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Research Article

Optimal Decision of Deferred Payment Supply Chain considering Bilateral Risk-Aversion Degree

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This paper aims to fill up the gap that the previous research has never explored, the deferred payment supply chain with a risk-averse supplier. To this end, the conditional value-at-risk (CVaR) was adopted as a criterion to measure the influence of retailer's deferred payment on supply chain performance. According to this criterion, the retailer's optimal order quantity and the supplier's optimal wholesale price per unit product were investigated under decentralized decision-making. Then, the existence of a unique optimal strategy was discussed for risk-averse supplier and retailer, and the values of risk-averse, initial capital, and wholesale price were calculated in detail. Finally, the theoretical results were testified through a numerical example. It is concluded that retailer's optimal order quantity is negatively correlated with the wholesale price, initial capital, and degree of risk aversion, so that the retailer can benefit through proper risk aversion; the supplier's expected profit decreases with the increase in the degree of risk aversion, yet the optimal wholesale price is determined by the degree of risk aversion of supplier and retailer. The research findings shed valuable new light on how to manage a supply chain involving risk-averse supplier and retailer.

1. Introduction

Facing the increasingly uncertain consumer demand, enterprises are competing to shorten the development cycle of new products. An inevitable outcome of the fierce competition is the issue of capital constraint, which casts a negative effect on supply chain members and upstream/downstream enterprises and, in turn, their income levels. Therefore, both upstream and downstream enterprises are eager to overcome the capital constraint. In pursuit of a solution to the constraint, many scholars have attempted to resolve the capital constraint of the retailer through endogenous financing. Some of them tried to help retailers get bank loans based on the credit of dominant suppliers, and some treated the goods in stock or receivable accounts as mortgage for financing. Taking the supply chain as a whole, the most representative measure of endogenous financing is the commercial credit; that is, the supplier provides such methods as deferred payment to alleviate the financial pressure on downstream retailers. Short-term commercial credit, a prevalent practice in developed countries, has become an essential channel for short-term internal financing [1, 2]. For instance, the Boeing 787 Dreamliner project contains a risk-sharing contract. Similar to deferred payment, the contract specifies that Boeing's strategic suppliers will not receive the payment until the aircraft is delivered to the customer. Deferred payment is also very popular in developing countries [3]. By the end of 2015, the total account receivable of China's industrial enterprises of medium scale and above stood at RMB 11 trillion yuan, and most of these enterprises eased the capital constraint of their upstream/downstream partners through short-term commercial credit mortgage. On a global scale, deferred payment contract accounts for 85% of global trade transactions. In recent years, much research has been done into deferred payment model based on single inventory. For example, the deferred payment model has been applied to evaluate supply chain decision and performance. In general, the existing literature tends to assume that both the supplier and the retailer are risk-neutral or only considers the risk aversion of the retailer. Actually, the supplier also faces various uncertain risks during decision-making, which affect the final decision and supply chain performance. In light of this, it is necessary to disclose the effect of risk-averse supplier and retailer on supply chain decision-making.

The remainder of this paper is organized as follows: Section 2 gives a brief review of relevant studies; Section 3 introduces the problem, preliminaries, and notations of this study; Section 4 examines the optimal strategies of risk-averse supplier and retailer under deferred payment; Section 5 provides numerical examples of the proposed strategy; Section 6 wraps up this research with some meaningful conclusions.

2. Literature Review

The existing studies on commercial credit mainly aim to find the optimal decision for credit term [3], order policy [4, 5], and optimal batch [6]. Recent years saw a boom in the research into deferred payment. For instance, Goyal [7] created an economic order quantity (EOQ) model with a constant demand rate under permissible deferred payment. Targeting the same issue, Chand and Ward [8] treated deferred payment as a price discount mechanism. Considering stock shortage and item corrosion, Aggarwal and Jaggi [9] further expanded the applicable scope of the EOQ model by setting out the optimal inventory policy. Later, Kim et al. [10] studied how suppliers manipulate the length of the credit period to maximize their profits. Huang [11] derived a theorem on selecting the optimal replenishment cycle under deferred payment and thus simplified the solution to Goyal's model. Huang and Hsu [12] modified Goyal's model to obtain three theorems on determining the optimal cycle length and optimal order quantity. To minimize the inventory system cost, Chung [13] investigated the retailer's inventory policy under two levels of trade credit. After that, Huang [14] developed the buyer's inventory model and obtained the buyer's optimal cycle time and optimal payment time under the supplier's trade credit policy and cash-discount policy. Teng [15] found that Goyal failed to differentiate between procurement cost and sales price and examined the impact of this difference on inventory decisions. Moreover, Chung and Huang [16] looked for the optimal ordering strategy of the retailer under limited storage capacity and deferred payment. Gupta and Wang [17] probed into deferred payment under stochastic demand. Dye and Ouyang [18, 19] employed particle swarm optimization to study the optimal pricing and inventory decision of the retailer under deferred payment. Jaber and Osman [20] discussed the coordinated management of supply chain inventory, with deferred payment as the decision variable shared by the buyer and the seller. Yang and Wee [21] established a supply chain inventory coordination model of perishable supply chain with limited replenishment rate and proved that deferred payment is a win-win strategy to achieve profit sharing. From the perspective of integrated supply chain, Chen and Kang [22], Chung and Liao [23], and Ouyang et al. [24] explored the optimal pricing, optimal order quantity, transport, and joint inventory of deferred payment in supply chain and put forward the corresponding solution methods. Furthermore, Arkan and Hejazi [25] studied the incentive of deferred payment under controllable lead time and order cost. Liping et al. [26] deliberated the incentive function of deferred payment provided by the manufacturer to the retailer through principal-agent model and designed the credit incentive contract of supply chain under both information symmetry and information asymmetry. Mingang and Gaohui [27] approached the coordination of two-stage supply chain of deferred payment from the angle of single-period stochastic demand, pointing out that deferred payment can increase the profits of the supply chain system under certain conditions and that the reasonable allocation mechanism enables the system to realize mutual benefit and win-win situation through in-depth integration and coordination. Inspired by Steinberg game model, Shu et al. [28] proposed the deferred payment strategy for the manufacturer and the optimal order decision for the seller. In addition, Yigang and Xiaowo [29] searched for the optimal ordering policy of the retailer under limited fund, deferred payment, and cash discount. Sarmah et al. [30] investigated the benefit allocation mechanism among supply chain members under deferred payment, as well as the fair transfer and surplus income distribution satisfying the profit objectives of all parties. Dayong and Jianwen [31] compared the reverse auction mechanism of the retailer in two scenarios: the fund shortage induced by financial institution loan and that arising from deferred payment. Zhu et al. [32] studied the pricing method of third-party logistics enterprises for inventory financing services under deferred payment. Chen and Wang [33] suggested that commercial credit can create value for supply chain and partially coordinate supply with capital constraint. Yang et al. [34] optimized the financing order decisions of a supply chain under the deferred payment of the retailer.

