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Research Article

A Mathematical Programming Model to Determine Objective Weights for the Interval Extension of TOPSIS

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Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) method has been extended in previous literature to consider the situation with interval input data. However, the weights associated with criteria are still subjectively assigned by decision makers. This paper develops a mathematical programming model to determine objective weights for the implementation of interval extension of TOPSIS. Our method not only takes into account the optimization of interval-valued Multiple Criteria Decision Making (MCDM) problems, but also determines the weights only based upon the data set itself. An illustrative example is performed to compare our results with that of existing literature.

1. Introduction

Decision makers are often confronted with a Multiple Criteria Decision Making (MCDM) problem that finds the best option among a finite set of feasible alternatives, usually taking into account multiple conflicting criteria [1]. The MCDM framework has been extensively applied in economy, engineering, marketing, management, military, and technology, and many other areas [2, 3]. In general, a MCDM problem with m alternatives, namely, $A_1, A_2, ..., A_m$, and n criteria, namely, $C_1, C_2, ..., C_n$, can be represented by the following decision matrix:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix}, \quad (1)$$

where $x_{ij},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n$ is the performance rating of alternative A_i with respect to criterion C_j, w_j is the weight of criterion C_j, and $\sum_{j=1}^{n} w_j = 1,\ w_j \geq 0$.

In line with Wang and Lee [4], the MCDM problems can be reasonably categorized into two groups: deterministic and uncertain MCDM problems with precise and fuzzy or interval input data and weights, respectively. Among the large variety of methods developed for solving MCDM problems, Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) was initially proposed by Hwang and Yoon [1], following the rationale that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest from the negative ideal solution. Behzadian et al. [5] conducted a state-of-the-art literature review to classify the extant TOPSIS research with respect to various applications. Specifically, there exist a large spectrum of papers that involve the extension of TOPSIS with interval parameters. Jahanshahloo et al. [6, 7] extended the TOPSIS method to solve MCDM problems with interval data and determined the most preferable alternative with both crisp numbers and interval scores. Tsaur [8] took into account decision makers' risk attitude towards the interval criteria values and developed a new TOPSIS method to normalize the collected data and rank the alternatives. Yue [9, 10] defined the positive and negative ideal solutions in a group decision environment, in which each individual decision result is expressed as an interval matrix and employed the Euclidean distance as the separation measure of each individual decision from the ideal solution and the relative closeness to the ideal solution. Zhang and Yu [11] developed a mathematical programming model to determine the weights of criteria, considering the MCDM problems with incomplete information of criteria under interval-valued intuitionistic fuzzy sets circumstance. To overcome the drawback that ideal solutions in forms of intervals are not attainable, Dymova et al. [12] presented a new direct approach to interval extension of TOPSIS method that is independent of heuristic assumptions and limitations of existing methods. Apart from the aforementioned efforts that have been made to handle TOPSIS with interval-valued data, Fan and Liu [13] untangled the group decision-making problems with uncertain preference information described as multigranularity linguistic terms, which were thereby transformed into trapezoidal fuzzy numbers. Fuzzy positive-ideal and negative-ideal solution solutions were defined to implement TOPSIS. Fan et al. [14] proposed a new method to solve stochastic MCDM problem, in which the consequences of alternatives associated with criteria were denoted by random variables with cumulative distribution functions. By means of TOPSIS, the ideal and anti-ideal points of the stochastic MCDM problem were determined as cumulative distribution function vectors. Differing from these papers, our study derives positive and negative ideal points based upon theoretic analysis on intervals and then seeks to eliminate decision bias through determining a set of objective weights associated with each criterion.

