# Advance Selling with Part Prepayment and Consumer Returns 

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#### Abstract

Part prepayment scheme can induce more consumers to purchase the product in advance but may also lead to the increase in consumer returns. This study develops a two-period theoretical model to examine the interaction between the part prepayment scheme and the return policy and its effect on the retailer's profit. Our analysis yields the following insights. First, part prepayment scheme can help the retailer to increase the demand without sacrificing the advance selling price. Second, the prepayment proportion and the consumers' hassle cost of return have a negative cross effect on the retailer's profit, which indicates that when the consumers are allowed to preorder the product with a relatively low proportion of prepayment, the retailer should impose more restrictions to increase the consumers' hassle cost of return. Third, the prepayment proportion and the ex-ante product information perceived by consumers also have a negative cross effect on the retailer's profit, which indicates that the retailer should consider the degree of product information disclosure when adopting the part prepayment scheme. We also extend the model to incorporate the retailer's handling cost of consumer returns and find that it is beneficial for the retailer to require a full prepayment when the handling cost is relatively high.


## 1. Introduction

Advance selling refers to the practice where firms offer consumers the opportunity to order new soon-to-be-released products or services in advance and guarantee prompt delivery on release [1]. During the past 20 years, advance selling has been proven to be a useful strategy for sellers to improve product demand forecasts [2], to achieve price differentiation based on the timing of consumer purchases [3], and to exploit consumer valuation uncertainty [4]. Due to these advantages, advance selling has been extensively adopted in practice (e.g., Apple, JD.com and Amazon.com).

In recent years, part prepayment scheme, which allows consumers to preorder the product with part prepayment, is widely used in advance selling settings to promote consumer purchases. E-retailers in Tmall.com, for example, adopt the advance selling strategy before the annual Singles' Day promotion activity and allow consumers to preorder product with part prepayment in the advance period and to pay the rest at the Singles' Day. Similar practices can be also found in JD.com, Ctrip.com, etc. According to the discounted utility model, people often underestimate the effect of future
results [5], which means that the pain of future expenditure is less painful than that of current spending. In such a case, the part prepayment scheme will induce more consumers to preorder the product in advance, which will inevitably affect the amount and distribution of demands over the two selling periods (the advance period and the spot period) and correspondingly the profit.

However, when a small proportion of prepayment stimulates consumers with relatively low valuations to impulsively preorder the product, it may also increase the amount of consumer returns since consumers with relatively lower valuations may have a relatively higher probability of dissatisfaction with the product; i.e., their net surplus of purchase is relatively low. As a result, the part prepayment scheme may also increase the number of consumer returns. As we know, a main feature of advance selling is the existence of time separation between product purchase and experience, which will increase the uncertainty of consumers' perception of product value and thus reduce their purchase intention. A lenient return policy may help to eliminate consumers' concern about product value uncertainty and to promote consumers' purchase but may also incur certain costs, including shipping,
repackaging, and associated handling costs when massive returns occur. In such a case, the interaction between the part prepayment scheme and the return policy will further affect the retailer's product demand and consumer returns and corresponding costs.

The above considerations raise the following research questions: (1) How will the interaction between the part prepayment scheme and the return policy affect the retailer's decisions and profit? (2) When is it beneficial for the retailer to adopt an advance selling with part prepayment or full prepayment scheme?

Despite the broad attention to advance selling in the past decades, the above issues have not been well addressed in the literature. To the best of our knowledge, our paper is among the first to investigate the interaction between advance selling with part prepayment and the return policy. For this purpose, we develop a theoretical model to consider a retailer selling a product to the market over two periods: the advance period and the spot period. In the advance period, the retailer offers the consumers with an advance selling with part prepayment scheme; that is, the consumers can preorder the product with a proportion of payment in the advance period and pay the rest at the beginning of the spot period. The product is released at the beginning of the spot period. Consumers can choose to return the product at full refund if they are dissatisfied with the product. We examine the interaction between the part prepayment scheme and the consumer returns policies from two aspects: the consumers' hassle cost of return and the retailer's handling cost of consumer returns. Through theoretical analysis, we highlight the following observations:
(i) The advance selling price is decreasing with the prepayment proportion, which indicates that the part prepayment scheme can be a useful tool for retailers to increase selling price without inducing a large decrease in demand. Further, it is beneficial for the retailer to set a relatively low (high) advance selling price when the consumers' hassle cost of return is relatively small (large). This finding explains why the retailer offered a discount of up to $49 \%$ when HarryPotter books, which have relatively low hassle cost, were released on Amazon.com in advance, whereas the Harry-Potter Blu-Ray was available for preorder with only $15 \%$ off discount.
(ii) The prepayment proportion and the consumers' hassle cost of return have a negative cross effect on the retailer's profit, which means that the retailer cannot provide a too tolerant return policy if the consumers are allowed to preorder the product with a relatively low proportion of prepayment and vice versa. This finding also indicates that the impact of return policy on the retailer's profit depends on the prepayment scheme, which enriches the research results in the fields of consumer return and advance selling.
(iii) The prepayment proportion and the ex-ante product information perceived by consumers also have a negative cross effect on the retailer's profit, which indicates that when the extent of ex-ante product
information perceived by consumers is relatively small, it is more beneficial for the retailer to adopt a relatively high proportion of prepayment and vice versa. For example, the prepayment proportion of Huawei's smartphone P20 is about $20 \%$ when it was just released and decreases to $2 \%$ two months later when consumers had more information about the product.
(iv) We also extend the model to consider the interaction between the part prepayment scheme and the retailer's handling cost of consumer returns and find that it is beneficial for the retailer to set a small proportion of prepayment when the ex-ante product information perceived by consumers is high enough or otherwise to require full prepayment especially when the handling cost is relatively high. This finding also provides explanations for many practices in reality. For example, JD.com allows consumers to preorder products with zero prepayment, and the retailers on Tmall.com usually set a prepayment proportion less than $20 \%$ of the product selling price. However, for fashionable electronic products, like Apple products and MIUI, full prepayment is mandatory if consumers intend to preorder them.

The remainder of this study is organized as follows. Section 2 provides a survey of the relevant literature. Section 3 formulates the decision behavior of the retailer and consumers. Section 4 presents the optimal advance selling decisions under various conditions. Section 5 extends the model to examine the interaction between the handling cost of consumer returns and the part prepayment scheme. Section 6 concludes the study.

## 2. Literature Review

Although our research questions have not been well addressed in the literature, our research is related to certain streams of prior studies on advance selling and return policy. We will review the most relevant studies in this section.
2.1. Advance Selling: Improving Demand Forecasting and Achieving Price Discrimination. Since Fisher and Raman [6] firstly introduce the idea of advance booking in the retail industry, advance selling has become an important tool for quick response, demand forecasting [2], inventory management [7], and supply chain management [8]. There has been a rich body of research in this branch, and the reader is referred to Choi and Sethi [9] for a comprehensive review of the literature.

Another line of researchers adopts advance selling as a tool to achieve price discrimination, when consumers are heterogeneous. Representatively, Dana [10] introduces consumer heterogeneity in production valuation into the traditional advance selling model and shows that consumers with relatively lower valuations have an incentive to buy in advance. Mccardle et al. [11] investigate the benefit of an advance booking discount program in which the retailer provides price discount to the advance purchasing consumers.

Boyaci and Özer [12] study the capacity planning strategy for a seller who collects purchasing commitments of pricesensitive consumers. Li et al. [13] investigate the advance selling price strategies for oligopolists when considering the effect of product diffusion and the competitive market. Further, some researchers consider the fact that advance selling with price discrimination can lead to strategic consumer behavior, e.g., Fay and Xie [1]; Li and Zhang [3]; Lim and Tang [14]; Yu et al. [15].

While the benefits and price discrimination effects of advance selling have been well documented in the above studies, these researches mainly focus on the advance selling with full prepayment scheme and have been paid little attention to the advance selling with part prepayment scheme. Among the few researches, Tang and Ang [16] build a two-period theoretical model to compare the full advance selling strategy and the part prepayment strategy. However, they have not considered the interaction between the part prepayment scheme and the return policy, which is the main consideration of our work.

### 2.2. Consumer Returns: The Consumers' Hassle Cost and the

 Retailer's Handling Cost. With the fast development of electronic commerce, return policy serves as an important tool for online retailers to eliminate consumers' concern about product value uncertainty and has drawn much attention in recent years. Generally, consumer return occurs when a consumer dissatisfies with the received product [17], and it may cause considerable return hassle for consumers (e.g., filling out the return request, repacking the item, paying for the shipping cost, etc.) and the handling cost for retailers (e.g., restocking fee, inspection cost, and sorting cost) [18, 19]. Considering these costs, some related studies focus on the retailer's optimal return policy (e.g., a full or partial refund for consumers) by assuming an exogenous consumer return rate[20, 21] and consumer return quantity [22]. Some recent works have also examined the return policy in advance selling settings. Representatively, Nasiry and Popescu [23] examine the optimal refund for consumer returns when consumers' regret negatively affects the retailer's profit and demonstrate that the partial refund policy can mitigate regret. By considering consumers' strategic and opportunistic behaviors, Li et al. [24] derive the retailer's optimal pricing and return policies (full refund and partial refund) for advance selling fashionable products. Yu et al. [25] consider strategic consumer reactions to the bias in two selling periods under a full refund policy and investigate the effect of the bias on the seller's optimal decisions. However, these studies examine the optimal return policies in the advance selling with full prepayment settings and have paid no attention to the advance selling with part prepayment scheme and its interaction with the consumer return policy.This paper attempts to examine the optimal advance selling strategy in the presence of part prepayment and consumer returns. To the best of our knowledge, our paper is among the first to investigate the interaction between advance selling with part prepayment scheme and consumer returns. Some interesting results have also been obtained, which can
provide some valuable suggestions or explanations for the practices in reality.

## 3. Model Description

We consider an online retailer selling a new product to a group of consumers in the market over two periods: the advance period and the spot period. The product is assumed to be fashion-like and is released at the beginning of the spot period. Consumers are strategic and decide whether to preorder the product in advance or wait for the spot. In the advance period, the consumers are allowed to preorder the product with part prepayment and delay the rest to the beginning of the spot period. After receiving the product, consumers can choose either to keep the product or to return it with full refund if they are dissatisfied with the product. To examine the interaction between the advance selling with part prepayment scheme and the consumer return policy, we assume that the retailer is rational and self-interested. Notations used in this paper are summarized in Table 1.

