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Research Article

Finite-Time Stabilization for Stochastic Inertial Neural Networks with Time-Delay via Nonlinear Delay Controller

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This paper pays close attention to the problem of finite-time stabilization related to stochastic inertial neural networks with or without time-delay. By establishing proper Lyapunov-Krasovskii functional and making use of matrix inequalities, some sufficient conditions on finite-time stabilization are obtained and the stochastic settling-time function is also estimated. Furthermore, in order to achieve the finite-time stabilization, both delayed and nondelayed nonlinear feedback controllers are designed, respectively, in terms of solutions to a set of linear matrix inequalities (LMIs). Finally, a numerical example is provided to demonstrate the correction of the theoretical results and the effectiveness of the proposed control design method.

1. Introduction

In recent years, more and more scholars have been attracted by neural networks due to their successful applications in associative memory [1, 2], pattern recognition [3], signal processing, optimization problems, and so forth [4]. These applications always rely on the dynamic behaviors of neural networks. Therefore, the investigation of dynamic trajectories is necessary for applied designation of neural networks. Hence, a large number of studies on stability [5–8], stabilization [9, 10], passivity [11], dissipativity [12, 13], synchronization [14, 15], and state estimation [16, 17] for neural networks have been reported.

On the other hand, many researchers have studied Hopfield neural networks [18], cell neural networks, recurrent neural networks [9, 19], Cohen-Grossberg neural networks, bidirectional associative memory neural networks, and Lotka-Volterra neural networks, as well as inertial neural networks [12, 14, 15, 20], which are more intricate than all kinds of prementioned neural networks with the standard resistor-capacitor variety [21]. The inertial term is taken as a critical tool to bring complex bifurcation behavior and chaos.

It has been confirmed that stochastic disturbances, which are unavoidable in actual applications of artificial neural networks, are probably one of the main sources leading to undesirable behaviors of dynamical systems, especially when a neural network is implemented for applications. Therefore, it is of great significance to study the stability and stabilization problems of neural networks with stochastic disturbances [22–24]. However, to the best of authors' knowledge, most of the researchers have either investigated the stability for stochastic neural networks with time-delay [25–28] or studied the stability for inertial neural networks with time-delay [20]. There are rare literatures that considered the finite-time stabilization for stochastic inertial neural networks with time-delay.

Inspired by the above comprehensive analysis, in this paper, we are devoted to investigating the finite-time stabilization for stochastic inertial neural networks with time-delay. First, by utilizing an appropriate variable substitution, a stochastic inertial neural network can be transformed into a first-order stochastic differential system. Then, some sufficient conditions on finite-time stability in probability are derived by means of establishing an appropriate Lyapunov function and applying inequality techniques. Moreover, the stochastic settling-time function is also given.

2. Problem Formulation and Preliminaries

2.1. Systems Description. Firstly, the inertial neural networks (INNs) without time-delay are considered, which is described as follows:

$$\frac{\mathrm{d}^2 x_i(t)}{\mathrm{d}t^2} = -a_i \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} - b_i x_i(t) + \sum_{j=1}^{n} c_{ij} f_j\left(x_j(t)\right) + I_i(t), \quad i = 1, 2, \ldots, n \tag{1}$$

where $x_i(t)$ is the state of the i-th neuron; the second derivative $\mathrm{d}^2 x_i(t)/\mathrm{d}t^2$ is the inertial term of INNs (1). $a_i > 0$, $b_i > 0$ are constants. c_{ij} denotes the connection weight between the i-th neuron and the j-th neuron. $f_j(\cdot)$ stands for activation function of the j-th neuron with $f_j(0) = 0 (j = 1, 2, \ldots, n)$. $I_i(t)$ is the external input on the i-th neuron.

The initial conditions of INNs (1) are

$$x_i(0) = \varphi_i(0), \quad \frac{\mathrm{d}x_i(0)}{\mathrm{d}t} = \psi_i(0), \quad i = 1, 2, \ldots, n, \tag{2}$$

where $\varphi_i(0)$ and $\psi_i(0)$ are real-valued continuous functions.

Suppose that the external input $I_i(t)$ is subject to the environmental noise and is described by $I_i(t) = u_i(t) + \beta_i(t, x_i(t))\dot{\omega}_i(t)$, where $u_i(t)$ is known as the control input and $\omega_i(t)$ is a one-dimensional white noise, which is also called Brown motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ and satisfied with

$$\mathbb{E}\left[\omega_i(t)\right] = 0, \quad \mathbb{E}\left[\omega_i^2(t)\right] = \mathrm{d}t, \quad \mathbb{E}\left[\omega_i(t)\,\omega_j(t)\right] = 0 \quad (i \neq j), \tag{3}$$

and $\beta_i(t, x_i(t))$ is the intensity function of the noise.

Then INNs (1) can be written the following stochastic inertial neural networks (SINNs):

$$\frac{\mathrm{d}^2 x_i(t)}{\mathrm{d}t^2} = -a_i \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} - b_i x_i(t) + \sum_{j=1}^{n} c_{ij} f_j\left(x_j(t)\right) + u_i(t) + \beta_i\left(t, x_i(t)\right)\dot{\omega}_i(t). \tag{4}$$

2.2. Problems Formulation. In general, making use of the variable transformation,

$$y_i(t) = \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} + \xi_i x_i(t), \quad i = 1, 2, \ldots, n, \tag{5}$$

then the SINNs (4) can be rewritten as

$$\begin{aligned} \mathrm{d}x_i(t) &= \left[-\xi_i x_i(t) + y_i(t)\right]\mathrm{d}t, \\ \mathrm{d}y_i(t) &= \left[-\widetilde{a}_i y_i(t) - \widetilde{b}_i x_i(t) + \sum_{j=1}^{n} c_{ij} f_j\left(x_j(t)\right) + u_i(t)\right]\mathrm{d}t \\ &\quad + \beta_i\left(t, x_i(t)\right)\mathrm{d}\omega_i(t), \end{aligned} \tag{6}$$

and the initial conditions are given as

$$x_i(0) = \varphi_i(0), \quad y_i(0) = -\xi_i(0)\,\varphi_i(0) + \psi_i(0), \quad i = 1, 2, \ldots, n, \tag{7}$$

where $\widetilde{a}_i = a_i - \xi_i$, $\widetilde{b}_i = b_i + \xi_i(\xi_i - a_i)$.

