

## Research Article

# Initial Alignment Error On-Line Identification Based on Adaptive Particle Swarm Optimization Algorithm

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To solve the problem of high accuracy initial alignment of strap-down inertial navigation system (SINS) for ballistic missile, an on-line identification method of initial alignment error based on adaptive particle swarm optimization (PSO) is proposed. Firstly, a complete navigation model of SINS is established to provide the accurate model basis for subsequent numerical optimization calculation. Then setting the initial alignment error as the optimization parameter and regarding the minimum deviation between SINS and GPS output as the objective function, the error parameter optimization model is designed. At the same time, the mutation idea of genetic algorithm (GA) is introduced into the PSO; thus the adaptive PSO is adopted to identify the initial alignment error on-line. The simulation results show that it is feasible to solve the initial alignment error identification problem of SINS by intelligent optimization algorithm. Compared with the standard PSO algorithm and the GA, the adaptive PSO algorithm has the fastest convergence speed and the highest convergence precision, and the initial pitch error and the initial yaw error precision are within  $10''$  and the initial azimuth error precision is within  $25''$ . The navigation accuracy of SINS is improved effectively. Finally, the feasibility of the adaptive PSO algorithm to identify the initial alignment error is further validated based on the test data.

## 1. Introduction

Initial alignment for strap-down inertial navigation system (SINS) plays an important role in the navigation operation of the ballistic missile. The main purpose of initial alignment is to establish the initial attitude matrix, and the quality of initial alignment will affect the navigation accuracy of SINS directly [1] and thus affect the missile firing accuracy ultimately. Therefore, improving the initial alignment accuracy of SINS is of great significance to improve the performance of ballistic missile weapon.

The propagation process of the initial alignment error of SINS is a complex nonlinear problem. The previous solution is to linearize the nonlinear problem, and the filtering algorithms based on Kalman filter are widely adopted [2–6]. A fast SINS initial alignment scheme based on the disturbance observer and Kalman filter is proposed to estimate the misalignment angles in [4], and an adaptive extended Kalman filter algorithm combined with innovation-based adaptive estimation is proposed in [7], while these filtering algorithms often have some disadvantages, such as the difficulty of

model establishing, poor observability of parameters, and long alignment time [8]. Consequently, this paper rejects the traditional research method based on analytic simplification, linearization and filtering, attempting to convert the initial alignment problem of SINS into parameter optimization identification problem. The complete nonlinear optimization model is established, and the intelligent optimization algorithm is used to realize the on-line identification of the initial alignment error of SINS.

To solve the initial alignment problem of inertial system, the application of genetic algorithm (GA) in the initial alignment of SINS on the static base is studied based on the intelligent optimization algorithm [9, 10]. The precision of the initial alignment error is about  $2'$  in [9], and the alignment accuracy needs to be improved. At the same time, the GA has the disadvantages of large computational capacity, low efficiency, and complicated coding, while the particle swarm optimization (PSO) is simple in structure, fast in convergence, and easy to implement and has the advantage of dealing with complex systems. The transfer alignment between the master inertial sensor and the slave inertial

sensor is realized by PSO, and the influence of the maneuver on alignment accuracy is analyzed in [11]. The PSO is applied into the parameter optimization of compass alignment circuit in SINS, and the performance of strap-down gyrocompass initial alignment is improved in [12]. In view of the above analysis and research results, the PSO algorithm is considered to solve the initial alignment problem of the ballistic missile SINS.

In this paper, a complete SINS navigation model is established, and then the error parameter optimization model is constructed based on the minimum deviation between the position parameters outputted by SINS and the position parameters measured by GPS. The mutation idea of GA is introduced into PSO, and the inertial weight and learning factors are improved to obtain adaptive PSO. Finally, the standard PSO, GA, and adaptive PSO are adopted to identify the initial alignment error with the flight software of a certain type of ballistic missile. At the same time, the test data are used to inspect the identification effect of intelligent optimization algorithm.

The rest of this paper is organized as follows. A navigation model of SINS, including initial alignment error model and error compensation model, is established in the second section. In the third section, an error parameter optimization model is constructed and the adaptive PSO is designed for the ballistic missile SINS. In the fourth section, the simulation for identification of initial alignment error is given to demonstrate the feasibility of the intelligent optimization algorithm. Finally, we conclude in the fifth section.

## 2. Establishment of SINS Navigation Model

The main coordinate frames used in this paper different from other references are defined as follows: the body coordinate of the ballistic missile is the orthogonal reference frame aligned with the inertial measurement unit (IMU) axes, and the origin locates the mass of the ballistic missile, the x-axis along the longitudinal direction forward, opposite the direction of gravity, the y-axis is perpendicular to the longitudinal direction upward, the z-axis along the transversal direction right, completing a right-handed system. Launch inertial coordinate (inertial coordinate) is a coordinate whose origin is the launch point, the x-axis points to target in the local level of launch point, and the y-axis is perpendicular to the launch point's local level (upward) and constitutes the right-handed Cartesian coordinates with the axes of x, y, z. The inertial coordinate is used as the navigation frame.

**2.1. Initial Alignment Error Model.** The initial alignment errors of SINS, including initial pitch angle error  $\Delta\varphi_0$ , initial yaw angle error  $\Delta\psi_0$ , and initial azimuth error  $\Delta\gamma_0$ , are caused by vertical degree, installation error, and aiming error of the ballistic missile. The initial attitude matrix  $A$  between the body coordinate and the inertial coordinate can be described as follows by using the quaternion  $(q_0, q_1, q_2, q_3)$ :

$A$

$$= \begin{bmatrix} q_1^2 + q_0^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_0q_2 + q_1q_3) \\ 2(q_1q_2 + q_0q_3) & q_0^2 + q_2^2 - q_1^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_0q_1 + q_2q_3) & q_0^2 + q_3^2 - q_1^2 - q_2^2 \end{bmatrix} \quad (1)$$