The implementation of TOPSIS requires incorporating relative weights of criterion importance, which are subjectively determined by decision makers and may significantly affect the results [15]. Deng et al. [16] complementarily modified TOPSIS by using Shannon entropy concept to determine objective weights associated with each criterion. Furthermore, various subjective and objective weights elicitation methods for fuzzy TOPSIS have been developed in literature [11, 17, 18]. Regarding the extension of TOPSIS with interval data, the weights associated with criteria are commonly determined in a subjective and arbitrary manner [6], which reveal the decision makers' judgement or intuition based on their knowledge and preferences, but are extremely difficult to reach a consensus [19]. This difficulty will be increased due to the absence of suitable decision makers but can be overcome by using an objective weights determination process [16]. However, to the best of our knowledge, the existing literature has unexpectedly ignored this piece of research. This study modifies the interval extension of TOPSIS by proposing a mathematical programming model to determine objective weights with respect to each criterion, in which a "virtual alternative" is created to represent the best weighted performance under all criteria. The rationale to elicit objective weights is originated from the logic of achieving a collective choice in group decision making [20, 21], but with distinct objective function. This stream of methods can effectively overcome the drawback of non-optimal aggregation process.

The rest of this paper proceeds as follows. Section 2 reviews the interval extension of TOPSIS. Section 3 develops a mathematical programming model to determine objective weights. Section 4 recalculates a previous example to demonstrate the effectiveness of our model. Section 5 concludes this work.

2. Interval Extension of TOPSIS

Taking into account the fact that the precise performance ratings of criteria are sometimes difficult to obtain and for the implementation of TOPSIS method, Jahanshahloo et al. [6] initially extended TOPSIS using intervals to denote input data and proposed an algorithmic procedure to solve. More specifically, input data of the aforementioned decision matrix are distributed across various intervals, that is, $x_{ij} \in [x_{ij}^L, x_{ij}^U]$. In this manner, the decision matrix with respect to a MCDM problem with interval parameters could be presented as follows:

$$\begin{bmatrix} [x_{11}^L, x_{11}^U] & [x_{12}^L, x_{12}^U] & [x_{13}^L, x_{13}^U] & \cdots & [x_{1n}^L, x_{1n}^U] \\ [x_{21}^L, x_{21}^U] & [x_{22}^L, x_{22}^U] & [x_{23}^L, x_{23}^U] & \cdots & [x_{2n}^L, x_{2n}^U] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ [x_{m1}^L, x_{m1}^U] & [x_{m2}^L, x_{m2}^U] & [x_{m3}^L, x_{m3}^U] & \cdots & [x_{mn}^L, x_{mn}^U] \end{bmatrix}, \quad (2)$$

and w_j is the weight of criterion C_j that satisfies $\sum_{j=1}^n w_j = 1, w_j \geq 0$.

The logic of TOPSIS is that the most preferred alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. Using the subjectively assigned weights with respect to each criterion, the positive and the negative ideal solutions consist of the best and the worst weighted performance ratings of all criteria, respectively.

The main steps to calculate the interval extension of TOPSIS could be summarized as follows [6, 7]:

(i) Normalizing the interval decision matrix using the following vector transformations to reduce the effect of data magnitude:

$$y_{ij}^L = \frac{x_{ij}^L}{\sqrt{\sum_{i=1}^m \left(\left(x_{ij}^L \right)^2 + \left(x_{ij}^U \right)^2 \right)}}, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n, \quad (3)$$

$$y_{ij}^U = \frac{x_{ij}^U}{\sqrt{\sum_{i=1}^m \left(\left(x_{ij}^L \right)^2 + \left(x_{ij}^U \right)^2 \right)}}, \quad i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n. \quad (4)$$

By doing so, the normalization of interval data $[x_{ij}^L, x_{ij}^U]$ is obtained as $[y_{ij}^L, y_{ij}^U]$, and $y_{ij}^L, y_{ij}^U \in [0, 1]$.

(ii) Constructing the weighted normalized interval decision matrix $V = ([v_{ij}^L, v_{ij}^U])_{m\times n}$, the elements of which are denoted as

$$v_{ij}^L = y_{ij}^L w_j,$$
$$v_{ij}^U = y_{ij}^U w_j, \quad (5)$$
$$i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n,$$

and w_j is subjectively determined by the decision maker.

