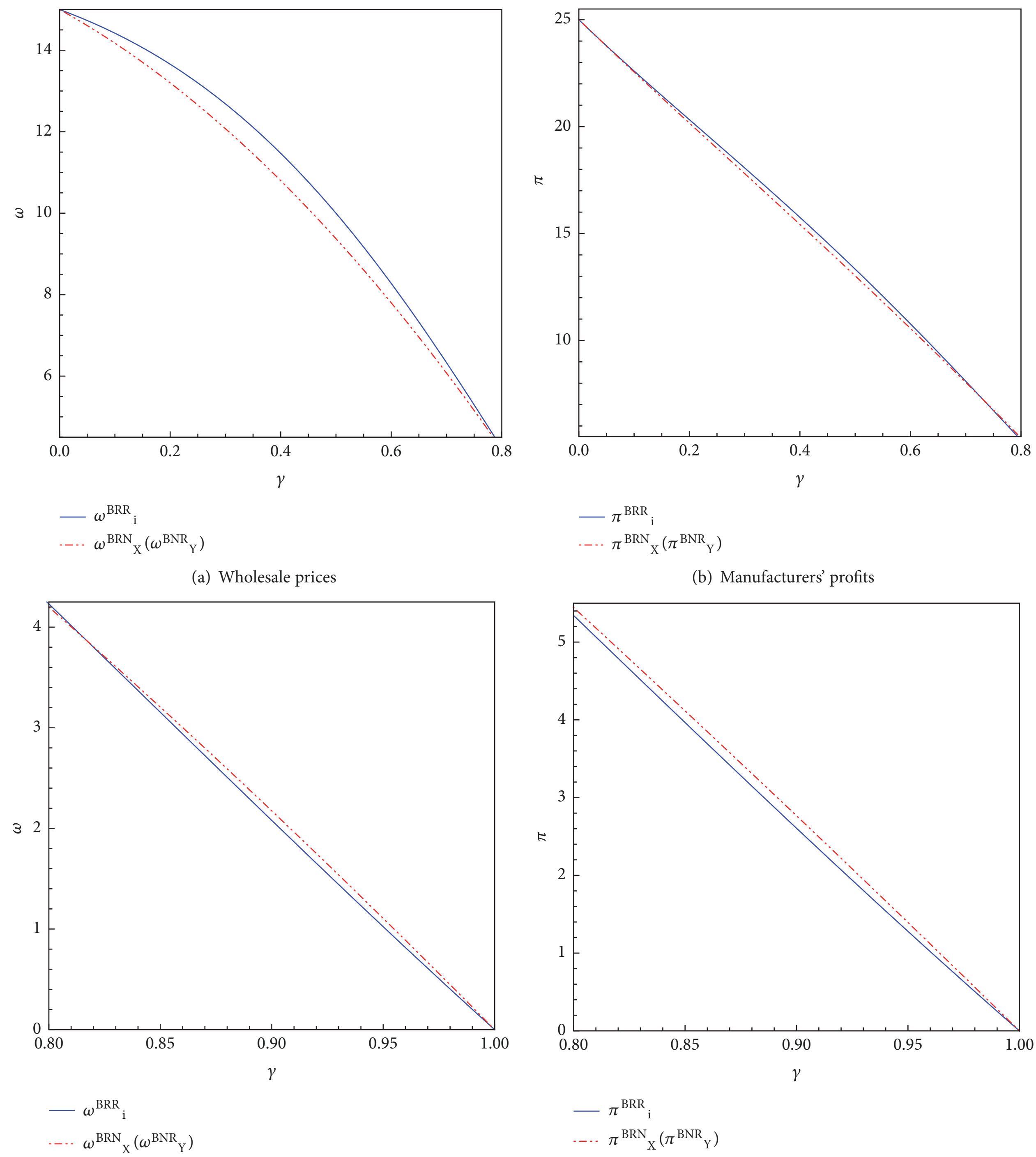
	$Z = \{NN\}$	$Z = \{RR\}$	$Z = \{RN\}$ (the results of $Z = \{NR\}$ is symmetric to $Z = \{RN\}$)
$\omega_X^{LZ}*$	$\frac{(1-\gamma)a}{2-\gamma}$	$\frac{am(5+2m(2-\gamma)-2\gamma)(1-\gamma)}{v_1(m,\gamma)}$	$\frac{am(5-4m)(1-\gamma)(2+\gamma)}{v_2(m,\gamma)}$
$\omega_Y^{LZ}*$	$\frac{(1-\gamma)a}{2-\gamma}$	$\frac{am(5+2m(2-\gamma)-2\gamma)(1-\gamma)}{v_1(m,\gamma)}$	$\frac{av_3(m,\gamma)(1-\gamma)}{v_2(m,\gamma)}$
$p_X^{LZ}*$	$\frac{(3-2\gamma)a}{2(2-\gamma)}$	$\frac{am[2\gamma(4-\gamma)-7+m(3-2\gamma)(2-\gamma)]}{v_1(m,\gamma)}$	$\frac{av_4(m,\gamma)}{2v_2(m,\gamma)}$
$p_Y^{LZ}*$	$\frac{(3-2\gamma)a}{2(2-\gamma)}$	$\frac{am[2\gamma(4-\gamma)-7+m(3-2\gamma)(2-\gamma)]}{v_1(m,\gamma)}$	$\frac{1}{2}\left(a+\frac{av_3(m,\gamma)(1-\gamma)}{v_2(m,\gamma)}\right)$
$r_X^{LZ}*$	–	$\frac{2a(1-\gamma)}{v_1(m,\gamma)}$	$\frac{2a(1-\gamma)(2+\gamma)}{v_2(m,\gamma)}$
$r_Y^{LZ}*$	–	$\frac{2a(1-\gamma)}{v_1(m,\gamma)}$	–
$D_X^{LZ}*$	$\frac{a}{2(2-\gamma)(1+\gamma)}$	$\frac{a(1-m)m(2-\gamma)}{v_1(m,\gamma)(1+\gamma)}$	$\frac{2a(-1-m)m(2+\gamma)}{v_2(m,\gamma)(1+\gamma)}$
$D_Y^{LZ}*$	$\frac{a}{2(2-\gamma)(1+\gamma)}$	$\frac{a(1-m)m(2-\gamma)}{v_1(m,\gamma)(1+\gamma)}$	$\frac{av_3(m,\gamma)(1-\gamma)}{2v_2(m,\gamma)(1+\gamma)}$
$\pi_X^{LZ}*$	$\frac{a^2(1-\gamma)}{2(2-\gamma)^2(1+\gamma)}$	$\frac{a^2(1-m)m(1-\gamma)}{v_1(m,\gamma)(1+\gamma)}$	$\frac{a^2m(-1+9m-16m^2+8m^3)(1-\gamma)(2+\gamma)^2}{v_2^2(m,\gamma)(1+\gamma)}$
$\pi_Y^{LZ}*$	$\frac{a^2(1-\gamma)}{2(2-\gamma)^2(1+\gamma)}$	$\frac{a^2(1-m)m(1-\gamma)}{v_1(m,\gamma)(1+\gamma)}$	$\frac{a^2v_3^2(m,\gamma)(1-\gamma)}{2v_2^2(m,\gamma)(1+\gamma)}$
$\pi_R^{LZ}*$	$\frac{a^2}{2(2-\gamma)^2(1+\gamma)}$	$\frac{4a^2(1-m)^2m^2(2-\gamma)^2}{v_1^2(m,\gamma)(1+\gamma)}$	$\frac{a^2v_5(m,\gamma)}{4v_2^2(m,\gamma)(1+\gamma)}$

(b) Sequential-move Stackelberg game ($L = \{X, Y\}$)

	$Z = \{NN\}$	$Z = \{RR\}$	$Z = \{RN\}$ $(L = X)/Z = \{NR\}$ $(L = Y)$	$Z = \{NR\}$ $(L = X)/$ $Z = \{RN\}$ $(L = Y)$
$\omega_X^{LZ}*$	$\frac{a(1-\gamma)(2+\gamma)}{2(2-\gamma^2)}$	$\frac{am(5-2m(2+\gamma)+3\gamma)(1-\gamma)}{v_6(m,\gamma)}$	$\frac{am(-10+8m+\gamma^2)(-2+\gamma+\gamma^2)}{v_7(m,\gamma)}$	$\frac{a(1-\gamma)v_3(m,\gamma)}{2v_6(m,\gamma)}$
$\omega_Y^{LZ}*$	$\frac{a(1-\gamma)(4+2\gamma-\gamma^2)}{4(2-\gamma^2)}$	$\frac{am(1-\gamma)v_8(m,\gamma)}{v_6(m,\gamma)s(m)}$	$\frac{a(1-\gamma)v_{15}(m,\gamma)}{v_7(m,\gamma)}$	$\frac{am(5-4m)(1-\gamma)v_{19}(m,\gamma)}{2v_6(m,\gamma)s(m)}$
$p_X^{LZ}*$	$\frac{a(6-\gamma-3\gamma^2)}{4(2-\gamma^2)}$	$\frac{am[-\gamma(1+4\gamma)+7+m(\gamma(1+3\gamma)-6)]}{v_6(m,\gamma)}$	$\frac{av_{16}(m,\gamma)}{2v_7(m,\gamma)}$	$\frac{1}{2}(a+\frac{a(1-\gamma)v_3(m,\gamma)}{2v_6(m,\gamma)})$

(b) Continued.

