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*Research Article*

# $H_\infty$ Control for Nonlinear Infinite Markov Jump Systems

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In this paper, we discuss the infinite horizon $H_\infty$ control problem for a class of nonlinear stochastic systems with state, control, and disturbance dependent noise. The jumping parameters are modelled as an infinite-state Markov chain. Based on the solvability of a set of coupled Hamilton-Jacobi inequalities (HJIs), the exponential mean square $H_\infty$ controller for the considered nonlinear stochastic systems is obtained. A numerical example is given to show the effectiveness of the proposed design method.

## 1. Introduction

During the past decades, as one of the most important robust control design, $H_\infty$ control has been extensively studied in both theory and practical applications [1]. From the time-domain viewpoint, $H_\infty$ control is to find a control law to eliminate the effect of external disturbance below a given level [2]. Due to the ability to model many real plants in practice, stochastic systems has gained much attention. In particular, stochastic $H_\infty$ control was firstly investigated in [3] for Itô systems, where a stochastic bounded real lemma was established in the form of linear matrix inequalities. References [4, 5], respectively, studied $H_\infty$ filtering and control for nonlinear stochastic systems via solving second-order nonlinear HJIs.

Stochastic systems with Markov jumps are powerful tool to describe physical systems which may encounter abrupt changes in their dynamics. In the theoretical study of stochastic Markov jump systems, stability and observability [6–11] and robust control [12–15] have been widely investigated. Recently, stable and control problems for nonlinear systems have become a hot research topic [16–22]. It should be pointed out that most of the aforementioned researches on Markov jump systems assume that Markov chain takes values in a finite set. However, Markov jump systems with infinite-state chains can be used to describe more plants in many real scenarios [23, 24]. Therefore, infinite Markov jump systems deserve our consideration. Recently, some papers on stability [25–27] and control problems [28, 29] of linear infinite Markov jump systems have appeared. To be specific, infinite horizon $H_2/H_\infty$ controller has been obtained by four coupled algebraic Riccati equations in [30]. Nevertheless, to the best of our knowledge, $H_\infty$ control problem for a class of nonlinear stochastic systems with infinite Markov jumps is still unsolved, let alone the case of $(x, u, v)$-dependent noise. This situation motivates us to carry out the present research.

This paper is concerned with the infinite horizon $H_\infty$ control problem for a class of nonlinear stochastic systems with infinite Markov jumps and $(x, u, v)$-dependent noise. The rest of the paper is organized as follows. Section 2 provides some useful definitions and lemmas. In Section 3, based on the generalized Itô-type formula and the technique of squares completion, an exponential mean square stable $H_\infty$ controller is designed in terms of a set of coupled HJIs. And a numerical example is provided to illustrate the applicability of the proposed design approach. Conclusions are made in Section 4.

Next, we adopt the following notations. $\mathscr{R}$ denotes the set of all real numbers and $\mathscr{R}_+$ is the set of all nonnegative real numbers. $\mathscr{R}^n$ and $\mathscr{R}^{m\times n}$ stand for $n$-dimensional real vector space and the vector space of all $m \times n$ matrices, respectively. For a matrix $A$, $A'$ represents the transpose and we denote $A \geq 0$ $(A > 0)$ the positive semidefinite (definite) symmetric matrix. Also, we make use of the notation of $\mathcal{S}_n$ and $I_n$ for the set of all $n \times n$ symmetric and identity matrices, respectively. The operator norm of $\mathscr{R}^{m\times n}$ or the Euclidean norm of $\mathscr{R}^n$ is

$\|\cdot\|$. By $l^2(\mathscr{R}_+;\mathscr{R}^m)$ we define the space of $\mathscr{R}^m$-valued, square integrable, and $\mathscr{F}_t$-measurable processes $\zeta=\{\zeta(t,w):\mathscr{R}_+\times\Omega\longrightarrow\mathscr{R}^m\}$ satisfying $\|\zeta\|^2=E\int_0^\infty\|\zeta(t)\|^2dt<\infty$. The class of functions $V(x)$ which are twice continuously differential with respect to $x\in\mathscr{U}$, except possibly at the point $x=0$, will be denoted by $C^2(\mathscr{U})$. $\mathscr{D}:=\{1,2,\ldots\}$.