Moreover, the controller $\nu_i(t)$ is considered; we have the following SINNs:

$$\begin{aligned} \mathrm{d}x_i(t) &= \left[-\xi_i x_i(t) + y_i(t) + \nu_i(t)\right]\mathrm{d}t, \\ \mathrm{d}y_i(t) &= \left[-\widetilde{a}_i y_i(t) - \widetilde{b}_i x_i(t) + \sum_{j=1}^{n} c_{ij} f_j\left(x_j(t)\right) + u_i(t)\right]\mathrm{d}t \\ &\quad + \beta_i\left(t, x_i(t)\right)\mathrm{d}\omega_i(t), \\ &\qquad i = 1, 2, \ldots, n. \end{aligned} \tag{8}$$

Denote

$$\begin{aligned} x(t) &= \left(x_1(t), x_2(t), \ldots, x_n(t)\right)^T, \\ y(t) &= \left(y_1(t), y_2(t), \ldots, y_n(t)\right)^T, \\ \Xi &= \operatorname{diag}\{\xi_1, \xi_2, \ldots, \xi_n\}, \\ \nu(t) &= \left(\nu_1(t), \nu_2(t), \ldots, \nu_n(t)\right)^T, \\ A &= \operatorname{diag}\{\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n\}, \\ B &= \operatorname{diag}\left\{\widetilde{b}_1, \widetilde{b}_2, \ldots, \widetilde{b}_n\right\}, \\ C &= \left(c_{ij}\right)_{n\times n}, \\ \beta(t, x(t)) &= \left(\beta_1\left(t, x_1(t)\right), \beta_2\left(t, x_2(t)\right), \ldots, \beta_n\left(t, x_n(t)\right)\right)^T, \\ \omega(t) &= \left(\omega_1(t), \omega_2(t), \ldots, \omega_n(t)\right), \\ u(t) &= \left(u_1(t), u_2(t), \ldots, u_n(t)\right)^T. \end{aligned} \tag{9}$$

Thus, the SINNs (8) can be written in vector form as

$$\begin{aligned} \mathrm{d}x(t) &= \left[-\Xi x(t) + y(t) + \nu(t)\right]\mathrm{d}t, \\ \mathrm{d}y(t) &= \left[-Ay(t) - Bx(t) + Cf(x(t)) + u(t)\right]\mathrm{d}t \\ &\quad + \beta(t, x(t))\,\mathrm{d}\omega(t). \end{aligned} \tag{10}$$

The control inputs to be designed are of the following form:

$$\begin{aligned} \nu(t) &= -K_1 x(t) - K_2 \operatorname{sign}(x(t)) |x(t)|^{\mu}, \\ u(t) &= -K_3 y(t) - K_4 \operatorname{sign}(y(t)) |y(t)|^{\mu}, \end{aligned} \tag{11}$$

where $\operatorname{sign}(x(t)) = (\operatorname{sign}(x_1(t)), \operatorname{sign}(x_2(t)), \ldots, \operatorname{sign}(x_n(t)))^T$, $\operatorname{sign}(y(t)) = (\operatorname{sign}(y_1(t)), \operatorname{sign}(y_2(t)), \ldots, \operatorname{sign}(y_n(t)))^T$, and K_1, K_2, K_3, K_4 are the control gain matrices to be determined. μ is a positive constant with $0 < \mu < 1$.

Remark 1. There are three cases for the value of μ. If $0 < \mu < 1$, the controllers $u(t), \nu(t)$ are continuous functions with respect to x and y, respectively, which bring about the continuity of SINNs (10) with respect to the systems state [29, 30], but the local Lipschitz condition is dissatisfied. If $\mu = 0$, $u(t), \nu(t)$ turn to be discontinuous ones, which have been studied in [31, 32]. If $\mu = 1$, then they become the typical stabilization issues which only can realize an asymptotical stabilization in infinite time [33, 34] due to the local Lipschitz conditions.

Remark 2. In fact, the control gain matrices K_1, K_2, K_3, K_4 in the controllers $\nu(t)$ and $u(t)$ play different roles in ensuring the finite-time stability of the SINNs (10) with (11), where K_1 and K_3 are used to guarantee the Lyapunov stability of the SINNs (10). And the convergence to zero of the SINNs (10) is determined by K_2 and K_4.

To achieve our main results, some assumptions, lemmas, and definitions are necessary to introduce firstly.

Assumption 3. The nonlinear activation function f satisfies $f(0) = 0$, and there exist some constants $m_{1i}^-, m_{1i}^+ (i = 1, 2, \ldots, n)$, such that

$$M_1^- \le \frac{f(x_1) - f(x_2)}{x_1 - x_2} \le M_1^+ \tag{12}$$

hold for all $x_1, x_2 \in \mathbb{R}$ and $x_1 \ne x_2$, where $M_1^- = \operatorname{diag}\{m_{11}^-, m_{12}^-, \ldots, m_{1n}^-\}$, $M_1^+ = \operatorname{diag}\{m_{11}^+, m_{12}^+, \ldots, m_{1n}^+\}$.

Remark 4. If we choose $m_{1i} = \max\{|m_{1i}^-|, |m_{1i}^+|\}$, the inequalities in Assumption 3 can be written as

$$\left| \frac{f(x_1) - f(x_2)}{x_1 - x_2} \right| \le M_1, \tag{13}$$

where $M_1 = \operatorname{diag}\{m_{11}, m_{12}, \ldots, m_{1n}\}$, which has been considerably studied.

Assumption 5. The intensity function $\beta(t, x(t))$ is a continuous function and is supposed to satisfy that

$$\operatorname{trace}\left[\beta^T(t, x(t)) \beta(t, x(t))\right] \le x^T(t) M_2^T M_2 x(t), \tag{14}$$

where M_2 is a known matrix with appropriate dimensions.

Definition 6. The SINNs (10) are said to be finite-time stabilizable by the controller (11); that is, the SINNs (10) are finite-time stable if, for any initial state $x(0)$, $y(0)$, there exists a finite-time function T_0 such that

$$\mathbb{E}\|x(t)\| = \mathbb{E}\|y(t)\| = 0, \quad \forall t \ge T_0, \tag{15}$$

where $T_0 = T_0(x(0), y(0), \omega) = \inf\{T \ge 0 : x(t) = y(t) = 0, \forall t \ge T\}$ is called the stochastic settling time function satisfying $E[T_0] < \infty$.

Lemma 7 (see [35]). *Suppose that SINNs (10) admit a unique solution. If there exist a C^2 function $V : \mathbb{R}^n \longrightarrow \mathbb{R}_+$, $\mathscr{K}_\infty$ class functions μ_1 and μ_2, and positive real constant $\eta > 0$ and $0 < \gamma < 1$, such that for all $x \in \mathbb{R}^n$ and $t \ge 0$,*

$$\begin{aligned} \mu_1(|x|) &\le V(x) \le \mu_2(|x|), \\ \mathscr{L}V(x) &\le -\eta (V(x))^{\gamma}, \end{aligned} \tag{16}$$

then the origin of SINNs (10) are stochastically finite-time stable, and $\mathbb{E}[T_0] < \mathbb{E}(V(x_0))^{1-\gamma}/\eta(1-\gamma)$.

