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Research Article

Time-Delayed Polynomial Grey System Model with the Fractional Order Accumulation

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In this work, a novel time-delayed polynomial grey prediction model with the fractional order accumulation is put forward, which is abbreviated as TDPFOGM(1,1), based on the new grey system theory to predict the small sample in comparison with the existing forecasting models. The new model takes into account the nonhomogeneous term and the priority of new information can be better reflected in the in-sample model. The data in this paper all come from the existing literatures. The results demonstrate that the TDPFOGM(1,1) model outperforms the TDPGM(1,1) and FOGM(1,1) model.

1. Introduction

Shale gas is one kind of fossil fuel which is clean, efficient, and cheap. With the shale gas revolution in the United States [1, 2], shale gas has been developed in many countries. And China has achieved some success [3]. The development of shale gas industry has been regarded as an important mission of the government [4, 5]. In such circumstances, it is very important to predict the natural gas production and consumption in China for the decision-makers, such as the government and energy companies.

Grey model proposed by Deng [6] is used for small sample forecasting. The grey prediction models play an important role in the grey system theory. These models are often named as grey models. GM(1,1), that is, first-order Grey model with one variable, has been successfully applied in many disciplines [7–13]. However, the existing GM(1,1) model cannot be accurate prediction for many actual systems. In recent literatures [14–23], the improved grey prediction models are popularly used for energy consumption. Ma and Liu [14] have proposed a novel time-delayed polynomial grey model to predict the natural gas consumption in China. Akay and Atak [15] have proposed a novel method based on the basic GM(1,1) model with rolling mechanism to forecast the electricity demand of Turkey. Wu and Shen [16] have proposed a grey-related least squares support vector machine optimization model and applied it, predicting the natural gas consumption demand. The authors in [17] have proposed a novel kernel regularized nonhomogeneous grey model and applied it, forecasting the petroleum production. Kumar and Jain [18] have used the Grey-Markov and GM(1,1) model with rolling mechanism to predict the energy consumption in India. In [19], a prediction method using grey model for cumulative plastic deformation under cyclic loads was demonstrated. Pao and Tsai [22] have used the GM(1,1) model to predict the energy consumption in Brazil compared with the ARIMA model.

Discrete grey model also has good effect on prediction. Ayvaz and Kusakci [24] have used the nonhomogeneous discrete grey model to predict the electricity consumption forecasting for Turkey. Xie and Pearman [25] have used the discrete GM(1,1) model (DGM(1,1)) to predict the energy consumption of China. Simultaneously, the fractional gray model is proposed. In [26], the authors have proposed a grey system model with the fractional order accumulation. Yuan et al. [27] have pointed out the advantages and disadvantages of GM(1,1) and the Autoregressive Integrated Moving Average (ARIMA) model and then proposed a novel hybrid model based on the GM(1,1) model and the ARIMA model to predict primary energy consumption in China. Wu et al. [28] have proposed a novel GM(1,1) model with the Principle of New Information Priority (NIGM(1,1)) to predict the natural gas consumption in China, and the results show that NIGM(1,1) outperforms several existing models. Mao et al. [29] present a new fractional grey model, in which first-order differential equations are transformed into fractional differential equations. And it has high modeling precision and can overcome the GM(1,1) model class ratio test restrictions. Yang et al. [30] modified optimized fractional grey model using the error feedback, and the performance is evaluated and greatly improved in modeling. Meng et al. [31] develop a discrete grey model with fractional operators, which also makes use of genetic algorithms to optimize the modeling parameter. Yang et al. [32] proposed that the geometric coordinate features are used by the coordinates of area and middle point lines and established the grey prediction model for interval grey number by the fractional order accumulation calculus. Wu et al. [33] developed a novel multivariable grey forecasting model that considered the total population to forecast the electricity consumption. Li et al. [34] proposed a novel grey forecasting model with full-order time power terms (FOTP-GM(1,1)). Wang et al. [35] propose a data grouping approach based grey modeling method DGGM (1,1) to predict quarterly hydropower production in China. All these researches indicate that the grey models are efficient to predict the consumption of many kinds of energy for many countries.

Motivated by the above, in this paper, we propose a novel time-delayed polynomial grey model with the fractional order accumulation. Simultaneously, the model considered the character time-delayed polynomial and fractional order. The main contributions of this paper are summarized as follows: (1) this paper is based on predecessor's research about grey model; a new model is put forward; (2) using the same data, the TDPFOGM(1,1) model makes predictions just as shown in figures. And the results were compared with the TDPGM(1,1) model and the FOGM(1,1) model; (3) more intuitively, the mean absolute percentage error (MAPE) is shown in the tables.