To sum up, the above studies are grounded on the risk neutrality of the supplier and the retailer. Nevertheless, the assumption that the decision-makers are risk-neutral goes against the actual situation. As mentioned before, the decision-makers are confronted with various uncertain risks, which affect the final decision and supply chain performance. Hence, the concept of risk aversion has been introduced to the supply chain research. For example, Gan et al. [35] coordinated a supply chain system, which consists of a risk-neutral supplier and a risk-averse retailer, through risk contract sharing mechanism. Xie et al. [36] analysed the supply chain model of risk aversion in light of three different supply chain strategies. Li et al. [37] studied a three-stage supply chain system containing a risk-neutral supplier and two risk-averse retailers and also revealed the impact of the retailer's risk sensitivity on the optimal strategy of the consumer. Similarly, Chen [38] looked for the optimal financial strategy for a two-stage supply chain of risk-neutral suppliers and risk-averse retailers and measured the effect of risk aversion of supply chain members. Besides, Chen [39] investigated the optimal decision mechanism of a two-stage supply chain containing a risk-neutral supplier and a risk-averse retailer. Ma et al. [40] examined the wholesale price and order quantity of a Nash game between a risk-neutral supplier and a risk-averse retailer. Wu et al. [41] discussed the quantity competition and price competition according to the conditional value-at-risk (CVaR) criteria, considered two requirement segmentation rules in risk-averse newsboy model, and disclosed the relationship between the expected profit and the risk-aversion degree. Abdel-Aal and Selim [42] dug into the CVaR risk-averse, multiproduct selective newsvendor problem. Following the mean CVaR criterion, Xie et al. [43] targeted a single-period supply chain with a newsvendor retailer and analysed the impact of risk preference on the optimal order quantity of the retailer under an independent setting. Chen et al. [44] took the CVaR as the decision criterion to derive the optimal pricing and ordering decisions. In addition, Jiang et al. [45] investigated a dual-channel supply chain consisting of a risk-neutral manufacturer and a risk-averse retailer.

Nonetheless, the above references only mention risk-averse retailer, but not risk-averse supplier. To make up for the gap, this paper takes both risk-averse retailer and risk-averse supplier into account and explores how optimal decision-making is affected by the risk aversion of both parties.

3. Problem Description and Preliminaries

The object of this research is a two-stage supply chain composed of a risk-averse supplier and a risk-averse retailer. Due to capital constraint, the retailer only pays the ready money to the supplier in the early stage and the remaining amount after the later sales (Figure 1). Let X be the stochastic market demand. Suppose $\overline{F}(x) = 1 - F(x)$, $h(x) = f(x)/\overline{F}(x)$, and $H(x) = xf(x)/\overline{F}(x)$, where $F(\cdot)$ is the probability distribution function and $f(\cdot)$ is the probability density function. To guarantee the existence of a unique solution to the objective solution, it is assumed that $F(\cdot)$ carries IFRD features. These features are evident in many distributions, such as normal distribution, exponential distribution, and uniform distribution.

The variables in this research are defined as follows:

k is the initial capital of the retailer.

p is selling price per unit product. Without loss of generality, the value of p was set to 1.

v_s is the risk value of the retailer under the given α_s.

v_r is the risk value of the supplier under the given α_r.

α_s is the risk-aversion degree of the retailer.

α_r is the risk-aversion degree of the supplier.

q is the order quantity of the retailer.

w is the wholesale price per unit product.

π_s is the profits of the supplier.

π_r is the profits of the retailer.

c is the cost per unit product.

For simplicity, the author put forward the following assumptions.

Assumption 1. The residual value of the unsold product is zero at the end of the selling season.

Assumption 2. The time cost of capital is negligible.

Assumption 3. Both the supplier and the retailer commit no breach of contract.

Assumption 4. There is no information asymmetry among the members of the supply chain.

The CVaR criterion was adopted to simulate risk aversion. Proposed by Rockafellar and Uryasev [46], the CVaR is a yardstick of the degree of risk aversion. During decision-making, the CVaR mainly identifies the average yield below a certain threshold α. The threshold value is negatively correlated with the degree of risk aversion of the decision-maker. The CVaR can be expressed as follows:

$$\begin{aligned} \mathrm{CVaR}_{\alpha}(\pi(x,y)) &= E\{\pi(x,y) \mid \pi(x,y) \le u_{\alpha}(y)\} \\ &= \frac{1}{\alpha}\int_{(x,y)\le u_{\alpha}} \pi(x,y)\, g(x,y)\, dy, \end{aligned} \tag{1}$$

where u is critical part; $u_{\alpha}(q) = \sup\{v \mid \mathrm{pr}\{\pi(w,q) \le v\} \le \alpha\}$, with v is being the risk value at the time of α. In practice, the CVaR is often expressed in a more general form:

$$\begin{aligned} &\mathrm{CVaR}_{\alpha}(\pi(w,q)) \\ &\quad = \max_{v\in R}\left\{v + \frac{1}{\alpha}E\left[\min\left(\pi(w,q) - v, 0\right)\right]\right\}. \end{aligned} \tag{2}$$

4. Model Construction and Solution

This section details the strategies of the risk-averse supplier and the risk-averse retailer. Under the risk-neutral and capital constraint assumptions, the retailer's decision model for deferred payment can be expressed as follows:

$$\max_{q>0} E\left[\pi_r(q)\right] = \max_{q>0} E\left[\min(x,q) - (wq-k)\right]^{+} - k. \tag{3}$$

The above equation can be rewritten as follows:

$$\begin{aligned} E\left[\pi_r(q)\right] &= \int_0^{\infty}\left[\min(x,q) - (wq-k)\right]^{+} f(x)\, dx \\ &\quad - B = (1-w)\,q - \int_{wq-k}^{q} F(x)\, dx. \end{aligned} \tag{4}$$