Lemma 8 (see [9]). *If $a_1, a_2, \ldots, a_n$ are positive number and $0 < r < p$, then*

$$\left(\sum_{i=1}^n a_i^p\right)^{1/p} \le \left(\sum_{i=1}^n a_i^r\right)^{1/r}. \tag{17}$$

3. Main Results

3.1. Finite-Time Stabilization Feedback Controller Design without Time-Delay

Theorem 9. *The controlled systems (10) with (11) are finite-time stable, if there exist some positive-definite matrices $P_1, P_2, M_3 \in \mathbb{R}^{n\times n}$ and some known constant matrices M_1, M_2 with compatible dimensions, such that*

$$\begin{pmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{pmatrix} < 0, \tag{18}$$

Moreover, the upper bound of the stochastic settling time for stabilization can be estimated as $\mathbb{E}\{T_0\} \le (\lambda_2(E\|x(0)\|^{1-\mu} + E\|y(0)\|^{1-\mu}))/\lambda_1(1-\mu)$,
where

$$\begin{aligned} \Theta_{11} &= -P_1\Xi - \Xi^T P_1 - P_1 K_1 - K_1^T P_1 + M_1^T M_3^{-1} M_1 \\ &\quad + M_2^T P_2 M_2, \\ \Theta_{12} &= P_1 - B^T P_2, \\ \Theta_{22} &= -P_2 A - A^T P_2 - P_2 K_3 - K_3^T P_2 + P_2 C M_3 C^T P_2, \\ \lambda_1 &= \min\{\lambda_{min}(P_1 K_2), \lambda_{min}(P_2 K_4)\}, \\ \lambda_2 &= \max\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}. \end{aligned} \tag{19}$$

Proof. Taking controller (11) into SINNs (10), it follows that

$$\begin{aligned} \mathrm{d}x(t) &= \left[-\left(\Xi + K_1\right)x(t) + y(t) \right. \\ &\quad \left. - K_2 \operatorname{sign}|x(t)|^{\mu}\right]\mathrm{d}t, \\ \mathrm{d}y(t) &= \left[-\left(A + K_3\right)y(t) - Bx(t) + Cf(x(t)) \right. \\ &\quad \left. - K_4 \operatorname{sign}\left(y(t)\right)\left|y(t)\right|^{\mu}\right]\mathrm{d}t + \beta(t, x(t))\,\mathrm{d}\omega(t). \end{aligned} \tag{20}$$

Next, we will prove that system (20) is finite-time stable in the sense of Definition 6.

Construct a Lyapunov function as

$$V(t) = x^T(t)P_1x(t) + y^T(t)P_2y(t), \tag{21}$$

where P_1 and P_2 are positive definite matrices. Then, calculate the time derivative of $V(t)$ along the trajectories of systems (20); we get

$$\mathrm{d}V(t) = \mathscr{L}V(t)\,\mathrm{d}t + 2y^T(t)P_2\beta(t, x(t))\,\mathrm{d}\omega(t), \tag{22}$$

where

$$\begin{aligned} \mathscr{L}V(t) &= 2x^T(t)P_1\left[-\left(\Xi + K_1\right)x(t) + y(t) \right. \\ &\quad \left. - K_2 \operatorname{sign}(x(t))|x(t)|^{\mu}\right] + 2y^T(t) \\ &\quad \cdot P_2\left[-\left(A + K_3\right)y(t) - Bx(t) + Cf(x(t)) \right. \\ &\quad \left. - K_4 \operatorname{sign}\left(y(t)\right)\left|y(t)\right|^{\mu}\right] \\ &\quad + \operatorname{trace}\left[\beta^T(t, x(t))P_2\beta(t, x(t))\right] = -2x^T(t)P_1\left(\Xi \right. \\ &\quad \left. + K_1\right)x(t) + 2x^T(t)P_1y(t) - 2y^T(t)P_2Bx(t) \\ &\quad - 2x^T(t)P_1K_2 \operatorname{sign}(x(t))|x(t)|^{\mu} - 2y^T(t)P_2K_4 \\ &\quad \cdot \operatorname{sign}\left(y(t)\right)\left|y(t)\right|^{\mu} - 2y^T(t)P_2\left(A + K_3\right)y(t) \\ &\quad + 2Y^TP_2Cf(x(t)) \\ &\quad + \operatorname{trace}\left[\beta^T(t, x(t))P_2\beta(t, x(t))\right]. \end{aligned} \tag{23}$$

From (13), we have

$$\begin{aligned} 2y^T(t)P_2Cf(x(t)) &= y^T(t)P_2Cf(x(t)) \\ &\quad + f^T(x(t))C^TP_2y(t) \\ &\le y^T(t)P_2CM_3C^TP_2y(t) \\ &\quad + f^T(x(t))M_3^{-1}f(x(t)) \\ &\le y^T(t)P_2CM_3C^TP_2y(t) \\ &\quad + x^T(t)M_1^TM_3^{-1}M_1x(t). \end{aligned} \tag{24}$$

From (14), we have

$$\begin{aligned} &\operatorname{trace}\left[\beta^T(t, x(t))P_2\beta(t, x(t))\right] \\ &\quad \le x^T(t)M_2^TP_2M_2x(t). \end{aligned} \tag{25}$$

Combining (23)-(25) and condition (18), one can follow that

$$\begin{aligned} \mathscr{L}V(t) &\le x^T(t) \\ &\quad \cdot \left[-2P_1\left(\Xi + K_1\right) + M_1^TM_3^{-1}M_1 + M_2^TP_2M_2\right]x(t) \\ &\quad + 2x^T(t)\left[P_1 - B^TP_2\right]y(t) + y^T(t) \\ &\quad \cdot \left[-2P_2\left(A + K_3\right) + P_2CM_3C^TP_2\right]y(t) - 2x^T(t) \\ &\quad \cdot P_1K_2 \operatorname{sign}(x(t))|x(t)|^{\mu} - 2y^T(t)P_2K_4 \\ &\quad \cdot \operatorname{sign}\left(y(t)\right)\left|y(t)\right|^{\mu} \\ &\le \begin{pmatrix} x^T(t) & y^T(t) \end{pmatrix}\begin{pmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{pmatrix}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} - 2x^T(t) \\ &\quad \cdot P_1K_2 \operatorname{sign}(x(t))|x(t)|^{\mu} - 2y^T(t)P_2K_4 \\ &\quad \cdot \operatorname{sign}\left(y(t)\right)\left|y(t)\right|^{\mu} \le -2x^T(t)P_1K_2 \operatorname{sign}(x(t)) \\ &\quad \cdot |x(t)|^{\mu} - 2y^T(t)P_2K_4 \operatorname{sign}\left(y(t)\right)\left|y(t)\right|^{\mu} \\ &\le -\lambda_{min}\left(P_1K_2 + K_2^TP_1\right)\sum_{i=1}^{n}\left|x_i(t)\right|^{\mu+1} \\ &\quad - \lambda_{min}\left(P_2K_4 + K_4^TP_2\right)\sum_{i=1}^{n}\left|y_i(t)\right|^{\mu+1}. \end{aligned} \tag{26}$$