The rest of this paper is organized as follows: the details of modeling procedures of the TDPFOGM(1,1) model are given in Section 2; the application of TDPFOGM(1,1) to small sample prediction is given in Section 3, including the comparison with the other commonly used prediction models presented based on the same data sample, and the conclusions are drawn in Section 4.

2. Description of the Problem

In this section, we will present the time-delayed polynomial grey system model with the fractional order accumulation, abbreviated as the TDPFOGM(1,1) or TDPGM$^{(p/q)}(1,1)$ model, including the principles and the computational steps.

2.1. Grey Model with the Fractional Order Accumulation. Let the rth ($r \in R_+$) order accumulated generating operator of the original nonnegative sequence $X^{(0)}$ be $X^{(r)}$, $r = 1, 2, \ldots, n$.

Set $\binom{p/q-1}{0} = 1$; $\binom{k-1}{k} = 0$, $k = 1, 2, \ldots, n$; then

$$X^{(r)}(k) = \sum_{i=1}^{k} \binom{k-i+r-1}{k-i} x^{(0)}(i), \quad k = 1, 2, \ldots, n \quad (1)$$

where $\binom{k-i+r-1}{k-i} = (r+k-i-1)(r+k-i-2)\cdots(r+1)r/(k-i)!$

Fractional derivatives accumulate the whole history of the system in weighted form. $x^{(1)}(k)$ in grey system theory denotes the weight of $x^{(0)}(i)$ $(i = 1, 2, \ldots, k)$ as 1. The larger r of $x^{(r)}(k)$ is the larger the weight of old data is; the smaller r of $x^{(r)}(k)$ is the smaller the weight of old data is. Reducing r can reduce the weights of old data, which can put more emphasis on the newer data (more details and the properties of the fractional order accumulation generation operation can be seen in [25]).

The original form of the GM$^{p/q}(1,1)$ model is as follows:

$$x^{(p/q)}(k) - x^{(p/q)}(k-1) + az^{(p/q)}(k) = b \quad (2)$$

If $r = p/q$, then (p/q) $(0 < p/q \leq 1)$ order inverse accumulated generating operator of $X^{(0)}$; we write

$$\begin{aligned} \alpha^{(p/q)}X^{(0)} &= \alpha^{(1)}X^{(1-p/q)}(k) = \{\alpha^{(1)}x^{(1-p/q)}(1), \\ &\alpha^{(1)}x^{(1-p/q)}(2), \ldots, \alpha^{(1)}x^{(1-p/q)}(n)\} \end{aligned} \quad (3)$$

2.2. The Representation of the TDPFOGM(1,1) Model. The new form of the TDPFOGM(1, 1) model is as follows:

$$\begin{aligned} &x^{(p/q)}(k) - x^{(p/q)}(k-1) + \lambda_1 z^{(p/q)}(k) \\ &= \lambda_2 \sum_{\tau=1}^{k} \tau^2 + \lambda_3 \sum_{\tau=1}^{k} \tau + \lambda_4 \end{aligned} \quad (4)$$

$$z^{(p/q)}(k) = \frac{x^{(p/q)}(k) + x^{(p/q)}(k+1)}{2}, \quad k = 1, 2, \ldots, n-1 \quad (5)$$

where $z^{(p/q)}(k)$ is called the background value. For TDPFOGM(1, 1) model, the first number "1" stands for "first order" and the second number "1" means "one sequence," as only one sequence is considered in this model. It is the traditional TDPGM(1, 1) model [14], when $p/q = 1$. The ordinary least squares estimate sequence of the TDPFOGM(1, 1) model is satisfied.

2.3. The Solution of the TDPFOGM(1,1) Model. $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$ is the given original sequence, and the unconstrained optimization equation (6) can describe the least squares criteria for the TDPFOGM(1,1) model:

$$\begin{aligned} \min_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} \sum_{k=2}^{n} \Bigg[& x^{(p/q)}(k) - x^{(p/q)}(k-1) + \lambda_1 z^{(p/q)}(k) \\ & - \lambda_2 \sum_{\tau=1}^{k} \tau^2 - \lambda_3 \sum_{\tau=1}^{k} \tau - \lambda_4 \Bigg]^2 \end{aligned} \quad (6)$$