Suppose that the market consists of a group of consumers with total mass normalized to one and each consumer demands at most one unit of the product during the two periods. Consumers' valuations on the product are uncertain; in particular, we assume that consumers' valuations on the product are independent and uniformly distributed in the consumer population: $v_{i} \sim U[0,1]$. Under advance selling settings, one important character is that consumers in the spot period can obtain more information from consumer reviews posted by advance consumers to facilitate their purchase. In such a case, consumers will strategically choose to preorder the product with part prepayment in the advance period or wait to buy until the spot period after acquiring more product information.

The timing sequence of game between the retailer and consumers is as follows. At the beginning of the advance period, the retailer preannounces the sale prices in the two periods $\left(p_{1}, p_{2}\right)$. Then consumers arrive and make their own purchasing decisions by comparing their utilities in two periods. If the consumer chooses to preorder the product in the advance period, he pays a part prepayment $\beta p_{1}$ in the advance period and delays the rest $(1-\beta) p_{1}$ to the spot period. At the beginning of the spot period, the product is released and the advance consumers will receive the product; meanwhile, the remaining consumers who choose to purchase in the spot period place orders and receive the product. After receiving the product, consumers decide whether to keep or return it. Figure 1 provides an illustration for the timing sequence of events.

Accordingly, the utility of consumer $i$ preordering the product in the advance period can be formulated as

$$
\begin{equation*}
u_{i 1}=\theta v_{i}-p_{1}+\delta(1-\beta) \tag{1}
\end{equation*}
$$

In (1), parameter $\theta(0 \leq \theta \leq 1)$ denotes the extent of ex-ante product information perceived by consumers in the advance period. The term $\delta(1-\beta)$ represents the utility generated by the part prepayment scheme to consumers, where $\delta(\delta \in[0,1])$ indicates consumer $i$ 's sensitivity to the part prepayment scheme. It is understandable that when

Table 1: Parameters and decision variables.

| Notation | Interpretation |
| :--- | ---: |
| $j$ | Superscript $j$ denotes the retailer's low, middle and high advance selling price strategy, respectively, $j \in[L, M, H]$. |
| $t$ | Subscript $t$ denotes the two periods, $t=1,2$. |
| $u_{i t}$ | Consumer $i$ 's utility of purchasing the product in period $t, u_{i t} \in[-1,2]$. |
| $v_{i}$ | Consumer $i$ 's valuation of the product, $v_{i} \sim U[0,1]$. |
| $\theta$ | Ex-ante product information perceived by consumers, $\theta \in[0,1]$. |
| $\delta$ | Consumers'sensitivity to the part-prepayment scheme, $\delta \in[0,1]$. |
| $\beta$ | Proportion of payment by consumers in the advance period, $\beta \in[0,1]$. |
| $p_{1}$ | Advance selling price of the product in the advance period. |
| $p_{2}$ | Spot selling price of the product in the spot selling period, $p_{2} \in[0,1]$. |
| $\lambda$ | Residual consumption value of the product after trial, $\lambda \in[0,1]$. |
| $h$ | A consumer's hassle cost when he chooses to return the product. |
| $c$ | Retailer's per unit handling cost of consumer returns. |
| $D_{t}$ | Demand for the product in each period, $t=1,2$. |
| $d_{t}$ | Number of consumer returns in each period, $t=1,2$. |



Figure 1: Illustration for the timing sequence of events.
$\beta=1$, the utility function in (1) is degenerated into the full prepayment scheme as described in Cachon and Swinney [26].

Consumers purchasing in the spot period can acquire more product information (e.g., through online reviews posted by advance consumers). Similar to Prasad et al. [27], the utility of consumer $i$ purchasing the product in the spot period is formulated as

$$
\begin{equation*}
u_{i 2}=v_{i}-p_{2} . \tag{2}
\end{equation*}
$$

Suppose that customers with nonnegative utilities will choose to buy the product. Meanwhile, consumers are strategic and decide whether and when to buy the product with the objective to maximize their own utilities. Let $u_{i 1}=u_{i 2}$, then we can obtain the indifference point $v_{i}=\bar{v}=\left(p_{2}-p_{1}+\delta(1-\right.$ $\beta)) /(1-\theta)$. For consumer $i$ if $u_{i 1} \geq 0$ and $u_{i 1} \geq u_{i 2}$ (that is, $v_{i} \geq v_{1}=\left(p_{1}-\delta(1-\beta)\right) / \theta$, and $\left.v_{i} \leq \bar{v}\right)$, he will preorder the product in the advance period; if $u_{i 2} \geq 0$ and $u_{i 2} \geq u_{i 1}$ (that is, $v_{i} \geq v_{2}=p_{2}$ and $v_{i} \geq \bar{v}$ ), he will buy the product in the spot period. Then we can obtain two scenarios:
(1) If $v_{1} \leq \bar{v}$ (i.e., $p_{1} \leq \theta p_{2}+\delta(1-\beta)$ ), we obtain that $v_{2} \leq \bar{v}$. Thus
(i) if $\bar{v}>1$ (i.e., $\left.p_{1}<p_{2}+\delta(1-\beta)-(1-\theta)\right)$, consumers with valuations on the interval $\left[v_{1}, 1\right]$ will buy the product in the advance period, and nobody will buy the product in spot period. Hence the advance demand equals $1-v_{1}$, and the spot demand equals zero;
(ii) if $\bar{v} \leq 1$ (i.e., $p_{2}+\delta(1-\beta)-(1-\theta) \leq p_{1} \leq$ $\left.\theta p_{2}+\delta(1-\beta)\right)$, consumers with valuations on the interval $\left(v_{1}, \bar{v}\right]$ will buy the product in the advance period, and those with valuations on the interval $[\bar{v}, 1]$ will buy the product in the spot period. Hence the advance demand equals $\bar{v}-v_{1}$, and the spot demand equals $1-\bar{v}$.
(2) If $v_{1}>\bar{v}$ (i.e., $\left.p_{1}>\theta p_{2}+\delta(1-\beta)\right)$, we obtain that $v_{2}>\bar{v}$. In this scenario, nobody will buy the product in the advance period, and consumers with valuations on the interval $\left[v_{2}, 1\right]$ will buy the product in the spot period. Hence the advance demand equals zero, and the spot demand equals $1-v_{2}$.
Denote by $p_{1 A}=p_{2}+\delta(1-\beta)-(1-\theta), p_{1 B}=\theta p_{2}+\delta(1-$ $\beta$ ). Combining the above two scenarios, we can formulate the
retailer's demands in the advance period $\left(D_{1}\right)$ and spot period $\left(D_{2}\right)$ as follows:

$$
\begin{align*}
& D_{1} \\
& = \begin{cases}1-\frac{p_{1}-\delta(1-\beta)}{\theta}, & \text { if } p_{1} \leq p_{1 A} \\
\frac{p_{2}-p_{1}+\delta(1-\beta)}{1-\theta}-\frac{p_{1}-\delta(1-\beta)}{\theta}, & \text { if } p_{1 A}<p_{1} \leq p_{1 B} \\
0, & \text { if } p_{1}>p_{1 B}\end{cases}  \tag{3}\\
& D_{2}= \begin{cases}0, & \text { if } p_{1} \leq p_{1 A} \\
1-\frac{p_{2}-p_{1}+\delta(1-\beta)}{1-\theta}, & \text { if } p_{1 A}<p_{1} \leq p_{1 B} \\
1-p_{2}, & \text { if } p_{1}>p_{1 B}\end{cases} \tag{4}
\end{align*}
$$

To facilitate the analysis and avoid trial cases, we make the following assumption throughout the paper to confine our analysis in the situation when the advance selling price is nonnegative.

Assumption 1. $p_{1 A} \geq 0$.
Meanwhile, the term $\delta(1-\beta)-(1-\theta)$ in the formulation of $p_{1 A}$ also represents the difference between the utility brought by the part prepayment scheme and the maximal value of additional product information generated in the advance period. The meaning is that a consumer will tend to delay his purchase until the spot period if his valuation $v_{i}$ is relatively high (e.g., the term $(1-\theta) v_{i}>\delta(1-\beta)$ means that the value of additional product information for consumer $i$ is larger than the utility brought by the part prepayment scheme).

Online markets allow buyers and retailers to overcome geographical and temporal barriers to buy products anytime, anywhere. However, online markets still face a barrier in perfectly conveying a product's characteristics [28]. Following Davis et al. [18] and Li et al. [24], we assume that consumers' valuations on the purchased product consist of two parts: trial value $(1-\lambda) v_{i}$ and residual consumption value $\lambda v_{i}(0<\lambda<1)$. If the residual consumption value is extremely low, consumers may choose to return the product even if the product is in a good shape. The time and effort needed by consumers to return the item (e.g., filling out the return request, repacking the item, and paying for the return shipping cost) are called the hassle cost [18]. We consider a full refund policy from the retailer when a consumer chooses to return the product. In specific, a consumer will choose to return the product if the residual consumption value of the product is lower than the refund minus the hassle cost; that is,

$$
\begin{equation*}
\lambda v_{i}<p_{t}-h \tag{5}
\end{equation*}
$$

In (5), the term $h$ represents the consumer's hassle cost of product return.

Denote by $d_{t}$ the amount of consumer returns in period $t$ ( $t=1,2$ ). Then combining equations (3), (4), and (5) we can derive that

$$
\begin{align*}
& d_{1}= \begin{cases}\left(\frac{p_{1}-h}{\lambda}-\frac{p_{1}-\delta(1-\beta)}{\theta}\right)^{+} & \text {if } p_{1} \leq p_{1 A} \\
\left(\frac{p_{1}-h}{\lambda}-\frac{p_{1}-\delta(1-\beta)}{\theta}\right)^{+} & \text {if } p_{1 A} \leq p_{1} \leq p_{1 B} \\
0 & \text { if } p_{1} \geq p_{1 B}\end{cases}  \tag{6}\\
& d_{2} \\
& = \begin{cases}0 & \text { if } p_{1} \leq p_{1 A} \\
\left(\frac{p_{2}-h}{\lambda}-\frac{p_{2}-p_{1}+\delta(1-\beta)}{1-\theta}\right)^{+} & \text {if } p_{1 A} \leq p_{1} \leq p_{1 B} \\
\frac{p_{2}-h}{\lambda}-p_{2} & \text { if } p_{1} \geq p_{1 B}\end{cases} \tag{7}
\end{align*}
$$

For ease of analyses, we normalize the retailer's procurement cost to zero and assume that the selling price in the spot period is exogenous and mainly determined by the competition in the market (i.e., the tag price). The retailer makes his advance selling price decision to maximize his overall profit over the two periods. In this section, we also assume that the returned product can be sold again with neglect-able storage cost, and in Section 5 we will extend the model to examine the impact of the retailer's handling cost of consumer returns on his profit. Then the retailer's optimization problem can be formulated as

$$
\begin{equation*}
\max \prod_{p_{1}}\left(p_{1}\right)=p_{1}\left(D_{1}-d_{1}\right)+p_{2}\left(D_{2}-d_{2}\right) . \tag{8}
\end{equation*}
$$

In the following section, we will further analyze the retailer's optimal pricing decision based on (8).