## 2. Preliminaries

Consider the following stochastic nonlinear system with infinite Markov jumps:

$$\begin{aligned} dx(t) &= [f_1(x(t),\eta_t)+g_1(x(t),\eta_t)u(t) \\ &\quad + h_1(x(t),\eta_t)v(t)]dt+[f_2(x(t),\eta_t) \\ &\quad + g_2(x(t),\eta_t)u(t)+h_2(x(t),\eta_t)v(t)]dw(t), \\ z(t) &= \begin{bmatrix} m(x(t),\eta_t) \\ u(t)\end{bmatrix}, \\ x(0) &= x_0\in\mathscr{R}^n,\quad t\in\mathscr{R}_+, \end{aligned} \tag{1}$$

where $x(t)\in\mathscr{R}^n$, $v(t)\in\mathscr{R}^{n_v}$, $u(t)\in\mathscr{R}^{n_u}$, and $z(t)\in\mathscr{R}^{n_z}$ stand for the system state, exogenous disturbance, control input, and measurement output, respectively. $w(t)$ is a standard one-dimensional Brownian motion on a probability space $(\Omega,\mathscr{F},\mathscr{P})$. Assume that $\mathscr{F}_t:=\sigma(w(s),0\le s\le t)\vee\sigma(\eta(s),0\le s\le t)\vee\mathscr{N}$, where $\mathscr{N}$ denotes the totality of $\mathscr{P}$-null sets and the $\sigma$-algebras $\sigma(w(s),0\le s\le t)$ and $\sigma(\eta(s),0\le s\le t)$ are mutually independent. We denote $\{\eta_t\}_{t\in\mathscr{R}_+}$ the right continuous, homogeneous Markov process on $\Omega$ taking values in the countably infinite state space $\mathscr{D}$ with generator $Q=(q_{ij})_{i,j\in\mathscr{D}}$ given by

$$P(\eta_{t+s}=j\mid\eta_t=i)=\begin{cases} q_{ij}s+o(s), & i\neq j, \\ 1+q_{ii}s+o(s), & i=j,\end{cases} \tag{2}$$

where $s>0$, $\lim_{s\longrightarrow 0}(o(s)/s)=0$, $q_{ij}\ge 0(i,j\in\mathscr{D},i\neq j)$ is the transition rate from mode $i$ at time $t$ to mode $j$ at time $t+s$ and $q_{ii}=-\sum_{j\in\mathscr{D},j\neq i}q_{ij}<\infty$ for all $i\in\mathscr{D}$. Suppose that $f_1,g_1,h_1,f_2,g_2,h_2$, and $m$ satisfy the local Lipschitz condition and the linear growth condition for any $i\in\mathscr{D}$, which guarantee that system (1) has a unique strong solution [13, 31]. Moreover, assume $f_1(0,i)=0$, $f_2(0,i)=0$, $i\in\mathscr{D}$.

Denote $\mathbb{E}_1^{m\times n}$ the Banach space of all sequences $\{E\mid E=(E(1),E(2),\cdots),E(i)\in\mathscr{R}^{m\times n}\}$ with the norm $\|E\|_1=\sum_{i=1}^\infty\|E(i)\|<\infty$. Likewise, define another Banach space $\mathbb{E}_\infty^{m\times n}$ with the norm $\|E\|_\infty=\sup_{i\in\mathscr{D}}\|E(i)\|$. Assume all coefficients of considered systems have a finite norm $\|\cdot\|_\infty$. If $m=n$, $\mathbb{E}_1^{m\times n}$ will be simplified as $\mathbb{E}_1^n$ and so does $\mathbb{E}_\infty^{m\times n}$. When $E(i)\in S_n$ and $E(i)\ge 0$, $i\in\mathscr{D}$, $\mathbb{E}_1^n(\mathbb{E}_\infty^n)$ is written as $\mathbb{E}_1^{n+}$(resp., $\mathbb{E}_\infty^{n+}$). For $L$, $M\in\mathbb{E}_1^{n+}$, $L\le M$ implies that $L(i)\le M(i)$, $i\in\mathscr{D}$. Therefore, we have $\|L\|_1\le\|M\|_1$.