The linear system in (7) can solve the above optimization problem:

$$[\lambda_1, \lambda_2, \lambda_3, \lambda_4]^T = \left(B^T B\right)^{-1} B^T Y \quad (7)$$

where

$$B = \begin{bmatrix} -z^{(p/q)}(2) & \frac{5}{2} & \frac{3}{2} & 1 \\ -z^{(p/q)}(3) & 9 & 4 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ -z^{(p/q)}(n) & \sum_{\tau=2}^{n} \frac{n^2 + (n-1)^2}{2} & \sum_{\tau=2}^{n} \frac{2n-1}{2} & 1 \end{bmatrix}, \tag{8}$$

$$Y = \begin{bmatrix} x^{(p/q)}(2) - x^{(p/q)}(1) \\ x^{(p/q)}(3) - x^{(p/q)}(2) \\ \vdots \\ x^{(p/q)}(n) - x^{(p/q)}(n-1) \end{bmatrix}$$

The equation

$$\frac{dx^{(p/q)}(t)}{dt} + \lambda_1 x^{(p/q)}(t) = \lambda_2 \sum_{\tau=1}^{t} \tau^2 + \lambda_3 \sum_{\tau=1}^{t} \tau + \lambda_4 \tag{9}$$

is called a whitenization differential equation of TDPFOGM(1,1), and the solution of the whitenization equation is given by the following equation:

$$\begin{aligned} \hat{x}^{(p/q)}(k) &= x^{(p/q)}(1) e^{-a(k-1)} \\ &+ \sum_{\tau=2}^{k} \frac{1}{2} \left[e^{-a(k-\tau)} f(\tau) + e^{-a(k-\tau+1)} f(\tau - 1) \right] \end{aligned} \tag{10}$$

where

$$f(t) = \lambda_2 \sum_{\tau=1}^{t} \tau^2 + \lambda_3 \sum_{\tau=1}^{t} \tau + \lambda_4 \tag{11}$$

Equation (10) is called the discrete response function, and within the initial condition $x^{(p/q)}(1) = x^{(0)}(1)$, the discrete response function can be obtained as follows:

$$\begin{aligned} \hat{x}^{(p/q)}(k) &= x^{(0)}(1) e^{-\lambda_1(k-1)} \\ &+ \sum_{t=2}^{k} \frac{1}{2} \left[e^{-\lambda_1(k-t)} f(t) + e^{-\lambda_1(k-t+1)} f(t-1) \right] \end{aligned} \tag{12}$$

The values of the series $x^{(p/q)}(k)$ can be computed using the response function (12), and the predicted values of the original series $x^{(0)}(k)$ can be obtained using the fractional order inverse accumulative generation operation as follows:

$$\begin{aligned} \hat{X}^{(0)} = \hat{X}^{(p/q)(-p/q)}(k) = \{ & x^{(p/q)(-p/q)}(1), \\ & x^{(p/q)(-p/q)}(2), \ldots, x^{(p/q)(-p/q)}(n) \} \end{aligned} \tag{13}$$

The derivation of the parameter estimation of TDPFOGM(1,1) is similar to the other first order grey prediction models, such as TDPGM(1,1) (see [14]).

Remark 1. The TDPFOGM(1,1) model has the properties of fractional order accumulation and the polynomial function, and it is not applicable for the periodic series and the random series and so forth according to its mathematical formulation. Exponential function and a discrete integral with an exponential and the polynomial function combination explain the discrete response function of the TDPFOGM(1,1) model. The priority of new information can be better reflected when the accumulation order number becomes smaller in the model.

2.4. The Computational Steps. According to the principles of time-delayed polynomial grey system model with the fractional order accumulation, the computational steps can be summarized as follows.

Step 1. Within a given original sequence, compute the series of the given time series using (1) and compute the background values using (5).

Step 2. Substitute the original sequence along with its series and the background values into (7) and compute the parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ by solving the linear system in (7).

Step 3. Substitute the parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ into the discrete response function in (12) and then compute the series $x^{(p/q)}(k)$.

Step 4. Compute the predicted values of $x^{(0)}(k)$ using the (13).

3. Model Application and Comparison

3.1. Raw Data Collection. The raw data of the natural gas consumption ($10^9 m^3$) of China are collected from [14] (page 20) as shown in Table 1. We build the prediction models using the data from 1995 to 2004 and validate the modeling accuracy using the data from 2005 to 2013.

We consider an example from paper [36]. The same sample is applied here to compare the precision. Actual values are presented in Table 2.