Under the risk-neutral condition, the supplier's decision-making model can be expressed as follows:

$$\begin{aligned} &\max_{0<w<1} E\left[\pi_s(w)\right] \\ &\quad = \max_{0<w<1} E\left\{\min\left[\min(x,q), (wq-k)\right] + k - cq\right\}. \end{aligned} \tag{5}$$

The above equation can be rewritten as follows:

$$E\left[\pi_s(w)\right] = (w-c)\,q - \int_0^{wq-k} F(x)\, dx. \tag{6}$$

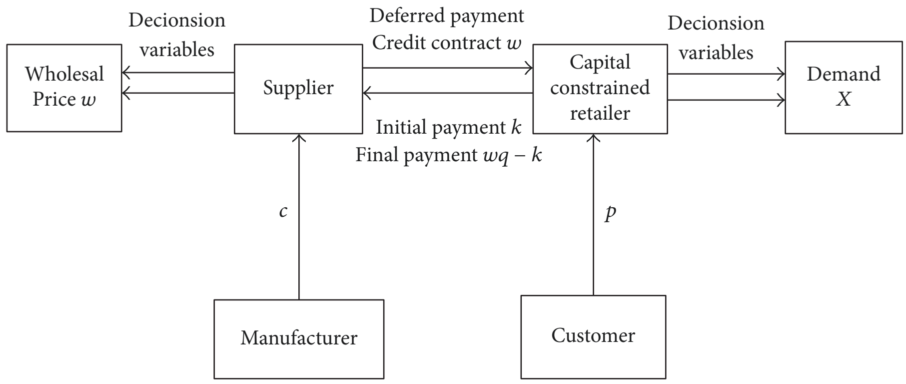


Figure 1: Payment process in the target supply chain.

Theorem 5. *In the decentralized case, if the stochastic market demand bears the IFRD features, there is an optimal* $v^* = (v_r^*, v_s^*)$ *such that* $(\max CVaR[\pi(q, v_r^*)], \max CVaR[\pi(w, v_s^*)])$*:*

$$v_r^* = \begin{cases} -k, & 0 < \alpha_r \leq F(wq-k) \\ F^{-1}(\alpha_r) - wq, & F(wq-k) < \alpha_r < F(q) \\ (1-w)q, & F(q) < \alpha_r < 1 \end{cases} \quad (7)$$

$$v_s^* = \begin{cases} (w-c)q, & F(wq-k) < \alpha_s < 1 \\ k - cq + F^{-1}(\alpha_s), & 0 < \alpha_r \leq F(wq-k). \end{cases}$$

Proof. (A) For the risk-averse retailer: substituting (3) into (2), we have

$$\begin{aligned} \mathrm{CVaR}(q, v_r) &= v_r - \frac{1}{\alpha_r} E[v_r - \pi_r(q)]^+ \\ &= v_r - \frac{1}{\alpha_r} \cdot E\left[v_r - \left[\min(x,q) - (wq-k)^+ + k\right]\right]^+ \\ &= v_r - \frac{1}{\alpha_r}\int_0^{wq-k} [v_r + k]^+ f(x)\,dx \\ &\quad - \frac{1}{\alpha_r}\int_{wq-k}^{q} \left[v_r - \left[x + (wq-k)^+ + k\right]^+\right] \\ &\quad \cdot f(x)\,dx - \frac{1}{\alpha_r}\int_q^{\infty} \left[v_r - [q + (wq-k)]^+ + k\right]^+ \\ &\quad \cdot f(x)\,dx. \end{aligned} \quad (8)$$

(1) If $v_r < -k$, then $\mathrm{CVaR}(q, v_r) = v_r$, $\partial \mathrm{CVaR}(q, v_r)/\partial v_r = 1 > 0$.

(2) If $-k \leq v_r < q - (wq-k) - k$, then

$$\begin{aligned} &\mathrm{CVaR}(q, v_r) \\ &= v_r - \frac{1}{\alpha_r}\int_0^{wq-k} (v_r + k) f(x)\,dx \\ &\quad - \frac{1}{\alpha_r}\int_{wq-k}^{v_r+wq} [v_r - x + (wq-k) + k] f(x)\,dx \\ &\left.\frac{\partial \mathrm{CVaR}(q, v_r)}{\partial v_r}\right|_{v=-k} = 1 - \frac{1}{\alpha_r}F(wq-k) \\ &\left.\frac{\partial \mathrm{CVaR}(q, v_r)}{\partial v_r}\right|_{v=(1-w)q} = 1 - \frac{1}{\alpha_r}F(q). \end{aligned} \quad (9)$$

(3) If $v_r > q - (wq-k) - k$, then

$$\begin{aligned} \mathrm{CVaR}(q, v_r) &= v_r - \frac{1}{\alpha_r}\int_0^{wq-k} (v_r + k) f(x)\,dx \\ &\quad - \frac{1}{\alpha_r}\int_{wq-k}^{q} [v_r - x + wq] f(x)\,dx \\ &\quad - \frac{1}{\alpha_r}\int_q^{\infty} [v_r - q + wq] f(x)\,dx. \end{aligned} \quad (10)$$

Find the first-order derivative of (10): $\partial \mathrm{CVaR}(q, v_r)/\partial v_r = 1 - 1/\alpha_r < 0$.

(B) For the risk-averse supplier: substituting (5) into (2), we have

$$\begin{aligned} &\mathrm{CVaR}(w, v_s) \\ &= v_s - \frac{1}{\alpha_s} \\ &\quad \cdot E[v_s - \min[\min(x,q), wq-k] - k + cq]^+ \end{aligned}$$

$$
\begin{aligned}
&= v_s - \frac{1}{\alpha_s}\int_0^{wq-k} [v_s - x - k + cq]^+ f(x)\,dx \\
&\quad - \frac{1}{\alpha_s}\int_{wq-k}^{\infty} [v_s - wq + cq]^+ f(x)\,dx.
\end{aligned} \tag{11}
$$

(1) If $v_s < k - cq$, then $\mathrm{CVaR}(w, v_s) = v_s$, $\mathrm{CVaR}(w, v_s)/\partial v_s = 1 > 0$.