For each $V\in C^2(\mathscr{R}^n\times\mathscr{D};\mathscr{R})$, an infinitesimal operator $\mathscr{L}V:\mathscr{R}^n\times\mathscr{D}\longrightarrow\mathscr{R}$ associated with system (1) is defined as follows [28, 31]:

$$\begin{aligned} \mathscr{L}V(x(t),i) &= \frac{\partial V(x(t),i)'}{\partial x}[f_1(x(t),i) \\ &\quad + g_1(x(t),i)u(t)+h_1(x(t),i)v(t)] \\ &\quad + \sum_{j=1}^\infty q_{ij}V(x(t),j)+\frac{1}{2}\operatorname{trace}\Big\{[f_2(x(t),i) \\ &\quad + g_2(x(t),i)u(t) \\ &\quad + h_2(x(t),i)v(t)]'\frac{\partial^2V(x(t),i)}{\partial x^2} \\ &\quad \cdot[f_2(x(t),i)+g_2(x(t),i)u(t) \\ &\quad + h_2(x(t),i)v(t)]\Big\},\quad i\in\mathscr{D}. \end{aligned} \tag{3}$$

To study the infinite horizon nonlinear stochastic $H_\infty$ control, the internal stability requirement is needed; thus we introduce the following definition.

*Definition 1* (see [13]). The unforced stochastic system with infinite Markov jumps,

$$dx(t)=f_1(x(t),\eta_t)dt+f_2(x(t),\eta_t)dw(t),\quad t\in\mathscr{R}_+, \tag{4}$$

is called exponentially mean square stable (EMSS) if there exist $\alpha>0$ and $\beta\ge 1$ such that

$$E\left[\|x(t)\|^2\right]\le\beta e^{-\alpha t}\|x_0\|^2 \tag{5}$$

for all $t\in\mathscr{R}_+$, $i\in\mathscr{D}$ and $x_0\in\mathscr{R}^n$.

*Definition 2.* For a given $\gamma>0$, the control $u^*(t)\in l^2(\mathscr{R}_+;\mathscr{R}^{n_u})$ is said to be an infinite horizon $H_\infty$ control of system (1), if

(i) $u^*(t)$ stabilizes system (1) internally; i.e. when $v(t)=0$, $u(t)=u^*(t)$, the trajectory of system (1) with any initial value $x(0)=x_0$ is EMSS

(ii) For any nonzero $v^*(t)\in l^2(\mathscr{R}_+;\mathscr{R}^{n_v})$ and zero initial state $x_0=0$, we always have

$$\|z(t)\|_{l^2(\mathscr{R}_+;\mathscr{R}^{n_u})}\le\gamma\|v(t)\|_{l^2(\mathscr{R}_+;\mathscr{R}^{n_v})}. \tag{6}$$

*Remark 3.* If (6) holds, it is easy to verify that (6) is equivalent to $\|L_\infty^{u^*}\|\le\gamma$, where the perturbation operator $\|L_\infty^{u^*}\|$ is defined by $L_\infty^{u^*}:l^2(\mathscr{R}_+;\mathscr{R}^{n_v})\longrightarrow l^2(\mathscr{R}_+;\mathscr{R}^{n_z})$ subject to system (1) with

$$\left\|L_\infty^{u^*}\right\|:=\sup_{\substack{v(\cdot)\in l^2(\mathscr{R}_+;\mathscr{R}^{n_v}),v\neq 0\\ \eta_0\in\mathscr{D},x_0=0}}\frac{\|z(t)\|_{l^2(\mathscr{R}_+;\mathscr{R}^{n_z})}}{\|v(t)\|_{l^2(\mathscr{R}_+;\mathscr{R}^{n_v})}}. \tag{7}$$

We provide some lemmas which are absolutely necessary to derive our main results.

**Lemma 4** (see [32]). *For $x, b \in \mathscr{R}^n$, $A \in \mathcal{S}_n$, $A^{-1}$ exists, we have*

$$x'Ax + x'b + b'x = \left(x + A^{-1}b\right)' A\left(x + A^{-1}b\right) - b'A^{-1}b. \tag{8}$$

The following lemma generalizes Theorem 5.8 [31] and Corollary 3.2.3 [13] to the infinite Markov jump and nonlinear systems, respectively. Its proof can be easily shown by analogous arguments.

**Lemma 5.** *Assume that there are a set of positive functions $V(x, \eta_t) \in C^2(\mathscr{R}^n \times \mathscr{D}; \mathscr{R}^+)$ and positive constants $c_1, c_2, c_3$ such that*

$$c_1 \|x\|^2 \leq V(x, i) \leq c_2 \|x\|^2 \tag{9}$$

*and*

$$\mathscr{L}V(x, i) \leq -c_3 \|x\|^2 \tag{10}$$

*for all $x \in \mathscr{R}^n$ and $i \in \mathscr{D}$. Then system (4) is EMSS.*

## 3. Infinite Horizon Nonlinear Stochastic $H_\infty$ Control

In this subsection, we attempt to obtain the sufficient condition for the infinite horizon nonlinear stochastic $H_\infty$ control problem of system (1).