The initial values of the quaternion  $(q_0, q_1, q_2, q_3)$  are

$$\begin{aligned} q_0 &= q_{00} - \frac{\gamma_0}{2} q_{20} \\ q_1 &= q_{10} + \frac{\gamma_0}{2} q_{30} \\ q_2 &= q_{20} + \frac{\gamma_0}{2} q_{00} \\ q_3 &= q_{30} - \frac{\gamma_0}{2} q_{10} \end{aligned} \quad (2)$$

where

$$\begin{aligned} q_{00} &= \frac{\sqrt{2}}{2} \left( 1 - \frac{\Delta\varphi_0}{2} \right) \\ q_{10} &= -\frac{\psi_0}{2} q_{00} \\ q_{20} &= \frac{\psi_0}{2} q_{30} \\ q_{30} &= \frac{\sqrt{2}}{2} \left( 1 + \frac{\Delta\varphi_0}{2} \right) \end{aligned} \quad (3)$$

From (2) and (3), we can see that the initial alignment error will affect the initial values of the quaternion and thus affect the calculation precision of the initial attitude matrix  $A$ .

**2.2. Error Compensation Model of SINS.** During the flight course of ballistic missile, the inertial measurement unit (IMU) of SINS, including gyroscope and accelerometer, can measure apparent acceleration and angular velocity in real time and output the data in pulse form. The pulse outputs of the accelerometer and the gyroscope under a navigation cycle are  $(\Delta N_{wx1}, \Delta N_{wy1}, \Delta N_{wz1})$  and  $(\Delta N_{bx1}, \Delta N_{by1}, \Delta N_{bz1})$ , respectively, in which the pulse number remains the same as the actual missile, and the outputs are integers.

After the IMU sends the pulse signals to the onboard computer, the error compensation calculation is completed by the onboard computer in real time. The equations of the error compensation calculation of the apparent velocity increment and the angular increment under a navigation cycle in body coordinate are shown in (4) and (5), respectively [13]:

$$\begin{bmatrix} \Delta W_{xb0} \\ \Delta W_{yb0} \\ \Delta W_{zb0} \end{bmatrix} = \begin{bmatrix} \frac{(\Delta N_{wx1} - K_{0x})}{K_{1x}} \\ \frac{(\Delta N_{wy1} - K_{0y})}{K_{1y}} \\ \frac{(\Delta N_{wz1} - K_{0z})}{K_{1z}} \end{bmatrix}$$

$$\begin{bmatrix} \Delta W_{xb} \\ \Delta W_{yb} \\ \Delta W_{zb} \end{bmatrix} = \begin{bmatrix} \Delta W_{xb0} \\ \Delta W_{yb0} \\ \Delta W_{zb0} \end{bmatrix} - \begin{bmatrix} 0 \\ E_{xy} \Delta W_{x10} \\ E_{xz} \Delta W_{x10} \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} \Delta \theta_{xb0} \\ \Delta \theta_{yb0} \\ \Delta \theta_{zb0} \end{bmatrix} = \begin{bmatrix} \frac{\Delta N_{bx1}}{K_x} \\ \frac{\Delta N_{by1}}{K_y} \\ \frac{\Delta N_{bz1}}{K_z} \end{bmatrix} - \begin{bmatrix} D_{0x} \\ D_{0y} \\ D_{0z} \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} \Delta \theta_{xb} \\ \Delta \theta_{yb} \\ \Delta \theta_{zb} \end{bmatrix} = \begin{bmatrix} \Delta \theta_{xb0} \\ \Delta \theta_{yb0} \\ \Delta \theta_{zb0} \end{bmatrix} - \begin{bmatrix} 0 & E_{yx} & E_{zx} \\ E_{xy} & 0 & E_{zy} \\ E_{xz} & E_{yz} & 0 \end{bmatrix} \begin{bmatrix} \Delta \theta_{xb0} \\ \Delta \theta_{yb0} \\ \Delta \theta_{zb0} \end{bmatrix}$$

$$- \begin{bmatrix} D_{1x} & D_{2x} & D_{3x} \\ D_{1y} & D_{2y} & D_{3y} \\ D_{1z} & D_{2z} & D_{3z} \end{bmatrix} \begin{bmatrix} \Delta W_{x1} \\ \Delta W_{y1} \\ \Delta W_{z1} \end{bmatrix}$$

where  $K_i, K_{0i}, K_{1i}, D_{0i}, D_{1i}, D_{2i}, D_{3i}$ , and  $E_{ij}$  are the tool error coefficient of IMU which are calibrated in the missile technical site.  $\Delta W_{ib0}$  and  $\Delta \theta_{ib0}$  are the calculated intermediate values of the apparent velocity and angle increment, respectively.  $\Delta W_{ib}$  and  $\Delta \theta_{ib}$  are the apparent velocity and the angle increment in body coordinate after the error compensation, respectively, where  $i, j = x, y, z$  represents the three directions of the  $x, y$ , and  $z$  axes.

**2.3. Calculation of Velocity and Position in Inertial Coordinate.** According to the error compensation model of SINS, the apparent velocity increment in body coordinate can be calculated. The apparent velocity increment in body coordinate is converted to inertial coordinate, and it can be presented as

$$\begin{bmatrix} \Delta W_{xa} \\ \Delta W_{ya} \\ \Delta W_{za} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \Delta W_{xb} \\ \Delta W_{yb} \\ \Delta W_{zb} \end{bmatrix} \quad (6)$$

where  $\Delta W_{xa}, \Delta W_{ya}$ , and  $\Delta W_{za}$  are the projections of apparent velocity increment in inertial coordinate and the matrix  $\mathbf{A}$  is calculated by (1), where the calculation equation of the quaternion is as follows:

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}_j = \begin{bmatrix} q_0 & -q_1 & -q_2 & -q_3 \\ q_1 & q_0 & -q_3 & q_2 \\ q_2 & q_3 & q_0 & -q_1 \\ q_3 & -q_2 & q_1 & q_0 \end{bmatrix}_{j-1} \begin{bmatrix} 1 - \frac{1}{8} \Delta \theta_j^2 \\ \left( \frac{1}{2} - \frac{1}{48} \Delta \theta_j^2 \right) \Delta \theta_{xb} \\ \left( \frac{1}{2} - \frac{1}{48} \Delta \theta_j^2 \right) \Delta \theta_{yb} \\ \left( \frac{1}{2} - \frac{1}{48} \Delta \theta_j^2 \right) \Delta \theta_{zb} \end{bmatrix}_j \quad (7)$$

where  $\Delta \theta_j = \sqrt{\Delta \theta_{xb}^2 + \Delta \theta_{yb}^2 + \Delta \theta_{zb}^2}$ . According to (7), the quaternion is calculated by the recursion method which based on the value of the previous moment, recursion to get the quaternion of the current moment. The initial value of the quaternion can be calculated by (2).