(2) If $k - cq < v_s < (w - c)q$, then

$$
\begin{aligned}
&\mathrm{CVaR}(w, v_s) \\
&\quad = v_s - \frac{1}{\alpha_s}\int_0^{v_s+cq-k} v_s f(x)\,dx \\
&\qquad - \frac{1}{\alpha_s}\int_0^{v_s+cq-k} [-x + cq - k] f(x)\,dx \\
&\left.\frac{\partial \mathrm{CVaR}(w, v_s)}{\partial v_s}\right|_{v=k-cq} = 1 > 0 \\
&\left.\frac{\partial \mathrm{CVaR}(w, v_s)}{\partial v_s}\right|_{v=(w-c)q} = 1 - \frac{1}{\alpha_s} F(wq - k).
\end{aligned} \tag{12}
$$

(3) If $v_s \geq (w - c)q$, then

$$
\begin{aligned}
&\mathrm{CVaR}(w, v_s) \\
&\quad = v_s - \frac{1}{\alpha_s}\int_0^{wq-k} (v_s - x + cq - k) f(x)\,dx \\
&\qquad - \frac{1}{\alpha_s}\int_{wq-k}^{\infty} [v_s - cq + wq] f(x)\,dx \\
&\frac{\mathrm{CVaR}(w, v_s)}{\partial v_s} = 1 - \frac{1}{\alpha_s} < 0.
\end{aligned} \tag{13}
$$

Since $\mathrm{CVaR}(w, v_s)$ is a concave function of v_s in $[k - cq, (w - c)q]$, $v_s = (w - c)q$ when $\alpha_s > F(wq - k)$ and $v_s = k - cq + F^{-1}(\alpha_s)$ when $\alpha_s < F(wq - k)$. □

Theorem 6. *In the decentralized case, if the stochastic market demand bears the IFRD features, the optimal order quantity of the risk-averse retailer under the CVaR criterion satisfies*

$$
w\overline{F}(wq^* - k) + (1 - \alpha_r)(1 - w) = \overline{F}(q^*). \tag{14}
$$

Proof. (1) If $v_r = -k$, then $\mathrm{CVaR}(q, v_r) = -k$,

$$
\frac{\mathrm{CVaR}(q, v_r)}{\partial k} = 0. \tag{15}
$$

(2) If $v_r = F^{-1}(\alpha_r)wq$, then

$$
\begin{aligned}
&\mathrm{CVaR}(q, v_r) \\
&\quad = F^{-1}(\alpha_r) - wq \\
&\qquad + \frac{1}{\alpha_r}\int_0^{wq-k} \left(wq - k - F^{-1}(\alpha_r)\right) f(x)\,dx \\
&\qquad - \int_{wq-k}^{F^{-1}(\alpha_r)} \left[F^{-1}(\alpha_r) - x\right] f(x)\,dx
\end{aligned} \tag{16}
$$

$$
\frac{\partial \mathrm{CVaR}(q, v_r)}{\partial q} = -w + \frac{w}{\alpha_r}\int_0^{wq-k} f(x)\,dx
$$

$$
\frac{\partial^2 \mathrm{CVaR}(q, v_r)}{\partial q^2} = \frac{w^2}{\alpha_r} f(wq - k) > 0.
$$

(3) If $v_r = (1 - w)q$, then

$$
\begin{aligned}
\mathrm{CVaR}(q, v_r) &= (1 - w)q - \frac{1}{\alpha_r}\int_0^{q} q f(x)\,dx \\
&\quad + \frac{1}{\alpha_r}\int_0^{wq-k} (wq - k) f(x)\,dx \\
&\quad + \frac{1}{\alpha_r}\int_{wq-k}^{q} x f(x)\,dx.
\end{aligned} \tag{17}
$$

Find the first-order derivative of (17):

$$
\begin{aligned}
\frac{\partial \mathrm{CVaR}(q, v_r)}{\partial q} &= (1 - w) + \frac{1}{\alpha_r}\int_0^{q} f(x)\,dx \\
&\quad + \frac{w}{\alpha_r}\int_0^{wq-k} f(x)\,dx.
\end{aligned} \tag{18}
$$

Note that $1 - 1/\alpha_s < 0$,

$$
\left(1 - \frac{1}{\alpha_r}\right)(1 - w) + \frac{1}{\alpha_r}\overline{F}(q) - \frac{w}{\alpha_r}\overline{F}(wq - k) = 0. \tag{19}
$$

Find the second-order derivative of (18):

$$
\begin{aligned}
&\frac{\partial^2 \mathrm{CVaR}(q, v_r)}{\partial q^2} = \frac{1}{\alpha_r}\overline{F}(wq - k) \\
&\quad \cdot \left\{ \frac{-(1/\alpha_r) f(q)}{(1 - 1/\alpha_r)(1 - w) + (1/\alpha_r)\overline{F}(q)} \right. \\
&\quad \left. + w\frac{f(wq - k)}{\overline{F}(wq - k)} \right\} < \frac{w}{\alpha_r}\overline{F}(wq - k) \\
&\quad \cdot \left\{ \frac{-(1/\alpha_r) f(q)}{(1/\alpha_r)\overline{F}(q)} + w\frac{f(wq - k)}{\overline{F}(wq - k)} \right\} = \frac{w}{\alpha_r}\overline{F}(wq \\
&\quad - k)\left[-h(q) + wh(wq - k)\right] < 0.
\end{aligned} \tag{20}
$$

□

Corollary 7. *In the decentralized case, if the stochastic market demand bears the IFRD features, the optimal order quantity q^* of the risk-averse retailer decreases with the increase in the wholesale price per unit product w under the CVaR criterion.*

Proof. Let $u(q^*, w) = w\overline{F}(wq^* - k) + (1-\alpha_r)(1-w) - \overline{F}(q^*)$. According to the implicit function theorem, we have

$$\frac{\partial q^*}{\partial w} = -\frac{\partial u(q^*, w)/\partial w}{\partial u(q^*, w)/\partial q^*} = -\frac{\overline{F}(wq^* - k) - wq^* f(wq^* - k) - (1-\alpha_r)}{-f(x) + w^2 f(wq^* - k)}. \tag{21}$$