## 4. Analysis

Following (3) and (4), the retailer's pricing strategies have three scenarios.

S1: Low Advance Selling Price Strategy (LAP). That is, if the retailer sets his advance selling price lower than $p_{1 A}$ (i.e., $p_{1} \leq$ $p_{1 A}$ ), consumers in the market are attracted by the low price and only buy the product in the advance period.

S2: Middle Advance Selling Price Strategy (MAP). That is, if the retailer sets his advance selling price between $p_{1 A}$ and $p_{1 B}$ (i.e., $p_{1 A}<p_{1} \leq p_{1 B}$ ), demands occur in both two periods.

S3: High Advance Selling Price Strategy (HAP). That is, if the retailer sets his advance selling price higher than $p_{1 B}$ (i.e., $p_{1}>p_{1 B}$ ), demand occurs only in the spot period. In this case, the demand in the spot period satisfies $D_{2}^{H}=1-p_{2}$, and the number of consumer returns satisfies $d_{2}^{H}=\left(p_{2}-h\right) / \lambda-p_{2}$.

In the following subsections, we first examine the optimal decisions under each type of pricing strategy and then further explore the overall optimal strategy for the retailer.
4.1. Low Advance Selling Price Strategy. In the LAP scenario, the demand and consumer return only occur in the advance period. Following (3) and (6) we obtain $D_{1}^{L}=1-\left(p_{1}-\delta(1-\right.$ $\beta)) / \theta$ and $d_{1}^{L}=\left(\left(p_{1}-h\right) / \lambda-\left(p_{1}-\delta(1-\beta)\right) / \theta\right)^{+}$. Figure 2 provides an illustration for this scenario.


Figure 2: Market situation under the low advance selling pricing strategy.

With the objective to maximize his profit, the retailer make his optimal decision following Theorem 2.

Theorem 2. In the LAP scenario (i.e., $0 \leq p_{1} \leq p_{1 A}$ ), the optimal advance selling price $\left(p_{1}^{L^{*}}\right)$ satisfies
(i) If $h \leq h_{1}$ and $h \leq H_{1}, p_{1}^{L^{*}}=(\lambda+h) / 2$;
(ii) If $h_{1} \leq h \leq H_{2}, p_{1}^{L^{*}}=p_{1 A}$,
where $h_{1}=2\left(p_{2}+\delta(1-\beta)-1+\theta\right)-\lambda, H_{1}=(2 \delta(1-\beta) \lambda+\lambda \theta-$ $\left.\lambda^{2}\right) /(\theta+\lambda)$, and $H_{2}=\left((\theta-\lambda) p_{2}+\theta \beta(1+\delta)-(\theta-\lambda)(1-\theta)\right) / \theta$.

All proofs are provided in Appendix.
The conditions $h \leq h_{1}$ and $h \leq H_{1}$ in Theorem 2 are to ensure the nonnegativity of consumer returns. It can be observed from Theorem 2 that when the consumer's hassle cost $h$ is less than a particular threshold, i.e., $h \leq \min \left\{h_{1}, H_{1}\right\}$, the retailer needs to trade-off the marginal profit and the actual sales amount $\left(D_{1}^{L}-d_{1}^{L}\right)$. As the hassle cost increases, i.e., $h_{1} \leq h \leq H_{2}$, it is optimal for the retailer to raise the advance selling price to the right point price on the LAP region, which means that in this case, the retailer can increase his profit by increasing the advance selling price. This is because, with higher hassle cost, consumer returns will decrease which in turn increases the actual sales amount $\left(D_{1}^{L}-d_{1}^{L}\right)$.

The following corollaries further examine the impact of exogenous parameters on the retailer's optimal decisions.

Corollary 3. $\partial p_{1}^{L^{*}} / \partial \beta \leq 0, \partial p_{1}^{L^{*}} / \partial h \geq 0$.
Corollary 3 shows that the advance selling price is decreasing with the prepayment proportion. This finding is meaningful, which indicates that the part prepayment scheme can be a useful tool for retailers to increase selling price without inducing a large decrease in demand. It is intuitive that when the retailer increases the selling price, the consumers' utility will decrease and so does the retailer's total demand. Hence the part prepayment policy can be used to offset the negative effect of price increase on the demand. This finding provides a valuable explanation for the reason why the part prepayment scheme is extensively adopted in reality.

Corollary 3 shows that the advance selling price is increasing in the consumer's hassle cost of product return. To gain competitive advantages and to reduce consumers' uncertainty about product value, many retailers in reality allow consumers to return the product with full refund within a certain period. Corollary 3 also indicates that the retailer
can reduce the amount of consumer returns by increasing the consumers' hassle cost of product return without sacrificing the refund. This observation provides explanations for many practices in reality. For example, many retailers in Tmall.com allow consumers to return the product with full refund within 7 days but require that the returned product should be originally packaged or be combined with the accessories like invoices, free gifts, etc.

The following corollary further demonstrates the properties of the demand and consumer returns with respect to the part prepayment scheme and the hassle cost.

Corollary 4. $D_{1}^{L^{*}}$ and $d_{1}^{L^{*}}$ are decreasing in $\beta$ and $h$. Further, $\left|\partial D_{1}^{L^{*}} / \partial \beta\right| \leq\left|\partial d_{1}^{L^{*}} / \partial \beta\right|$ and $\left|\partial D_{1}^{L^{*}} / \partial h\right| \leq\left|\partial d_{1}^{L^{*}} / \partial h\right|$.

The above corollary shows that the amount of consumer returns will decrease faster than the demand in the prepayment proportion and the consumer's hassle cost. This indicates that an extremely lower level of prepayment proportion or an extremely tolerant return policy may incur substantial amount of consumer returns. In such a case, when consumer returns cause relatively large cost for the retailer, it may not be always beneficial for the retailer to decrease the prepayment proportion or to offer a tolerant return policy to increase the product demand. We will extend the current model to further discuss the impact of the retailer's handling cost of consumer returns on the pricing, demand, and profit in Section 5.
4.2. Middle Advance Selling Price Strategy. In the MAP scenario, the demand and consumer returns occur in both two periods. Figure 3 provides an illustration for the demands and returns under this scenario.

Following (3) to (6) we obtain that $D_{1}^{M}=\left(p_{2}-p_{1}+\delta(1-\right.$ $\beta)) /(1-\theta)-\left(p_{1}-\delta(1-\beta)\right) / \theta, D_{2}^{M}=1-\left(p_{2}-p_{1}+\delta(1-\right.$ $\beta)) /(1-\theta), d_{1}^{M}=\left(\left(p_{1}-h\right) / \lambda-\left(p_{1}-\delta(1-\beta)\right) / \theta\right)^{+}$, and $d_{2}^{M}=\left(\left(p_{2}-h\right) / \lambda-\left(p_{2}-p_{1}+\delta(1-\beta)\right) /(1-\theta)\right)^{+}$, respectively. Hence the optimal advance selling decision under the MAP scenario follows Theorem 5.

Theorem 5. In the MAP scenario (i.e., $p_{1 A}<p_{1} \leq p_{1 B}$ ), the optimal advance selling price $p_{1}^{M *}$ satisfies
(i) if $h \leq h_{2}$ and $h \leq H_{2}, p_{1}^{M *}=p_{1 A}$;
(ii) if $h_{2} \leq h \leq h_{3}$ and $h \leq \min \left\{H_{3}, H_{4}\right\}, p_{1}^{M *}=(h(1-$ $\left.\theta)+\lambda\left(p_{2}+\delta(1-\beta)\right)\right) / 2(1+\lambda-\theta)$;
(iii) if $h_{3} \leq h \leq(1-\lambda) p_{2}, p_{1}^{M *}=p_{1 B}$,


Figure 3: Market situation under the MAP scenario.
with $h_{2}=\left((2(1-\theta)+\lambda)\left(p_{2}+\delta(1-\beta)\right)-2(1+\lambda-\theta)(1-\theta)\right) /(1-\theta)$, $h_{3}=\left((2 \theta(1-\theta)+\lambda(2 \theta-1)) p_{2}+(2(1-\theta)+\lambda) \delta(1-\beta)\right) /(1-\theta)$, $H_{2}=\left((\theta-\lambda) p_{2}+\theta \beta(1+\delta)-(\theta-\lambda)(1-\theta)\right) / \theta$, and $H_{3}=$ $\left(p_{2} \lambda(\theta-\lambda)+\delta(1-\beta) \lambda(2-\theta+\lambda)\right) /(\theta(1-\theta)+\lambda(1+\theta))$.

In Theorem 5, the conditions $h \leq \min \left\{H_{3}, H_{4}\right\}$ and $h \leq H_{2}$ are to ensure the nonnegativity of the amount of consumers returns in each period. It can be observed from Theorem 5 that when $h \leq h_{2}$ or $h \geq h_{3}$, the optimal pricing decisions will degenerate into those in the LAP or HAP scenario. When $h_{2} \leq h \leq h_{3}$, there are both demands in the advance and spot periods. The properties of the pricing decisions with respect to the part prepayment scheme and consumer returns in this case are similar to those in the LAP strategy. Additionally, some new properties are also found as described in the following corollaries.

Corollary 6. When $\theta(-5+3 \theta-2 \lambda)+2+\lambda<0, \partial d_{T}^{M *} / \partial \beta>0$; otherwise, $\partial d_{T}^{M *} / \partial \beta \leq 0$.