Due to $0 < \mu < 1$, together with Lemma 8, one has

$$\left(\sum_{i=1}^{n}\left|x_i(t)\right|^{\mu+1}\right)^{1/(\mu+1)} \ge \left(\sum_{i=1}^{n}\left|x_i(t)\right|^{2}\right)^{1/2}, \tag{27}$$

and then

$$\begin{aligned} \sum_{i=1}^{n}\left|x_i(t)\right|^{\mu+1} &\ge \left(\sum_{i=1}^{n}\left|x_i(t)\right|^{2}\right)^{(\mu+1)/2} \\ &= \left[x^T(t)x(t)\right]^{(\mu+1)/2}. \end{aligned} \tag{28}$$

Similarly, we have

$$\begin{aligned} \sum_{i=1}^{n}\left|y_i(t)\right|^{\mu+1} &\ge \left(\sum_{i=1}^{n}\left|y_i(t)\right|^{2}\right)^{(\mu+1)/2} \\ &= \left[y^T(t)y(t)\right]^{(\mu+1)/2}. \end{aligned} \tag{29}$$

So, we have

$$\begin{aligned} &\mathscr{L}V(t) \\ &\quad \le -\lambda_{min}\left(P_1K_2 + K_2^TP_1\right)\left[x^T(t)x(t)\right]^{(\mu+1)/2} \\ &\quad - \lambda_{min}\left(P_2K_4 + K_4^TP_2\right)\left[y^T(t)y(t)\right]^{(\mu+1)/2}. \end{aligned} \tag{30}$$

Now, taking the expectations on both sides of (22), and letting $\lambda_1 = \min\{\lambda_{min}(P_1K_2), \lambda_{min}(P_2K_4)\}$, $\lambda_2 = \max\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}$, we can get

$$\begin{aligned}
&\mathbb{E}\{\mathrm{d}V(t)\} \\
&\le -2\lambda_1\mathbb{E}\left\{\left[x^T(t)x(t)\right]^{(\mu+1)/2} + \left[y^T(t)y(t)\right]^{(\mu+1)/2}\right\} \\
&\le -2\lambda_1\lambda_2^{-(\mu+1)/2}\mathbb{E}\left\{V(t)^{(\mu+1)/2}\right\}, \\
&\mathbb{E}\left\{V^{(\mu+1)/2}(0)\right\} = (\mathbb{E}\{V(0)\})^{(\mu+1)/2}.
\end{aligned} \tag{31}$$

From Lemma 7, we get that the controlled systems (20) are finite-time stable, and the upper bounded stochastic settling time can be estimated by

$$\begin{aligned}
\mathbb{E}\{T_0\} &= \frac{\lambda_2^{(\mu+1)/2}\mathbb{E}[V(0)]^{(1-\mu)/2}}{2\lambda_1((1-\mu)/2)} \\
&\le \frac{\lambda_2^{(\mu+1)/2}\lambda_2^{(1-\mu)/2}\left(\mathbb{E}\|x(0)\|_2^{1-\mu} + \mathbb{E}\|y(0)\|_2^{1-\mu}\right)}{\lambda_1(1-\mu)} \\
&= \frac{\lambda_2\left(\mathbb{E}\|x(0)\|^{1-\mu} + \mathbb{E}\|y(0)\|^{1-\mu}\right)}{\lambda_1(1-\mu)}.
\end{aligned} \tag{32}$$

This completes the proof. □

Summing up the above analysis, some sufficient conditions on finite-time stability for the SINNs (10) with (11) are obtained. In the following, we mainly focus on the design of finite-time stabilizing controllers by transforming the sufficient conditions into solvable linear matrix inequalities.

Theorem 10. *If there exist some positive define matrices X_1, X_2, M_3, matrices Y_1,Y_2 with appropriate dimensions, for fixed control gain matrices K_2 and K_4, such that*

$$\begin{pmatrix} \Phi_{11} & \Phi_{12} & X_1M_1^T & X_1M_2^T & 0 \\ * & \Phi_{22} & 0 & 0 & CM_3 \\ * & * & -M_3 & 0 & 0 \\ * & * & * & -X_2 & 0 \\ * & * & * & * & -M_3 \end{pmatrix} < 0, \tag{33}$$

where

$$\begin{aligned}
\Phi_{11} &= -\Xi X_1 - X_1\Xi^T - Y_1 - Y_1^T, \\
\Phi_{12} &= X_2 - X_1B^T, \\
\Phi_{22} &= -AX_2 - X_2A^T - Y_2 - Y_2^T,
\end{aligned} \tag{34}$$

then the finite-time stabilization problem is solvable for the stochastic inertial neural networks (4) and the control gain matrices $K_1 = Y_1X_1^{-1}$, $K_3 = Y_2X_2^{-1}$.

Proof. Setting $P_1^{-1} = X_1$, $P_2^{-1} = X_2$, $K_1X_1 = Y_1$, $K_3X_2 = Y_2$, (18) can be written as

$$\begin{pmatrix} \Theta_{111} & \Theta_{112} \\ * & \Theta_{122} \end{pmatrix} < 0, \tag{35}$$

where

$$\begin{aligned}
\Theta_{111} &= -X_1^{-1}\Xi - \Xi^TX_1^{-1} - X_1^{-1}K_1 - K_1^TX_1^{-1} \\
&\quad + M_1^TM_3^{-1}M_1 + M_2^TX_2^{-1}M_2, \\
\Theta_{112} &= X_1^{-1} - B^TX_2^{-1}, \\
\Theta_{122} &= -X_2^{-1}A - A^TX_2^{-1} - X_2^{-1}K_3 - K_3^TX_2^{-1} \\
&\quad + X_2^{-1}CM_3C^TX_2^{-1}.
\end{aligned} \tag{36}$$

Then, left- and right-multiplying inequality (35) by the block-diagonal matrix diag$\{X_1, I\}$, which follows

$$\begin{pmatrix} X_1 & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} \Theta_{111} & \Theta_{112} \\ * & \Theta_{122} \end{pmatrix}\begin{pmatrix} X_1 & 0 \\ 0 & I \end{pmatrix} = \begin{pmatrix} \Theta_{211} & \Theta_{212} \\ * & \Theta_{222} \end{pmatrix} < 0, \tag{37}$$

where

$$\begin{aligned}
\Theta_{211} &= -\Xi X_1 - X_1\Xi^T - K_1X_1 - X_1K_1^T \\
&\quad + X_1M_1^TM_3^{-1}M_1X_1 + X_1M_2^TX_2^{-1}M_2X_1, \\
\Theta_{212} &= I - X_1B^TX_2^{-1}, \\
\Theta_{222} &= -X_2^{-1}A - A^TX_2^{-1} - X_2^{-1}K_3 - K_3^TX_2^{-1} \\
&\quad + X_2^{-1}CM_3C^TX_2^{-1},
\end{aligned} \tag{38}$$

and left- and right-multiplying inequality (37) by the block-diagonal matrix diag$\{I, X_1\}$, we can obtain

$$\begin{pmatrix} I & 0 \\ 0 & X_1 \end{pmatrix}\begin{pmatrix} \Theta_{211} & \Theta_{212} \\ * & \Theta_{222} \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & X_1 \end{pmatrix} = \begin{pmatrix} \Theta_{311} & \Theta_{312} \\ * & \Theta_{322} \end{pmatrix} < 0, \tag{39}$$

where

$$\begin{aligned}
\Theta_{311} &= -\Xi X_1 - X_1\Xi^T - K_1X_1 - X_1K_1^T \\
&\quad + X_1M_1^TM_3^{-1}M_1X_1 + X_1M_2^TX_2^{-1}M_2X_1, \\
\Theta_{312} &= X_2 - X_1B^T, \\
\Theta_{322} &= -AX_2 - X_2A^T - K_3X_2 - X_2K_3^T + CM_3C^T,
\end{aligned} \tag{40}$$