	$Z = \{NN\}$	$Z = \{RR\}$	$Z = \{RN\}\ (L = X)/Z = \{NR\}\ (L = Y)$	$Z = \{NR\}\ (L = X)/$ $Z = \{RN\}\ (L = Y)$
$p_Y^{LZ}*$	$\dfrac{a(12-2\gamma-7\gamma^2+\gamma^3)}{8(2-\gamma^2)}$	$\dfrac{amv_9(m,\gamma)}{v_6(m,\gamma)s(m)}$	$\dfrac{av_{17}(m,\gamma)}{2v_7(m,\gamma)}$	$\dfrac{av_{20}(m,\gamma)}{4v_6(m,\gamma)s(m)}$
$r_X^{LZ}*$	–	$\dfrac{2a(1+\gamma)(1-\gamma)}{v_6(m,\gamma)}$	$\dfrac{2a(-2+\gamma+\gamma^2)(\gamma^2-2)}{v_7(m,\gamma)}$	–
$r_Y^{LZ}*$	–	$\dfrac{2a(1-\gamma)[2m(2+\gamma)^2(1-m)-(1+\gamma)]}{v_6(m,\gamma)s(m)}$	–	$\dfrac{a(1-\gamma)v_{19}(m,\gamma)}{v_6(m,\gamma)s(m)}$
$D_X^{LZ}*$	$\dfrac{a(2+\gamma)}{8(1+\gamma)}$	$\dfrac{am(1-m)v_{10}(m,\gamma)}{v_6(m,\gamma)s(m)(1+\gamma)}$	$\dfrac{2a(1-m)m(2+\gamma)(2-\gamma^2)}{v_7(m,\gamma)(1+\gamma)}$	$\dfrac{av_3(m,\gamma)}{4s(m)(1+\gamma)}$
$D_Y^{LZ}*$	$\dfrac{a(4+2\gamma-\gamma^2)}{8(2-\gamma^2)(1+\gamma)}$	$\dfrac{am(1-m)v_{11}(m,\gamma)}{v_6(m,\gamma)s(m)(1+\gamma)}$	$\dfrac{av_{15}(m,\gamma)}{2v_7(m,\gamma)(1+\gamma)}$	$\dfrac{am^2v_{21}(m,\gamma)}{v_6(m,\gamma)s(m)(1+\gamma)}$
$\pi_X^{LZ}*$	$\dfrac{a^2[(1-\gamma)(2+\gamma)^2]}{16(2-\gamma^2)(1+\gamma)}$	$\dfrac{2a^2m(1-m)(1-\gamma)v_{12}(m,\gamma)}{v_6(m,\gamma)s(m)(1+\gamma)}$	$\dfrac{a^2(1-m)m(1-\gamma)(2+\gamma)^2}{v_7(m,\gamma)(1+\gamma)}$	$\dfrac{a^2(1-\gamma)v_3^2(m,\gamma)}{8v_6(m,\gamma)s(m)(1+\gamma)}$
$\pi_Y^{LZ}*$	$\dfrac{a^2(1-\gamma)(4+2\gamma-\gamma^2)^2}{16(2-\gamma^2)^2(1+\gamma)}$	$\dfrac{2a^2mv_{13}(m,\gamma)}{v_6^2(m,\gamma)s(m)(1+\gamma)}$	$\dfrac{a^2(1-\gamma)v_{15}^2(m,\gamma)}{2v_7^2(m,\gamma)(1+\gamma)}$	$\dfrac{a^2(1-m)m(1-\gamma)v_{17}^2(m,\gamma)}{4v_6^2(m,\gamma)s(m)(1+\gamma)}$
$\pi_R^{LZ}*$	$\dfrac{a^2g(\gamma)}{64(2-\gamma^2)^2(1+\gamma)}$	$\dfrac{2a^2(-1+m)^2m^2v_{14}(m,\gamma)}{v_6(m,\gamma)s^2(m)(1+\gamma)}$	$\dfrac{a^2v_{18}(m,\gamma)}{4v_7^2(m,\gamma)(1+\gamma)}$	$\dfrac{a^2v_{22}(m,\gamma)}{16v_6^2(m,\gamma)s^2(m)(1+\gamma)}$

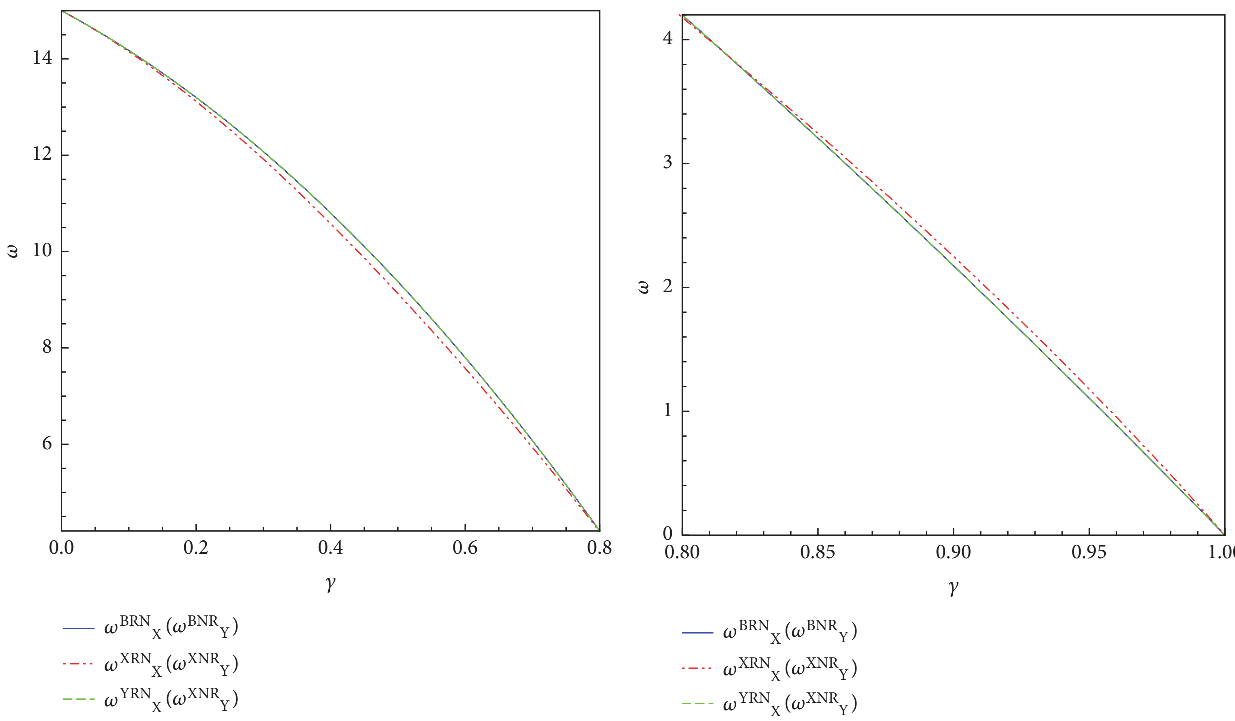


(a) Wholesale prices (b) Manufacturers' profits (c) Wholesale prices (d) Manufacturers' profits

Figure 1: Equilibrium wholesale prices and manufacturers' profits under $L = B$ versus γ (a = 10, $m = 1/2$).

more intense competition will prohibit manufacturers from offering a rebate; consequently the retailer may be worse off, so he should be prudent to intensify the manufacturer competition. That is to say, the profits for retailer are up to manufacturers' rebate decision.

Similarly, we reveal the optimal solutions of sequential-move Stackelberg game scenario in Table 1(b), where either manufacturer X moves first as a leader ($L = X$), or manufacturer Y moves first as a leader ($L = Y$) in the second-stage game. For the succinctness and integrity of analysis, this paper just presents the rank orders of prices and rebate values decision, as well as demands and profits under sequential-move Stackelberg game in Appendix.



(a) Wholesale prices (b) Wholesale prices

FIGURE 2: Equilibrium wholesale prices under $L = \{B, X, Y\}$ versus γ $(a = 10, m = 1/2)$.

3.2. Impact of Leadership Strategy. Before examining the effect of leadership strategy on manufacturers and retailer, we compare the prices and rebate values decision of sequential-move Stackelberg game with that of Bertrand-Nash game.

With equilibrium solutions in Table 1, the comparisons of wholesale prices and rebate values for manufacturers under $L = \{B, X, Y\}$ are summarized in Lemmas 4 and 5.

Lemma 4. *For $L = \{B, X, Y\}$,*

(1) $\omega_i^{BNN} < \omega_X^{YNN}(\omega_Y^{XNN}) < \omega_X^{XNN}(\omega_Y^{YNN})$;

(2) $\omega_X^{BZ}(\omega_Y^{BZ}) \le \omega_X^{XZ}(\omega_Y^{YZ}) < \omega_X^{YZ}(\omega_Y^{XZ})$;

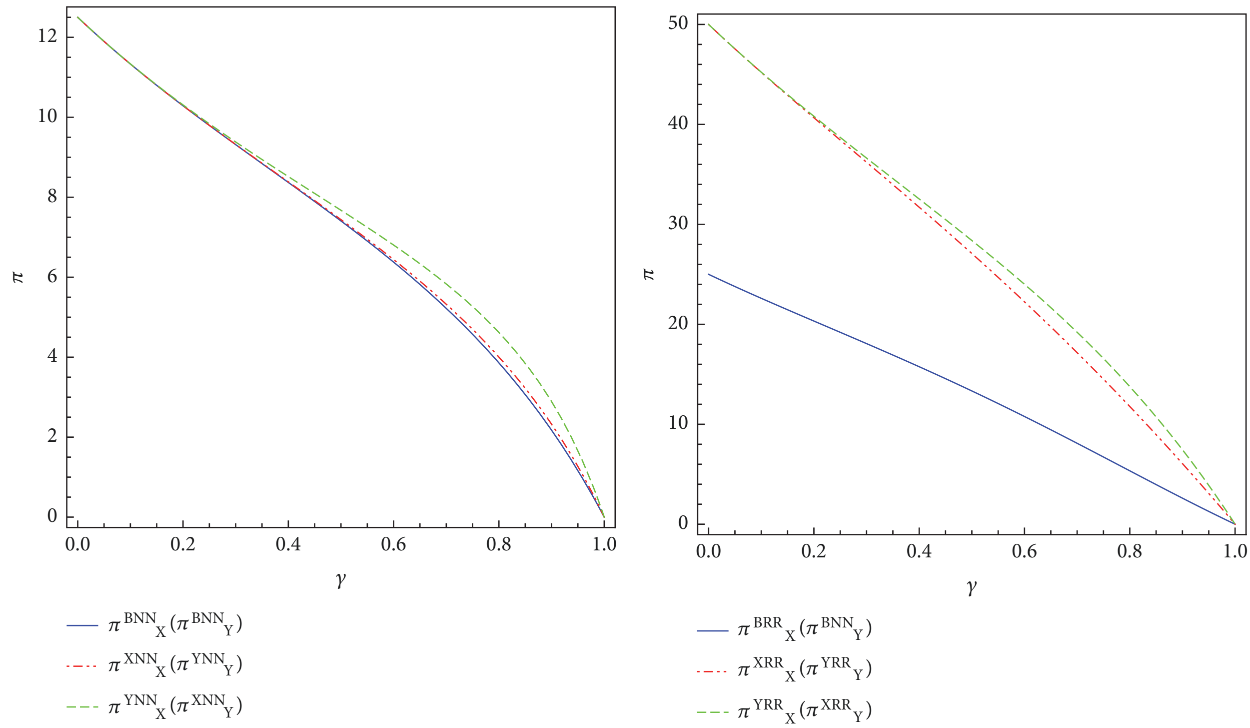
(3) $\omega_X^{BRN}(\omega_Y^{BNR}) = \omega_X^{YRN}(\omega_Y^{XNR})$.

From parts (1) and (2), where $Z = \{RR\}$ or $Z = \{NR\}$ for manufacturer X ($Z = \{RN\}$ for manufacturer Y), the wholesale prices under sequential-move Stackelberg game are not lower than the ones under Bertrand-Nash game; that is, both manufacturers will reduce wholesale prices if they decide to move simultaneously. The insight is indicative of the fact that Nash game intensifies the competition between the two manufacturers, whereas Stackelberg game softens it. From part (3), the rank order between the wholesale prices of taking a leadership position and acting as a follower competitor or moving simultaneously is subject to γ, shown in Figure 2. It can be inferred that when one manufacturer decides to offer a rebate while another does not, the level of competition is critical to the choice of leadership strategy.