Intuitively, we might expect that the total amount of consumer returns will decrease in the prepayment proportion since a relatively low prepayment will induce more consumers with relatively low valuations to buy the product, which may lead to more consumer returns (i.e., $\partial d_{T}^{M *} / \partial \beta \leq 0$ ). However, as shown by Corollary 6, when the total amount of consumer returns in the MAP strategy it may even increase in the prepayment proportion under some situations (i.e., $\left.\partial d_{T}^{M *} / \partial \beta>0\right)$. In specific, the condition $\theta(-5+3 \theta-2 \lambda)+2+$ $\lambda<0$ indicates that the extent of ex-ante product information perceived by consumers is relatively high (e.g., $\theta \geq 2 / 3$ ) or the residual value of the returned product is relatively large. In this case, a relatively large prepayment proportion may increase the consumer returns. This is because that the increase of prepayment proportion will lead more consumers to wait until the spot period. However, the price in the spot period is relatively high, which will ultimately lead to more consumer returns.

Corollary 7. The retailer's profit in the MAP scenario has following properties:
(1) $\partial^{2} \prod^{M *} / \partial \beta \partial h \leq 0$;
(2) When $p_{1}^{M *}=p_{1 A}, \partial^{2} \Pi^{M *} / \partial \beta \partial \theta>0$; otherwise, $\partial^{2} \Pi^{M *} / \partial \beta \partial \theta \leq 0$.

Corollary 7(1) further shows the cross effect of the part prepayment scheme and the consumer's hassle cost on the retailer's profit. This finding indicates that the part prepayment scheme and the hassle cost have negative cross effect on the retailer's profit. That is, the retailer cannot provide a too tolerant return policy if the consumers are allowed to preorder the product with a relatively low proportion of prepayment and vice versa. The reason is understandable that when the consumers are allowed to preorder the product with an extremely low proportion of prepayment, consumers with extremely low valuations will also preorder the product, which may greatly increase the amount of consumer returns if the return policy is too tolerant. This finding differs from the findings in prior studies such as Xie and Gerstner [29], which show that it will be more superior for the retailer to set sufficiently small hassle cost of return when advance selling with the buy-back clause; hence it enriches the impact of hassle cost of return on advance selling strategies.

Corollary 7(2) shows that when $h_{2} \leq h \leq h_{3}$, the part prepayment scheme and the extent of ex-ante product information perceived by consumers also have negative cross effect on the retailer's profit. This finding also provides valuable implications for managers in reality. Specifically, if the retailer intends to use the part prepayment scheme to increase demand without sacrificing the retail price (as described by Corollary 3); he can choose to offer more product information to reduce the consumers' valuation uncertainty about the product. Similarly, if the consumers are required to preorder the product with full prepayment, the retailer can strategically choose to offer less product information before purchase. This finding also coincides with the practices in reality. Take the smartphone, for instance, Huawei's P20 smartphone was released on March 27, 2018, and the selling price is RMB 4988. Meanwhile, consumers can pay RMB 999 to preorder the P20 smartphone in advance at Tmall.com. However, consumers only need to pay RMB 100 to preorder the P20 smartphone in the 'June 18 promotion' activity, about two months after the release of the P20 smartphone. Overall, when the Huawei's P20 was just released, the proportion of the prepayment is about $20 \%$, which decreases to $2 \%$ two months later. Hence one explanation for the Huawei's practice is that, before the release of the smartphone, the extent of ex-ante product information perceived by consumers is relatively low, hence Huawei adopts a relatively high proportion of prepayment. With the increase of extent of ex-ante product information due to information sharing between consumers [30], Huawei decreases the prepayment proportion.


Figure 4: Illustrations for the retailer's advance selling pricing strategy.
4.3. Comparisons and Managerial Insights. In this subsection, we further explore the overall optimal pricing strategies based on the above analyses, as described in the following proposition.

Proposition 8. The optimal advance selling strategy for the retailer $\left(p_{1}^{*}\right)$ satisfies the following:
(1) If $h \leq h_{2}$ and $h \leq \min \left\{H_{1}, H_{2}\right\}$, it is beneficial for the retailer to choose the LAP strategy. In this case, if $h \leq h_{1}, p_{1}^{*}=(\lambda+h) / 2 ;$ if $h_{1} \leq h \leq h_{2}, p_{1}^{*}=p_{1 A}$.
(2) If $h_{2} \leq h \leq h_{3}$ and $h \leq \min \left\{H_{3}, H_{4}\right\}$, it is beneficial for the retailer to choose the MAP strategy, and the optimal advance selling price satisfies $p_{1}^{*}=\left(h(1-\theta)+\lambda\left(p_{2}+\right.\right.$ $\delta(1-\beta))) / 2(1+\lambda-\theta)$.
(3) If $h_{3} \leq h \leq(1-\lambda) p_{2}$, it is beneficial for the retailer to choose the HAP strategy, and $p_{1}^{*}=p_{1 B}$.

In Proposition 8, the conditions $h \leq \min \left\{H_{1}, H_{2}\right\}$ and $h \leq \min \left\{H_{3}, H_{4}\right\}$ are provided to avoid the trivial cases when the number of consumer returns is negative. Overall, Proposition 8 provides an illustration for the retailer's pricing strategies with respect to the consumers' hassle cost and indicates it is beneficial for the retailer to set a relatively low (high) advance selling price when the consumers' hassle cost of return is relatively small (large). This finding coincides with the practices in reality. For example, when Harry-Potter books, which have relatively low hassle cost, were released on Amazon.com in advance, the retailer offered a discount of up to $49 \%$; in contrast, the Harry-Potter Blu-Ray was available for preorder with only $15 \%$ off discount [24].

Specifically, when the consumers' hassle cost of return is relatively low (e.g., $h \leq h_{2}$ ), it will lead to more consumer returns. As a result, it may be beneficial for the retailer to adopt the LAP strategy (i.e., decreasing the advance selling
price) to reduce consumer returns. When the consumers' hassle cost of return is extremely high (e.g., $h_{3} \leq h \leq$ ( $1-$ d) $p_{2}$ ), it is beneficial for the retailer to set an extremely high advance selling price to lead all consumers to purchase in the spot period. This is understandable that when the amount of consumer returns is low (e.g., hassle cost of return is high), retailer can increase his profit by increasing the advance selling price. This finding also differs from the findings in prior studies such as Shulman et al. [31] which indicates that the retailer's selling price is decreasing in the hassle cost of consumer return.

Notably, the conditions in Proposition 8 can also be transformed to those regarding $\beta$. Specifically, when the prepayment proportion $\beta$ is relatively low, it is more beneficial to adopt advance selling; whereas when $\beta$ is extremely high, it is more beneficial for the retailer to set a high advance selling price to lead more consumers to purchase in the spot period. To better illustrate these results, a numerical example is conducted as shown in Figure 4 by setting $\delta=0.5, \theta=0.55$, $p_{2}=0.4$, and $\lambda=0.5$.

As shown in Figure 4, it is beneficial for the retailer to set $p_{1}^{*}=(\lambda+h) / 2$ in the area of Segment 1 ; in the area of Segment 2 , the optimal selling price follows $p_{1}^{*}=p_{1 A}$; in Segment 3, the optimal price follows $p_{1}^{*}=\left(h(1-\theta)+\lambda\left(p_{2}+\delta(1-\beta)\right)\right) / 2(1+$ $\lambda-\theta)$; and in Segment 4, the optimal price follows $p_{1}^{*}=p_{1 B}$.

## 5. Extension and Discussion on the Impact of Handling Cost

In previous sections, we normalize the retailer's handling cost of consumer returns to zero to focus on the interaction between the advance selling with part prepayment scheme and the consumer's hassle cost of return. In this section, we extend the model to examine the impact of the retailer's handling cost of returns on the retailer's part prepayment
advance selling strategy and profit. We use the superscript " $R$ " to represent the variables and parameters in this situation. To facilitate the theoretical analysis, we normalize the consumer's hassle cost of returns to zero in the following analyses. Denote by $c$ the retailer's handling cost per each unit of consumer return. In such a case, the retailer's profit function follows:

$$
\begin{align*}
\max _{p_{1}} \prod^{R}\left(p_{1}^{R}\right)= & p_{1}^{R}\left(D_{1}^{R}-d_{1}^{R}\right)+p_{2}^{R}\left(D_{2}^{R}-d_{2}^{R}\right)  \tag{9}\\
& -c\left(d_{1}^{R}+d_{2}^{R}\right)
\end{align*}
$$

Then we can obtain the following theorem.
Theorem 9. When considering the handling cost of consumer returns, the retailer's optimal advance selling decision ( $p_{1}^{R *}$ ) follows:
(i) if $\beta_{1} \leq \beta \leq \beta_{2}$ and $B_{2} \leq \beta \leq B_{1}, p_{1}^{R *}=\left(\left(p_{2}+\delta(1-\right.\right.$ $\beta)) \theta \lambda+c(\theta(\theta-1)+\lambda(1-2 \theta))) / 2 \theta(1-\theta+\lambda)$;
(ii) if $\beta \leq \beta_{1}$ and $\beta \leq B_{3}, p_{1}^{R *}=p_{1 A}$;
(iii) if $\beta \geq \beta_{2}, p_{1}^{R *}=p_{1 B}$,
where $\beta_{1}=\left((1-\theta) \theta\left(c+2\left(p_{2}+\delta+\theta-1\right)\right)\right)+\lambda(c(2 \theta-1)+$ $\left.\left.\theta\left(p_{2}+\delta-2(1-\theta)\right)\right)\right) / \delta \theta(2(1-\theta)+\lambda)$ and $\beta_{2}=((1-\theta) \theta(c+$ $\left.\left.2\left(\theta p_{2}+\delta\right)\right)+\lambda\left(c(2 \theta-1)+p_{2} \theta(2 \theta-1)+\delta \theta\right)\right) / \delta \theta(2(1-\theta)+\lambda)$.