By integrating the apparent velocity increment in inertial coordinate, the recursive value of the velocity and position in inertial coordinate at any moment of the missile can be obtained, and the calculation equation is as follows:

$$\begin{bmatrix} v_{xa} \\ v_{ya} \\ v_{za} \end{bmatrix}_j = \begin{bmatrix} v_{xa} \\ v_{ya} \\ v_{za} \end{bmatrix}_{j-1} + \begin{bmatrix} \Delta W_{xa} \\ \Delta W_{ya} \\ \Delta W_{za} \end{bmatrix}_j + \begin{bmatrix} g_{xa} \\ g_{ya} \\ g_{za} \end{bmatrix}_j \frac{\Delta T}{2}$$

$$+ \begin{bmatrix} g_{xa} \\ g_{ya} \\ g_{za} \end{bmatrix}_{j-1} \frac{\Delta T}{2} \quad (8)$$

$$\begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}_j = \begin{bmatrix} x_a \\ y_a \\ z_a \end{bmatrix}_{j-1} + \begin{bmatrix} v_{xa} \\ v_{ya} \\ v_{za} \end{bmatrix}_{j-1} \Delta T$$

$$+ \left( \begin{bmatrix} \Delta W_{xa} \\ \Delta W_{ya} \\ \Delta W_{za} \end{bmatrix}_j + \begin{bmatrix} g_{xa} \\ g_{ya} \\ g_{za} \end{bmatrix}_{j-1} \Delta T \right) \frac{\Delta T}{2} \quad (9)$$

where  $v_{xa}, v_{ya}, v_{za}$  are the projections of velocity in inertial coordinate;  $x_a, y_a, z_a$  are the projections of position in inertial coordinate;  $g_{xa}, g_{ya}, g_{za}$  are the projections of gravity acceleration in inertial coordinate, which can be computed by the ellipsoid gravity acceleration model;  $\Delta T$  is the navigation cycle.

### 3. Error Parameter Optimization Model and Algorithm Design

#### 3.1. Establishment of Error Parameter Optimization Model

**3.1.1. Select the Optimization Variable.** Set the initial alignment error of SINS as the optimization parameter; that is,

$$\mathbf{X} = (x_1, x_2, x_3)^T = (\Delta \varphi_0, \psi_0, \gamma_0)^T \quad (10)$$

**3.1.2. Determine the Objective Function.** Regard the minimum deviation between the position parameters outputted by SINS and the position parameters measured by GPS as the objective function, namely,

$$J(\mathbf{X}) = \min \sum_{i=1}^{Num} \sqrt{\delta x(i)^2 + \delta y(i)^2 + \delta z(i)^2}$$

$$\delta x(i) = x_{INS}(i) - x_{GPS}(i) \quad (11)$$

$$\delta y(i) = y_{INS}(i) - y_{GPS}(i)$$

$$\delta z(i) = z_{INS}(i) - z_{GPS}(i)$$

where  $x_{INS}(i)$ ,  $y_{INS}(i)$ , and  $z_{INS}(i)$  are the position parameters outputted by SINS at the navigation cycle  $i$ , which can be calculated by (9);  $x_{GPS}(i)$ ,  $y_{GPS}(i)$ , and  $z_{GPS}(i)$  are the position parameters measured by GPS at the navigation cycle  $i$ ;  $\delta x(i)$ ,  $\delta y(i)$ , and  $\delta z(i)$  are the position deviations.  $Num$  is the number of navigation cycle used to optimize alignment, and  $Num \cdot \Delta T \leq T_s$ ,  $T_s$  is the total simulation test time.

**3.2. Design of Adaptive Particle Swarm Optimization Algorithm.** PSO is an intelligent optimization algorithm for finding optimal region of search spaces through the interaction of individuals in a swarm [14], and it has been widely applied in the fields of aeronautics and astronautics because of its advantages such as fast convergence, simple structure, and strong versatility [15–17]. However, the standard PSO algorithm has the disadvantage of premature convergence and low efficiency in optimization iteration. Consequently, the standard PSO is improved to the adaptive PSO, and the improved strategy is as follows:

(1) The mutation idea is introduced into PSO algorithm based on GA algorithm. Mutation operation is an important means to increase population diversity in GA algorithm, which can expand search space and avoid falling into local optimization. Therefore, the mutation idea is introduced into PSO algorithm, and through the mutation operation of particles, the population can jump out of the current local optimal position in foraging process and search for a larger space range. Thus, the global search ability is enhanced to overcome the shortcoming of premature convergence of PSO.

(2) The learning factors and inertia weight of PSO are designed as dynamic adjustment form to improve the convergence speed and overcome the disadvantage of low efficiency of PSO in the late optimization. By dynamically adjusting the values of the learning factors and inertia weight of PSO algorithm, it is ensured that PSO has strong global search ability at the initial stage of optimization and a fast search speed at the later stage of optimization.

The basic optimization flow of initial alignment error parameter by adaptive PSO is shown in Figure 1, and the specific steps for optimization calculations are as follows.

**Step 1** (initialize the population). Set particle population size  $N$ , maximum iterations  $M$ , maximum position  $x_{\max}$ , and minimum position  $x_{\min} = -x_{\max}$ . The initial position and the initial velocity of the particles are randomly generated, and each particles fitness value of initialization population is calculated to determine the individual best and the global best of the particles.