Lemma 5. *For $L = \{B, X, Y\}$, $r_X^{XZ}(r_Y^{YZ}) < r_X^{BZ}(r_Y^{BZ}) \le r_X^{YZ}(r_Y^{XZ})$.*

Lemma 5 is unsurprisingly intuitive that manufacturers prefer to take the advantage of follower to determine the rebate values for adjusting the profit margin $(\omega - mr)$ flexibly and accordingly inducing a rosy perceived price to stimulate more demand from the rebate-seeking segment. Although in our paper setting manufacturers determine the wholesale prices and rebate values at the same time, the rebate values may be set a little later than wholesale prices in reality. Similarly, Nash game can trigger more intensity between the two manufacturers than Stackelberg game.

As the wholesale prices and rebate values are acknowledged, the rank orders of retail prices and demands for consumers can be implied that, for any $Z = \{NN, RN, NR, RR\}$, the impact of leadership strategy on retail prices is identical to the wholesale prices, except that $p_X^{YRN}(p_Y^{XNR})$ is exactly higher than $p_X^{XRN}(p_Y^{YNR})$ for all values of γ, as retail prices are codetermined by wholesale prices and rebate values. With regard to demands for consumers, for any $Z = \{NN, RN, NR, RR\}$, $D_X^{XZ}(D_Y^{YZ}) \le D_X^{BZ}(D_Y^{BZ}) \le D_X^{YZ}(D_Y^{XZ})$, which is similar to the rank order of rebate values and indicates the insight above. With these results of prices and rebate values decision presented in Lemmas 4 and 5 under the three game scenarios ($L = \{B, X, Y\}$), we observe the following proposition about the impact of leadership strategy on manufacturers' profits and the retailer's profits, shown in Figures 3 and 4.



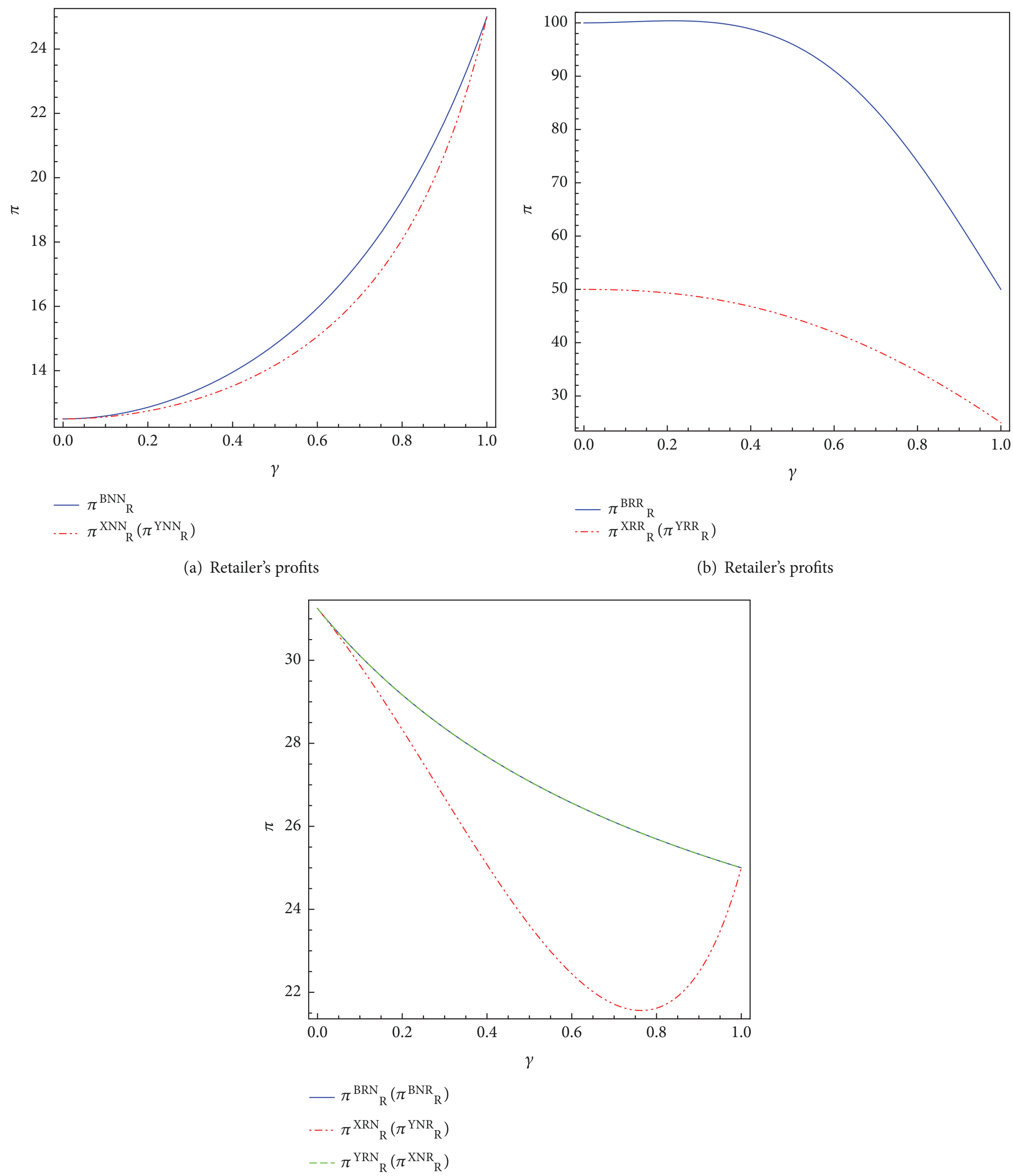
(a) Manufacturers' profits (b) Manufacturers' profits

(c) Manufacturers' profits (d) Manufacturers' profits

Figure 3: Equilibrium manufacturers' profits under $L = \{B, X, Y\}$ versus γ $(a = 10, m = 1/2)$.

Proposition 6. *For any* Z, $\pi_X^{BZ}(\pi_Y^{BZ}) \leq \pi_X^{XZ}(\pi_Y^{YZ}) < \pi_X^{YZ}(\pi_Y^{XZ})$, *except for* $Z = \{RN\}(Z = \{NR\})$.

Proposition 6 investigates the impact of leadership strategy on manufacturers' profits. The rank order of these is intuitive according to the analysis above. In most cases of Z, it is widely shared that manufacturers can reap huge fruits when acting as a follower-competitor, and moving simultaneously will induce the lowest profits; that is, Nash game will intensify the competition between two manufacturers to lower the



(a) Retailer's profits

(b) Retailer's profits

(c) Retailer's profits

FIGURE 4: Equilibrium retailer's profits under $L = \{B, X, Y\}$ versus γ $(a = 10, m = 1/2)$.

profits, but Stackelberg game softens it to increase the profits. While under $Z = \{RN\}$ for manufacturer X ($Z = \{NR\}$ for manufacturer Y) as depicted in Figure 3(d), the manufacturer who offers a rebate can seize the initiative to determine the highest wholesale price and the lowest rebate values to make more profits with high level of competition. Consequently, when one manufacturer knows her rival does not offer a rebate, she should take the advantage of competition to move first for favorable profits. In addition, one in particular is that, unlike Nash strategy, the manufacturers will not cease

Table 2: Payoff matrix of rebate decision.

Manufacturer X\Manufacturer Y	$Z_Y = R$	$Z_Y = N$
$Z_X = R$	$\pi_X^{LRR} - F_X, \pi_Y^{LRR} - F_Y$	$\pi_X^{LRN} - F_X, \pi_Y^{LRN}$
$Z_X = N$	$\pi_X^{LNR}, \pi_Y^{LNR} - F_Y$	π_X^{LNN}, π_Y^{LNN}

offering a rebate as the competition becomes more intense under sequential-move Stackelberg strategy (i.e., $\omega_i^{LRN}(\pi_i^{LRN}) < \omega_i^{LRR}(\pi_i^{LRR})$, $L = \{X, Y\}$).

Proposition 7. *For any $Z = \{NN, RN, NR, RR\}$, $\pi_R^{XZ}(\pi_R^{YZ}) \leq \pi_R^{BZ}(\pi_R^{BZ})$.*

Proposition 7 indicates the impact of leadership strategy on retailer's profits. The rank order of them is opposite to manufacturer's. In other words, the retailer could make a larger profit with more intense competition induced by the Nash game between the two competing manufacturers compared with Stackelberg game. More interestingly, the retailer's profit is first decreasing and then increasing as competition becomes more intense depicted in Figure 4(c), when manufacturer i moves first on rebate offered. This can be explained as follows. More intense competition induces the wholesale price (ω_Y^{XRN}) to decrease and the demand (D_Y^{XRN}) to increase sharply, which generates much higher profits from product j than product i ($i, j = X, Y$). What is more, under Stackelberg game, the retailer could take positive actions to intensify the manufacturer competition without concern about either manufacturer ceasing offering a rebate as competition between two manufacturers grows more intense, which is different from Nash game.

From Propositions 6 and 7, and the above discussion, our results show that leadership strategy has a great impact on manufacturers and retailer; particularly, the Bertrand-Nash game could damage the manufacturers' profits compared with the sequential-move Stackelberg game, whereas benefiting the retailer. The insight is obvious that the same leadership strategy could impose opposite influences on manufacturers and retailer, and conflict of interest always exists between manufacturer and retailer.

4. Rebate Decision

To distinguish the impact of leadership strategy on rebate decision, we first study the Bertrand-Nash game scenario ($L = B$); that is, the two competing manufacturers simultaneously decide whether to incur a fixed cost to offer a rebate or not. Without loss of generality, we assume $F_Y \leq F_X$. We use the same approach by Ha et al. [19] to institute a normal game with the manufacturers as game players, and a payoff matrix shown in Table 2. Similarly, we presume that a manufacturer will offer a rebate if she is neutral.

Let $H_1 \equiv \pi_X^{BRN}(\pi_Y^{BNR}) - \pi_i^{BNN}$, $H_2 \equiv \pi_i^{BRR} - \pi_X^{BNR}(\pi_Y^{BRN})$, and $H_3 \equiv \pi_i^{BRR} - \pi_i^{BNN}$, where H_1, H_2, H_3 are the functions of α, m, γ. We can present $0 < H_1 < H_2$ and $0 < H_3 < H_2$ from Proposition 6; particularly, the comparison of H_1 and H_3 is correlated to γ. The following proposition demonstrates the equilibrium rebate structure and it is depicted in Figure 5(a).