In Theorem 9, the conditions $B_{2} \leq \beta \leq B_{1}$ and $\beta \leq B_{3}$ are provided to avoid trivial cases when the amount of consumer returns is negative. Specifically, $B_{1}=(c(\theta-\lambda)(\theta(-1+\theta-2 \lambda)+$ $\left.\lambda)+\theta \lambda\left(\delta(2-\theta+\lambda)+p_{2}(\theta-\lambda)\right)\right) / \delta \theta \lambda(2-\theta-\lambda), B_{2}=\left(\left(\left(2 c+p_{2}+\right.\right.\right.$ $\left.\delta) \theta-c) \lambda^{2}-2 p_{2}(1-\theta)^{2} \theta+(c+2 \delta)(1-\theta) \theta \lambda\right) / \delta \theta \lambda(2-2 \theta+\lambda)$, and $B_{3}=1+\left(1-\theta-p_{2}\right)(\lambda-\theta) / \delta \theta$.

It can be observed from Theorem 9 that when considering the retailer's handling cost of consumer returns, the optimal advance selling strategy and corresponding pricing decision are heavily dependent on the handling cost and the part prepayment scheme. The following corollaries further examine the properties of the retailer's pricing decision and profit with respect to the part prepayment scheme and the handling cost of consumer returns.

Corollary 10. The retailer's optimal advance selling price $p_{1}^{R *}$ satisfies the following properties:
(1) $\partial p_{1}^{R *} / \partial \beta \leq 0$;
(2) If $\theta<\bar{\theta}, \partial p_{1}^{R *} / \partial c<0$; otherwise $\partial p_{1}^{R *} / \partial c \geq 0$, with $\bar{\theta}=\left(2 \lambda+1-\sqrt{4 \lambda^{2}+1}\right) / 2$.

Corollary 10 (1) shows that the retailer's advance selling price is decreasing in the proportion of prepayment in the advance period, which is consistent with the results in Corollary 3. Notably, Corollary 10 (2) shows that when the consumers receive relatively few ex-ante product information in the advance period (i.e., $\theta<\bar{\theta}$ ), the optimal advance selling price is decreasing in the retailer's handling cost of consumer returns. This is because that, when the consumers receive
relatively few ex-ante product information in the advance period, they tend to buy the product in the spot period, as a result, the demand in the advance period will decrease. In this case, the retailer may choose to increase the advance demand for the product and to reduce the consumer returns by reducing the advance selling price, especially when the handling cost is relatively high.

When $\theta \geq \bar{\theta}$, the advance selling price is increasing in the handling cost. This is understandable that in this case the retailer has to increase the advance selling price to compensate the handling cost caused by consumer returns. This finding is similar to that presented by Xiao and Shi [32], who demonstrate that the retailer's optimal price depends on the handling cost from the perspective of supply chain information asymmetry.

The following propositions further demonstrate the impact of the part prepayment scheme and the handling cost on the retailer's profit.

Proposition 11. When $\beta_{1} \leq \beta \leq \beta_{2}$ and $B_{2} \leq \beta \leq B_{1}$, there exists $\bar{\beta}$ satisfying that if $\beta<\bar{\beta}, \partial \prod^{R *} / \partial \beta<0$, otherwise $\partial \prod^{R *} / \partial \beta \geq 0$, with $\bar{\beta}=\left(\left(p_{2}+\delta\right) \theta \lambda-c(2+\theta(-5+3 \theta-\right.$ $2 \lambda)+\lambda)) / \delta \lambda \theta$.

This observation indicates that it is more beneficial for the retailer to set an extremely low prepayment proportion or to require full prepayment in the advance period. This finding provides explanations for many practices in reality. For example, JD.com allows consumers to preorder products with zero prepayment, and the retailers on Tmall.com usually set a prepayment proportion less than $20 \%$ of the product selling price. However, the fashionable electronic products, like Apple products and MIUI, should be preordered with full prepayment.

To further explore when it is more beneficial to set an extremely low prepayment proportion or to require full prepayment, we draw the following finding.

Proposition 12. Define $\Delta \Pi=\prod^{R}(\beta=1)-\prod^{R}(\beta=0)$, then $\Delta \prod$ has the following properties:
(i) If $2+\theta(-5+3 \theta-2 \lambda)+\lambda<0, \Delta$ П is decreasing in $c$ and $\Delta \prod<0$.
(ii) If $2+\theta(-5+3 \theta-2 \lambda)+\lambda \geq 0, \Delta \prod$ is increasing in $c$, and when $c>\widetilde{c}, \Delta \prod>0$, with $\widetilde{c}=\left(2 p_{2}+\delta\right) \theta \lambda / 2(2+$ $\theta(-5+3 \theta-2 \lambda)+\lambda)$.

Proposition 12 shows that when the consumers can receive enough product information in the advance period (i.e., $\theta$ is relatively large), it is more beneficial for the retailer to adopt the free reservation scheme or to set a relatively small prepayment proportion. This is because that when $\theta$ is large enough, the consumers have more confidence to buy in the advance period, and thus the free reservation scheme can greatly increase the total demand. When $\theta$ is relatively small and the handling cost is relatively high, it is more beneficial for the retailer to adopt the full prepayment scheme. This is

Table 2: Optimal decisions and relevant expressions in the LAP scenario.

| $h$ | $h \leq \min \left\{h_{1}, H_{1}\right\}$ | $h_{1} \leq h \leq H_{2}$ |
| :--- | :---: | :---: |
| $p_{1}^{L^{*}}$ | $p_{1}^{L E}$ | $p_{1 A}$ |
| $\prod^{L *}$ | $\frac{(h+\lambda)^{2}}{4 \lambda}$ | $\left(p_{2}+\theta+\delta(1-\beta)-1\right)\left(1-\frac{-h+p_{2}+\theta+\delta(1-\beta)-1}{\lambda}\right)$ |
| $D_{1}^{L^{*}}$ | $1-\frac{h+\lambda-2 \delta(1-\beta)}{2 \theta}$ | $\frac{1-p_{2}}{\theta}$ |
| $d_{1}^{L^{*}}$ | $\frac{\lambda(\theta-\lambda+2 \delta(1-\beta))-h(\lambda+\theta)}{2 \theta \lambda}$ | $\frac{\left(1-\theta-p_{2}\right) \lambda+\theta\left(p_{2}+\delta(1-\beta)-(1-\theta)\right)-h \theta}{\theta \lambda}$ |

because that when $\theta$ is relatively small, the large purchase in the advance period will increase the consumer returns; hence the retailer may prefer the full prepayment scheme.

## 6. Conclusion

Part prepayment scheme, which allows consumers to preorder the product with part prepayment, has recently been widely used in advance selling settings to promote consumer purchases. However, this scheme may also lead to the increase of consumer returns. This study develops a two-period theoretical model to examine the interaction between the part prepayment scheme and the return policy and its effect on the retailer's profit. Our analysis yields the following insights. First, the advance selling price is decreasing with the prepayment proportion, which means that the part prepayment scheme can help the retailer to increase the demand without sacrificing the advance selling price. This finding provides an explanation for why the part prepayment scheme is extensively adopted in reality. Further, it is beneficial for the retailer to set a relatively low (high) advance selling price when the consumers' hassle cost of return is relatively small (large). Second, the prepayment proportion and the consumers' hassle cost of return have a negative cross effect on the retailer's profit, which means that the retailer should set a relatively strict return policy if the consumers can preorder the product with a relatively small prepayment proportion and vice versa. Third, the prepayment proportion and the ex-ante product information perceived by consumers also have a negative cross effect on the retailer's profit, which indicates that when the extent of ex-ante product information perceived by consumers is relatively small, it is more beneficial for the retailer to adopt a relatively high proportion of prepayment and vice versa. We also extend the model to incorporate the retailer's handling cost of consumer returns and find that it is beneficial for the retailer to set a small prepayment proportion when the ex-ante product information perceived by consumers is high enough, or otherwise to require a full prepayment especially when the handling cost is relatively high. These findings also provide explanations for many practices in reality.

This paper examines the interaction between the advance selling with part prepayment scheme and the consumer return policy under the assumption that there is only one retailer in the supply chain. When there are (more than) two
competitors simultaneously sell products in the market, the conclusions in this paper need be respeculated. Also, this paper only considers the full refund return policy, and how will the part prepayment scheme interact with the partial refund return policy is an interesting topic that deserves our future attention. Additionally, with the fast generation of big data, examining the proposed theoretical strategies with the practical data will also be an important and exciting topic for future research.

## Appendix

## Proofs of Theorems and Propositions

## A. Low Advance Selling Price Strategy

## See Table 2.

Proof of Theorem 2. In the case of LAP strategy, the range of advance selling price is $0 \leq p_{1} \leq p_{1 A}$, and the market consists of only the advance consumers. The profit function of the retailer is

$$
\begin{align*}
\max & \prod^{L}\left(p_{1}^{L}\right)=\left(D_{1}^{L}-d_{1}^{L}\right) p_{1}^{L}  \tag{A.1}\\
\text { s.t. } & 0 \leq p_{1}<p_{2}+\delta(1-\beta)-(1-\theta)
\end{align*}
$$

By taking the first-order and second-order derivatives of (A.1), we have $\partial \prod^{L}\left(p_{1}^{L}\right) / \partial p_{1}^{L}=\left(h-2 p_{1}+\lambda\right) / \lambda$ and $\partial^{2} \prod^{L}\left(p_{1}^{L}\right) / \partial p_{1}^{L 2}=-2 / \lambda$. Hence $\prod^{L}\left(p_{1}^{L}\right)$ is concave in $p_{1}^{L}$ and achieves its maximal value at $p_{1}^{L E}$ where $p_{1}^{L E}=(\lambda+h) / 2$.

The maximum points of the LAP strategy are as follows:
(i) If $h \leq h_{1}$, the profit function is concave in the LAP strategy's region. From the first-order derivative, we have the extreme point $p_{1}^{L E}$, and the maximum profit of the retailer $\prod^{L}\left(p_{1}^{L E}\right)=(h+\lambda)^{2} / 4 \lambda$. The constraint that the amount of consumer returns is nonnegative follows $\left(p_{1}-\delta(1-\beta)\right) / \theta \leq\left(p_{1}-h\right) / \lambda$, which can be transformed into $h \leq H_{1}$.
(ii) If $h \geq h_{1}$. The profit function is increasing in the advance selling price in the LAP strategy's region, and the optimal price of this case is the right boundary of
the region $p_{1 A}$. The maximum profit of the retailer is $\prod^{L}\left(p_{1 A}\right)=\left(p_{2}+\theta+\delta(1-\beta)-1\right)\left(1-\left(-h+p_{2}+\theta+\right.\right.$ $\delta(1-\beta)-1) / \lambda)$.
To ensure the nonnegativity of consumer returns, we have $h \leq H_{2}$, with $h_{1}=2\left(p_{2}+\delta(1-\beta)-1+\theta\right)-\lambda, H_{1}=(2 \delta(1-$ $\left.\beta) \lambda+\lambda \theta-\lambda^{2}\right) /(\theta+\lambda)$, and $H_{2}=\left((\theta-\lambda) p_{2}+\theta \beta(1+\delta)-(\theta-\right.$ $\lambda)(1-\theta)) / \theta$.