**Theorem 6.** *For a given disturbance attenuation level $\gamma > 0$, if there exist a set of positive functions $V(x, i) \in C^2(\mathscr{R}^n \times \mathscr{D}; \mathscr{R}^+)$, $V(0, i) = 0$, and $\partial^2 V(x, i)/\partial x^2 \geq 0$ for all nonzero $x \in \mathscr{R}^n$, $i \in \mathscr{D}$ with the properties of*

$$\begin{gathered} c_1 \|x\|^2 \leq V(x, i) \leq c_2 \|x\|^2, \\ -\|m(x, i)\|^2 \leq -c_3 \|x\|^2, \end{gathered} \tag{11}$$

*for some positive constants $c_1, c_2, c_3$ such that $V(x, i)$ solves the coupled HJIs,*

$$\begin{aligned}
\Upsilon_i &= \frac{\partial V(x,i)'}{\partial x} f_1(x,i) + \frac{1}{2} f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) \\
&\quad + m(x,i)' m(x,i) + \sum_{j=1}^{\infty} q_{ij} V(x,j) \\
&\quad - \frac{1}{4}\left[f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} h_2(x,i) + \frac{\partial V(x,i)'}{\partial x} h_1(x,i)\right]\left[-\gamma^2 I + h_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} h_2(x,i)\right]^{-1} \\
&\quad \cdot \left[h_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + h_1(x,i)' \frac{\partial V(x,i)}{\partial x}\right] \\
&\quad - \frac{1}{4}\left[f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i) + \frac{\partial V(x,i)'}{\partial x} g_1(x,i)\right]\left[I + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right]^{-1} \\
&\quad \cdot \left[g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\right] \leq 0, \\
&- \gamma^2 I + h_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} h_2(x,i) < 0, \quad i \in \mathscr{D},
\end{aligned} \tag{12}$$

*then*

$$\begin{aligned} u^*(x,i) = &-\frac{1}{2}\left[I + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right]^{-1} \\ &\cdot \left[g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\right] \end{aligned} \tag{13}$$

*is an infinite horizon $H_\infty$ control of system (1).*

*Proof.* We first verify that (6) holds. For any $T > 0$ and initial state $x_0 = 0, \eta_0 = i$, note that the generalized Itô-type formula [28] and (3) yield

$$\begin{aligned}
E\left[V(x(T), \eta_T) - V(x_0, \eta_0) \mid \eta_0 = i\right] &= E\left\{\int_0^T \mathscr{L}V(x, \eta_t)\, dt \mid \eta_0 = i\right\} \\
= E\Bigg\{\int_0^T \Bigg[&\frac{\partial V(x, \eta_t)'}{\partial x}\left(f_1(x, \eta_t) + g_1(x, \eta_t) u + h_1(x, \eta_t) v\right) + \sum_{j=1}^{\infty} q_{ij} V(x, j) \\
&+ \frac{1}{2}\left(f_2(x, \eta_t) + g_2(x, \eta_t) u + h_2(x, \eta_t) v\right)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2}\left(f_2(x, \eta_t) + g_2(x, \eta_t) u + h_2(x, \eta_t) v\right) + \|m(x, \eta_t)\|^2 + \|u\|^2
\end{aligned}$$

$$- \|z\|^2 - \gamma^2 \|v\|^2 + \gamma^2 \|v\|^2 \Bigg] dt \mid \eta_0 = i \Bigg\}$$

$$= E\left\{\int_0^T \left[\Lambda_1(v, x, \eta_t) + \Lambda_2(x, \eta_t) + \Lambda_3(u, x, \eta_t) + \sum_{j=1}^{\infty} q_{ij} V(x, j) - \|z\|^2 + \gamma^2 \|v\|^2 + \frac{1}{2}\left(u' g_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} h_2(x, \eta_t) v + v' h_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} g_2(x, \eta_t) u\right)\right] dt \mid \eta_0 = i\right\}, \quad (14)$$