(iii) Identifying positive and negative ideal solutions as follows:

$$A^+ = \{v_1^+, v_2^+, \ldots, v_n^+\} = \left\{\left(\max_i v_{ij}^U,\ j \in J_b\right), \left(\min_i v_{ij}^L,\ j \in J_c\right)\right\}, \quad (6)$$

$$A^- = \{v_1^-, v_2^-, \ldots, v_n^-\} = \left\{\left(\min_i v_{ij}^L,\ j \in J_b\right), \left(\max_i v_{ij}^U,\ j \in J_c\right)\right\}, \quad (7)$$

where J_b and J_c represent the sets of benefit-type and cost-type criteria, respectively.

(iv) Using the n-dimensional Euclidean distance concept to compute the corresponding separation of each alternative form the positive and the negative ideal solutions:

$$d_i^+ = \sqrt{\sum_{j\in J_b}\left(v_{ij}^L - v_j^+\right)^2 + \sum_{j\in J_c}\left(v_{ij}^U - v_j^+\right)^2}, \quad i = 1, 2, \ldots, m, \quad (8)$$

$$d_i^- = \sqrt{\sum_{j\in J_b}\left(v_{ij}^U - v_j^-\right)^2 + \sum_{j\in J_c}\left(v_{ij}^L - v_j^-\right)^2}, \quad i = 1, 2, \ldots, m. \quad (9)$$

(v) Defining the closeness of alternative A_j to the ideal alternative A^+ as

$$RC_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad i = 1, 2, \ldots, m. \quad (10)$$

(vi) Ranking alternatives based on the relative closeness to the ideal alternative. Because RC_i is increasing with respect to d_i^- and decreasing with respect to d_i^+, a larger RC_i indicates alternative i should be preferred. Therefore, alternatives should be ranked in a monotonously decreasing order.

However, the subjective elicitation of weights is usually restricted to decision makers' experience and knowledge and highly dispersed across different decision makers and thereby suffers from systematic biases [22]. Specifically, when the problem involves a committee of multiple decision makers of distinct interests, a consensus about the criteria weights may be extremely difficult to achieve [16].

3. A Mathematical Programming Model

This section aims at developing a mathematical programming model to determine objective weights associated with each criterion, in the presence of interval-valued input data. Recall the aforementioned weighted normalized decision matrix $V = ([v_{ij}^L, v_{ij}^U])_{m\times n}$; we construct a "virtual alternative", A^*, which is composed of the best performance ratings with respect to all criteria. That is, $A^* = A^+ = (v_1^+, v_2^+, \ldots, v_n^+)$, where

$$v_j^+ = \left\{\left(\max_i v_{ij}^U,\ j \in J_b\right), \left(\min_i v_{ij}^L,\ j \in J_c\right)\right\} \quad (11)$$

$$= \left\{\left(\max_i y_{ij}^U w_j,\ j \in J_b\right), \left(\min_i y_{ij}^L w_j,\ j \in J_c\right)\right\} \quad (12)$$

$$= \left\{\left(\max_i y_{ij}^U,\ j \in J_b\right), \left(\min_i y_{ij}^L,\ j \in J_c\right)\right\} w_j \quad (13)$$

$$= y_j^* w_j. \quad (14)$$

$y_j^* = \{(\max_i y_{ij}^U,\ j \in J_b), (\min_i y_{ij}^L,\ j \in J_c)\}$ can be reasonably considered at the ideal performance rating of criterion j. More specifically, it is reasonable to formulate a "virtual alternative" using crisp values for the interval-valued decision matrix [23].

In accordance with Ma et al. [19] and Wu et al. [23], the disparity between the virtual alternative and each alternative can be measured by the following square distance functions:

$$f_i = \sum_{j=1}^{n}\left[\left(y_j^* - y_{ij}^L\right)^2 + \left(y_j^* - y_{ij}^U\right)^2\right] w_j^2, \quad i = 1, 2, \ldots, m. \quad (15)$$

The smaller f_i is, the better the alternative i will be. Therefore, the determination of w_i requires the simultaneous optimization of all disparities, that is, $\min\{f_1, f_2, \ldots, f_m\}$.