**Step 2** (update particle swarm velocity and position). The equation for calculating the velocity and position of particles is as follows:

$$\begin{aligned} v_{ij}(k+1) &= w(k) v_{ij}(k) + r_1 c_1(k) (P_{ij}(k) - x_{ij}(k)) \\ &\quad + r_2 c_2(k) (P_{gj}(k) - x_{ij}(k)) \\ x_{ij}(k+1) &= x_{ij}(k) + v_{ij}(k+1) \end{aligned} \quad (12)$$

where  $x_{ij}(k)$  and  $v_{ij}(k)$  ( $j = 1, 2, 3$ ) are the position and velocity of the  $j$ th dimension of particle  $i$  at iteration  $k$ ;  $w(k)$  is the inertia weight;  $r_1$  and  $r_2$  are the random numbers distributed in the range  $[0, 1]$ ;  $c_1(k)$  and  $c_2(k)$  are the learning factors;  $P_{ij}(k)$  and  $P_{gj}(k)$  are the individual best and the global best at iteration  $k$ , respectively.

In order to enhance the global exploration ability in the early stage of optimization and improve the convergence speed at the later stage of optimization, the inertia weight is designed to be the dynamic adjustment mode. The inertial weight has a large value at the beginning, and the weight decreases with the increase of the number of iterations. Consequently, the inertia weight  $w(k)$  can be designed as

$$w(k) = w_{\max} - \left( \frac{k}{M} \right)^2 (w_{\max} - w_{\min}) \quad (13)$$

Learning factors  $c_1(k)$  and  $c_2(k)$  are the factors which can control degree of self-learning and group learning of particles, respectively. The particle swarm needs a large self-learning ability to enhance the global search effect in the early stage of optimization, and it needs a large group learning ability to speed up the convergence speed in the late optimization. Therefore, with the increase of the number of iterations, learning factors  $c_1(k)$  continues to decrease and learning factors  $c_2(k)$  increases gradually and the learning factors can be described as

$$\begin{aligned} c_1(k) &= c_{1\max} - \left( \frac{k}{M} \right)^2 (c_{1\max} - c_{1\min}) \\ c_2(k) &= c_{2\min} + \left( \frac{k}{M} \right)^2 (c_{2\max} - c_{2\min}) \end{aligned} \quad (14)$$

**Step 3.** Calculate fitness value of population, and update individual best and global best. According to (11), the fitness value  $f(x_i(k+1))$  of particles is calculated at iteration  $k+1$  and compared with the previous fitness value  $f(P_i(k))$  of individual best. If  $f(x_i(k+1)) < f(P_i(k))$ , the individual best is updated, namely,  $P_i(k+1) = x_i(k+1)$ . Similarly, comparing the fitness value of the individual best  $f(P_i(k+1))$  at iteration  $k+1$  with the previous fitness value of the global best  $f(P_g(k))$ , if  $f(P_i(k+1)) < f(P_g(k))$ , the global best of particles is updated, namely,  $P_g(k+1) = P_i(k+1)$ .

**Step 4.** Mutation operation and update the individual best and the global best again.

(A) *Select Mutation Particles.* The fitness values of particle swarm at iteration  $k+1$  were sorted by descending order, and the numbers  $N_m$  of particles arranged in front and with large fitness value were selected as the mutation objects.

(B) *Mutation Operation.* First, generate the random number  $r$ , and then compare the sizes of the random number  $r$  and  $r_m$ , where  $r_m$  is the mutation probability of particle. If the generated random number  $r$  is less than  $r_m$ , the mutation operation is performed as follows:

$$x_{ij}(k) = (2r - 1) (x_{\max}(j) - x_{\min}(j)) \quad (15)$$

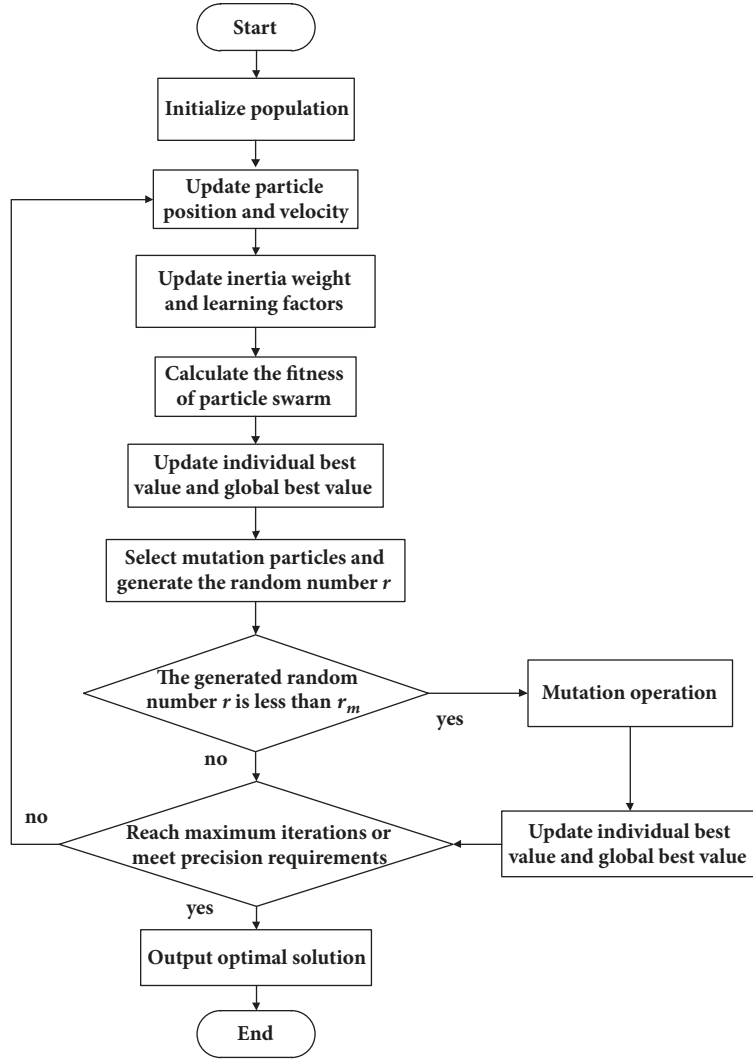


FIGURE 1: Calculation flow chart of adaptive PSO.

where  $r$  is the random number distributed in the range  $[0, 1]$  and  $\lambda$  is the mutation coefficient. Otherwise, go to Step 5 for the next calculation.