where

$$\begin{aligned}
\Lambda_1(v, x, \eta_t) &= v' \left(-\gamma^2 I + \frac{1}{2} \cdot h_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} h_2(x, \eta_t)\right) v \\
&\quad + \frac{1}{2}\left(f_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} h_2(x, \eta_t) + \frac{\partial V(x, \eta_t)'}{\partial x} h_1(x, \eta_t)\right) v + \frac{1}{2} \cdot v' \left(h_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} f_2(x, \eta_t) + h_1(x, \eta_t)' \frac{\partial V(x, \eta_t)}{\partial x}\right), \\
\Lambda_2(x, \eta_t) &= \frac{\partial V(x, \eta_t)'}{\partial x} f_1(x, \eta_t) + \frac{1}{2} \cdot f_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} f_2(x, \eta_t) + m(x, \eta_t)' m(x, \eta_t), \\
\Lambda_3(u, x, \eta_t) &= u' \left(I + \frac{1}{2} \cdot g_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} g_2(x, \eta_t)\right) u \\
&\quad + \frac{1}{2}\left(f_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} g_2(x, \eta_t) + \frac{\partial V(x, \eta_t)'}{\partial x} g_1(x, \eta_t)\right) u + \frac{1}{2} \cdot u' \left(g_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} f_2(x, \eta_t) + g_1(x, \eta_t)' \frac{\partial V(x, \eta_t)}{\partial x}\right).
\end{aligned} \quad (15)$$

Invoking $\partial^2 V(x, i)/\partial x^2 \geq 0$, $i \in \mathscr{D}$, we deduce that

$$\frac{1}{2}\left(-u' g_2(x, \eta_t)' + v' h_2(x, \eta_t)'\right) \frac{\partial^2 V(x, \eta_t)}{\partial x^2} \left(-g_2(x, \eta_t) u + h_2(x, \eta_t) v\right) \geq 0, \quad (16)$$

which shows that

$$\begin{aligned}
&\frac{1}{2}\left(u' g_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} h_2(x, \eta_t) v + v' h_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} g_2(x, \eta_t) u\right) \\
&\leq \frac{1}{2}\left(u' g_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} g_2(x, \eta_t) u + v' h_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} h_2(x, \eta_t) v\right).
\end{aligned} \quad (17)$$

Hence,

$$\begin{aligned}
E[V(x(T), \eta_T) - V(x_0, \eta_0) \mid \eta_0 = i] &\leq E\left\{\int_0^T \left[\Lambda_1(v, x, \eta_t) + \Lambda_2(x, \eta_t) + \Lambda_3(u, x, \eta_t) + \sum_{j=1}^{\infty} q_{ij} V(x, j) - \|z\|^2 \right.\right. \\
&\quad \left.\left. + \gamma^2 \|v\|^2 + \frac{1}{2}\left(u' g_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} g_2(x, \eta_t) u + v' h_2(x, \eta_t)' \frac{\partial^2 V(x, \eta_t)}{\partial x^2} h_2(x, \eta_t) v\right)\right] dt \mid \eta_0 = i\right\} \\
&= E\left\{\int_0^T \left[\overline{\Lambda}_1(v, x, \eta_t) + \Lambda_2(x, \eta_t) + \overline{\Lambda}_3(u, x, \eta_t) + \sum_{j=1}^{\infty} q_{ij} V(x, j) - \|z\|^2 + \gamma^2 \|v\|^2\right] dt \mid \eta_0 = i\right\},
\end{aligned} \quad (18)$$

where

$$\begin{aligned}
\overline{\Lambda}_1(v,x,\eta_t) &= v'\Bigg(-\gamma^2 I + h_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}h_2(x,\eta_t)\Bigg)v \\
&\quad + \frac{1}{2}\Bigg(f_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}h_2(x,\eta_t) + \frac{\partial V(x,\eta_t)'}{\partial x}h_1(x,\eta_t)\Bigg)v + \frac{1}{2} \\
&\quad \cdot v'\Bigg(h_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}f_2(x,\eta_t) + h_1(x,\eta_t)'\frac{\partial V(x,\eta_t)}{\partial x}\Bigg), \\
\overline{\Lambda}_3(u,x,\eta_t) &= u'\Bigg(I + g_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}g_2(x,\eta_t)\Bigg)u \\
&\quad + \frac{1}{2}\Bigg(f_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}g_2(x,\eta_t) + \frac{\partial V(x,\eta_t)'}{\partial x}g_1(x,\eta_t)\Bigg)u + \frac{1}{2} \\
&\quad \cdot u'\Bigg(g_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}f_2(x,\eta_t) + g_1(x,\eta_t)'\frac{\partial V(x,\eta_t)}{\partial x}\Bigg).
\end{aligned} \tag{19}$$