Proposition 8. *(1) If $H_1 < F_Y$ and $H_2 < F_X$, $Z = \{N, N\}$ will be the unique equilibrium; (2) if $F_Y < H_1$ and $F_X < H_2$, $Z = \{R, R\}$ will be the unique equilibrium; (3) if $H_1 \leq F_i \leq H_2$, there will be two equilibria, $Z = \{R, R\}$ and $Z = \{N, N\}$. If $H_1 < H_3 < F_i < H_2$, $Z = \{N, N\}$ will be Pareto optimal, and $Z = \{R, R\}$ will be Pareto optimal if $H_1 < F_i < H_3 < H_2$; (4) if $H_2 < F_X$ and $F_Y \leq H_1$, $Z = \{N, R\}$ will be the unique equilibrium; (5) with more intention, $H_3 < H_1$. (a) If $H_3 < H_1 < F_i$, $Z = \{N, N\}$ will be Pareto optimal; (b) if $H_3 < F_i < H_1$, there will be the classical prisoners' dilemma (where $F_Y < F_X$).*

Proposition 8 investigates that neither manufacturers will offer a rebate when their fixed costs are high; both of them will offer a rebate when they incur low fixed costs. Otherwise, only the manufacturer incurring a lower fixed cost will offer rebate. Particularly the unique equilibrium with $H_1 < H_3$ is not absolutely the same as the one with $H_3 < H_1$. That is to say, when the competition is less intense, the unique equilibrium is $Z = \{R, R\}$ with less high fixed costs; otherwise the unique equilibrium is $Z = \{N, N\}$. When the competition is more intense, $Z = \{N, N\}$ is unique optimal with higher fixed costs; otherwise $Z = \{R, R\}$ is unique optimal; however, both of the two manufacturers would be better off with $Z = \{N, N\}$. This is the classical prisoners' dilemma, which indicates that when a manufacturer offers rebate, the competitive manufacturer had to act so; otherwise she will be worse off.

Next, we work on the sequential-move game scenario ($L = \{X, Y\}$); that is, the two competing manufacturers move sequentially to decide whether to offer rebate. Similarly, let $H_{1,i}^L \equiv \pi_i^{LRN} - \pi_i^{LNN}$, $H_{2,i}^L \equiv \pi_i^{LRR} - \pi_i^{LNR}$, and $H_{3,i}^L \equiv \pi_i^{LRR} - \pi_i^{LNN}$, where $H_{1,i}^L, H_{2,i}^L, H_{3,i}^L$ are the functions of α, m, γ. We can show $0 < H_{1,i}^L < H_{3,i}^L < H_{2,i}^L$ from the manufacturers' profits under $L = \{X, Y\}$. The following proposition demonstrates the equilibrium rebate structure under $L = X$ (the results of $L = Y$ are symmetric to $L = X$) and it is depicted in Figure 5(b).

Proposition 9. *(1) If $H_{1,Y}^X < F_Y$ and $H_{2,X}^X < F_X$, $Z = \{N, N\}$ will be the unique equilibrium; (2) if $F_Y < H_{1,Y}^X$ and $F_X < H_{2,X}^X$, $Z = \{R, R\}$ will be the unique equilibrium; (3) if $H_{1,i}^X \leq F_i \leq H_{2,i}^X$, $Z = \{N, N\}$ and $Z = \{R, R\}$ will be the only two equilibria. $Z = \{N, N\}$ will be Pareto optimal, if $H_{1,i}^X < H_{3,i}^X < F_i < H_{2,i}^X$, and $Z = \{R, R\}$ will be Pareto optimal if $H_{1,i}^X < F_i < H_{3,i}^X < H_{2,i}^X$; (4) if $H_{2,X}^X < F_X$ and $F_Y \leq H_{1,Y}^X$, $Z = \{N, R\}$ will be the unique equilibrium (where $F_Y < F_X$).*

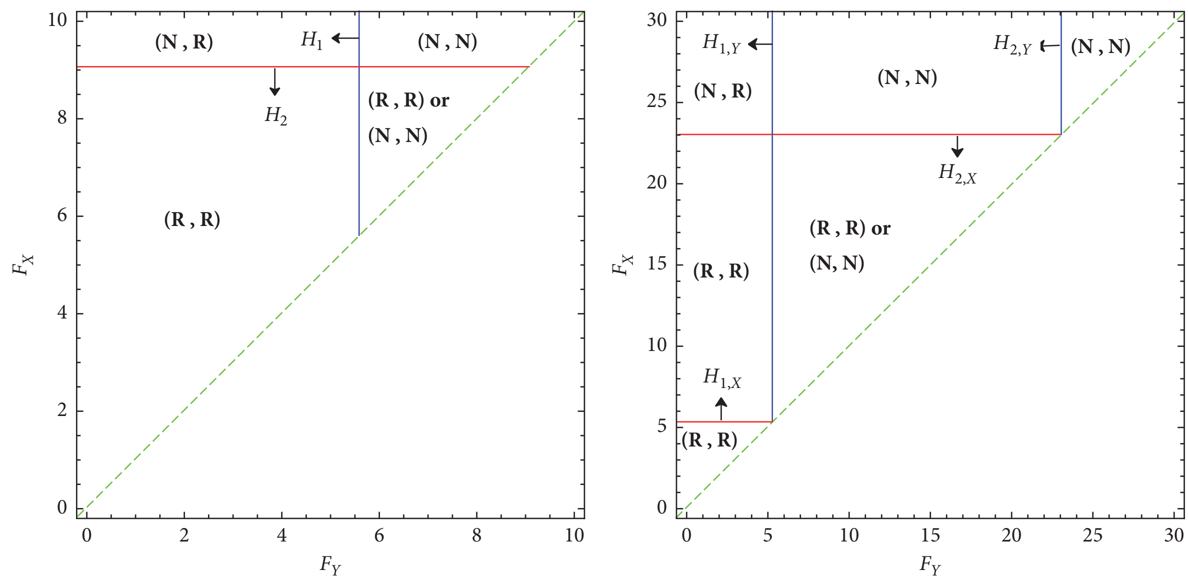
Table 3

Notation	Expression
$g(\gamma)$	$\gamma^4(1+3\gamma)-4\gamma^2(4+5\gamma)+32(1+\gamma)$
$s(m)$	$-8m^2+8m-1$
$v_1(m,\gamma)$	$2m(1-m)(-2+\gamma)^2-1+\gamma$
$v_2(m,\gamma)$	$4m(1-m)(4-\gamma^2)-2+\gamma^2$
$v_3(m,\gamma)$	$m(1-m)(8+4\gamma)-1-\gamma$
$v_4(m,\gamma)$	$4m^2(-6+\gamma+2\gamma^2)-2m(-14+3\gamma+5\gamma^2)-\gamma$
$v_5(m,\gamma)$	$32m^3(m-2)(2+\gamma)^2-8m(2+3\gamma+\gamma^2)+8m^2(18+19\gamma+5\gamma^2)+1+\gamma$
$v_6(m,\gamma)$	$m(m-1)(8-4\gamma^2)-1+\gamma^2$
$v_7(m,\gamma)$	$m(1-m)(32-16\gamma^2)-(\gamma^2-2)^2$
$v_8(m,\gamma)$	$8m^3(\gamma(2-\gamma)+4)-2m^2(\gamma(20-7\gamma)+36)$ $+m(\gamma(13-4\gamma)+22)-(1+\gamma)(5-2\gamma)$
$v_9(m,\gamma)$	$4m^3(12-\gamma(2-(7-\gamma)\gamma))-2m^2(52-\gamma(8-(31-3\gamma)\gamma))$ $+m(2-\gamma(9-(39-4\gamma)\gamma))+(1+\gamma)(2\gamma(4-\gamma)-7)$
$v_{10}(m,\gamma)$	$4m(1-m)(2+\gamma)(2-\gamma^2)-(1+\gamma)(2-\gamma)$
$v_{11}(m,\gamma)$	$4m(1-m)(4+(\gamma-2)\gamma)-(1+\gamma)(2-\gamma)$
$v_{12}(m,\gamma)$	$m(1-m)(8+8\gamma+2\gamma^2)-1-\gamma$
$v_{13}(m,\gamma)$	$4m^5(-1+\gamma)(-4-2\gamma+\gamma^2)^2-12m^4(-1+\gamma)(-4-2\gamma+\gamma^2)^2+2m^3(-104+133\gamma^2-35\gamma^4+6\gamma^5)$ $-4m^2(-24+33\gamma^2-10\gamma^4+\gamma^5)+m(-17+28\gamma^2-11\gamma^4)+(-1+\gamma^2)^2$
$v_{14}(m,\gamma)$	$8m^4(32+32\gamma-16\gamma^2-20\gamma^3+\gamma^4+3\gamma^5)-16m^3(32+32\gamma-16\gamma^2-20\gamma^3+\gamma^4+3\gamma^5)$ $+4m^2(80+80\gamma-42\gamma^2-49\gamma^3+4\gamma^4+7\gamma^5)-4m(16+16\gamma-10\gamma^2-9\gamma^3+2\gamma^4+\gamma^5)+(2+\gamma-\gamma^2)^2$
$v_{15}(m,\gamma)$	$4m(1-m)(4+2\gamma-\gamma^2)-2-2\gamma+\gamma^2+\gamma^3$
$v_{16}(m,\gamma)$	$8m^2(-6+\gamma+3\gamma^2)+2m(28-6\gamma-16\gamma^2+\gamma^3+\gamma^4)+\gamma(-2+\gamma^2)$
$v_{17}(m,\gamma)$	$4m(1-m)(-12+2\gamma+7\gamma^2-\gamma^3)+6-7\gamma^2+2\gamma^4$
$v_{18}(m,\gamma)$	$16m^4(32+32\gamma-16\gamma^2-20\gamma^3+\gamma^4+3\gamma^5)-32m^3(32+32\gamma-16\gamma^2-20\gamma^3+\gamma^4+3\gamma^5)$ $+8m^2(72+76\gamma-34\gamma^2-50\gamma^3-\gamma^4+8\gamma^5+\gamma^6)-8m(8+12\gamma-2\gamma^2-10\gamma^3-3\gamma^4+2\gamma^5+\gamma^6)+(1+\gamma)(-2+\gamma^2)^2$
$v_{19}(m,\gamma)$	$4m(1-m)(4+2\gamma-\gamma^2)-2-\gamma+\gamma^2$
$v_{20}(m,\gamma)$	$16m^4(12-2\gamma-7\gamma^2+\gamma^3)-8m^3(52-10\gamma-31\gamma^2+5\gamma^3)$ $+4m^2(62-11\gamma-39\gamma^2+6\gamma^3)-2m(14+\gamma-12\gamma^2+\gamma^3)+\gamma(1-\gamma^2)$
$v_{21}(m,\gamma)$	$m^4(16+8\gamma-4\gamma^2)+m^3(-32-16\gamma+8\gamma^2)+m(-2-\gamma+\gamma^2)+(18+9\gamma-5\gamma^2)$
$v_{22}(m,\gamma)$	$256m^8(32+32\gamma-16\gamma^2-20\gamma^3+\gamma^4+3\gamma^5)-1024m^7(32+32\gamma-16\gamma^2-20\gamma^3+\gamma^4+3\gamma^5)$ $+128m^6(408+412\gamma-210\gamma^2-265\gamma^3+14\gamma^4+41\gamma^5)-128m^5(328+340\gamma-182\gamma^2-235\gamma^3+14\gamma^4+39\gamma^5)$ $+32m^4(558+610\gamma-359\gamma^2-482\gamma^3+35\gamma^4+90\gamma^5)-64m^3(62+74\gamma-51\gamma^2-72\gamma^3+7\gamma^4+16\gamma^5)$ $+8m^2(1+\gamma)^2(60-43\gamma-40\gamma^2+27\gamma^3)-8m(1+\gamma)^3(4-7\gamma+3\gamma^2)+(-1+\gamma)^2(1+\gamma)^3$

Proposition 9 develops that the equilibrium rebate structure under sequential-move Stackelberg game scenario is approximately the same as that under Bertrand-Nash game scenario, except for the absence of classical prisoners' dilemma under $L=\{X,Y\}$. The comparison of $H_{1,i}^L$ and $H_{3,i}^L$ is not referred to how intense the competition is between the two manufacturers; that is, $H_{1,i}^L < H_{3,i}^L$ for all values of γ; and both manufacturers will not be in the prisoners' dilemma with intense competition under sequential-move Stackelberg strategy.