(C) *Update Particle's Best.* For the new particles generated by mutation, update the individual best and the global best again.

*Step 5 (end condition).* If reaching maximum iterations, namely,  $k > M$ , or the search results satisfy the accuracy requirement, the calculation is stopped and the optimal parameter of initial alignment error is output. Otherwise, return Step 2 for the next generation calculation.

#### 4. Simulation Experiment and Result Analysis

The simulation study is presented to confirm the feasibility of initial alignment error on-line identification based on adaptive PSO in this section. Firstly, the simulation conditions are set up, then the simulation experiment and results are analyzed. At last, the test data is introduced.

*4.1. Simulation Condition Settings.* The simulation conditions of the initial alignment error optimization model for SINS are set as follows:

(1) Setting initial pitch angle error is  $90''$ , initial yaw angle error is  $-90''$  and initial azimuth error is  $-150''$ . The navigation cycle  $\Delta T$  is 0.1 s, the number of navigation cycles  $Num$  is 500, and the total simulation test time  $T_s$  is 50 s.

(2) Tool error of SINS. The constant and random drifts of gyro are chosen as  $0.02^\circ/\text{h}$  and  $0.01^\circ/\text{h/Hz}$ , respectively. The constant and random drifts of accelerators are chosen as  $100 \mu\text{g}$  and  $50 \mu\text{g/Hz}$ , respectively.

(3) GPS navigation error. The position error is 5.0 m, and the velocity error is 0.1 m/s.

(4) Simulation conditions of PSO algorithm. The maximum iterations  $M$  is 100, the population size  $N$  is 40, and the maximum position  $x_{\max} = (300 \ 300 \ 500)$ . The inertia weights  $w_{\max}$  and  $w_{\min}$  are 0.8 and 0.4, respectively. The learning factors  $c_{1\max}$  and  $c_{2\max}$  are all 3.0, and the learning factors  $c_{1\min}$  and  $c_{2\min}$  are all 0.5. The mutation numbers of particles  $N_m$  are 10, the mutation probability  $r_m$  is 0.2, and the mutation coefficient  $\lambda$  is 0.5.



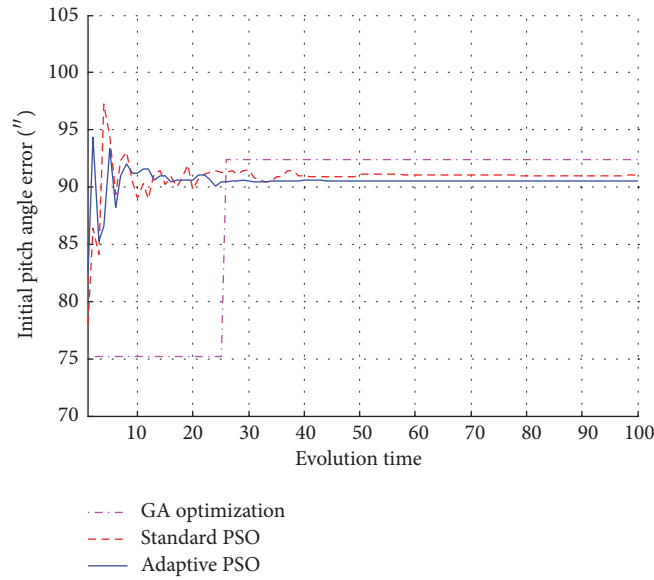


FIGURE 2: Convergence process of initial pitch angle error.

**4.2. Simulation Results and Analyses.** Based on the above error parameter optimization model and taking the flight software of a certain ballistic missile as the simulation experiment environment, the standard PSO algorithm, the GA, and the adaptive PSO algorithm are adopted to optimize the initial alignment error parameter of SINS, and the convergence process of the initial pitch angle error, the initial yaw angle error, and the initial azimuth angle error are shown in Figures 2–4, and the fitness convergence diagrams of the three algorithms are shown in Figure 5. The optimization results of initial alignment error are shown in Table 1. It shows the accuracy and convergence speed of three different algorithms.

It is shown from Table 1 that the GA, the standard PSO, and the adaptive PSO can be used to optimize the initial alignment error parameter; the maximum residual error of the initial alignment error of three algorithms is not more than  $30''$ , the optimization calculation time is less than 4.0 s, and the convergence speed is fast. It shows that it is feasible and effective in identifying the initial alignment error of SINS by using intelligent optimization algorithm.

From Figures 2–5, we can see that the adaptive PSO has a fastest convergence speed than the GA and the standard PSO. It is shown from Table 1 that the residual errors of initial pitch angle and the initial yaw angle calculated by the adaptive PSO are less than  $10''$ , and the residual error of the initial azimuth is less than  $25''$ , which shows that the adaptive PSO can improve the convergence accuracy of the initial alignment error.

The initial alignment error parameter calculated by the adaptive PSO is compensated, and then the navigation parameters of the SINS are recalculated. The deviation between the navigation parameters obtained by error compensation and the actual navigation parameters of the missile is called the optimization residual. By simulation, the SINS position error, GPS position error and optimization residual

of position are shown in Figure 6, and the SINS velocity error, GPS velocity error, and optimization residual of velocity are shown in Figure 7. The root mean square (RMS) statistic results of navigation parameter error are listed in Table 2.