Applying Lemma 4 to $\overline{\Lambda}_1(v,x,\eta_t)$ and $\overline{\Lambda}_3(u,x,\eta_t)$, we conclude that

$$\begin{aligned}
\overline{\Lambda}_1(v,x,\eta_t) &= (v+F_1)'\Bigg(-\gamma^2 I + h_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}h_2(x,\eta_t)\Bigg)(v+F_1), \\
&\quad - \frac{1}{4}\Bigg(f_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}h_2(x,\eta_t) + \frac{\partial V(x,\eta_t)'}{\partial x}h_1(x,\eta_t)\Bigg)\cdot\Bigg(-\gamma^2 I \\
&\quad + h_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}h_2(x,\eta_t)\Bigg)^{-1} \\
&\quad \cdot\Bigg(h_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}f_2(x,\eta_t) + h_1(x,\eta_t)'\frac{\partial V(x,\eta_t)}{\partial x}\Bigg),
\end{aligned} \tag{20}$$

$$\begin{aligned}
\overline{\Lambda}_3(u,x,\eta_t) &= (u+F_2)'\Bigg(I + g_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}g_2(x,\eta_t)\Bigg)(u+F_2) \\
&\quad - \frac{1}{4}\Bigg(f_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}g_2(x,\eta_t) + \frac{\partial V(x,\eta_t)'}{\partial x}g_1(x,\eta_t)\Bigg)\cdot\Bigg(I \\
&\quad + g_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}g_2(x,\eta_t)\Bigg)^{-1} \\
&\quad \cdot\Bigg(g_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}f_2(x,\eta_t) + g_1(x,\eta_t)'\frac{\partial V(x,\eta_t)}{\partial x}\Bigg),
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
F_1 &= \frac{1}{2}\Bigg[-\gamma^2 I + h_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}h_2(x,\eta_t)\Bigg]^{-1} \\
&\quad \cdot\Bigg[h_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}f_2(x,\eta_t) + h_1(x,\eta_t)'\frac{\partial V(x,\eta_t)}{\partial x}\Bigg],
\end{aligned} \tag{22}$$

$$\begin{aligned}
F_2 &= \frac{1}{2}\Bigg[I + g_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}g_2(x,\eta_t)\Bigg]^{-1} \\
&\quad \cdot\Bigg[g_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}f_2(x,\eta_t) + g_1(x,\eta_t)'\frac{\partial V(x,\eta_t)}{\partial x}\Bigg].
\end{aligned} \tag{23}$$

Recalling (12) and substituting (20) and (21) into (18) yield that

$$\begin{aligned}
&E\left[V(x(T),\eta_T) - V(x_0,\eta_0) \mid \eta_0 = i\right] \\
&\quad \le E\Bigg\{\int_0^T \Bigg[(v+F_1)' \\
&\quad \cdot\Bigg(-\gamma^2 I + h_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}h_2(x,\eta_t)\Bigg) \\
&\quad \cdot(v+F_1) + (u+F_2)' \\
&\quad \cdot\Bigg(I + g_2(x,\eta_t)'\frac{\partial^2 V(x,\eta_t)}{\partial x^2}g_2(x,\eta_t)\Bigg)(u+F_2) \\
&\quad - \|z\|^2 + \gamma^2\|v\|^2\Bigg]dt \mid \eta_0 = i\Bigg\}.
\end{aligned} \tag{24}$$

In view of (12), for $x_0 = 0$, if we choose $u = u^* = -F_2$, then it follows from (24) that

$$E\left\{\int_0^T \|z\|^2\,dt \mid \eta_0 = i\right\} < \gamma^2 E\left\{\int_0^T \|v\|^2\,dt \mid \eta_0 = i\right\}. \quad (25)$$

Taking the limit for $T \longrightarrow \infty$ in the above, it is easy to show (6) by Definition 2.

Next, we remain to show that when $v = 0$, $u = u^*$, the trajectory of system (1) with any initial value $x(0) = x_0$ is EMSS. To this end, for $i \in \mathscr{D}$, let $\mathscr{L}_{u^*}$ be the infinitesimal operator of system (1) with $v = 0$, $u = u^*$; then