According to Lagrange mean value theorem, we have

$$\begin{aligned} 1 - F(wq^* - k) - w^2 f(wq^* - k) - (1-\alpha_r) &= \lim_{\beta\to+\infty}\left[f(\delta)(\beta - wq^* + k) - w^2 f(wq^* - k)\right. \\ &\quad \left. - (1-\alpha_r)\right], \end{aligned} \tag{22}$$

where $q^* \in (wq^* - k, +\infty)$ and $f(\delta) > 0$, $\beta \to +\infty$. Thus,

$$1 - F(wq^* - k) - w^2 f(wq^* - k) - (1-\alpha_r) > 0. \tag{23}$$

Since $w^2 f(wq^* - k) - f(q^*) < 0$, $(\partial q^*)/\partial w < 0$, and the optimal order quantity q^* of the risk-averse retailer decreases with the increase in the wholesale price per unit product w. □

Corollary 8. *In the decentralized case, if the stochastic market demand bears the IFRD features, the optimal order quantity q^* of the risk-averse retailer increases with the risk-aversion degree of the supplier α_r under the CVaR criterion.*

Proof. Let $u(q^*, \alpha_r) = w\overline{F}(wq^* - k) + (1-\alpha_r)(1-w) - \overline{F}(q^*)$. Considering that $1-w > 0$, $f(q^*) - w^2 f(wq^* - k) > 0$, we have the following equation according to the implicit function theorem:

$$\frac{\partial q^*}{\partial \alpha_r} = -\frac{\partial u(q^*, \alpha_r)/\partial \alpha_r}{\partial u(q^*, \alpha_r)/\partial q^*} = \frac{1-w}{f(q^*) - w^2(wq^* - k)}. \tag{24}$$

Hence, $\partial q^*/\alpha_r > 0$; that is, the optimal order quantity q^* of the risk-averse retailer increases with the risk-aversion degree of the supplier α_r. □

Corollary 9. *In the decentralized case, if the stochastic market demand bears the IFRD features, the optimal order quantity q^* of the risk-averse retailer increases with the retailer's initial capital k.*

Proof. Let $u(q^*, k) = w\overline{F}(wq^* - k) + (1-\alpha_r)(1-w) - \overline{F}(q^*)$. Considering that $w^2 f(wq^* - k) - f(q^*) < 0$, we have the following equation according to the implicit function theorem:

$$\frac{\partial q^*}{\partial k} = -\frac{\partial u(q^*, k)/\partial k}{\partial u(q^*, k)/\partial q^*} = \frac{wf(wq^* - k)}{w^2 f(wq^* - k) - f(q^*)}. \tag{25}$$

Thus, $\partial q^*/\partial k > 0$; that is, the optimal order quantity q^* of the risk-averse retailer increases with the retailer's initial capital k. □

Theorem 10. *In the decentralized case, if the stochastic market demand bears the IFRD features and $w \in [\underline{w}, \overline{w}]$, the supplier's expected profit is an increasing function of w when $0 < \alpha_s < F(wq^* - k)$ under the CVaR criterion and unimodal in w when $F(wq^* - k) < \alpha_s < 1$ under the same criterion. Thus, the optimal wholesale price is $\overline{w}$ if $\overline{w} < \widetilde{w}$, $\widetilde{w}$ if $\underline{w} < \overline{w} < \widetilde{w}$, and $\underline{w}$ if $\widetilde{w} < \underline{w}$, where $\widetilde{w}$ satisfies*

$$\begin{aligned} &\frac{\overline{F}(q^*)\left[1 - (1/\alpha_s) F(wq^* - k)\right]}{1 - \alpha_r + \overline{F}(wq^* - k)\left[1 - wq^* h(wq^* - k)\right]} \\ &\quad \cdot \left[1 - H(q^*) - \frac{(1-\alpha_r)}{\overline{F}(q^*)}\right] = c. \end{aligned} \tag{26}$$

Proof. (1) If $v_s^* = (w - c)q^*$, we have

$$\text{CVaR}(w, v_s^*) = (w - c) q^* - \frac{1}{\alpha_s}\int_0^{wq^* - k} F(x)\,dx. \tag{27}$$

Find first-order derivative of the above equation:

$$\begin{aligned} \frac{\partial \text{CVaR}(w, v_s^*)}{\partial w} &= q^*\left[1 - \frac{1}{\alpha_s} F(wq^* - k)\right] \\ \frac{\partial \text{CVaR}(w, v_s^*)}{\partial q^*} &= w\left[1 - \frac{1}{\alpha_s} F(wq^* - k)\right] \\ \frac{d\text{CVaR}(w, v_s^*)}{dw} &= \frac{\partial \text{CVaR}(w, v_s^*)}{\partial w} + \frac{\partial \text{CVaR}(w, v_s^*)}{\partial w} \cdot \frac{\partial q^*}{\partial w} \\ &= \frac{\partial q^*}{\partial w}\left\{\frac{q^*\left[1 - (1/\alpha_s) F(wq^* - k)\right]}{\partial q^*/\partial w} + w\left[1 - \frac{1}{\alpha_s} F(wq^* - k)\right] - c\right\} \\ &= \frac{\partial q^*}{\partial w}\left\{\frac{1 - (1/\alpha_s) F(wq^* - k)\left[q^* f(q^*) + w(1-\alpha_r) - w\overline{F}(wq^* - k)\right]}{1 - \alpha_r - \overline{F}(wq^* - k)\left[1 - wq^* h(wq^* - k)\right]} - c\right\}. \end{aligned} \tag{28}$$

Substituting $w\overline{F}(wq^* - k) = -(1-\alpha_r)(1-w) + \overline{F}(q^*)$ into the above equation, we have

$$\frac{d\mathrm{CVaR}(w, v_s^*)}{dw} = \frac{\partial q^*}{\partial w}\left\{\frac{q^*\left[1-(1/\alpha_s)F(wq^*-k)\right]\left[q^* f(q^*) + w(1-\alpha_r) + (1-\alpha_r)(1-w) - \overline{F}(q^*)\right]}{1-\alpha_r - \overline{F}(wq^*-k)\left[1-wq^* h(wq^*-k)\right]} - c\right\}$$
$$= \frac{\partial q^*}{\partial w}\left\{\frac{\overline{F}(q^*)\left[1-(1/\alpha_s)F(wq^*-k)\right]\left[1-H(q^*)-(1-\alpha_r)/\overline{F}(q^*)\right]}{1-\alpha_r-\overline{F}(wq^*-k)\left[1-wq^*h(wq^*-k)\right]}\right\} = \frac{\partial q^*}{\partial w}\left[\varepsilon(w) - c\right]. \tag{29}$$

Corollary 7 shows that $\partial q^*/\partial w < 0$. Thus, the sign of $d\mathrm{CVaR}(w, v_s^*)/dw$ depends on $\varepsilon(w) - c$. According to [38, 39], it can be easily obtained that $d\varepsilon(w)/dw > 0$. If $\overline{w}$ satisfies $\varepsilon(w) - c = 0$, $d\mathrm{CVaR}(w, v_s^*)/dw > 0$ when $w < \overline{w}$ and $d\mathrm{CVaR}(w, v_s^*)/dw < 0$ when $w > \overline{w}$. Therefore, $\mathrm{CVaR}(w, v_s^*)$ is unimodal in w.