Proof of Corollary 3. From Table 2, we can take the derivations of optimal advance selling price w.r.t. $\beta$ and $h$, respectively:
(1) if $h \leq \min \left\{h_{1}, H_{1}\right\}, \partial p_{1}^{L^{*}} / \partial \beta=0$; if $h_{1} \leq h \leq H_{2}$, $\partial p_{1}^{L^{*}} / \partial \beta=-\delta$, hence $\partial p_{1}^{L^{*}} / \partial \beta<0 ;$
(2) if $h \leq \min \left\{h_{1}, H_{1}\right\}, \partial p_{1}^{L^{*}} / \partial h=1 / 2$ and hence $\partial p_{1}^{L^{*}} / \partial h>0$; if $h_{1} \leq h \leq H_{2}, \partial p_{1}^{L^{*}} / \partial h=0$.

Hence $p_{1}^{L^{*}}$ is decreasing in $\beta$ and is increasing in $h$.
Proof of Corollary 4. From Table 2, we can take the derivations of demands and returns decisions w.r.t. $\beta$, respectively:
(1) if $h \leq \min \left\{h_{1}, H_{1}\right\}, \partial D_{1}^{L^{*}} / \partial \beta=-\delta / \theta \leq 0, \partial d_{1}^{L^{*}} / \partial \beta=$ $-\delta / \theta \leq 0$, and $\left|\partial D_{1}^{L^{*}} / \partial \beta\right|=\left|\partial d_{1}^{L^{*}} / \partial \beta\right| ;$
(2) if $h_{1} \leq h \leq H_{2}, \partial D_{1}^{L^{*}} / \partial \beta=0, \partial d_{1}^{L^{*}} / \partial \beta=-\delta / \lambda \leq 0$, and $\left|\partial D_{1}^{L^{*}} / \partial \beta\right| \leq\left|\partial d_{1}^{L^{*}} / \partial \beta\right|$.

From Table 2, we can take the derivations of demands and returns decisions w.r.t. $h$, respectively:
(1) if $h \leq \min \left\{h_{1}, H_{1}\right\}, \partial D_{1}^{L^{*}} / \partial h=-1 / 2 \theta<0, \partial d_{1}^{L^{*}} / \partial h=$ $-(\theta+\lambda) / 2 \theta \lambda \leq 0$, and $\left|\partial D_{1}^{L^{*}} / \partial h\right| \leq\left|\partial d_{1}^{L^{*}} / \partial h\right| ;$
(2) if $h_{1} \leq h \leq H_{2}, \partial D_{1}^{L^{*}} / \partial h=0, \partial d_{1}^{L^{*}} / \partial h=-1 / \lambda<0$, and $\left|\partial D_{1}^{L^{*}} / \partial h\right| \leq\left|\partial d_{1}^{L^{*}} / \partial h\right|$.

Combining the above results, we can obtain Corollary 4.

$$
\begin{align*}
& \prod_{1}^{M}\left(p_{1}^{M E}\right) \\
& \quad=\frac{h^{2}(1-\theta)^{2}+2 h p_{2}(1-\theta)(2(1-\theta)+3 \lambda)+p_{2}\left(-4 p_{2}(1-\theta)^{2}+4(1-\theta)\left(1-\theta-p_{2}\right) \lambda+\left(4-4 \theta+p_{2}\right) \lambda^{2}\right)}{4 \lambda(1-\theta)(1+\lambda-\theta)}  \tag{B.3}\\
& \quad+\frac{2 h \delta(1-\beta) \lambda(1-\theta)+2 p_{2} \delta(1-\beta) \lambda^{2}}{4 \lambda(1-\theta)(1+\lambda-\theta)}+\frac{\lambda \delta^{2}(1-\beta)^{2}}{4(1-\theta)(1+\lambda-\theta)} .
\end{align*}
$$

To ensure the nonnegativity of the amount of consumer returns, we have $\left(p_{1}-\delta(1-\beta)\right) / \theta \leq$ $\left(p_{1}-h\right) / \lambda$, which can be transformed into $h \leq H_{3}$. Similarly, there is $\left(p_{2}-p_{1}+\delta(1-\right.$ $\beta)) /(1-\theta) \leq\left(p_{2}-h\right) / \lambda$, which is equal to $h \leq H_{4}$.

## B. Middle Advance Selling Price Strategy

See Table 3.
Proof of Theorem 5. When $p_{1 A}<p_{1} \leq p_{1 B}$, there are consumers purchasing in both the advance and spot periods. The retailer's profit function is

$$
\begin{align*}
\max & \prod^{M}\left(p_{1}^{M}\right) \\
& =p_{1}^{M}\left(D_{1}^{M}-d_{1}^{M}\right)+p_{2}\left(D_{2}^{M}-d_{2}^{M}\right)  \tag{B.1}\\
\text { s.t. } & p_{2}+\delta(1-\beta)-(1-\theta)<p_{1} \\
& \leq \theta p_{2}+\delta(1-\beta)
\end{align*}
$$

The first-order and second-order derivatives of (B.1) are $\partial \prod^{M}\left(p_{1}^{M}\right) / \partial p_{1}^{M}=\left(h(1-\theta)-2 p_{1}(1-\theta+\lambda)+\lambda\left(p_{2}+\delta(1-\right.\right.$ $\beta))) / \lambda(1-\theta)$ and $\partial^{2} \prod^{M}\left(p_{1}^{M}\right) / \partial p_{1}^{M 2}=-2 /(1-\theta)-2 / \lambda$. Hence (B.1) achieves its maximal value at $p_{1}^{M E}$ and $p_{1}^{M E}=(h(1-\theta)+$ $\left.\lambda\left(p_{2}+\delta(1-\beta)\right)\right) / 2(1+\lambda-\theta)$.

Then we will first analyze the maximum points and for each of the three cases defined by the hassle cost as follows:
(i) If $h \leq h_{2}$, the profit function is decreasing in the advance selling price. Hence the optimal point of this case is $p_{1 A}$, and the optimal profit of the retailer is

$$
\begin{align*}
& \Pi\left(p_{1 A}\right)=p_{1}\left(D_{1}-d_{1}\right)=\left(p_{2}+\theta+\delta(1-\beta)-1\right) \\
& \quad \cdot\left(1-\frac{p_{2}+\theta+\delta(1-\beta)-1-h}{\lambda}\right) . \tag{B.2}
\end{align*}
$$

To ensure the number of returns is nonnegative, we have $h \leq H_{2}$.
(ii) If $h_{2} \leq h \leq h_{3}$, the profit function is concave in the advance selling price; thus the extreme point $p_{1}^{M E}$ is the optimal solution of the profit function. The corresponding profit is
(iii) If $h \geq h_{3}$, the profit function is increasing in the advance selling price, thus the optimal price is $p_{1 B}$, and the maximum profit of the retailer is $\Pi^{M}\left(p_{1 B}\right)=$ $p_{2}\left(h-p_{2}+\lambda\right) / \lambda$. In this situation, the condition $h \leq$ $(1-\lambda) p_{2}$ is provided to ensure the nonnegativity of the amount of consumer returns.

Table 3: Optimal decisions in the MAP scenario.
(a) Optimal decisions in the MAP scenario when $h \leq h_{2}$ and $h \leq H_{2}$.

| $h$ | $h \leq \min \left\{h_{2}, H_{2}\right\}$ |
| :--- | :---: |
| $p_{1}^{M *}$ | $p_{1 A}$ |
| $\prod\left(p_{1}^{M *}\right)$ | $\left(p_{2}+\theta+\delta(1-\beta)-1\right)\left(1-\frac{-h+p_{2}+\theta+\delta(1-\beta)-1}{\lambda}\right)$ |
| $D_{1}^{M *}$ | $\frac{1-p_{2}}{\theta}$ |
| $d_{1}^{M *}$ | $\frac{\left(1-\theta-p_{2}\right) \lambda+\left(p_{2}+\delta(1-\beta)-(1-\theta)\right)-h \theta}{\theta \lambda}$ |

(b) Optimal decisions in the MAP scenario when $h_{3} \leq h<(1-\lambda) p_{2}$.

| $h$ | $h_{3} \leq h<(1-\lambda) p_{2}$ |
| :--- | :---: |
| $p_{1}^{M *}$ | $p_{1 B}$ |
| $\Pi^{M *}$ | $\frac{p_{2}\left(h-p_{2}+\lambda\right)}{\lambda}$ |
| $D_{2}^{M *}$ | $1-p_{2}$ |
| $d_{2}^{M *}$ | $\frac{p_{2}(1-\lambda)-h}{\lambda}$ |

(c) Optimal decisions in the MAP scenario when $h_{2} \leq h \leq h_{3}$ and $h \leq \min \left\{H_{3}, H_{4}\right\}$.

\begin{tabular}{|c|c|}
\hline $h$ \& $h_{2} \leq h \leq h_{3}$ and $h \leq \min \left\{H_{3}, H_{4}\right\}$ <br>
\hline $p_{1}^{M *}$ \& $p_{1}^{M E}$ <br>
\hline \& $h^{2}(1-\theta)^{2}+2 h p_{2}(1-\theta)(2(1-\theta)+3 \lambda)+p_{2}\left(-4 p_{2}(1-\theta)^{2}+4(1-\theta)\left(1-\theta-p_{2}\right) \lambda+\left(4-4 \theta+p_{2}\right) \lambda^{2}\right)$ <br>
\hline $\Pi^{M *}$ \& $$
\begin{array}{rc}
4 \lambda(1-\theta)(1+\lambda-\theta) \\
2 h \delta \lambda(1-\beta)(1-\theta)+2 p_{2} \delta(1-\beta) \lambda^{2} & \lambda \delta^{2}(1-\beta)^{2}
\end{array}
$$ <br>
\hline $D_{1}^{M *}$ \& $$
\begin{gathered}
4 \lambda(1-\theta)(1+\lambda-\theta) \\
p_{2}(2 \theta(1+\lambda-\theta)-\lambda)-h(1-\theta) \\
\hline
\end{gathered}
$$ <br>
\hline $D_{2}^{M *}$ \& $$
\begin{aligned}
& 2 \theta(1-\theta)(1+\lambda-\theta) \\
& 1-\frac{\left(p_{2}+\delta(1-\beta)\right)(2-2 \theta+\lambda)-h(1-\theta)(1+\lambda-\theta)}{2 \theta(1-\theta)}
\end{aligned}
$$ <br>
\hline $D_{2}^{\text {M* }}$