$$\begin{aligned}
\mathscr{L}_{u^*}V(x,i)\big|_{v=0} &= \frac{\partial V(x,i)'}{\partial x}\left(f_1(x,i)+g_1(x,i)u^*\right) \\
&\quad + \sum_{j=1}^{\infty} q_{ij}V(x,j) + \frac{1}{2}\left(f_2(x,i)+g_2(x,i)\right. \\
&\quad \left.\cdot u^*\right)' \frac{\partial^2 V(x,i)}{\partial x^2}\left(f_2(x,i)+g_2(x,i)u^*\right) \\
&= \frac{\partial V(x,i)'}{\partial x} f_1(x,i) + \frac{1}{2} f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) \\
&\quad + \sum_{j=1}^{\infty} q_{ij}V(x,j) + \frac{\partial V(x,i)'}{\partial x} g_1(x,i)u^* + \frac{1}{2} \\
&\quad \cdot f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)u^* \\
&\quad + \frac{1}{2}\left(g_2(x,i)u^*\right)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) \\
&\quad + \frac{1}{2}\left(g_2(x,i)u^*\right)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)u^*, \\
&= \frac{\partial V(x,i)'}{\partial x} f_1(x,i) + \frac{1}{2} f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) \\
&\quad + \sum_{j=1}^{\infty} q_{ij}V(x,j) + \Psi_{1i} + \Psi_{2i},
\end{aligned} \quad (26)$$

*where*

$$\begin{aligned}
\Psi_{1i} &= \frac{\partial V(x,i)'}{\partial x} g_1(x,i)u^* + \frac{1}{2} f_2(x, \\
&\quad i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)u^* + \frac{1}{2}\left(g_2(x,i)\right. \\
&\quad \left.\cdot u^*\right)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) = -\frac{1}{2} \\
&\quad \cdot \frac{\partial V(x,i)'}{\partial x} g_1(x,i)\left[I + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right]^{-1} \\
&\quad \cdot \left[g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i)\right. \\
&\quad \left. + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\right] - \frac{1}{4} f_2(x, \\
&\quad i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\left[I + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right]^{-1} \\
&\quad \cdot \left[g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\right] \\
&\quad - \frac{1}{4}\left[f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i) + \frac{\partial V(x,i)'}{\partial x} g_1(x,i)\right] \\
&\quad \cdot \left[I\right. \\
&\quad \left. + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right]^{-1} g_2(x, \\
&\quad i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i),
\end{aligned} \quad (27)$$

and

$$\begin{aligned}
\Psi_{2i} &= \frac{1}{2}\left(g_2(x,i)u^*\right)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)u^* \\
&= \frac{1}{8}\left[f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i) + \frac{\partial V(x,i)'}{\partial x} g_1(x,i)\right] \\
&\quad \cdot \left[I\right. \\
&\quad \left. + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right]^{-1} \cdot g_2(x, \\
&\quad i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\left[I + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right]^{-1} \\
&\quad \cdot \left[g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\right].
\end{aligned} \quad (28)$$

By direct calculations, one obtains that

$$\begin{aligned}
\Psi_{1i} &= -\frac{1}{2}\left[f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right. \\
&\quad \left. + \frac{\partial V(x,i)'}{\partial x} g_1(x,i)\right]\left[I\right. \\
&\quad \left. + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right]^{-1} \\
&\quad \cdot \left[g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i)\right. \\
&\quad \left. + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\right],
\end{aligned} \quad (29)$$

and

$$\begin{aligned}
\Psi_{2i} &\le \frac{1}{8}\left[f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\right. \\
&\quad \left. + \frac{\partial V(x,i)'}{\partial x} g_1(x,i)\right]\left[I\right.
\end{aligned}$$

$$+ g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\Bigg]^{-1} \cdot \Bigg[ g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\Bigg]. \tag{30}$$

Implementing (29) and (30) into (16) and taking into account (11) and (12), we have

$$\begin{aligned}\mathcal{L}_{u^*} V(x,i)\big|_{v=0} &\le \frac{\partial V(x,i)'}{\partial x} f_1(x,i) + \frac{1}{2} \cdot f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + \sum_{j=1}^{\infty} q_{ij} V(x,j) - \frac{3}{8}\Bigg[ f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i) \\ &+ \frac{\partial V(x,i)'}{\partial x} g_1(x,i)\Bigg] \cdot \Bigg[ I + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\Bigg]^{-1} \cdot \Bigg[ g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\Bigg] \\ &\le \frac{\partial V(x,i)'}{\partial x} f_1(x,i) + \frac{1}{2} \cdot f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + \sum_{j=1}^{\infty} q_{ij} V(x,j) - \frac{1}{4}\Bigg[ f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i) \\ &+ \frac{\partial V(x,i)'}{\partial x} g_1(x,i)\Bigg] \cdot \Bigg[ I + g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} g_2(x,i)\Bigg]^{-1} \cdot \Bigg[ g_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + g_1(x,i)' \frac{\partial V(x,i)}{\partial x}\Bigg] \\ &\le -m(x,i)' m(x,i) - \frac{1}{4}\Bigg[ f_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} h_2(x,i) + \frac{\partial V(x,i)'}{\partial x} h_1(x,i)\Bigg] \cdot \Bigg[ \gamma^2 I \\ &- h_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} h_2(x,i)\Bigg]^{-1} \cdot \Bigg[ h_2(x,i)' \frac{\partial^2 V(x,i)}{\partial x^2} f_2(x,i) + h_1(x,i)' \frac{\partial V(x,i)}{\partial x}\Bigg] \le -\|m(x,i)\|^2 \le -c_3\|x\|^2.\end{aligned} \tag{31}$$