Corollary 10. *Suppose $F_X = F_Y = F$. (1) When $F < H_{1,i}^L$, $Z=\{R,R\}$ is the unique equilibrium; (2) when $H_{1,i}^L \le F \le H_{2,i}^L$, $Z=\{N,N\}$ and $Z=\{R,R\}$ are the only two equilibria, and if $H_{3,i}^L < F$, $Z=\{N,N\}$ is Pareto optimal; otherwise $Z=\{R,R\}$ is Pareto optimal; (3) when $H_{2,i}^L < F$, $Z=\{N,N\}$ is the unique equilibrium.*

However, compared with Figure 5(a), Figure 5(b) shows great differences between different leadership strategies. In Figure 5(a), the largest region is $Z=\{R,R\}$ and $Z=\{N,N\}$ has the smallest region under $L=B$, which is consistent with Proposition 6. By contrast, the definite region of $Z=\{R,R\}$ under $L=X$ consequently decreases as the definite region of $Z=\{N,N\}$ and the indefinite region greatly increase, which does not accord with the conclusion that manufacturers would make the most profits under $Z=\{R,R\}$. Both of the two competing manufacturers could be less likely to decide to



(a) L=B

(b) L=X

FIGURE 5: Equilibrium rebate decision under $L = B$ and $L = X$ ($a = 10, m = 1/2, \gamma = 1/2$).

offer a rebate when they play the sequential-move Stackelberg game; to a great extent, manufacturers' profits depend on their fixed cost, whereas the manufacturers make more profit with the Nash strategy allowing them to compensate the fixed cost.

5. Conclusion

In this research, we develop leadership strategy in manufacturer rebate competition with heterogeneous consumers. Compared with previous research, we take the redemption rate into the consumer utility functions, which is more realistic. We first study the prices and rebate values decision under different leadership strategies, and next we focus on the profits for manufacturers and retailer, and the impact of leadership strategy on them. Then we investigate the manufacturer rebate decision under these leadership strategies and their effect on it.

Our analysis reveals some initial and counterintuitive results. The manufacturers will not phase out their rebate programs as the competition between them becomes intense under sequential-move Stackelberg strategy, compared with Bertrand-Nash strategy. We also observe that although sequential-move Stackelberg strategy could eliminate the classic prisoners' dilemma in manufacturer rebate decision, which is also influenced by fixed cost control, as a result, manufacturers could gain more under sequential-move Stackelberg strategy if they well control the fixed costs.

Our findings present some interesting managerial insights to practitioners. With rebate competition, except for the scenario of one manufacturer offering a rebate, the manufacturers could be better off as a price follower rather than a price leader. Compared with Bertrand-Nash strategy, the retailer could take more positive actions to intensify the competition between manufacturers without being worse off, because of manufacturers' continuously offering rebates in such a condition under sequential-move Stackelberg strategy.

Our model has a few limitations, such as zero redemption cost for the rebate-sensitive consumers and a constant redemption rate for consumers and manufacturers. These values can be nonzero or nonconstant and subject to a certain distribution. In addition, our paper just considers the leadership strategy between the two competing manufacturers, but does not take the price leadership between manufacturers and retailer into consideration with a symmetric or asymmetric relationship, as well as the scenario that the retailer could offer a rebate or both of the manufacturers and retailer offer a rebate cooperatively. We leave these issues for future investigation.

Appendix

A. Proof A

(1) For $Z = \{NN, RN, NR, RR\}$, and with demand functions for consumers (3), (4), (6), and (7), we can derive the profit function of manufacturers

$$\begin{aligned}\pi_i^{LZ} &= \frac{1}{2}\omega_i^{LZ} \\ &\quad * \frac{(1-\gamma)a - p_i^{LZ} + m * r_i^{LZ} + \gamma\left(p_j^{LZ} - m * r_j^{LZ}\right)}{1-\gamma^2} \\ &\quad + \frac{1}{2}\left(\omega_i^{LZ} - m r_i^{LZ}\right) \\ &\quad * \frac{(1-\gamma)a - p_i^{LZ} + r_i^{LZ} + \gamma\left(p_j^{LZ} - r_j^{LZ}\right)}{1-\gamma^2}, \\ &\qquad i, j = X, Y.\end{aligned} \tag{A.1}$$

(1) If $L = B$, the Hessian matrix is negative due to $\partial^2\pi_i^{BZ}/\partial\omega_i^{BZ^2} = -1/(1-\gamma^2) < 0$, $\partial^2\pi_i^{BZ}/\partial r_i^{BZ^2} = -(3-m)m/4(1-\gamma^2) < 0$ and $(\partial^2\pi_i^{BZ}/\partial\omega_i^{BZ^2})(\partial^2\pi_i^{BZ}/\partial r_i^{BZ^2}) - (\partial^2\pi_i^{BZ}/\partial\omega_i^{BZ}\partial r_i^{BZ})(\partial^2\pi_i^{BZ}/\partial r_i^{BZ}\partial\omega_i^{BZ}) = (-8m^2+8m-1)/16(1-\gamma^2) > 0$ for any $m \in ((2-\sqrt{2})/4, (2+\sqrt{2})/4)$ and $\gamma \in [0,1)$. Thus the profit of manufacturer i satisfies the second-order condition for a maximum. We conclude that there exists a unique optimal pair $(\omega_i^{BZ}, r_i^{BZ})$, which is given by resolving $\partial\pi_i^{BZ}/\partial\omega_i^{BZ} = 0$ and $\partial\pi_i^{BZ}/\partial r_i^{BZ} = 0$ simultaneously (if rebates are offered). We obtain

$$\begin{aligned}
\omega_i^{BNN} &= \frac{(1-\gamma)a}{2-\gamma},\\
\omega_X^{BRN} &= \frac{am(5-4m)(1-\gamma)(2+\gamma)}{v_2(m,\gamma)},\\
r_X^{BRN} &= \frac{2a(1-\gamma)(2+\gamma)}{v_2(m,\gamma)};\\
\omega_Y^{BRN} &= \frac{av_3(m,\gamma)(1-\gamma)}{v_2(m,\gamma)},\\
\omega_X^{BNR} &= \frac{av_3(m,\gamma)(1-\gamma)}{v_2(m,\gamma)};\\
\omega_Y^{BNR} &= \frac{am(5-4m)(1-\gamma)(2+\gamma)}{v_2(m,\gamma)},\\
r_Y^{BNR} &= \frac{2a(1-\gamma)(2+\gamma)}{v_2(m,\gamma)},\\
\omega_i^{BRR} &= \frac{am(5+2m(2-\gamma)-2\gamma)(1-\gamma)}{v_1(m,\gamma)},\\
r_i^{BRR} &= \frac{2a(1-\gamma)}{v_1(m,\gamma)}.
\end{aligned} \tag{A.2}$$

(2) If $L = X$, for given $(\omega_X^{XZ}, r_X^{XZ})$, manufacturer Y's profit function is concave in ω_Y^{XZ} and r_Y^{XZ} as $\partial^2\pi_Y^{XZ}/\partial\omega_Y^{XZ^2} = -1/(1-\gamma^2) < 0$ and $\partial^2\pi_Y^{XZ}/\partial r_Y^{XZ^2} = -(3-m)m/4(1-\gamma^2) < 0$, for which the Hessian matrix is negative for any $m \in ((2-\sqrt{2})/4, (2+\sqrt{2})/4)$ and $\gamma \in [0,1)$, suggesting that there exists a unique optimal solution $(\omega_Y^{XZ}, r_Y^{XZ})$, which can be obtained by solving $\partial\pi_Y^{XZ}/\partial\omega_Y^{XZ} = 0$ and $\partial\pi_Y^{XZ}/\partial r_Y^{XZ} = 0$ simultaneously (if rebates are offered). Then,

$$\begin{aligned}
\omega_Y^{XNN} &= \frac{1}{2}\left(a - a\gamma + \gamma\omega_X^{XNN}\right),\\
\omega_Y^{XRN} &= \frac{1}{4}\left(2a - 2a\gamma - \gamma r_X^{XRN} - m\gamma r_X^{XRN} + 2\gamma\omega_X^{XRN}\right),\\
\omega_Y^{XNR} &= -\frac{\left(-5m+4m^2\right)\left(-a+a\gamma-\gamma\omega_X^{XNR}\right)}{1-8m+8m^2},\\
r_Y^{XNR} &= \frac{2\left(-a+a\gamma-\gamma\omega_X^{XNR}\right)}{1-8m+8m^2},\\
\omega_Y^{XRR} &= -\frac{m\left(10a-8am-10a\gamma+8am\gamma-3\gamma r_X^{XRR}+m\gamma r_X^{XRR}+10\gamma\omega_X^{XRR}-8m\gamma\omega_X^{XRR}\right)}{2\left(1-8m+8m^2\right)},\\
r_Y^{XRR} &= -\frac{2a-2a\gamma-\gamma r_X^{XRR}+3m\gamma r_X^{XRR}-4m^2\gamma r_X^{XRR}+2\gamma\omega_X^{XRR}}{1-8m+8m^2}
\end{aligned} \tag{A.3}$$