$D_{1}^{M *}$ \& $$
\begin{aligned}
& \left.2 \theta(1+\lambda-\theta)-p_{2} \lambda-(1-\theta)(1+\lambda) h-\theta\right) \\
& \underline{2}+(2-2 \theta+\lambda) \delta(1-\beta) \\
& \hline
\end{aligned}
$$ <br>

\hline $D_{T}^{M *}$

$d_{1}^{M *}$ \& $$
\begin{gathered}
2 \theta(1+\lambda-\theta) \\
p_{2} \lambda(\theta-\lambda)-h(\lambda+\theta(1+\lambda-\theta)) \\
\hline
\end{gathered}
$$ <br>

\hline $d_{1}{ }^{\text {a }}$

$d^{M *}$ \& $$
\begin{gathered}
2 \theta \lambda(1+\lambda-\theta) \\
p_{2}-h \\
\underline{2}-(2-2 \theta+\lambda)\left(p_{2}+\delta(1-\lambda)\right)-(1-\theta) h
\end{gathered}
$$ <br>

\hline $d_{2}{ }^{\text {a }}$
$d_{T}^{M *}$ \&  <br>
\hline $d_{T}$ \& $2 \theta \lambda(1-\theta)(1+\lambda-\theta)+\frac{2 \theta(1-\theta)(1+\lambda-\theta)}{}$ <br>
\hline
\end{tabular}

In the above situations, $h_{2}=\left((2(1-\theta)+\lambda)\left(p_{2}+\delta(1-\right.\right.$ $\beta))-2(1+\lambda-\theta)(1-\theta)) /(1-\theta), h_{3}=((2 \theta(1-\theta)+\lambda(2 \theta-$ 1)) $\left.p_{2}+(2(1-\theta)+\lambda) \delta(1-\beta)\right) /(1-\theta), H_{3}=\left(p_{2} \lambda(\theta-\lambda)+\right.$ $\delta(1-\beta) \lambda(2-\theta+\lambda)) /(\theta(1-\theta)+\lambda(1+\theta))$, and $H_{4}=((2(1-$ $\left.\left.\theta)^{2}-\lambda^{2}\right) p_{2}-(2(1-\theta)+\lambda) \delta(1-\beta) \lambda\right) /(2(1-\theta)+\lambda)(1-\theta)$.
Proof of Corollary 6 . Following Table 3, by taking the derivatives of consumer returns w.r.t. $\beta$ we have
(1) If $h \leq h_{2}$ and $h \leq H_{2}$, there is $d_{T}^{M^{*}}=d_{1}^{M^{*}}$ and $\partial d_{T}^{M^{*}} / \partial \beta=-\frac{\delta}{\lambda} \leq 0$.
(2) If $h_{2} \leq h \leq h_{3}$ and $h \leq \min \left\{H_{3}, H_{4}\right\}$, there is $d_{T}^{M^{*}}=$ $d_{1}^{M^{*}}+d_{2}^{M^{*}}$ and $\partial d_{T}^{M *} / \partial \beta=-\delta(2+\theta(-5+3 \theta-2 \lambda)+$ $\lambda) / 2 \theta(1-\theta)(1-\theta+\lambda)$. When $\theta(-5+3 \theta-2 \lambda)+2+\lambda<0$, we have $\partial d_{T}^{M *} / \partial \beta>0$.
(3) If $h_{3} \leq h \leq(1-\lambda) p_{2}$, there is $d_{T}^{M^{*}}=d_{2}^{M^{*}}$ and $\partial d_{T}^{M^{*}} / \partial \beta=0$.

Combining the above results, we can obtain Corollary 6.

Proof of Corollary 7. By taking the second-order cross-partial derivatives of $\partial \prod^{M *} / \partial \beta$ w.r.t. $h, \theta$, and $\delta$, respectively,
(1) if $h \leq h_{2}$ and $h \leq H_{2}, \partial^{2} \prod^{M *} / \partial \beta \partial h=-\delta / \lambda<0$;
if $h_{2} \leq h \leq h_{3}$ and $h \leq \min \left\{H_{3}, H_{4}\right\}, \partial^{2} \Pi^{M *} / \partial \beta \partial h=$ $-\delta / 2(1+\lambda-\theta)<0$;
if $h_{3} \leq h<(1-\lambda) p_{2}, \partial^{2} \prod^{M *} / \partial \beta \partial h=0$;
(2) if $h \leq h_{2}$ and $h \leq H_{2}, \partial^{2} \Pi^{M *} / \partial \beta \partial \theta=2 \delta / \lambda>0$;

Table 4: Optimal decisions of the extension model.
(a) Optimal decisions of the extension model when $\beta \leq \beta_{1}$ and $\beta \leq B_{3}$.

| $\beta$ | $\beta \leq \beta_{1}$ and $\beta \leq B_{3}$ |
| :--- | :---: |
| $p_{1}^{R *}$ | $\frac{c\left(p_{2}-(1-\theta)\right)}{\theta}-\frac{c\left(p_{2}+\delta(1-\beta)-(1-\theta)\right)}{\lambda}-\frac{\left(p_{2}+\delta(1-\beta)-(1-\theta)\right)\left(p_{2}+\delta(1-\beta)-(1-\theta)-\lambda\right)}{\lambda}$ |
| $\prod^{R *}$ | $\frac{1-p_{2}}{\theta}$ |
| $D_{1}^{R *}$ | $\frac{p_{2}+\delta(1-\beta)-(1-\theta)}{\lambda}-\frac{p_{2}-(1-\theta)}{\theta}$ |
| $d_{1}^{R *}$ |  |

(b) Optimal decisions of the extension model when $\beta_{1} \leq \beta<\beta_{2}$ and $\mathrm{B}_{2}<\beta<\mathrm{B}_{1}$.

| $h$ | $\beta_{1} \leq \beta<\beta_{2}$ and $\mathrm{B}_{2}<\beta<\mathrm{B}_{1}$ |
| :---: | :---: |
| $p_{1}^{R *}$ | $\begin{gathered} p_{1}^{R E} \\ \left(c^{2}\left(\theta^{2}+\lambda-\theta(1+2 \lambda)\right)^{2}+\theta^{2}\left((1-\beta)^{2} \delta^{2} \lambda^{2}+2 p_{2} \lambda\left(2+2 \theta^{2}+\lambda(2+\delta-\delta \beta)-2 \theta(2+\lambda)\right)\right.\right. \end{gathered}$ |
| $\Pi^{R *}$ | - $\left.4 \theta^{2}-4 \lambda+\lambda^{2}+4 \theta(2+\lambda)\right)$ ) $2 c \theta\left(p_{2}\left(2 \theta^{3}-\lambda^{2}+\theta(2+\lambda)-\theta^{2}(4+\lambda)\right)+(1-\beta) \delta \lambda\left(2+3 \theta^{2}+\lambda-\theta(5+2 \lambda)\right)\right)$ ) |
| $D_{1}^{R *}$ | $\begin{gathered} 4 \theta^{2} \lambda(1-\theta)(1-\theta+\lambda) \\ \frac{\theta(1-\theta)\left(c+2\left(\delta-\delta \beta+p_{2} \theta\right)\right)+\left(c(2 \theta-1)+\theta\left(\delta-\delta \beta+p_{2}(2 \theta-1)\right)\right) \lambda}{2 \theta^{2}(1-\theta)(1-\theta+\lambda)} \end{gathered}$ |
| $D_{2}^{R *}$ | $\begin{gathered} 1-\frac{\left(p_{2}+\delta-\delta \beta\right) \theta(2-2 \theta+\lambda)+c(\theta(1-\theta)+\lambda(2 \theta-1))}{2 \theta(1-\theta)(1-\theta+\lambda)} \\ (1-\theta) \theta(c+2(\delta-\delta \beta+\theta))-\left(c-2 c \theta+\left(p_{2}-\delta(1-\beta)-2 \theta\right) \theta\right) \lambda \end{gathered}$ |
| $D_{T}^{R *}$ | $\begin{gathered} 2 \theta^{2}(1-\theta+\lambda) \\ c(\theta-\lambda)(\theta(-1+\theta-2 \lambda)+\lambda)+\theta \lambda\left((\delta-\delta \beta)(2-\theta+\lambda)+p_{2}(\theta-\lambda)\right) \\ \hline \end{gathered}$ |
| $d_{2}^{R *}$ | $\begin{gathered} 2 \theta^{2} \lambda(1-\theta+\lambda) \\ \frac{p_{2}}{\lambda}-\frac{\left(p_{2}+\delta-\delta \beta\right) \theta(2-2 \theta+\lambda)+c(\theta(1-\theta+2 \lambda)-\lambda)}{2 \theta(1-\theta)(1-\theta+\lambda)} \end{gathered}$ |
| $d^{R *}$ | $\underline{-c(\theta(\theta-1-2 \lambda)+\lambda)^{2}+\theta\left(\delta(1-\beta) \lambda(2+\theta(-5+3 \theta-2 \lambda)+\lambda)+p_{2}\left(2(1-\theta)^{2} \theta+(1-\theta) \theta \lambda-\lambda^{2}\right)\right)}$ |
|  | 2 $2 \theta^{2} \lambda(1-\theta)(1-\theta+\lambda)$ |

(c) Optimal decisions of the extension model when $\beta \geq \beta_{2}$.