3.2. Evaluation Indices of the Modeling Accuracy. The mean absolute percentage error (MAPE) is used to evaluate the overall forecast performance of the prediction models, which is defined as follows:

$$\text{MAPE} = \frac{1}{n} \sum_{k=1}^{n} \left| \frac{x^{(0)}(k) - \hat{x}^{(0)}(k)}{x^{(0)}(k)} \right| \times 100\,(\%) \tag{14}$$

3.3. Results and Analysis

Case 1. We consider the data from Table 1. The same sample is applied here to compare the precision. Actual values and the fitting values of four compared models are presented in Table 3 and the predicted values are also plotted in Figure 1. From Table 3, TDPGM$^{(0.25)}(1,1)$ yielded lower MAPE compared with the traditional TDPGM(1,1), GM(1,1), and GM$^{(0.25)}(1,1)$.

Table 1: The raw data of the natural gas consumption of China.

Year	Natural gas consumption
1995	177.53
1996	188.05
1997	176.13
1998	218.7
1999	215.34
2000	245.03
2001	274.3
2002	291.84
2003	339.08
2004	396.72
2005	467.63
2006	561.41
2007	705.23
2008	812.94
2009	895.2
2010	1069.41
2011	1305.3
2012	1463
2013	1705.37

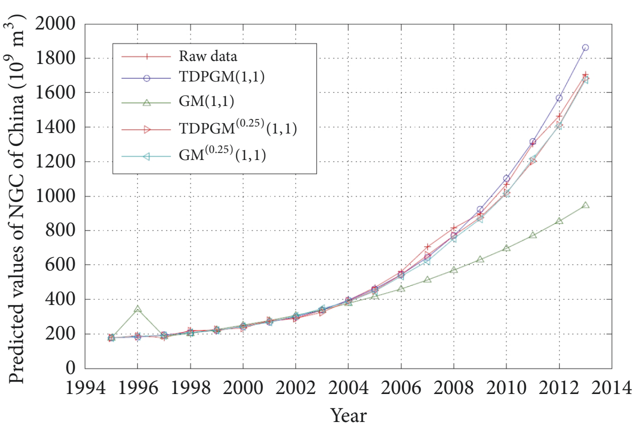

Figure 1: Prediction results by TDPGM(1,1), GM(1,1), GM$^{(0.25)}$ (1,1), and TDPGM$^{(0.25)}$ (1,1).

Case 2. We consider the data from Table 2. The same sample is applied here to compare the precision. Actual values and the fitting values of four compared models are presented in Table 4 and the predicted values are also plotted in Figure 2. From Table 4, TDPGM$^{(0.5)}(1,1)$ yielded lower MAPE compared with the traditional GM(1,1) and GM$^{(0.5)}(1,1)$.

Remark 2. In this paper, we established the novel grey prediction model that considers the fractional order accumulation while considering the polynomial based on the TDPGM(1,1) model which is different from the existing literature [14]. There are three main reasons; firstly, in [14], the authors proposed a time-delay polynomial grey model, but fractional order accumulation is not mentioned. Secondly, The TDPFOGM(1,1) model is a further study on the basis of

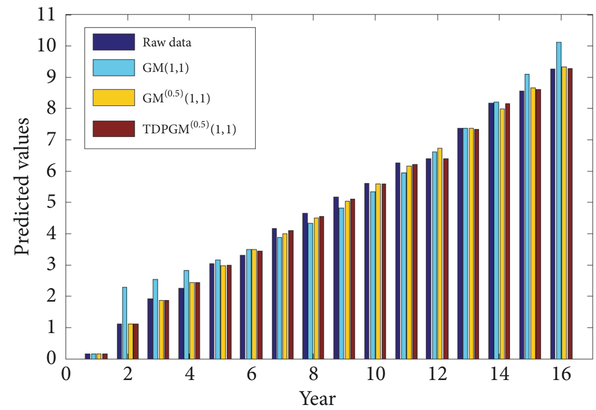

Figure 2: Prediction results by GM(1,1), GM$^{(0.5)}$ (1,1), and TDPGM$^{(0.5)}$ (1,1).

the TDPGM(1,1) model and TDPGM(1,1) model is a special case when fractional order $p/q = 1$. Finally, The prediction performance of TDPFOGM(1,1) model is better than that of the TDPGM(1,1) model according to the numerical results and analysis by comparing the TDPFOGM(1,1) and TDPGM(1,1) models based on the same data sample.

4. Conclusion

Small sample forecasting is a difficult and important problem. We have improved the grey prediction model after studying the grey system model with the fractional order accumulation and the time-delayed polynomial grey model. Based on a series of analysis and derivation for grey prediction system, the following conclusions could be drawn:

(1) Time-delayed polynomial grey system model with the fractional order accumulation (TDPFOGM(1,1)) is established and it can be used for forecasting.