By Schur complement, (33) implies the above inequality (39) holds. This completes the proof. □

3.2. Finite-Time Stabilization Feedback Controller Design with Time-Delay. In the above section, we discussed the finite-time stabilization for stochastic inertial neural networks without time-delay. However, when designing a neural network or implementing it, the occurrence of time-delay is unavoidable. It may cause instability and oscillation [36–38]. Therefore, in order to reduce the conservatism, in this section, we will study the finite-time stabilization for stochastic inertial neural networks with time-delay.

Consider the following SINNs with time-delay,

$$\begin{aligned}\frac{\mathrm{d}^2 x_i(t)}{\mathrm{d}t^2} &= -a_i \frac{\mathrm{d}x_i(t)}{\mathrm{d}t} - b_i x_i(t) \\ &\quad + \sum_{j=1}^{n} c_{ij} f_j\left(x_j(t)\right) + \sum_{j=1}^{n} c_{ij} f_j\left(x_j(t)\right) \\ &\quad + \sum_{j=1}^{n} d_{ij} f_j\left(x_j\left(t - h_j(t)\right)\right) + u_i(t) \\ &\quad + \beta_i\left(t, x_i(t)\right) \dot{\omega}_i(t),\end{aligned} \tag{41}$$

where $h_j(t)$ is the time-varying delay of j-th neuron with $0 \le h_j(t) \le h$.

Denote

$$\begin{aligned} D &= \left(d_{ij}\right)_{n\times n}, \\ h(t) &= \left(h_1(t), h_2(t), \ldots, h_n(t)\right)^T. \end{aligned} \tag{42}$$

Then we have

$$\begin{aligned} \mathrm{d}x(t) &= \left[-\Xi x(t) + y(t) + \nu(t)\right] \mathrm{d}t, \\ \mathrm{d}y(t) &= \left[-Ay(t) - Bx(t) + Cf(x(t))\right. \\ &\quad \left. + Df(x(t - h(t))) + u(t)\right] \mathrm{d}t \\ &\quad + \beta(t, x(t))\, \mathrm{d}\omega(t). \end{aligned} \tag{43}$$

The nonlinear delay-feedback controller is designed as the following form:

$$\begin{aligned} \nu(t) &= -K_1 x(t) - K_2 \operatorname{sign}(x(t)) |x(t)|^{\mu}, \\ u(t) &= -K_3 y(t) - K_4 \operatorname{sign}\left(y(t)\right) \left|y(t)\right|^{\mu} \\ &\quad - DM_1 \operatorname{sign}(x(t)) |x(t - h(t))|, \end{aligned} \tag{44}$$

where K_1, K_2, K_3, K_4 are gain matrices to be determined, and $M_1 = \operatorname{diag}\{m_{11}, m_{12}, \ldots, m_{1n}\}$, $m_{1i} = \max\{|m_{1i}^-|, |m_{1i}^+|\}$.

Theorem 11. *The SINNs with time-delay (43) with (44) are finite-time stable, if there exist some positive-definite matrices $P_1, P_2 \in \mathbb{R}^{n\times n}$ such that*

$$\begin{pmatrix} \Theta_{11} & \Theta_{12} \\ * & \Theta_{22} \end{pmatrix} < 0, \tag{45}$$

where

$$\begin{aligned} \Theta_{11} &= -P_1\Xi - \Xi^T P_1 - P_1 K_1 - K_1^T P_1 + M_1^T M_3^{-1} M_1 \\ &\quad + M_2^T P_2 M_2, \\ \Theta_{12} &= P_1 - B^T P_2, \\ \Theta_{22} &= -P_2 A - A^T P_2 - P_2 K_3 - K_3^T P_2 + P_2 C M_3 C^T P_2. \end{aligned} \tag{46}$$

Moreover, the upper bound of the stochastic setting time for stabilization can be estimated as $\mathbb{E}\{T_0\} \le (\lambda_2(\mathbb{E}\{\|x(0)\|\}^{1-\mu} + \mathbb{E}\{\|y(0)\|\}^{1-\mu}))/\lambda_1(1-\mu)$ with $\lambda_1 = \min\{\lambda_{\min}(P_1K_2), \lambda_{\min}(P_2K_4)\}$ and $\lambda_2 = \max\{\lambda_{\max}(P_1), \lambda_{\max}(P_2)\}$.

Proof. Construct a Lyapunov function:

$$V(t) = x^T(t) P_1 x(t) + y^T(t) P_2 y(t). \tag{47}$$

Calculating the Itô differential of $V(t)$ along with (43), we can obtain

$$\begin{aligned} \mathscr{L}V(t) &= 2x^T(t) P_1 \left[-\left(\Xi + K_1\right) x(t) + y(t)\right. \\ &\quad \left. - K_2 \operatorname{sign}(x(t)) |x(t)|^{\mu}\right] + 2y^T(t) \\ &\quad \cdot P_2 \left[-\left(A + K_3\right) y(t) - Bx(t) + Cf(x(t))\right. \\ &\quad \left. - K_4 \operatorname{sign}\left(y(t)\right) \left|y(t)\right|^{\mu}\right] \\ &\quad + \operatorname{trace}\left[\beta^T(t, x(t)) P_2 \beta(t, x(t))\right] = -2x^T(t) \\ &\quad \cdot P_1 \left(\Xi + K_1\right) x(t) + 2x^T(t) P_1 y(t) - 2y^T(t) \\ &\quad \cdot P_2 Bx(t) - 2x^T(t) P_1 K_2 \operatorname{sign}(x(t)) |x(t)|^{\mu} \\ &\quad - 2y^T(t) P_2 K_4 \operatorname{sign}\left(y(t)\right) \left|y(t)\right|^{\mu} - 2y^T(t) P_2 \left(A\right. \\ &\quad \left. + K_3\right) y(t) + 2Y^T P_2 Cf(x(t)) + 2y^T P_2 D \\ &\quad \times \left[\left|f(x(t - h(t)))\right|\right. \\ &\quad \left. - M_1 \operatorname{sign}(x(t)) |x(t - h(t))|\right] \\ &\quad + \operatorname{trace}\left[\beta^T(t, x(t)) P_2 \beta(t, x(t))\right] \le -2x^T(t) \\ &\quad \cdot P_1 \left(\Xi + K_1\right) x(t) + 2x^T(t) P_1 y(t) - 2y^T(t) \\ &\quad \cdot P_2 Bx(t) - 2x^T(t) P_1 K_2 \operatorname{sign}(x(t)) |x(t)|^{\mu} \\ &\quad - 2y^T(t) P_2 K_4 \operatorname{sign}\left(y(t)\right) \left|y(t)\right|^{\mu} - 2y^T(t) P_2 \left(A\right. \\ &\quad \left. + K_3\right) y(t) + 2Y^T P_2 Cf(x(t)) \\ &\quad + \operatorname{trace}\left[\beta^T(t, x(t)) P_2 \beta(t, x(t))\right]. \end{aligned} \tag{48}$$

We can see that the right of inequality (48) equals (23). Hence, the rest of the proof is the same as that of Theorem 9 and it is omitted here. □

Similar to the proof of Theorem 10, we have the following result.