(2) If $v_s = k - cq^* + F^{-1}(\alpha_s)$, we have

$$\mathrm{CVaR}(w, v_s^*) = k - cq^* + F^{-1}(\alpha_s) - \frac{1}{\alpha_s}\int_0^{F^{-1}(\alpha_s)} F(x)\,dx. \tag{30}$$

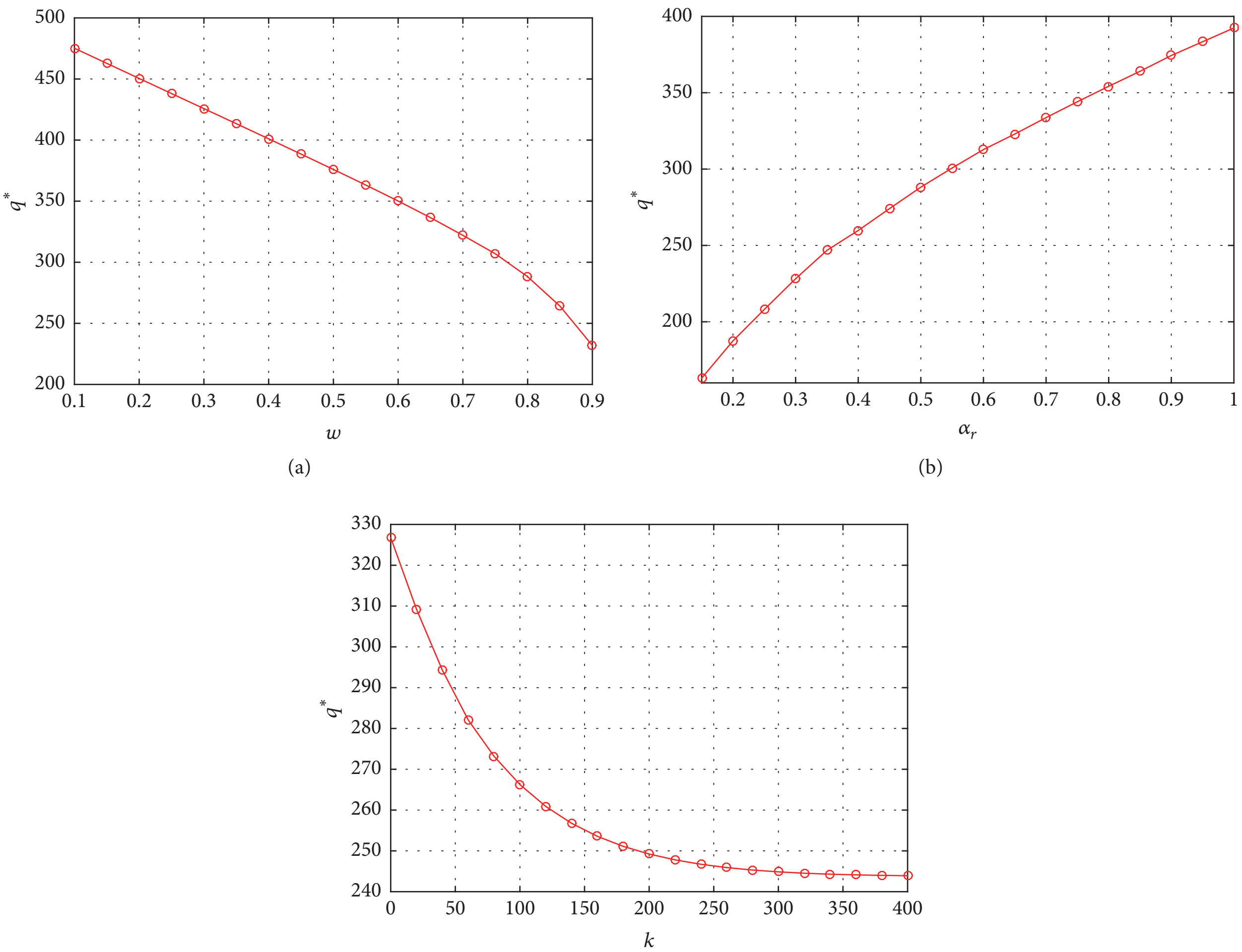
Find the first-order derivative of the above equation. Corollary 7 shows that $\partial q^*/\partial w < 0$. Thus, $\partial\mathrm{CVaR}(w, v_s^*)/\partial w = -c \cdot \partial q^*/\partial w > 0$. This means the supplier's expected profit is an increasing function of w. □