Substituting (A.3) into π_X^{XZ}, we can prove that the Hessian matrix is negative due to

$$\begin{aligned}
\frac{\partial^2\pi_X^{XZ_XN}}{\partial\omega_X^{XZ_XN^2}} &= -\frac{2-\gamma^2}{2(1-\gamma^2)} < 0,\\
\frac{\partial^2\pi_X^{XZ_XN}}{\partial r_X^{XZ_XN^2}} &= -\frac{m\left(6-\gamma^2-m\left(2+\gamma^2\right)\right)}{8(1-\gamma^2)} < 0,\\
\frac{\partial^2\pi_X^{XZ_XN}}{\partial\omega_X^{XZ_XN^2}}\frac{\partial^2\pi_X^{XZ_XN}}{\partial r_X^{XZ_XN^2}}
&- \frac{\partial^2\pi_X^{XZ_XN}}{\partial\omega_X^{XZ_XN}\partial r_X^{XZ_XN}}\frac{\partial^2\pi_X^{XZ_XN}}{\partial r_X^{XZ_XN}\partial\omega_X^{XZ_XN}}\\
&= \frac{\left(-2+16m-16m^2+\gamma^2\right)\left(2-\gamma^2\right)}{64\left(-1+\gamma^2\right)^2} > 0
\end{aligned} \tag{A.4}$$

for any $m \in ((2-\sqrt{2})/4, (2+\sqrt{2})/4)$ and $\gamma \in [0,1)$,

$$\begin{aligned}
&\text{and } \frac{\partial^2 \pi_X^{XZ_X R}}{\partial \omega_X^{XZ_X R^2}} = \frac{1-\gamma^2+4m\left(-2+\gamma^2\right)-4m^2\left(-2+\gamma^2\right)}{\left(1-8m+8m^2\right)\left(-1+\gamma^2\right)} < 0, \\
&\frac{\partial^2 \pi_X^{XZ_X R}}{\partial r_X^{XZ_X R^2}} = \frac{(-3+m)\,m\left(-1+\gamma^2-4m\left(-2+\gamma^2\right)+4m^2\left(-2+\gamma^2\right)\right)}{4\left(1-8m+8m^2\right)\left(-1+\gamma^2\right)} < 0, \\
&\frac{\partial^2 \pi_X^{XZ_X R}}{\partial \omega_X^{XZ_X R^2}} \frac{\partial^2 \pi_X^{XZ_X R}}{\partial r_X^{XZ_X R^2}} - \frac{\partial^2 \pi_X^{XZ_X R}}{\partial \omega_X^{XZ_X R} \partial r_X^{XZ_X R}} \frac{\partial^2 \pi_X^{XZ_X R}}{\partial r_X^{XZ_X R} \partial \omega_X^{XZ_X R}} \\
&\quad = \frac{\left(15-4m-4m^2\right)\left(-1+\gamma^2-4m\left(-2+\gamma^2\right)+4m^2\left(-2+\gamma^2\right)\right)^2}{16\left(1-8m+8m^2\right)^2\left(-1+\gamma^2\right)^2} > 0
\end{aligned} \tag{A.5}$$

for any $m \in ((2-\sqrt{2})/4, (2+\sqrt{2})/4)$ and $\gamma \in [0,1)$, suggesting that there exists a unique optimal solution $(\omega_X^{XZ}, r_X^{XZ})$, which can be obtained by solving $\partial \pi_X^{XZ}/\partial \omega_X^{XZ} = 0$ and $\partial \pi_X^{XZ}/\partial r_X^{XZ} = 0$ simultaneously.

$$\begin{aligned}
\omega_X^{XNN} &= \frac{a(1-\gamma)(2+\gamma)}{2\left(2-\gamma^2\right)}, \\
\omega_X^{XRN} &= \frac{am\left(-10+8m+\gamma^2\right)\left(-2+\gamma+\gamma^2\right)}{v_7(m,\gamma)}, \\
r_X^{XRN} &= \frac{2a\left(-2+\gamma+\gamma^2\right)\left(\gamma^2-2\right)}{v_7(m,\gamma)}, \\
\omega_X^{XNR} &= \frac{a(1-\gamma)\,v_3(m,\gamma)}{2v_6(m,\gamma)}, \\
\omega_X^{XRR} &= \frac{am\left(5-2m(2+\gamma)+3\gamma\right)(1-\gamma)}{v_6(m,\gamma)}, \\
r_X^{XRR} &= \frac{2a(1+\gamma)(1-\gamma)}{v_6(m,\gamma)}.
\end{aligned} \tag{A.6}$$

With (A.6) and (A.3), we can obtain

$$\begin{aligned}
\omega_Y^{XNN} &= \frac{a(1-\gamma)\left(4+2\gamma-\gamma^2\right)}{4\left(2-\gamma^2\right)}, \\
\omega_Y^{XRN} &= \frac{a(1-\gamma)\,v_{15}(m,\gamma)}{v_7(m,\gamma)}, \\
\omega_Y^{XNR} &= \frac{am(5-4m)(1-\gamma)\,v_{19}(m,\gamma)}{2v_6(m,\gamma)\,s(m)}, \\
r_Y^{XRN} &= \frac{a(1-\gamma)\,v_{19}(m,\gamma)}{v_6(m,\gamma)\,s(m)}, \\
\omega_Y^{XRR} &= \frac{am(1-\gamma)\,v_8(m,\gamma)}{v_6(m,\gamma)\,s(m)}, \\
r_Y^{XRR} &= \frac{2a(1-\gamma)\left[2m(2+\gamma)^2(1-m)-(1+\gamma)\right]}{v_6(m,\gamma)\,s(m)}.
\end{aligned} \tag{A.7}$$

(3) If $L = Y$, because of the symmetry, for given $(\omega_Y^{YZ}, r_Y^{YZ})$, we can prove manufacturer X's profit function is concave in ω_X^{YZ} and r_X^{YZ} similarly and the Hessian matrix is negative, suggesting that there exists a unique optimal solution $(\omega_X^{YZ}, r_X^{YZ})$, which can be obtained by solving $\partial \pi_X^{YZ}/\partial \omega_X^{YZ} = 0$ and $\partial \pi_X^{YZ}/\partial r_X^{YZ} = 0$ simultaneously (if rebates are offered). Then,

$$\begin{aligned}
\omega_X^{YNN} &= \frac{1}{2}\left(a-a\gamma+\gamma\omega_Y^{YNN}\right), \\
\omega_X^{YRN} &= -\frac{\left(-5m+4m^2\right)\left(-a+a\gamma-\gamma\omega_Y^{YRN}\right)}{1-8m+8m^2}, \\
r_X^{YRN} &= \frac{2\left(-a+a\gamma-\gamma\omega_X^{YRN}\right)}{1-8m+8m^2}, \\
\omega_X^{YNR} &= \frac{1}{4}\left(2a-2a\gamma-\gamma r_Y^{YNR}-m\gamma r_Y^{YNR}+2\gamma\omega_Y^{YNR}\right), \\
\omega_X^{YRR} &= -\frac{m\left(10a-8am-10a\gamma+8am\gamma-3\gamma r_Y^{YRR}+m\gamma r_Y^{YRR}+10\gamma\omega_Y^{YRR}-8m\gamma\omega_Y^{YRR}\right)}{2\left(1-8m+8m^2\right)}, \\
r_X^{YRR} &= -\frac{2a-2a\gamma-\gamma r_Y^{YRR}+3m\gamma r_Y^{YRR}-4m^2\gamma r_Y^{YRR}+2\gamma\omega_Y^{YRR}}{1-8m+8m^2}
\end{aligned} \tag{A.8}$$

Substituting (A.3) into π_Y^{YZ}, we can prove that π_Y^{YZ} is concave in $(\omega_Y^{YZ}, r_Y^{YZ})$, suggesting that there exists a unique optimal solution $(\omega_Y^{YZ}, r_Y^{YZ})$, which can be obtained by solving $\partial\pi_Y^{YZ}/\partial\omega_Y^{YZ} = 0$ and $\partial\pi_Y^{YZ}/\partial r_Y^{YZ} = 0$ simultaneously.

$$\begin{aligned}
\omega_Y^{YNN} &= \frac{a(1-\gamma)(2+\gamma)}{2(2-\gamma^2)},\\
\omega_Y^{YRN} &= \frac{a(1-\gamma)v_3(m,\gamma)}{2v_6(m,\gamma)},\\
\omega_Y^{YNR} &= \frac{am\left(-10+8m+\gamma^2\right)\left(-2+\gamma+\gamma^2\right)}{v_7(m,\gamma)},\\
r_Y^{YNR} &= \frac{2a\left(-2+\gamma+\gamma^2\right)\left(\gamma^2-2\right)}{v_7(m,\gamma)},\\
\omega_Y^{YRR} &= \frac{am(5-2m(2+\gamma)+3\gamma)(1-\gamma)}{v_6(m,\gamma)},\\
r_X^{XRR} &= \frac{2a(1+\gamma)(1-\gamma)}{v_6(m,\gamma)}.
\end{aligned} \tag{A.9}$$

With (A.8) and (A.9), we can obtain

$$\begin{aligned}
\omega_X^{YNN} &= \frac{a(1-\gamma)\left(4+2\gamma-\gamma^2\right)}{4(2-\gamma^2)},\\
\omega_X^{YRN} &= \frac{am(5-4m)(1-\gamma)v_{19}(m,\gamma)}{2v_6(m,\gamma)s(m)},\\
r_X^{YRN} &= \frac{a(1-\gamma)v_{19}(m,\gamma)}{v_6(m,\gamma)s(m)},\\
\omega_X^{YNR} &= \frac{a(1-\gamma)v_{15}(m,\gamma)}{v_7(m,\gamma)},\\
\omega_X^{YRR} &= \frac{am(1-\gamma)v_8(m,\gamma)}{v_6(m,\gamma)s(m)},\\
r_X^{YRR} &= \frac{2a(1-\gamma)\left[2m(2+\gamma)^2(1-m)-(1+\gamma)\right]}{v_6(m,\gamma)s(m)}.
\end{aligned} \tag{A.10}$$

B. Proof B

Proof of Lemma 1. With wholesale prices and profits of the two manufacturers shown in Table 1(a), for $L = B$ and $a > 0$, $m \in ((2-\sqrt{2})/4, (2+\sqrt{2})/4)$, $\gamma \in [0,1)$,

$$\begin{aligned}
&\omega_i^{BRR} - \omega_i^{BNN} = \frac{a(1-\gamma)(1-\gamma+m(2-\gamma))}{\left(\gamma-1+2m(-2+\gamma)^2(1-m)\right)(2-\gamma)} > 0,\\
&\omega_X^{BRN} - \omega_i^{BNN} = \frac{a(1-\gamma)\left(2-\gamma^2+m\left(4-\gamma^2\right)\right)}{(2-\gamma)(-2+\gamma^2+4m(4-\gamma^2)(1-m))} > 0,\\
&\omega_i^{BNN} - \omega_X^{BNR} = \frac{a(1-\gamma)\gamma}{(2-\gamma)(-2+\gamma^2+4m(4-\gamma^2)(1-m))} > 0;\\
&\pi_i^{BRR} - \pi_i^{BNN} = \frac{a^2(-1+\gamma)^2}{2\left(\gamma-1+2m(-2+\gamma)^2(1-m)\right)(-2+\gamma)^2(1+\gamma)} > 0,\\
&\pi_X^{BRN} - \pi_i^{BNN} = \frac{a^2(1-\gamma)}{2(1+\gamma)}\left(\frac{2(1-m)m(8(1-m)m-1)(2+\gamma)^2}{\left(-2+\gamma^2+4(-1+m)m(-4+\gamma^2)\right)^2} - \frac{1}{(-2+\gamma)^2}\right) > 0,\\
&\pi_i^{BNN} - \pi_X^{BNR} = \frac{a^2(1-\gamma)\gamma\left(-4-\gamma+2\gamma^2+8m\left(4-\gamma^2\right)(1-m)\right)}{2(-2+\gamma)^2(1+\gamma)(-2+\gamma^2+4m(4-\gamma^2)(1-m))^2} > 0.
\end{aligned} \tag{B.1}$$