Theorem 12. *If there exist some positive define matrices X_1, X_2, matrices Y_1, Y_2 with appropriate dimensions, for fixed control gain matrices K_2 and K_4, such that*

$$\begin{pmatrix} \Phi_{11} & \Phi_{12} & X_1 M_1^T & X_1 M_2^T & 0 \\ * & \Phi_{22} & 0 & 0 & CM_3 \\ * & * & -M_3 & 0 & 0 \\ * & * & * & -X_2 & 0 \\ * & * & * & * & -M_3 \end{pmatrix} < 0, \tag{49}$$

where

$$\begin{aligned}\Phi_{11} &= -\Xi X_1 - X_1\Xi^T - Y_1 - Y_1^T,\\ \Phi_{12} &= X_2 - X_1 B^T,\\ \Phi_{22} &= -AX_2 - X_2A^T - Y_2 - Y_2^T,\end{aligned} \tag{50}$$

then the finite-time stabilization problem is solvable for the stochastic inertial neural networks (41) and the control gain matrices $K_1 = Y_1X_1^{-1}$, $K_3 = Y_2X_2^{-1}$.

4. Illustrative Example

Consider the following stochastic inertial neural networks with time-delay:

$$\begin{aligned}\frac{d^2x_i(t)}{dt^2} &= -a_i\frac{dx_i(t)}{dt} - b_ix_i(t) + \sum_{j=1}^{n}c_{ij}f_j\left(x_j(t)\right)\\ &\quad + \sum_{j=1}^{n}c_{ij}f_j\left(x_j(t)\right)\\ &\quad + \sum_{j=1}^{n}d_{ij}f_j\left(x_j\left(t-h_j(t)\right)\right)\\ &\quad + \beta_i\left(t, x_i(t)\right)\dot{\omega}_i(t),\end{aligned} \tag{51}$$

which are equivalent to the following vector form:

$$\begin{aligned}dx(t) &= \left[-\Xi x(t) + y(t) + v(t)\right]dt,\\ dy(t) &= \left[-Ay(t) - Bx(t) + Cf(x(t))\right.\\ &\quad \left. + Df(x(t-h(t))) + u(t)\right]dt\\ &\quad + \beta(t, x(t))\,d\omega(t),\end{aligned} \tag{52}$$

where

$$\Xi = \begin{pmatrix}1 & 0\\ 0 & 3\end{pmatrix},$$

$$A = \begin{pmatrix}2 & 0\\ 0 & 3\end{pmatrix},$$

$$B = \begin{pmatrix}4 & 0\\ 0 & 6\end{pmatrix},$$

$$C = \begin{pmatrix}2 & -5\\ -1 & -3\end{pmatrix},$$

$$D = \begin{pmatrix}1 & -1\\ -1 & -2\end{pmatrix},$$

$$\beta(t, x(t)) = \begin{pmatrix}\sin(x_1 + x_2) & 0\\ 0 & 2\cos 2(x_1 - x_2)\end{pmatrix},$$

$$f(x(t)) = \begin{pmatrix}\tanh(x_1)\\ \tanh(x_2)\end{pmatrix},$$

$$h(t) = \begin{pmatrix}0.25\sin(t) + 0.75\\ 0.25\sin(t) + 0.75\end{pmatrix},$$

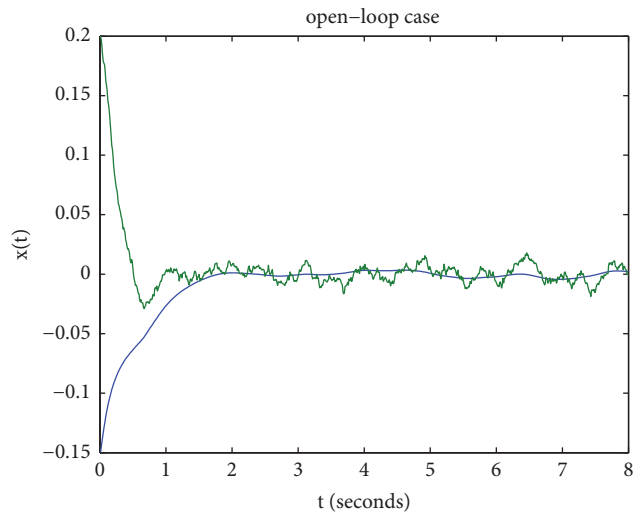

Figure 1: The state trajectories for open-loop SINNs (52) with initial condition $x(0) = (-0.15, 0.2)^T$, $y(0) = (0.3, 0.3)^T$.

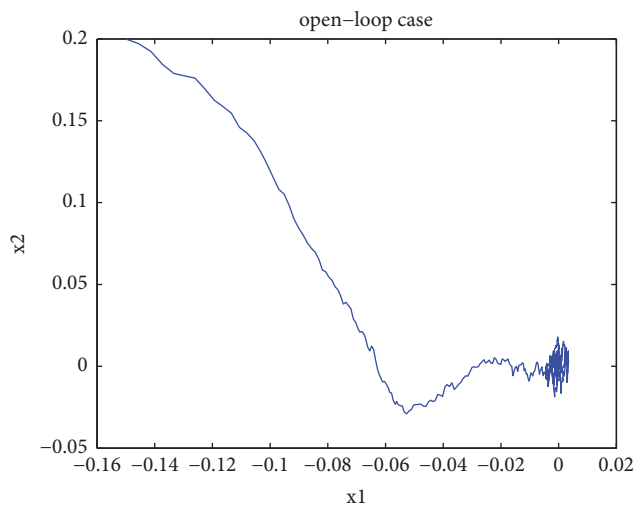

Figure 2: The phrase trajectories for open-loop SINNs (52) with initial condition $x(0) = (-0.15, 0.2)^T$, $y(0) = (0.3, 0.3)^T$.

$$M_1 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},$$

$$M_2 = \begin{pmatrix}1 & 1\\ 2 & -2\end{pmatrix}. \tag{53}$$

Setting the initial values $x(0) = (-0.15, 0.2)^T$, $y(0) = (0.3, 0.3)^T$, the state trajectories and phrase trajectories of the open-loop system are shown in Figures 1 and 2, respectively. Moreover, take 10 sets of numbers randomly as the initial values of $x(0)$ and $y(0)$ and satisfy $x(0) \in (-1, 1)$, $y(0) \in (-3, 3)$. Then the corresponding state trajectories and phrase trajectories of the open-loop system are shown in Figures 3 and 4, respectively. Obviously, the stochastic inertial neural networks with time-delay (51) are not finite-time stabilization.