5. Numerical Examples

This section performs numerical experiment on Matlab to disclose the impacts of initial capital, risk-aversion degree, and wholesale price on retailer's optimal decision, profit of supply chain members, and profit of supply chain system. Suppose that the demand for a product in the market obeys the normal distribution of $X \sim (500, 200^2)$, the initial capital of the retailer is $k = 50$, the wholesale price per unit product is $w = 0.8$, the supplier's risk-aversion degree is $\alpha_s = 0.5$, and the retailer's risk-aversion degree is $\alpha_r = 0.5$. Firstly, two of these variables were kept constant to reveal the impact of another variable on the retailer's optimal order quantity. For example, the initial capital of the retailer k and the retailer's risk-aversion degree α_r were kept unchanged to disclose the effect of discount price on the retailer's optimal order quantity. The results in Figure 2(a) testify the correctness of Corollary 7. Figures 2(b) and 2(c), respectively, present the impact of the retailer's risk-aversion degree on the optimal order quantity under constant initial capital and discount price and the impact of initial capital on the optimal order quantity under the constant risk-aversion degree and discount price. It can be seen that the retailer's optimal order quantity decreases monotonically with the initial capital and the discount price. The risk-aversion degree is negatively correlated with the optimal order quantity. Figure 3 illustrates the effect of retailer's order quantity on its profit at different risk-aversion degrees according to the CVaR criterion. It is clear that the risk-averse retailer tends to make a conservative decision and pursue a relatively low profit. The finding is consistent with the fact. Figure 4 displays the variation in the retailer's profit with risk-aversion degrees. As can be seen from this figure, the retailer can reap more profit after taking proper risk-aversion measures. Figure 5 shows the effect of discount price on supplier profit at different risk-aversion degrees. When $\overline{F}(wq^* - k) < \alpha_s < 1$, the expected profit of the supplier is an unimodal function in w; when $0 < \alpha_s < \overline{F}(wq^* - k)$, the expected profit of the supplier increases with w. The results echo with the contents in Theorem 10. Figure 6 depicts the impact of the initial capital k, wholesale price per unit product w, the supplier's risk-aversion degree α_r, and the retailer's risk-aversion degree α_r on supply chain members and the profit of the supply chain system. In the case of a fixed wholesale price, the supplier's risk evasion is not conductive to either sides of the supply chain. Figure 7(a) describes the impact of α_r on the supplier's expected profits when $\overline{F}(wq^* - k) < \alpha_s < 1$, while Figure 7(b) shows that impact when $0 < \alpha_s < \overline{F}(wq^* - k)$. It can be learned that the risk-averse retailer tends to make a conservative decision and pursue a relatively low profit. Therefore, decision-makers should formulate reasonable operation strategies according to these variables, such as the retailer's initial capital and risk-aversion degree.

6. Conclusions

This paper studies the deferred payment of a two-stage supply chain containing a risk-averse supplier and a risk-averse retailer under stochastic market demand and establishes a two-stage supply chain decision model considering order and wholesale price. The CVaR was adopted as a criterion to measure the influence of retailer's deferred payment on supply chain performance. According to this criterion, the retailer's optimal order quantity and the supplier's optimal wholesale price per unit product were investigated under decentralized decision-making. Then, the existence of a unique optimal strategy was discussed for risk-averse supplier and retailer, and the values of risk-averse, initial capital, and wholesale price were calculated in detail. Finally, the theoretical results were testified through a numerical example. It is concluded that retailer's optimal order quantity is negatively correlated with the wholesale price, initial capital, and degree of risk aversion, so that the retailer can benefit through proper risk aversion; the supplier's expected profit decreases with



(c)

FIGURE 2: Impact of w, α_s, and k on the retailer's optimal order quantity.

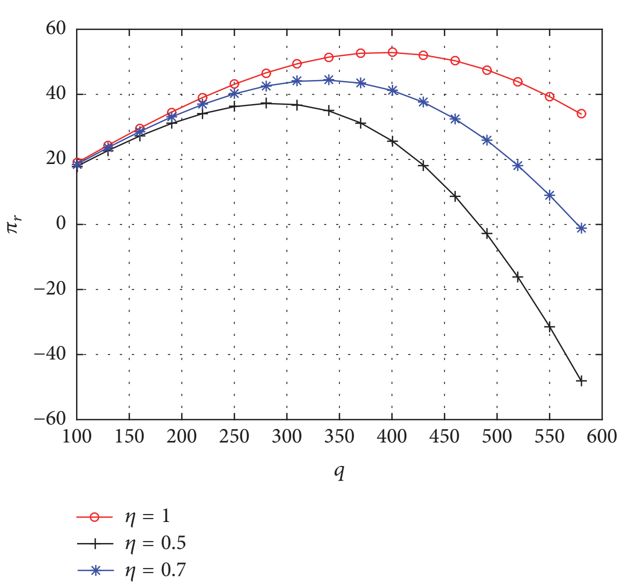


FIGURE 3: Impact of α_r and q on the retailer's profit.

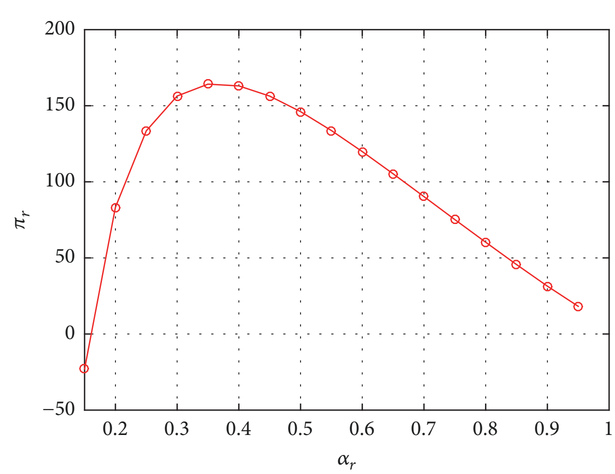


FIGURE 4: Impact of α_r on the retailer's profit.

the increase in the degree of risk aversion, yet the optimal wholesale price is determined by the degree of risk aversion of supplier and retailer.

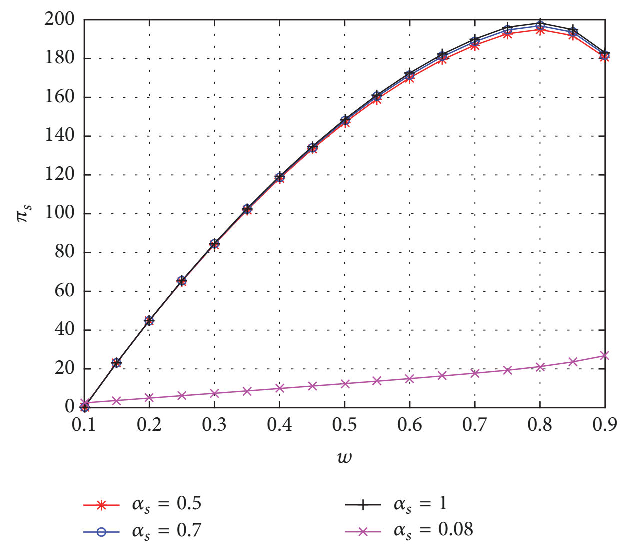


FIGURE 5: Impact of w and α_s on the supplier's expected profit.