(2) The anticipated effect using the TDPFOGM(1,1) model is better than the TDPGM model and the FOGM model under the same data condition.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Table 2: The raw data of the example.

Year	1	2	3	4	5	6	7	8
Actual value	0.16	1.11	1.92	2.24	3.03	3.3	4.16	4.64
Year	9	10	11	12	13	14	15	16
Actual value	5.18	5.6	6.25	6.39	7.35	8.18	8.57	9.27

Table 3: Numerical results by the TDPGM(1,1), GM(1,1), TDPGM$^{(0.25)}(1,1)$, and GM$^{(0.25)}(1,1)$.

Year	Natural gas consumption	TDPGM(1,1)	Eror(%)	GM(1,1)	Eror(%)	TDPG$^{(0.25)}(1,1)$	Eror(%)	GM$^{(0.25)}(1,1)$	Eror(%)
1995	177.53	177.53	0	177.53	0	177.53	0	177.53	0
1996	188.05	180.88	3.81	343.87	82.86	186.45	0.85	185.64	1.28
1997	176.13	193.11	9.46	184.26	4.62	190.65	8.24	189.35	7.51
1998	218.7	206.39	5.63	204.11	6.67	215.94	1.26	207.48	5.13
1999	215.34	222.06	3.12	226.11	5.00	223.14	3.62	221.26	2.75
2000	245.03	241.61	1.40	250.47	2.22	235.75	3.79	240.34	1.91
2001	274.3	266.66	2.79	277.45	1.15	276.62	0.85	268.57	2.09
2002	291.84	299.00	2.45	307.34	5.31	289.28	0.88	305.29	4.61
2003	339.08	340.60	0.45	340.46	0.41	326.92	3.59	347.21	2.40
2004	396.72	393.61	0.79	377.14	4.94	391.64	1.28	384.94	2.97
MAPE (%)			3.01		11.32		**2.44**		3.06
2005	467.63	460.40	1.55	417.77	10.66	451.32	3.49	448.38	4.12
2006	561.41	543.58	3.18	462.78	17.57	543.84	3.13	533.06	5.05
2007	705.23	646.00	8.40	512.64	27.31	658.56	6.62	624.31	11.47
2008	812.94	770.82	5.18	567.87	30.15	771.85	5.05	751.51	7.56
2009	895.2	921.47	2.93	629.05	29.73	876.45	2.09	867.73	3.07
2010	1069.41	1101.76	3.03	696.83	34.84	1020.62	4.56	1016.15	4.98
2011	1305.3	1315.85	0.81	771.90	40.86	1203.47	7.80	1215.32	6.89
2012	1463	1568.31	7.20	855.07	41.55	1412.62	3.44	1406.28	3.88
2013	1705.37	1864.18	9.31	947.19	44.46	1685.16	1.19	1675.61	1.75
MAPE (%)			4.62		30.79		**4.15**		5.42

Table 4: Numerical results by the GM(1,1), GM$^{(0.5)}(1,1)$, and TDPGM$^{(0.5)}(1,1)$.

Year	Actual value	GM(1,1)	Error (%)	GM$^{(0.5)}(1,1)$	Error (%)	TDPGM$^{(0.5)}(1,1)$	Error (%)
1	0.155	0.155	0.00	0.155	0.00	0.155	0.00
2	1.11	2.29	106.31	1.11	0.00	1.11	0.00
3	1.92	2.54	32.29	1.86	3.12	1.87	2.60
4	2.24	2.83	26.34	2.44	8.93	2.43	8.48
5	3.03	3.15	3.96	2.97	1.98	2.99	1.32
6	3.30	3.50	6.06	3.49	5.76	3.45	4.55
7	4.16	3.89	6.49	3.99	4.09	4.10	1.44
8	4.64	4.33	6.68	4.51	2.80	4.55	1.94
9	5.18	4.81	7.14	5.04	2.70	5.11	1.35
10	5.60	5.35	4.46	5.59	0.18	5.59	0.18
11	6.25	5.95	4.80	6.15	1.60	6.21	0.64
12	6.39	6.62	3.60	6.73	5.32	6.41	0.31
13	7.35	7.36	0.14	7.35	0.00	7.34	0.14
14	8.18	8.19	0.12	7.99	2.32	8.61	0.24
15	8.57	9.10	6.18	8.66	1.05	8.61	0.47
16	9.27	10.12	9.17	9.34	0.76	9.29	0.22
MAPE (%)		14.07		2.53		**1.49**	

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