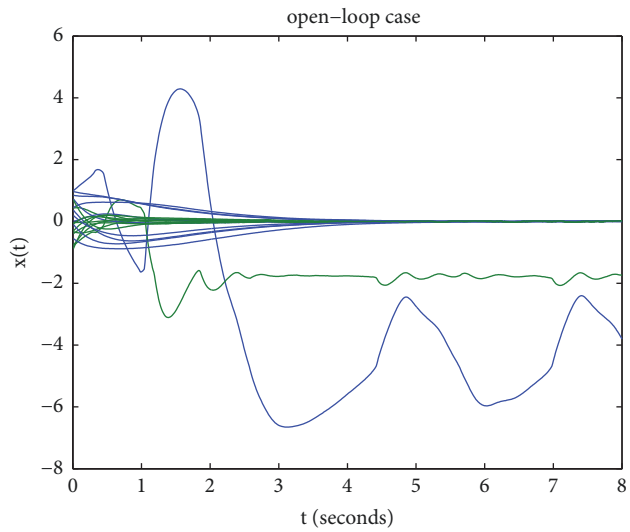

FIGURE 3: The state trajectories for open-loop SINNs (52) with 10 different initial conditions $x(0) \in (-1, 1)$, $y(0) \in (-3, 3)$.

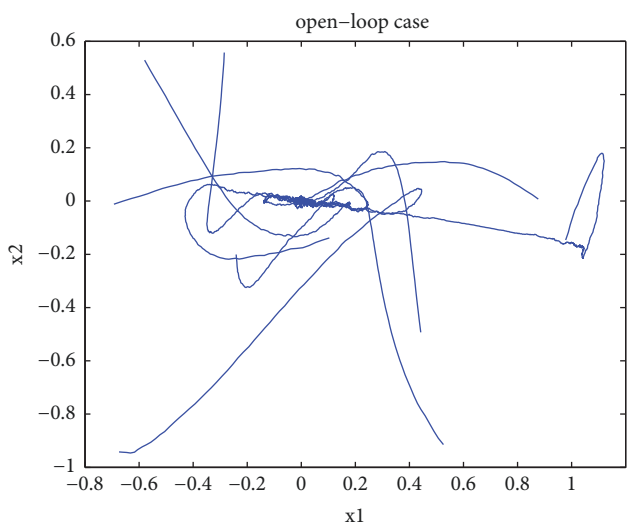

FIGURE 4: The phrase trajectories for open-loop SINNs (52) with 10 different initial conditions $x(0) \in (-1, 1)$, $y(0) \in (-3, 3)$.

Hence, we need to design the delay-feedback controller as (44) for system (51), where the parameter μ is chosen as 0.6, and the initial values $x(0) = (-1, 1)^T$, $y(0) = (3, -3)^T$, $K_2 = \left(\begin{smallmatrix} 5 & 2 \\ 2 & 1 \end{smallmatrix}\right)$, $K_4 = \left(\begin{smallmatrix} 3 & 2 \\ 2 & 4 \end{smallmatrix}\right)$. The solution of (49) is derived by resorting to Matlab LMI Control Toolbox:

$$X_1 = \begin{pmatrix} 0.0793 & 0.0242 \\ 0.0242 & 0.0712 \end{pmatrix},$$

$$X_2 = \begin{pmatrix} 0.5705 & 0.0492 \\ 0.0492 & 0.6263 \end{pmatrix},$$

$$M_3 = \begin{pmatrix} 0.2766 & 0.0585 \\ 0.0585 & 0.0399 \end{pmatrix},$$

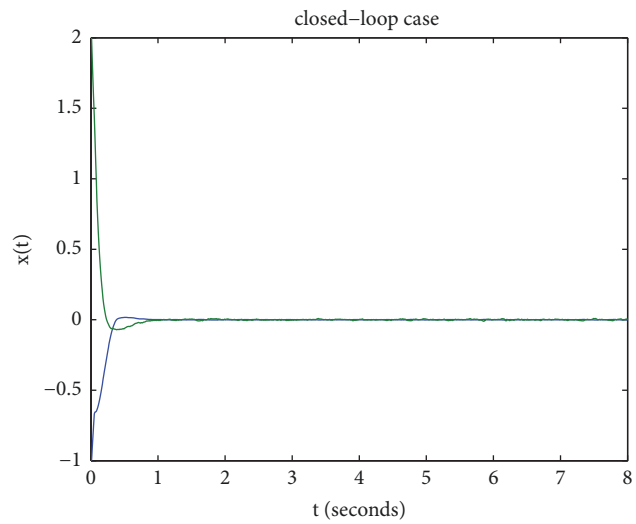

FIGURE 5: The state trajectories for closed-loop SINNs (52) with initial condition $x(0) = (-1, 1)^T$, $y(0) = (-3, 3)^T$.

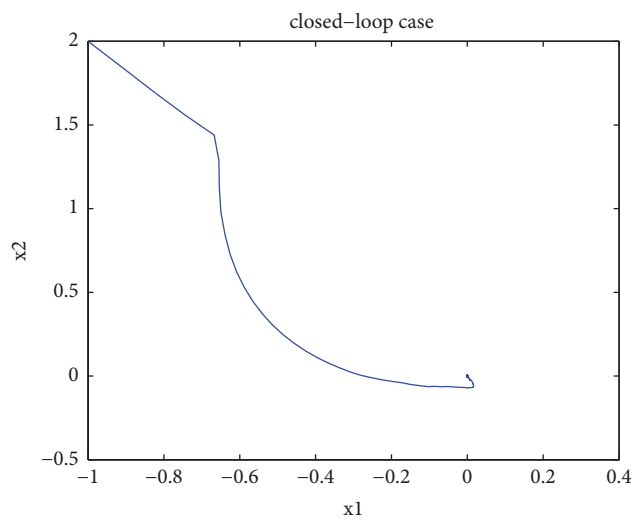

FIGURE 6: The phrase trajectories for closed-loop SINNs (52) with initial condition $x(0) = (-1, 1)^T$, $y(0) = (-3, 3)^T$.

$$Y_1 = \begin{pmatrix} 0.3609 & -9.9199 \\ 9.8791 & 0.2501 \end{pmatrix},$$

$$Y_2 = \begin{pmatrix} -0.5432 & -5.9469 \\ 5.4102 & -1.0974 \end{pmatrix},$$

$$K_1 = \begin{pmatrix} 52.5297 & 152.1518 \\ 137.7642 & -43.3383 \end{pmatrix},$$

$$K_3 = \begin{pmatrix} -0.1341 & -9.4851 \\ 9.6993 & -2.5141 \end{pmatrix}. \tag{54}$$

We can get $P_1 = \left(\begin{smallmatrix} 14.0697 & -4.7821 \\ -4.7821 & 15.6703 \end{smallmatrix}\right)$, $P_2 = \left(\begin{smallmatrix} 1.7648 & -0.1386 \\ -0.1386 & 1.6076 \end{smallmatrix}\right)$, $T_0 \leq 3.2211$. The state trajectories and phrase trajectories of close-loop system are shown in Figures 5 and 6, respectively.