Using a linear equal weighted summation method [24], the aforementioned multiple objective optimization problem can be converted into the following single objective minimization problem:

$$\min \quad F = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(y_j^* - y_{ij}^L\right)^2 + \left(y_j^* - y_{ij}^U\right)^2\right] w_j^2 \quad (16)$$

$$s.t. \quad \sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0. \quad (17)$$

The optimal weights obtained from (17) are regarded as "objective" because they are derived from the input data set itself and immune to the preferences of decision makers. Moreover, the rationale to elicit weights by minimizing

Table 1: Normalized interval decision matrix.

	C_1	C_2	C_3	C_4
A_1	[0.0856,0.1645]	[0.5176,0.6001]	[0.1974,0.2865]	[0.0706,0.5086]
A_2	[0.1495,0.3038]	[0.1972,0.2037]	[0.0283,0.3768]	[0.1760,0.2320]
A_3	[0.0164,0.0336]	[0.2198,0.2329]	[0.1720,0.2010]	[0.0152,0.0373]
A_4	[0.1451,0.2999]	[0.0750,0.0758]	[0.0036,0.0090]	[0.0046,0.0067]
A_5	[0.0100,0.0206]	[0.0278,0.0318]	[0.1352,0.1353]	[0.0129,0.0271]
A_6	[0.0794,0.1635]	[0.0783,0.0799]	[0.3036,0.3643]	[0.3403,0.4300]
A_7	[0.0265,0.0586]	[0.0643,0.0787]	[0.2531,0.3365]	[0.0409,0.1832]
A_8	[0.2999,0.6211]	[0.1345,0.1475]	[0.0107,0.0113]	[0.0043,0.0063]
A_9	[0.0418,0.0848]	[0.1231,0.1336]	[0.0855,0.1805]	[0.0782,0.1669]
A_{10}	[0.1249,0.2425]	[0.0869,0.0925]	[0.0203,0.0221]	[0.0274,0.0409]
A_{11}	[0.0778,0.1593]	[0.0594,0.0657]	[0.0151,0.0196]	[0.0684,0.1073]
A_{12}	[0.0519,0.1078]	[0.0549,0.0608]	[0.1117,0.2027]	[0.0880,0.3169]
A_{13}	[0.1127,0.2302]	[0.0616,0.0667]	[0.0495,0.0602]	[0.0146,0.0292]
A_{14}	[0.0719,0.1473]	[0.1186,0.1604]	[0.1829,0.3030]	[0.2033,0.3330]
A_{15}	[0.0247,0.0500]	[0.0230,0.0231]	[0.0112,0.0134]	[0.0207,0.0345]

the overall difference for the ideal performance rating is motivated from Fu et al. [25, 26]. For the convenience of solving this quadratic programming model, we construct a Lagrange function as follows:

$$L = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[\left(y_j^* - y_{ij}^L\right)^2 + \left(y_j^* - y_{ij}^U\right)^2\right] w_j^2 - \lambda\left(\sum_{j=1}^{n} w_j - 1\right), \tag{18}$$

and the Hessian matrix of L with respect to λ is a $n\times n$ diagonal matrix, the diagonal elements of which are

$$\frac{\partial^2 L}{\partial w_j^2} = 2\sum_{i=1}^{m}\left[\left(y_j^* - y_{ij}^L\right)^2 + \left(y_j^* - y_{ij}^U\right)^2\right] > 0. \tag{19}$$

The well-known Hessian theorem suggests that this Lagrange function L has a minimum objective value, which can be obtained by simultaneously setting $\partial L/\partial\lambda = 0$, $\partial L/\partial w_j = 0$. Thereby, the optimal weights of criteria importance are reported as

$$w_j^* = \frac{1}{\left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\left[\left(y_j^* - y_{ij}^L\right)^2 + \left(y_j^* - y_{ij}^U\right)^2\right]\right)^{-1}\right]\left[\sum_{i=1}^{m}\left[\left(y_j^* - y_{ij}^L\right)^2 + \left(y_j^* - y_{ij}^U\right)^2\right]\right]}. \tag{20}$$

Objective weights obtained from a quadratic programming model can be incorporated into (5) to determine the weighted normalized matrix $V = ([v_{ij}^L, v_{ij}^U])_{m\times n}$. These weights are derived only based on the data set itself, thus effectively reduce decision bias, and add objectiveness to the solutions.