□

Proof of Lemma 2. With rebate values of manufacturers, retail prices, and demands of consumers shown in Table 1(a), for $L = B$ and $a > 0$, $m \in ((2-\sqrt{2})/4, (2+\sqrt{2})/4)$, $\gamma \in [0,1)$,

$$r_i^{BRR} - r_X^{BRN} = 2a(1-\gamma)\left(\frac{1}{\gamma-1+2m(-2+\gamma)^2(1-m)} - \frac{2+\gamma}{-2+\gamma^2+4m(4-\gamma^2)(1-m)}\right) > 0;$$

$$p_i^{BRR} - p_X^{BRN} = \frac{a(1-\gamma)\gamma\left(-1+8m^2+m\left(6-4\gamma^2\right)+4m^3\left(-4+\gamma^2\right)\right)}{2\left(\gamma-1+2m(-2+\gamma)^2(1-m)\right)(-2+\gamma^2+4m(4-\gamma^2)(1-m))} > 0,$$

$$\text{where } \gamma < \sqrt{\frac{1-6m-8m^2+16m^3}{-4m+4m^3}},$$

$$p_X^{BRN} - p_i^{BNN} = \frac{a(1-\gamma)\left(3-\gamma^2+m\left(4-\gamma^2\right)\right)}{(2-\gamma)\left(-2+\gamma^2+4m\left(4-\gamma^2\right)(1-m)\right)} > 0,$$

$$p_i^{BNN} - p_X^{BNR} = \frac{a(1-\gamma)\gamma}{2(2-\gamma)\left(-2+\gamma^2+4m\left(4-\gamma^2\right)(1-m)\right)} > 0;$$

$$D_X^{BRN} - D_i^{BRR} = \frac{a(1-m)m\gamma^3}{\left(\gamma-1+2m(-2+\gamma)^2(1-m)\right)(1+\gamma)\left(-2+\gamma^2+4m\left(4-\gamma^2\right)(1-m)\right)} > 0,$$

$$D_i^{BRR} - D_i^{BNN} = \frac{a(1-\gamma)}{2\left(\gamma-1+2m(-2+\gamma)^2(1-m)\right)(2-\gamma)(1+\gamma)} > 0,$$

$$D_i^{BNN} - D_X^{BNR} = \frac{a\gamma}{2(2-\gamma)(1+\gamma)\left(-2+\gamma^2+4m\left(4-\gamma^2\right)(1-m)\right)} > 0. \tag{B.2}$$

□

Proof of Lemma 3. With retail prices and demands of consumers shown in Table 1(a), for $L = B$ and $a > 0$, $m \in ((2-\sqrt{2})/4, (2+\sqrt{2})/4)$, $\gamma \in [0,1)$,

$$\pi_R^{BRR} - \pi_R^{BRN} = -\frac{25\left(-44+36\gamma+4\gamma^3-7\gamma^4+3\gamma^5\right)}{4(1+\gamma)\left(2-2\gamma+\gamma^2\right)^2} > 0, \tag{B.3}$$

$$\pi_R^{BRN} - \pi_R^{BNN} = \frac{25\left(12-8\gamma-7\gamma^2+3\gamma^3\right)}{4(-2+\gamma)^2(1+\gamma)} > 0$$

□

Proof of Lemmas 4 (part(1) and part(2)) and 5. With wholesale prices and rebate values of the two manufacturers shown in Table 1, for $L = \{B, X, Y\}$, the comparisons of them, respectively, are depicted in Figures 6 and 7.

Although some results of the above are obtained by assuming $a = 10$, $m = 1/2$ (identical to Ha et al. [19] for better comparison), one can easily find from the equilibrium results that the results are not affected by different a and m values. □

The rank orders of prices and rebate values decision, as well as demands and profits under sequential-move Stackelberg game:

For $L = \{X, Y\}$,

(1) Wholesale price:

$\omega_X^{LNR}(\omega_Y^{LRN}) < \omega_X^{LNN}(\omega_Y^{LNN}) < \omega_X^{LRN}(\omega_Y^{LNR}) < \omega_X^{LRR}(\omega_Y^{LRR})$.

(2) Rebate value:

$r_X^{LRN}(r_Y^{LNR}) < r_X^{LRR}(r_Y^{LRR})$.

(3) Retail price:

$p_X^{LNR}(p_Y^{LRN}) < p_X^{LNN}(p_Y^{LNN}) < p_X^{LRN}(p_Y^{LNR}) < p_X^{LRR}(p_Y^{LRR})$.

(4) Demand:

(1) $D_X^{XNR}(D_Y^{YRN}) < D_X^{XRR}(p_Y^{YRR}) < D_X^{XRN}(D_Y^{YNR})$;

(2) $D_X^{XNR}(D_Y^{YRN}) < D_X^{XNN}(D_Y^{YNN}) < D_X^{XRN}(D_Y^{YNR})$;

(3) $D_X^{YNR}(D_Y^{XRN}) < D_X^{YNN}(D_Y^{XNN}) < D_X^{YRR}(D_Y^{XRR}) = D_X^{YRN}(D_Y^{XNR})$.

(5) Profit:

(1) $\pi_X^{XNR}(\pi_Y^{YRN}) < \pi_X^{XNN}(\pi_Y^{YNN}) < \pi_X^{XRN}(\pi_Y^{YNR}) < \pi_X^{XRR}(\pi_Y^{YRR})$

(2) $\pi_X^{YNR}(\pi_Y^{XRN}) < \pi_X^{YRN}(\pi_Y^{XNR}) < \pi_X^{YRR}(\pi_Y^{XRR})$;

(3) $\pi_X^{YNR}(\pi_Y^{XRN}) < \pi_X^{YNN}(\pi_Y^{XNN}) < \pi_X^{YRR}(\pi_Y^{XRR})$;

(4) $\pi_R^{XNN}(\pi_R^{YNN}) < \pi_R^{XRN}(\pi_R^{YNR}) < \pi_R^{XNR}(\pi_R^{YRN}) < \pi_R^{XRR}(\pi_R^{YRR})$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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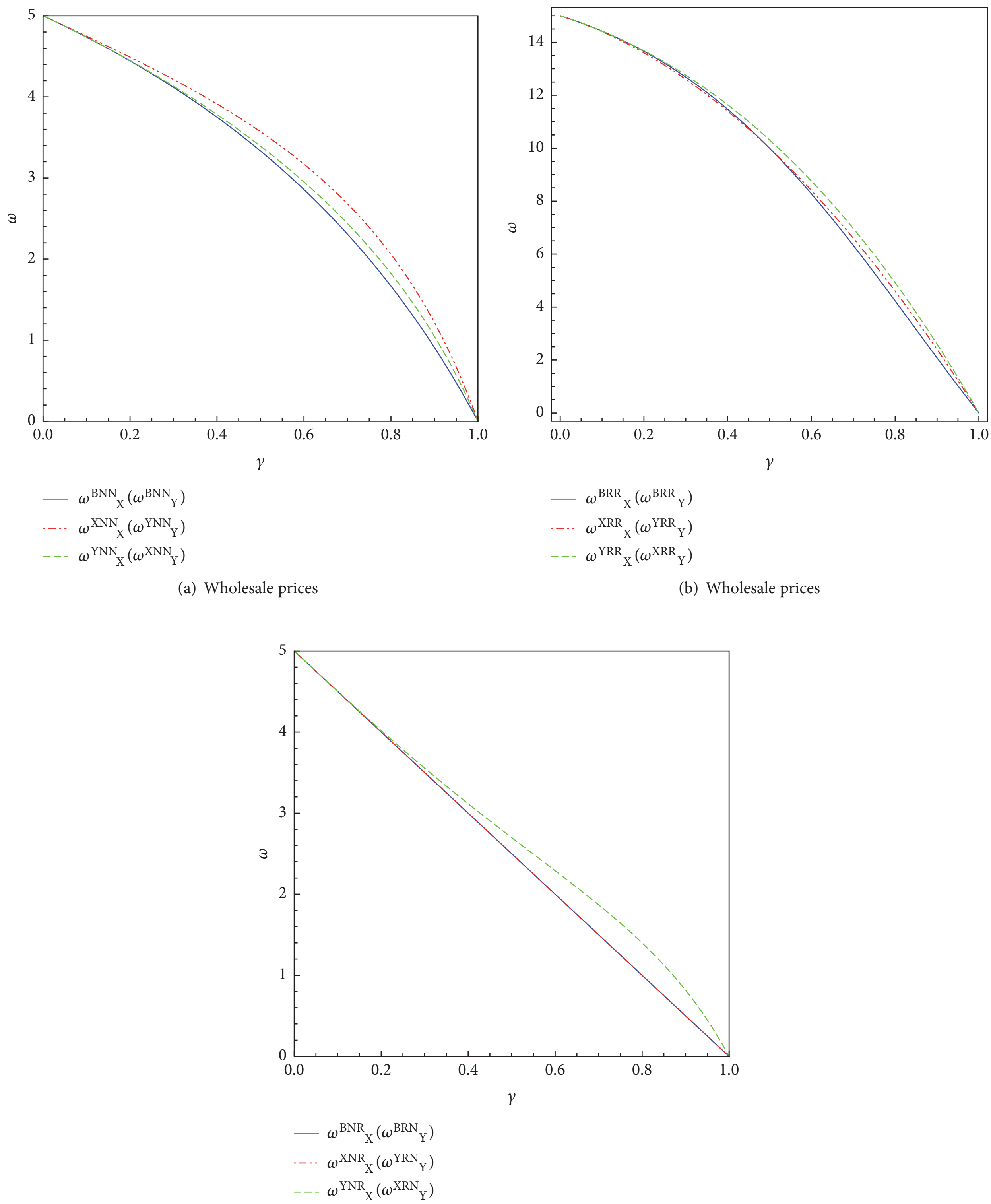
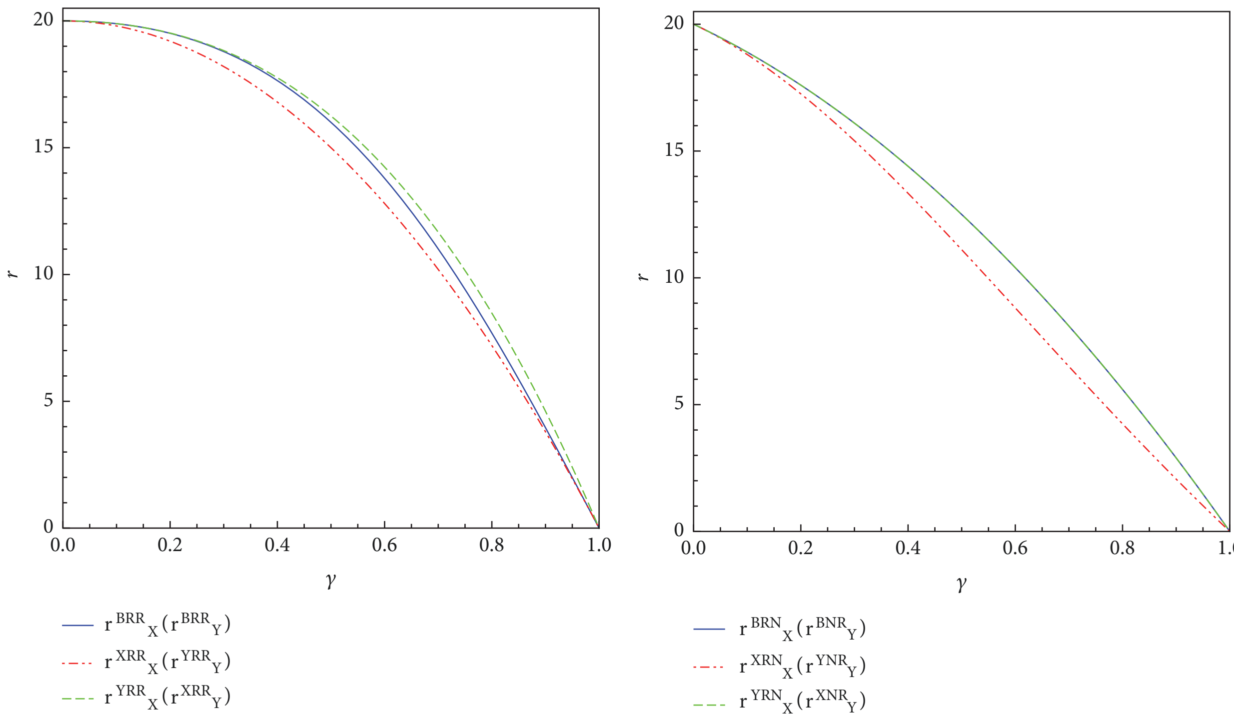