From Figures 6 and 7, we can find out that the optimization residuals of position and velocity are not only far less than SINS, but also significantly less than GPS navigation system. It is shown from Table 2 that the RMS position errors of SINS are within 10 m, and the RMS velocity errors of SINS are within 1 m/s while the RMS position errors are less than 1 m, and the RMS velocity errors are less than 0.1 m/s, after the initial alignment error is identified and compensated by the adaptive PSO algorithm. Obviously, compared with SINS, the RMS errors of the navigation parameters compensated by the adaptive PSO are reduced by 10 times, and the navigation accuracy is greatly improved. The simulation results indicate that the adaptive PSO can effectively identify the initial alignment error and improve the navigation accuracy of SINS.

**4.3. Test Data Validation.** In order to verify the effectiveness of adaptive PSO algorithm to identify initial alignment error, the data collected from the test are analyzed. Among them, the gyroscope constant drift and the accelerometer constant bias are about  $0.01^\circ/\text{h}$  and  $100\mu\text{g}$ , respectively; SINS data update frequency is 10.0Hz; GPS data update frequency is 1.0Hz. The actual measured output values of SINS and GPS were collected and recorded during the test. The deviation between the position and the speed of SINS and GPS output is called the position error and the velocity error of SINS, then the position error and the velocity error curve of SINS are shown in Figures 8 and 9, respectively.

Based on the above test data, the adaptive PSO algorithm is used to optimize the initial alignment error parameters, and the results of the optimization parameters are shown in Table 3.

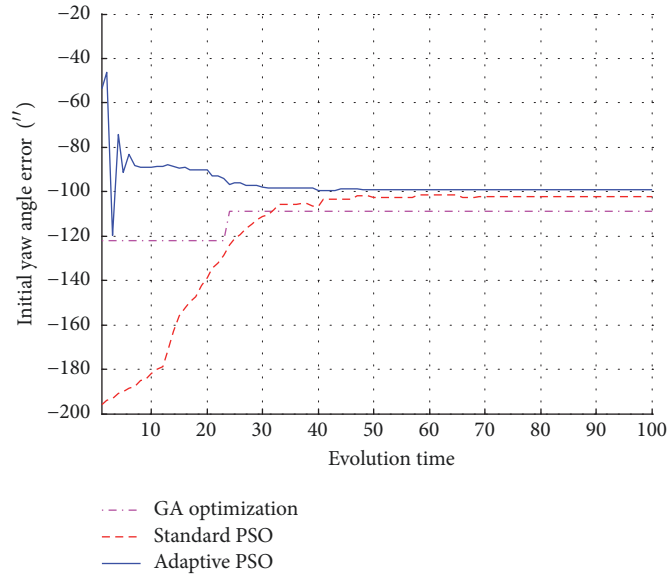


FIGURE 3: Convergence process of initial yaw angle error.

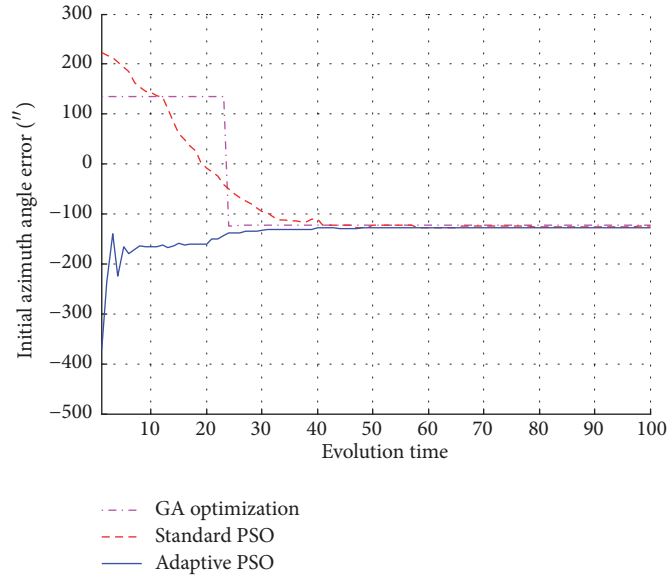


FIGURE 4: Convergence process of initial azimuth angle error.

TABLE 1: Optimization results of initial alignment error.

Optimization algorithm	Parameter	Initial alignment error			Calculation time/(s)
		$\Delta\varphi_0/('')$	$\Delta\psi_0/('')$	$\Delta\gamma_0/('')$	
GA optimization	Optimization value	92.42	-108.67	-122.62	3.85
	Residual error	2.42	-18.67	27.38	
Standard PSO	Optimization value	91.03	-102.35	-124.53	3.46
	Residual error	1.03	-12.35	25.47	
Adaptive PSO	Optimization value	90.51	-99.29	-127.56	3.48
	Residual error	0.51	-9.29	22.44	

TABLE 2: RMS statistical results of navigation parameter errors.

Error type	Navigation parameter	Pure SINS error	Compensation error
Position error (m)	$x_a$	6.9748	0.1730
	$y_a$	2.4885	0.3683
	$z_a$	10.7109	0.0960
Velocity error (m/s)	$v_{ax}$	0.3748	0.0147
	$v_{ay}$	0.1790	0.0306
	$v_{az}$	0.6336	0.0044

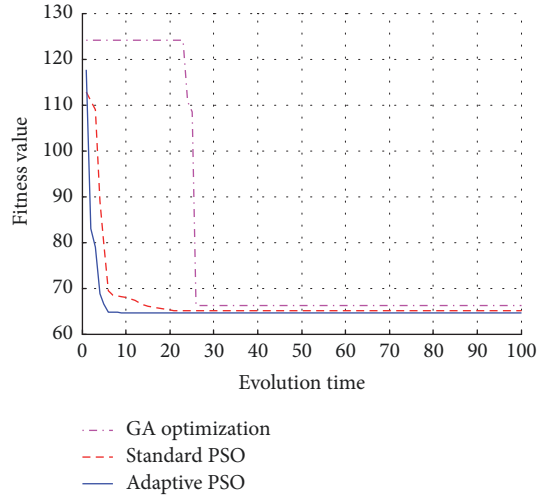


FIGURE 5: Fitness convergence diagram.

TABLE 3: Results of optimization parameters.