Based on Lemma 5, it results that system (1) with $v = 0, u = u^*$ is EMSS. The proof is complete. □

*Remark 7.* It should be pointed that, in Theorem 6, if we take $\Upsilon_i < 0$ in (12), system (1) is internally stable (globally asymptotically stable in probability) even without condition (11). Then the controller defined in (13) is still an $H_\infty$ controller (globally asymptotically stable in probability).

Below, we will give an example to show the effectiveness of our above developed $H_\infty$ design method.

*Example 8.* Consider a one-dimensional stochastic nonlinear system with infinite Markov jumps and the parameters as follows:

$$\begin{aligned} f_1(x,i) &= -\frac{2ix}{i+1}, \\ g_1(x,i) &= \frac{1}{i+1}, \\ h_1(x,i) &= \frac{1}{4}, \\ f_2(x,i) &= \frac{x}{i+1}, \\ g_2(x,i) &= 1, \\ h_2(x,i) &= 1, \\ m(x,i) &= \frac{ix}{2(i+1)}. \end{aligned} \tag{32}$$

Let $\{\eta_t\}_{t\in\mathcal{R}_+}$ be a Poisson process with parameter $\lambda > 0$. It is obvious that $\{\eta_t\}_{t\in\mathcal{R}_+}$ is a homogeneous Markov process with the countably infinite state space, and its infinitesimal matrix $Q = (q_{ij})$ is given by $-q_{ii} = q_{i,i+1} = \lambda$ and $q_{ij} = 0, i \in \mathcal{D}, j \in \mathcal{D}/\{i, i+1\}$.

Assume the disturbance attenuation level $\gamma = \sqrt{2}$ and $\lambda = 1$. Then setting $V(x,i) = ix^2/2(i+1)$, we solve the coupled HJIs (12), it is easy to verify that the conditions of Theorem 6 are satisfied; thus, via Theorem 6, we have

$$\Upsilon_i = \frac{-15.5i^5 - 53.5i^4 - 30i^3 - 2.75i^2 + 8i + 2}{4(i+1)^2(i+2)(2i+1)} < 0. \tag{33}$$

So the $H_\infty$ controller is

$$u^*(x,i) = -\frac{ix}{(i+1)(i+2)}. \tag{34}$$

With the initial conditions $x_0 = 0.5$ and the exogenous disturbance $v(t) = e^{-t}\sin t$, Figure 1 shows the state response.

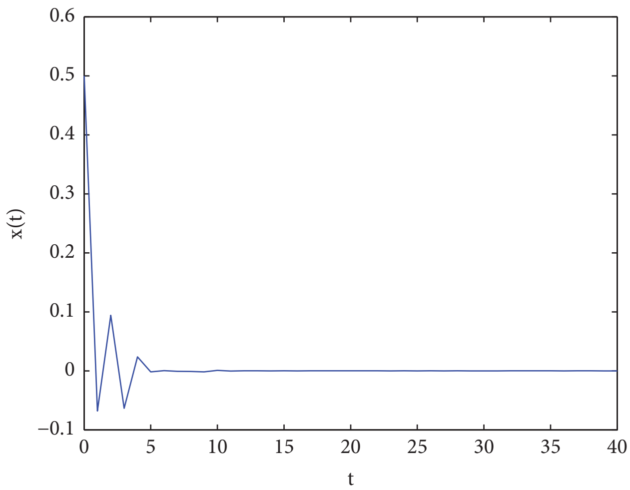


Figure 1: System state response in Example 8.

## 4. Conclusion

For a class of nonlinear stochastic systems with infinite Markov jumps and $(x, u, v)$-dependent noise, a sufficient condition for infinite horizon $H_\infty$ control problem has been obtained in terms of coupled HJIs, and the effectiveness of the proposed design method is demonstrated by a numerical example. There are some further research directions including the investigation on $H_2/H_\infty$ control and $H_\infty$ filter problems for nonlinear infinite Markov jump systems.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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