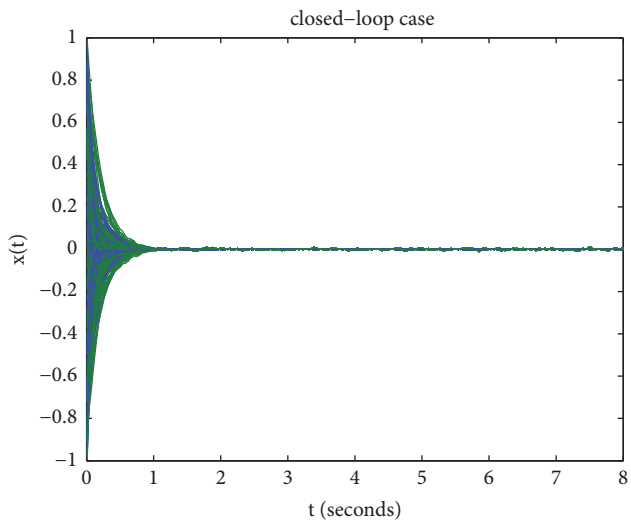

Figure 7: The state trajectories for closed-loop SINNs (52) with 100 different initial conditions $x(0) \in (-1, 1)$, $y(0) \in (-3, 3)$.

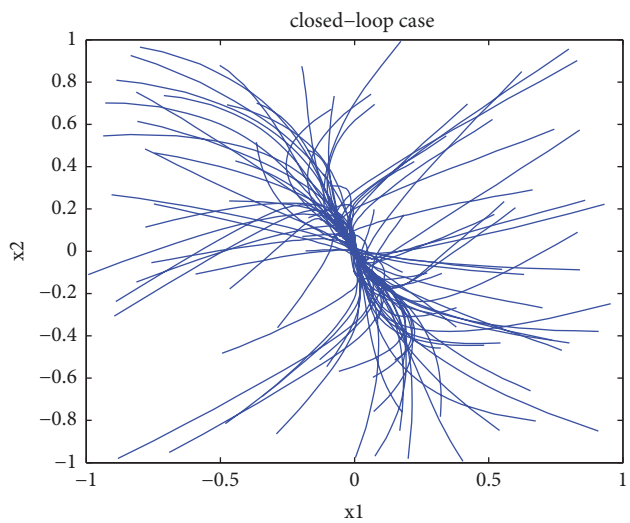

Figure 8: The phrase trajectories for closed-loop SINNs (52) with 100 different initial conditions $x(0) \in (-1, 1)$, $y(0) \in (-3, 3)$.

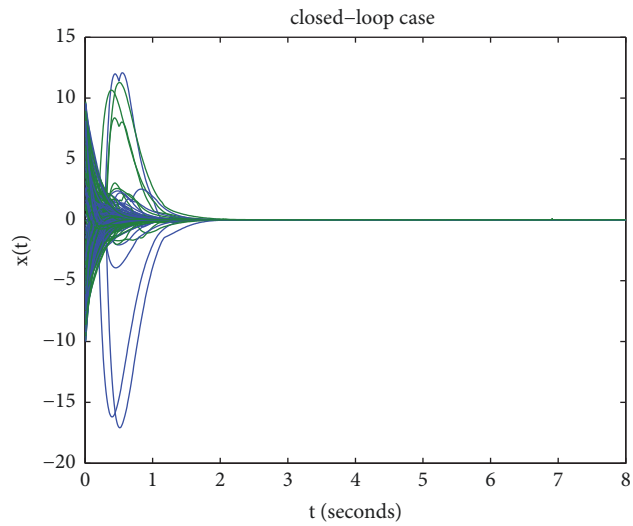

Figure 9: The state trajectories for closed-loop SINNs (52) with 100 different initial conditions $x(0) \in (-10, 10)$, $y(0) \in (-30, 30)$.

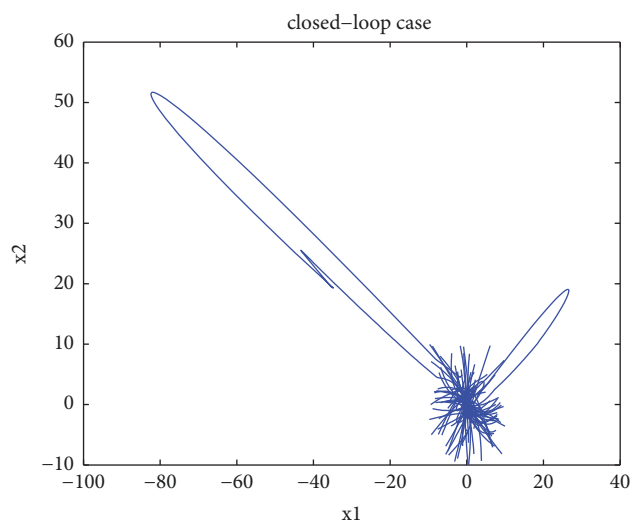

Figure 10: The phrase trajectories for closed-loop SINNs (52) with 100 different initial conditions $x(0) \in (-10, 10)$, $y(0) \in (-30, 30)$.

In order to make the result of the simulation more convincing, we take 100 sets of numbers randomly as the initial values of $x(0)$ and $y(0)$ and satisfy $x(0) \in (-1, 1)$, $y(0) \in (-3, 3)$. Then the corresponding state trajectories are shown in Figure 7 and the corresponding phrase trajectories are shown in Figure 8. Obviously, the stochastic inertial neural networks with time-delay (51) are finite-time stabilization. Moreover, when $x(0) \in (-10, 10)$, $y(0) \in (-30, 30)$, we have state trajectories in Figure 9 and phrase trajectories in Figure 10, which also figure out that the stochastic inertial neural networks with time-delay (51) are finite-time stabilization.

5. Conclusions

In this work, by constructing a proper Lyapunov function, the finite-time stabilization problem has been addressed for stochastic inertial neural networks with or without time-delay. Provided that a set of LMIs are feasible, a suitable delayed or nondelayed nonlinear feedback controller can be designed such that finite-time stability in probability can be ensured for the system under study. An example has been given to demonstrate the correctness of the theoretical results and the effectiveness of the proposed methods.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

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