| $\beta$ | $\beta \geq \beta_{2}$ |
| :--- | :---: |
| $p_{1}^{R *}$ | $p_{1 B}$ |
| $\prod^{R *}$ | $p_{2}\left(1+c-\frac{c+p_{2}}{\lambda}\right)$ |
| $D_{2}^{R *}$ | $1-p_{2}$ |
| $d_{2}^{R *}$ | $p_{2}\left(\frac{1}{\lambda}-1\right)$ |

$$
\begin{aligned}
& \text { if } h_{2} \leq h \leq h_{3} \text { and } h \leq \min \left\{H_{3}, H_{4}\right\}, \partial^{2} \Pi^{M *} / \partial \beta \partial \theta= \\
& -\delta\left(h(1-\theta)^{2}+\left(p_{2}+\delta(1-\beta)\right)(2-2 \theta+\lambda) \lambda\right) / 2(1- \\
& \theta)^{2}(1+\lambda-\theta)^{2}<0 ; \\
& \text { if } h_{3} \leq h<(1-\lambda) p_{2}, \partial^{2} \Pi^{M *} / \partial \beta \partial \theta=0 .
\end{aligned}
$$

## C. Extension and Discussions

See Table 4.
Proof of Theorem 9. The retailer's optimization problem can be formulated as
max $\prod_{n}^{n}\left(p_{i}^{R}\right)$

$$
=p_{1}^{R}\left(D_{1}^{R}-d_{1}^{R}\right)+p_{2}\left(D_{2}^{R}-d_{2}^{R}\right)
$$

$$
-c\left(d_{1}^{R}+d_{2}^{R}\right)
$$

$$
\text { s.t. } \quad p_{2}+\delta(1-\beta)-(1-\theta)<p_{1}
$$

$$
\leq \theta p_{2}+\delta(1-\beta)
$$

$$
(\theta-\lambda) p_{1} \geq-\lambda \delta(1-\beta)
$$

$$
\lambda p_{1} \geq \lambda \delta(1-\beta)+(\theta+\lambda-1) p_{2}
$$

By taking the first-order and second-order derivatives of (C.1) we have $\partial \prod^{R} / \partial p_{1}^{R}=\left(\delta(1-\beta)+p_{2}-2 p_{1}-c\right) /(1-\theta)+$ $c / \theta-\left(c+2 p_{1}\right) / \lambda$ and $\partial^{2} \prod^{R} / \partial p_{1}^{R^{2}}=-2 /(1-\theta)-2 / \lambda<0$.

Hence $\prod^{R}\left(p_{1}^{R}\right)$ is concave in $p_{1}^{R}$ and achieves its maximal value at $p_{1}^{R E}$ where $p_{1}^{R E}=\left(\left(p_{2}+\delta(1-\beta)\right) \theta \lambda+c(\theta(\theta-1)+\right.$ $\lambda(1-2 \theta))) / 2 \theta(1-\theta+\lambda)$.

Recall that $p_{1 A} \leq p_{1} \leq p_{1 B}$ we have the following:
(i) If $\beta \leq \beta_{1}$, there is $p_{1}^{R E}<p_{1 A}$, so that $p_{1}^{R *}=p_{1 A}$. In this situation, the condition $\beta \leq B_{3}$ is provided to ensure the nonnegativity of consumer returns.
(ii) If $\beta_{1} \leq \beta \leq \beta_{2}$, there is $p_{1 A}<p_{1}^{R E}<p_{1 B}$, so that $p_{1}^{R *}=p_{1}^{R E}$. The constraint $\left(p_{1}-\delta(1-\beta)\right) / \theta \leq p_{1} / \lambda$ is to ensure that the consume returns in the advance period is nonnegative, which can be transformed into $\beta \leq B_{1}$. Similarly, the condition $\left(p_{2}-p_{1}+\delta(1-\beta)\right) /(1-$ $\theta) \leq p_{2} / \lambda$ is to ensure that the consume returns in the spot period is nonnegative, which can be transformed into $\beta \geq \mathrm{B}_{2}$; thus we have $\mathrm{B}_{2} \leq \beta \leq \mathrm{B}_{1}$.
(iii) If $\beta \geq \beta_{2}$, there is $p_{1}^{R E} \geq p_{1 B}$, so that $p_{1}^{R *}=p_{1 B}$.

In the above situations, $\beta_{1}=\left((1-\theta) \theta\left(c+2\left(p_{2}+\delta+\theta-\right.\right.\right.$ $\left.1)))+\lambda\left(c(2 \theta-1)+\theta\left(p_{2}+\delta-2(1-\theta)\right)\right)\right) / \delta \theta(2(1-\theta)+\lambda)$, $\beta_{2}=\left((1-\theta) \theta\left(c+2\left(\theta p_{2}+\delta\right)\right)+\lambda\left(c(2 \theta-1)+p_{2} \theta(2 \theta-1)+\right.\right.$ $\delta \theta)) / \delta \theta(2(1-\theta)+\lambda), B_{1}=(c(\theta-\lambda)(\theta(-1+\theta-2 \lambda)+\lambda)+$ $\left.\theta \lambda\left(\delta(2-\theta+\lambda)+p_{2}(\theta-\lambda)\right)\right) / \delta \theta \lambda(2-\theta-\lambda), B_{2}=\left(\left(\left(2 c+p_{2}+\right.\right.\right.$ $\left.\delta) \theta-c) \lambda^{2}-2 p_{2}(1-\theta)^{2} \theta+(c+2 \delta)(1-\theta) \theta \lambda\right) / \delta \theta \lambda(2-2 \theta+\lambda)$, and $B_{3}=1+\left(1-\theta-p_{2}\right)(\lambda-\theta) / \delta \theta$.

Proof of Corollary 10 . Following Table 4, by taking the derivatives of optimal advance selling price w.r.t. $\beta$ we have the following:
(1) If $\beta \leq \min \left\{\beta_{1}, B_{3}\right\}$, so that $p_{1}^{R *}=p_{1 A}, \partial p_{1}^{R *} / \partial \beta=$ $-\delta \leq 0$.
(2) If $\beta_{1} \leq \beta \leq \beta_{2}$ and $B_{2} \leq \beta \leq B_{1}, p_{1}^{R *}=p_{1}^{R E}$. Hence, $\partial p_{1}^{R *} / \partial \beta=-\delta \lambda / 2(1-\theta+\lambda) \leq 0$.
(3) If $\beta \geq \beta_{2}$, so that $p_{1}^{R *}=p_{1 B}$. Hence, $\partial p_{1}^{R *} / \partial \beta=-\delta \leq$ 0.

Similarly, by taking the derivatives of optimal advance selling price w.r.t. $c$ we have the following:
(1) If $\beta \leq \min \left\{\beta_{1}, B_{3}\right\}, p_{1}^{R *}=p_{1 A}$, which is independent of the retailer's handling cost.
(2) If $\beta_{1} \leq \beta \leq \beta_{2}$ and $B_{2} \leq \beta \leq B_{1}, p_{1}^{R *}=p_{1}^{R E}$. Hence, $\partial p_{1}^{R *} / \partial c=(\theta(1-\theta+2 \lambda)-\lambda) / 2 \theta(1-\theta+\lambda)$. When $\theta \in\left[0,\left(2 \lambda+1-\sqrt{4 \lambda^{2}+1}\right) / 2\right], \partial p_{1}^{R *} / \partial c \leq 0$; when $\theta \in\left[\left(2 \lambda+1-\sqrt{4 \lambda^{2}+1}\right) / 2,1\right], \partial p_{1}^{R *} / \partial c \geq 0$.
(3) If $\beta \geq \beta_{2}, p_{1}^{R *}=p_{1 B}$. Hence, $\partial p_{1}^{R *} / \partial c=0$.

Combining the above results we have Corollary 10.
Proof of Proposition 11 . Following Table 4, by taking the derivatives of optimal profit w.r.t. $\beta$ we have, when $\beta_{1} \leq \beta<$ $\beta_{2}$ and $\mathrm{B}_{2} \leq \beta \leq \mathrm{B}_{1}, p_{1}^{R *}=p_{1}^{R E}$. Hence, $\partial \prod^{R *} / \partial \beta=\delta\left(-\left(p_{2}+\right.\right.$
$\delta(1-\beta)) \theta \lambda+c(2+\theta(-5+3 \theta-2 \lambda)+\lambda)) / 2 \theta(1-\theta)(1-\theta+\lambda)$ and $\partial^{2} \prod^{R *} / \partial \beta^{2}=\delta^{2} \lambda / 2(1-\theta)(1-\theta+\lambda)>0$.

When $\beta=\bar{\beta}, \partial \prod^{R *} / \partial \beta=0, \bar{\beta}=\left(\left(p_{2}+\delta\right) \theta \lambda-c(2+\theta(-5+\right.$ $3 \theta-2 \lambda)+\lambda)) / \delta \lambda \theta$.
Proof of Proposition 12. If $\beta_{1} \leq \beta<\beta_{2}$ and $\mathrm{B}_{2} \leq \beta \leq \mathrm{B}_{1}$, there is $\partial^{2} \prod^{R *} / \partial \beta^{2}=\delta^{2} \lambda / 2(1-\theta)(1-\theta+\lambda)>0$.

Define $\Delta \Pi=\Pi^{R}(\beta=1)-\prod^{R}(\beta=0)$, so that $\Delta \Pi=$ $\left(2 c \delta(2+\theta(-5+3 \theta-2 \lambda)+\lambda)-\delta \lambda \theta\left(2 p_{2}+\delta\right)\right) / 4 \theta(1-\theta)(1+\lambda-\theta)$ and $\partial \Delta \prod / \partial c=\delta(2+\theta(-5+3 \theta-2 \lambda)+\lambda) / 2 \theta(1-\theta)(1+\lambda-\theta)$.

When $c=\widetilde{c}$, there is $\Delta \Pi=0$, where $\widetilde{c}=\left(2 p_{2}+\delta\right) \theta \lambda / 2(2+$ $\theta(-5+3 \theta-2 \lambda)+\lambda)$. Hence
(1) if $2+\theta(-5+3 \theta-2 \lambda)+\lambda>0, \Delta \prod$ is increasing in $c$; there is $c>\tilde{c}, \Delta \Pi>0$.
(2) If $2+\theta(-5+3 \theta-2 \lambda)+\lambda<0, \Delta \prod$ is decreasing in $c$; and $\Delta \prod<0$.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

In this paper, names of some real-world companies are mentioned. The authors declare that there are no conflicts of interest in citing these companies. The authors need to cite them because they are well-known examples to illustrate the real-world relevance of the analysis.

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