Optimization parameter	Lower bound	Upper bound	Optimization value
$\Delta\varphi_0/('')$	-90.00	90.00	90.00
$\Delta\psi_0/('')$	-90.00	90.00	73.46
$\Delta\gamma_0/('')$	-180.00	180.00	-104.04

TABLE 4: RMS statistical results of errors.

Error type	Navigation parameter	Initial error	Optimization error
Position error (m)	$x_a$	6.2535	5.4109
	$y_a$	10.3145	8.2356
	$z_a$	8.6935	7.5713
Velocity error (m/s)	$v_{ax}$	0.4784	0.2770
	$v_{ay}$	0.7956	0.6541
	$v_{az}$	0.2769	0.2540

The initial alignment error calculated by the adaptive PSO is compensated, and then the navigation parameters of SINS are recalculated. After calculation and compensation, the position and velocity error curves are shown in Figures 10 and 11 respectively, and the error RMS statistic results are shown in Table 4.

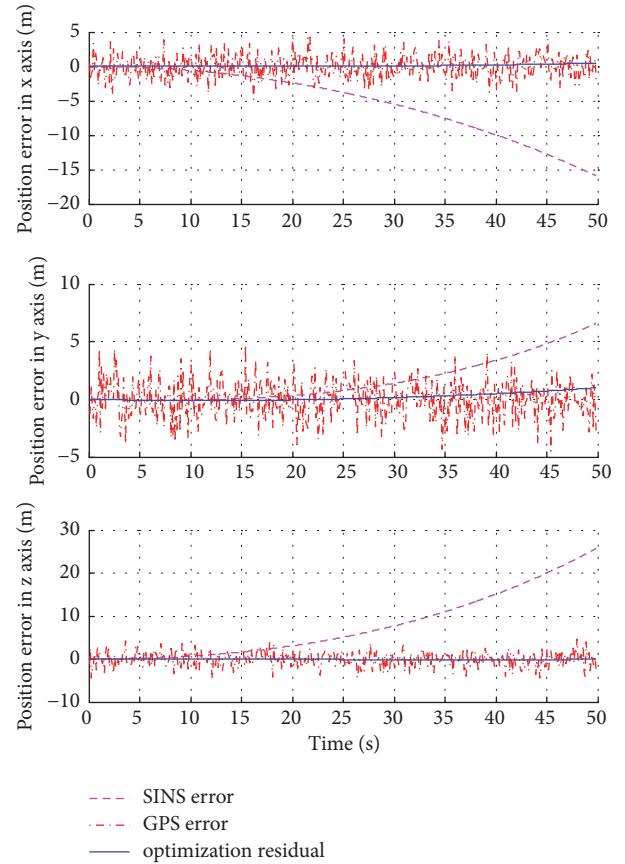


FIGURE 6: Position error simulation curve.

From Figures 10 and 11, we can see that the optimization errors of the position and velocity are less than the initial errors of the position and velocity of the test data, after the initial alignment error is compensated. It is shown from Table 4 that the RMS errors of the position and velocity of optimization calculation are less than the RMS errors of the test data. Therefore, it is proved that the adaptive PSO algorithm is effective in identifying the initial alignment error and can improve the navigation accuracy of missile flight.

## 5. Conclusion

The initial alignment error identification of ballistic missile SINS is studied in this paper. The real and complete navigation model of SINS is established, which provides an accurate model basis for the initial alignment error identification. At



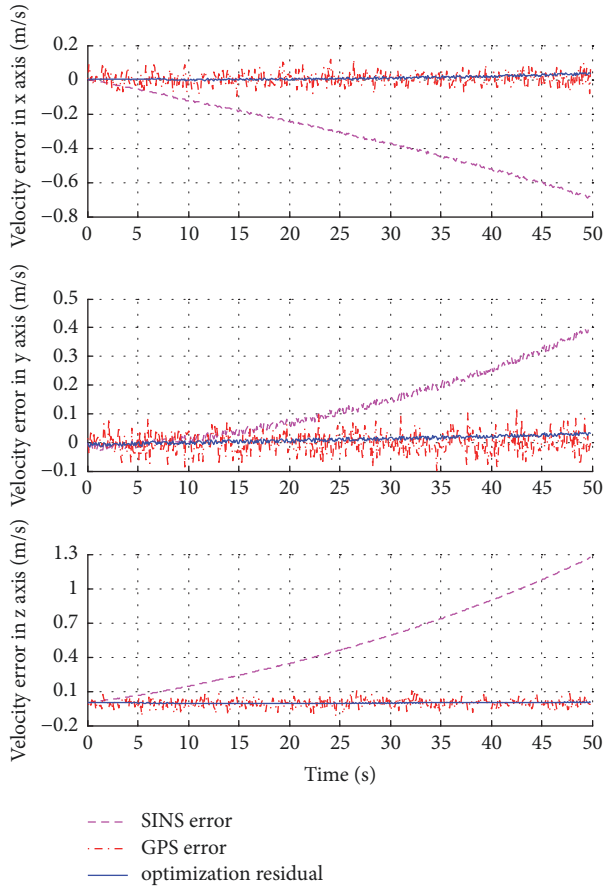


FIGURE 7: Velocity error simulation curve.