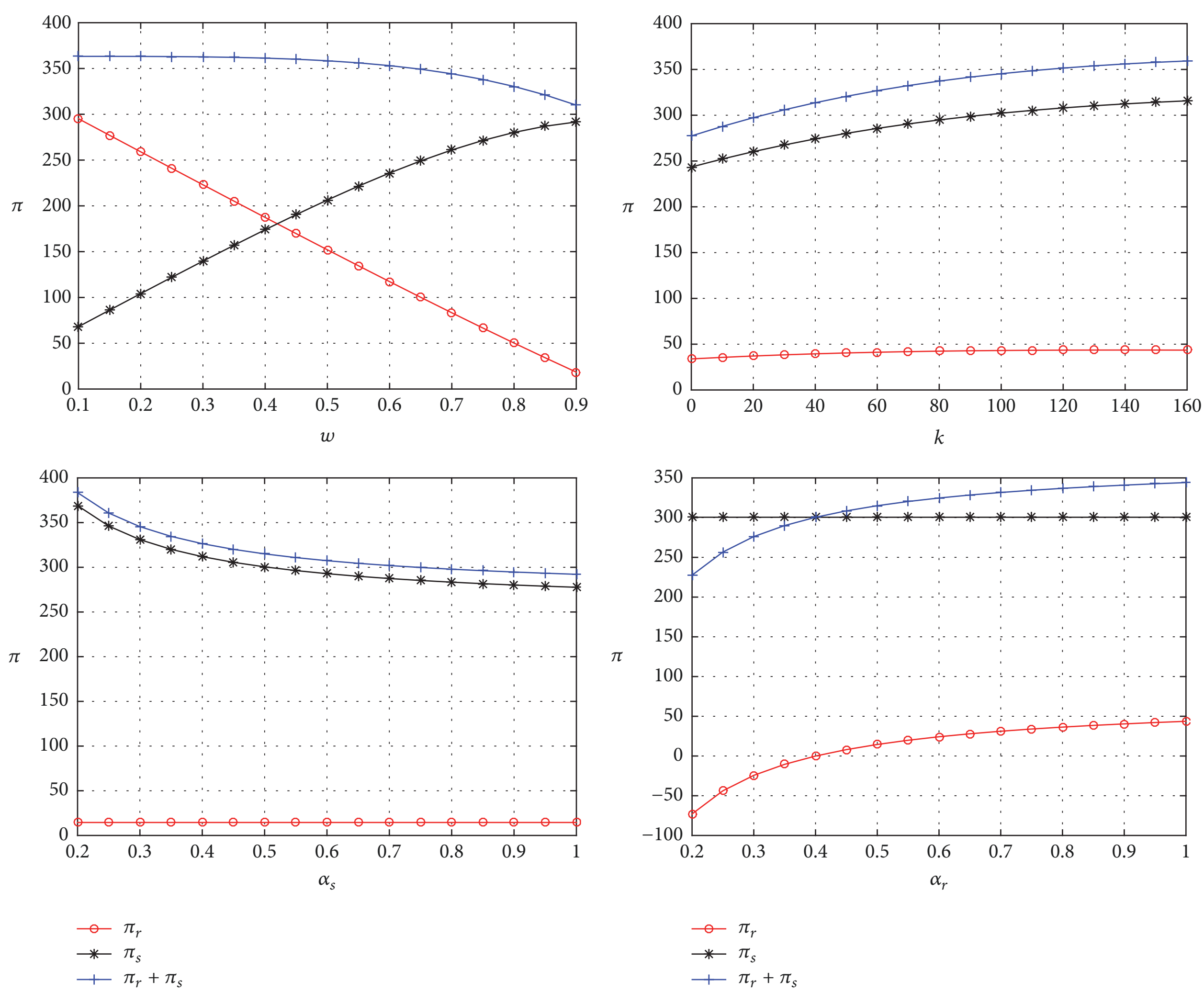
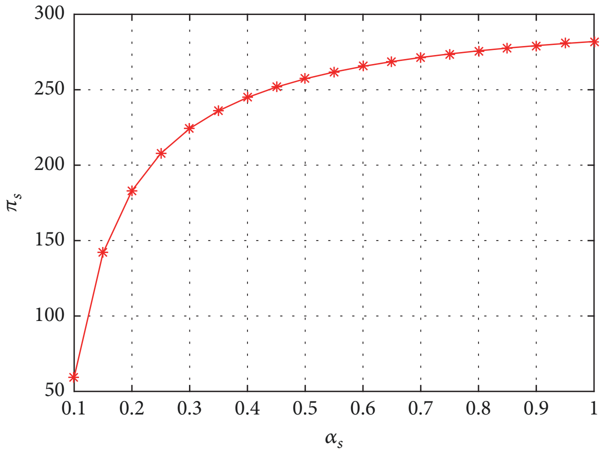
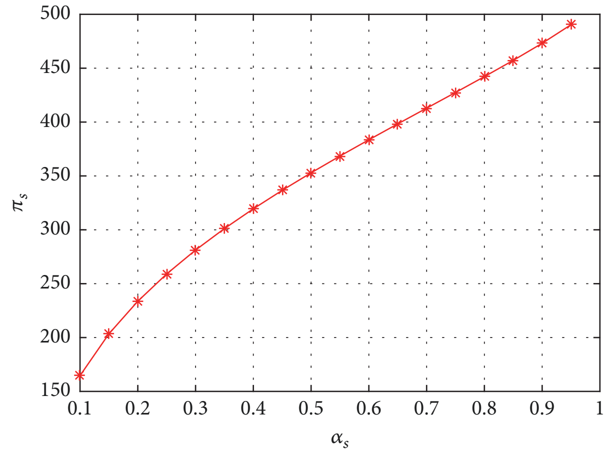


FIGURE 6: Impact of k, w, α_s, and α_r on the profit of supply chain members.



(a)



(b)

Figure 7: Impact of α_s on the supplier's expected profit.

In general, the proposed model sheds valuable new light on how to manage a supply chain involving risk-averse suppler and retailer. However, there are still some limitations in the current research. First, it is assumed that there is no information asymmetry among supply chain members; second, the author only considered the profits of the supplier and retailer under risk-averse assumption, failing to take account of their expected profits. Hence, the future research will adopt the mean CVaR criteria to explore the optimal decisions of the retailer and the supplier, study the effect of information asymmetry on the optimal decision-making, and develop even more complicated models.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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