(a) Wholesale prices

(b) Wholesale prices

(c) Wholesale prices

Figure 6: Comparison of wholesale prices under $L = \{B, X, Y\}$ versus γ $(a = 10, m = 1/2)$.



(a) Rebate values

(b) Rebate values

FIGURE 7: Comparison of rebate values under $L = \{B, X, Y\}$ versus γ $(a = 10, m = 1/2)$.

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References

[1] E. Gerstner and J. D. Hess, "Pull promotions and channel coordination," *Marketing Science*, vol. 14, no. 1, pp. 43–60, 1995.

[2] D. Xing and T. Liu, "Sales effort free riding and coordination with price match and channel rebate," *European Journal of Operational Research*, vol. 219, no. 2, pp. 264–271, 2012.

[3] PR Newswire, *One in every two retailers using rebates to drive customer responsiveness*, 2011a.

[4] Parago, *Let's make a deal: 2014 UK shopper behavior study*, 2014, http://www.slideshare.net/Parago/uk-shopperweb.

[5] D. Darlin, *Rebates on the way to expiring*, Rebates on the way to expiring, New York Times, 2006.

[6] G. Aydin and E. L. Porteus, "Manufacturer-to-retailer versus manufacturer-to-consumer rebates in a supply chain," in *Retail Supply Chain Management: Quantitative Models and Empirical Studies*, N. Agrawal and S. A. Smith, Eds., pp. 237–270, Springer, Boston, MA, 2009.

[7] O. C. Demirag, O. Baysar, P. Keskinocak, and J. L. Swann, "The effects of customer rebates and retailer incentives on a manufacturer's profits and sales," *Naval Research Logistics (NRL)*, vol. 57, no. 1, pp. 88–108, 2010.

[8] O. C. Demirag, Y. Chen, and J. Li, "Customer and retailer rebates under risk aversion," *International Journal of Production Economics*, vol. 133, no. 2, pp. 736–750, 2011.

[9] O. C. Demirag, P. Keskinocak, and J. Swann, "Customer rebates and retailer incentives in the presence of competition and price discrimination," *European Journal of Operational Research*, vol. 215, no. 1, pp. 268–280, 2011.

[10] Q. Geng and S. Mallik, "Joint mail-in rebate decisions in supply chains under demand uncertainty," *Production Engineering Research and Development*, vol. 20, no. 4, pp. 587–602, 2011.

[11] F. J. Arcelus, R. Gor, and G. Srinivasan, "Price, rebate and order quantity decisions in a newsvendor framework with rebate-dependent recapture of lost sales," *International Journal of Production Economics*, vol. 140, no. 1, pp. 473–482, 2012.

[12] E. Gerstner and J. D. Hess, "A theory of channel price promotions," *American Economic Association*, vol. 81, no. 4, pp. 872–886, 1991.

[13] Y. Chen, S. Moorthy, and Z. J. Zhang, "Research note - Price discrimination after the purchase: Rebates as state-dependent discounts," *Management Science*, vol. 51, no. 7, pp. 1131–1140, 2005.

[14] L. Qiang and S. Moorthy, "Coupons versus rebates," *Marketing Science*, vol. 26, no. 1, pp. 67–82, 2007.

[15] X. Chen, C.-L. Li, B.-D. Rhee, and D. Simchi-Levi, "The impact of manufacturer rebates on supply chain profits," *Naval Research Logistics (NRL)*, vol. 54, no. 6, pp. 667–680, 2007.

[16] S.-H. Cho, K. F. McCardle, and C. S. Tang, "Optimal pricing and rebate strategies in a two-level supply chain," *Production Engineering Research and Development*, vol. 18, no. 4, pp. 426–446, 2009.

[17] M. Khouja and Z. Jing, "The effect of delayed incentives on supply chain profits and consumer surplus," *Production Engineering Research and Development*, vol. 19, no. 2, pp. 172–197, 2010.

[18] J. Huang, M. Leng, and M. Parlar, "Demand functions in decision modeling: a comprehensive survey and research directions," *Decision Sciences*, vol. 44, no. 3, pp. 557–609, 2013.

[19] A. Y. Ha, W. Shang, and Y. Wang, "Manufacturer Rebate Competition in a Supply Chain with a Common Retailer," *Production Engineering Research and Development*, vol. 26, no. 11, pp. 2122–2136, 2017.

[20] J.-Y. Lee and L. Ren, "Vendor-managed inventory in a global environment with exchange rate uncertainty," *International Journal of Production Economics*, vol. 130, no. 2, pp. 169–174, 2011.

[21] E. Almehdawe and B. Mantin, "Vendor managed inventory with a capacitated manufacturer and multiple retailers: Retailer versus manufacturer leadership," *International Journal of Production Economics*, vol. 128, no. 1, pp. 292–302, 2010.

[22] E. Gal-Or, "First mover and second mover advantages," *International Economic Review*, vol. 26, no. 3, pp. 649–653, 1985.

[23] R. J. Deneckere and D. Kovenock, "Price Leadership," *Review of Economic Studies*, vol. 59, no. 1, pp. 143–162, 1992.

[24] R. Amir and A. Stepanova, "Second-mover advantage and price leadership in Bertrand duopoly," *Games and Economic Behavior*, vol. 55, no. 1, pp. 1–20, 2006.

[25] J. C. Bárcena-Ruiz, "Endogenous timing in a mixed duopoly: Price competition," *Journal of Economics/ Zeitschrift fur Nationalokonomie*, vol. 91, no. 3, pp. 263–272, 2007.

[26] Y. Nakamura and T. Inoue, "Endogenous timing in a mixed duopoly: price competition with managerial delegation," *Managerial and Decision Economics*, vol. 30, no. 5, pp. 325–333, 2009.

[27] R. Zhang, B. Liu, and W. Wang, "Pricing decisions in a dual channels system with different power structures," *Economic Modelling*, vol. 29, no. 2, pp. 523–533, 2012.

[28] J. S. Seaton and M. Waterson, "Identifying and characterising price leadership in British supermarkets," *International Journal of Industrial Organization*, vol. 31, no. 5, pp. 392–403, 2013.

[29] K. Matsui, "When should a manufacturer set its direct price and wholesale price in dual-channel supply chains?" *European Journal of Operational Research*, vol. 258, no. 2, pp. 501–511, 2017.

[30] H. Chung and E. Lee, "Asymmetric relationships with symmetric suppliers: Strategic choice of supply chain price leadership in a competitive market," *European Journal of Operational Research*, vol. 259, no. 2, pp. 564–575, 2017.

[31] R. Zhang and B. Liu, "Group buying decisions of competing retailers with emergency procurement," *Annals of Operations Research*, vol. 257, no. 1-2, pp. 317–333, 2017.

[32] J. Chen, B. Chen, and W. Li, "Who should be pricing leader in the presence of customer returns?" *European Journal of Operational Research*, vol. 265, no. 2, pp. 735–747, 2018.

[33] T. Li, R. Zhang, S. Zhao, and B. Liu, "Low carbon strategy analysis under revenue-sharing and cost-sharing contracts," *Journal of Cleaner Production*, vol. 212, no. 1, pp. 1462–1477, 2019.

[34] S. Dowrick, "von Stackelberg and Cournot duopoly: choosing roles," *The RAND Journal of Economics*, vol. 17, no. 2, pp. 251–260, 1986.

[35] J. H. Hamilton and S. M. Slutsky, "Endogeneous timing in duopoly games: stackelberg or Cournot equilibria," *Games and Economic Behavior*, vol. 2, no. 1, pp. 29–46, 1990.

[36] S. Huck, W. Müller, and H.-T. Normann, "To commit or not to commit: Endogenous timing in experimental duopoly markets," *Games and Economic Behavior*, vol. 38, no. 2, pp. 240–264, 2002.

[37] S. D. Mago and E. Dechenaux, "Price leadership and firm size asymmetry: An experimental analysis," *Experimental Economics*, vol. 12, no. 3, pp. 289–317, 2009.

[38] D. Hirata and T. Matsumura, "Price leadership in a homogeneous product market," *Journal of Economics/ Zeitschrift fur Nationalokonomie*, vol. 104, no. 3, pp. 199–217, 2011.

[39] T. Wang, D. J. Thomas, and N. Rudi, "The effect of competition on the efficient-responsive choice," *Production Engineering Research and Development*, vol. 23, no. 5, pp. 829–846, 2014.

[40] B. Niu, Y. Wang, and P. Guo, "Equilibrium pricing sequence in a co-opetitive supply chain with the ODM as a downstream rival of its OEM," *OMEGA - The International Journal of Management Science*, vol. 57, pp. 249–270, 2015.

[41] J. Tasoff and R. Letzler, "Everyone believes in redemption: Nudges and overoptimism in costly task completion," *Journal of Economic Behavior & Organization*, vol. 107, pp. 107–122, 2014.

[42] G. Cai, Y. Dai, and W. Zhang, "Modeling Multichannel Supply Chain Management with Marketing Mixes: A Survey," in *Social Science Electronic Publishing*, 2015.


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