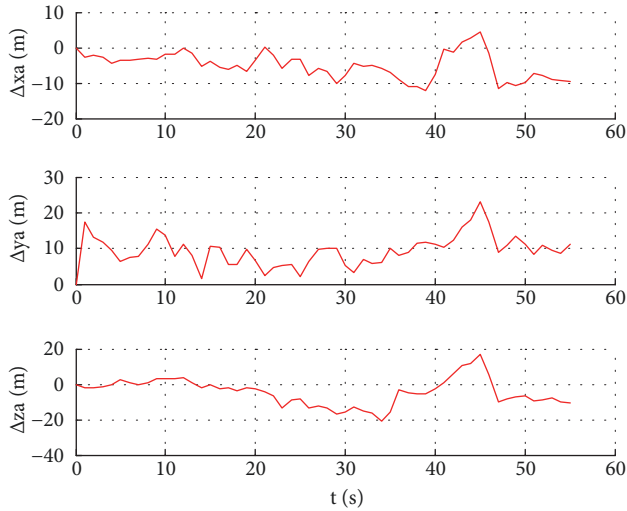


FIGURE 8: Position error curve of SINS.

the same time, the error parameter optimization model is designed, and the initial alignment error is identified on-line by the intelligent optimization algorithm. What is more, the inertia weight and learning factors of PSO are designed as dynamic adjustment form to improve search speed and search accuracy, and the mutation operation of the GA is

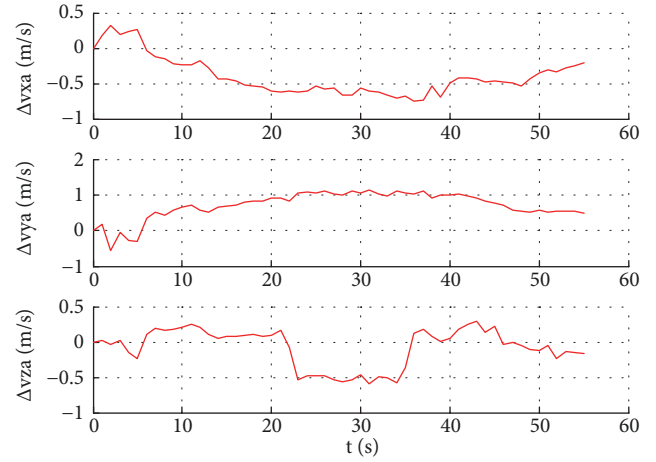


FIGURE 9: Velocity error curve of SINS.

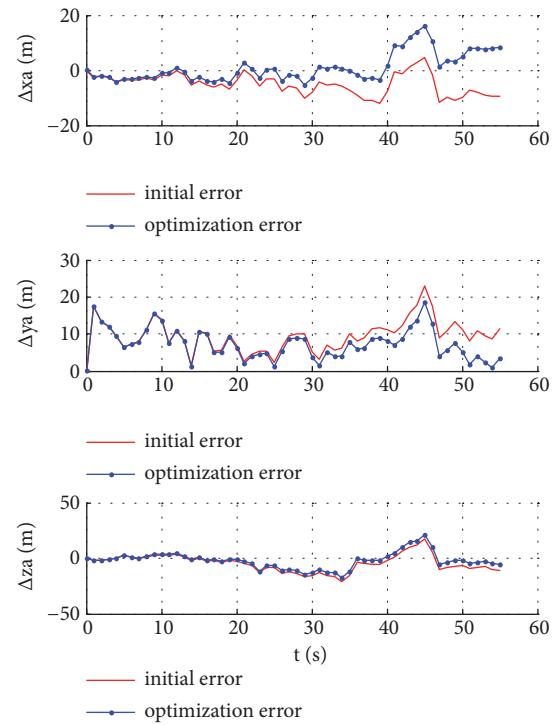


FIGURE 10: Position optimization error curve.

introduced into the PSO algorithm to jump out local optimal value and enhance global convergence ability.

The simulation results show that the intelligent optimization algorithm is efficient to solve the problem of initial alignment error identification. Of course, the results show that the adaptive PSO algorithm has fastest search efficiency and highest convergence accuracy than the standard PSO algorithm and the GA, and the residuals of the initial pitch angle and the initial yaw angle are less than  $10''$ , and the residual of the initial azimuth is less than  $25''$ . Finally, the validity of the adaptive PSO algorithm to identify the initial alignment error is validated based on the test data. Therefore,

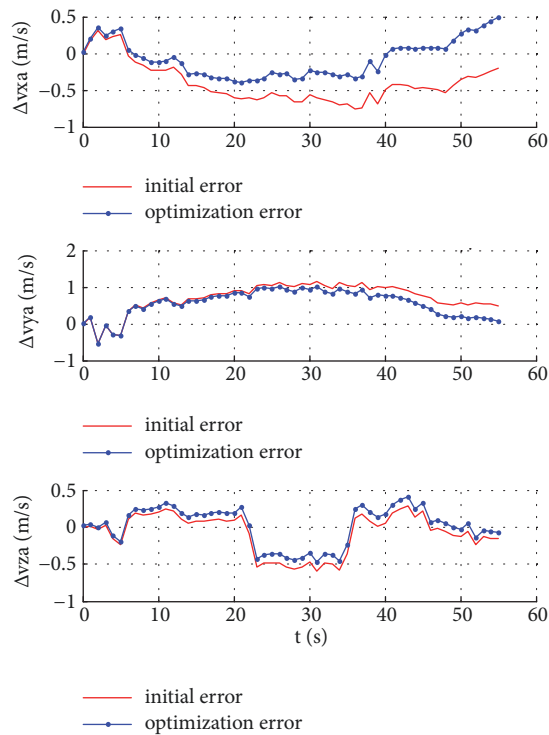


FIGURE 11: Velocity optimization error curve.

the content of this paper has a certain reference value for the improvement of the initial alignment accuracy of the ballistic missile SINS.

## Data Availability

The test data used to support the findings of this study have not been made available because the data are currently under embargo, and requests for data will be considered by the corresponding author at the right time.

## Conflicts of Interest

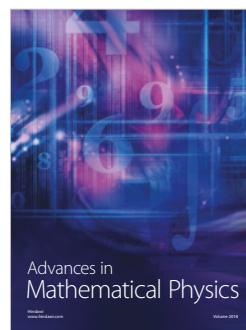
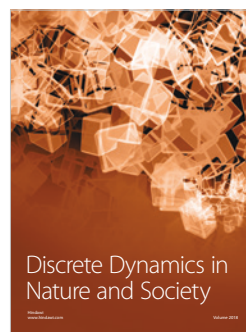
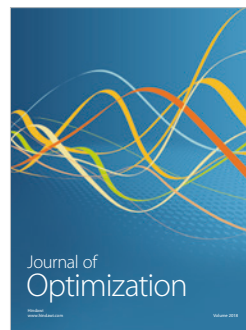
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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