4. Numerical Illustration

In this section, we apply the proposed mathematical programming model to determine objective weights of evaluation criteria as investigated in literature [6], in which 15 bank branches $(A_1, A_2, \ldots, A_{15})$ in Iran are examined and compared with respect to 4 financial ratios (C_1, C_2, C_3, C_4), using the TOPSIS with interval data. We directly draw the normalized interval decision matrix from Jahanshahloo et al. [6] and report it as Table 1.

Using the proposed mathematical programming model, we obtain the objective weights with respect to 4 criteria as

$$\begin{aligned} w_1 &= 0.1649, \\ w_2 &= 0.1689, \\ w_3 &= 0.4233, \\ w_4 &= 0.2429. \end{aligned} \tag{21}$$

Therefore, the weighted normalized interval matrix could be presented as Table 2.

According to the results reported in Table 2, the positive and the negative ideal solutions are then determined as

$$\begin{aligned} A^+ &= \{0.1025, 0.1013, 0.1595, 0.1236\}, \\ A^- &= \{0.0017, 0.0039, 0.0016, 0.0010\}. \end{aligned} \tag{22}$$

Table 2: Normalized interval decision matrix.

	C_1	C_2	C_3	C_4
A_1	[0.0141,0.0271]	[0.0874,0.1013]	[0.0836,0.1213]	[0.0172,0.1236]
A_2	[0.0247,0.0501]	[0.0333,0.0344]	[0.0120,0.1595]	[0.0406,0.0564]
A_3	[0.0027,0.0055]	[0.0371,0.0393]	[0.0728,0.0851]	[0.0037,0.0091]
A_4	[0.0239,0.0495]	[0.0127,0.0128]	[0.0016,0.0038]	[0.0011,0.0016]
A_5	[0.0017,0.0034]	[0.0047,0.0054]	[0.0572,0.0573]	[0.0031,0.0066]
A_6	[0.0131,0.0270]	[0.0132,0.0135]	[0.1285,0.1542]	[0.0827,0.1405]
A_7	[0.0044,0.0097]	[0.0109,0.0133]	[0.1072,0.1425]	[0.0100,0.0445]
A_8	[0.0495,0.1025]	[0.0227,0.0249]	[0.0046,0.0048]	[0.0010,0.0015]
A_9	[0.0069,0.0140]	[0.0280,0.0226]	[0.0362,0.0764]	[0.0190,0.0406]
A_{10}	[0.0206,0.0400]	[0.0147,0.0156]	[0.0086,0.0094]	[0.0067,0.0099]
A_{11}	[0.0128,0.0263]	[0.0100,0.0111]	[0.0064,0.0083]	[0.0166,0.0261]
A_{12}	[0.0086,0.0178]	[0.0093,0.0103]	[0.0473,0.0858]	[0.0214,0.0770]
A_{13}	[0.0186,0.0380]	[0.0104,0.0113]	[0.0210,0.0225]	[0.0036,0.0071]
A_{14}	[0.0119,0.0243]	[0.0200,0.0271]	[0.0774,0.1283]	[0.0494,0.0809]
A_{15}	[0.0041,0.0082]	[0.0039,0.0039]	[0.0047,0.0057]	[0.0050,0.0084]

Table 3: Distance from the ideal and the negative ideal solutions.

	d_j^+	d_j^-
A_1	0.1584	0.1987
A_2	0.1983	0.1769
A_3	0.1896	0.0192
A_4	0.2323	0.0487
A_5	0.2109	0.0560
A_6	0.1356	0.1864
A_7	0.1829	0.1480
A_8	0.2191	0.1030
A_9	0.2043	0.0876
A_{10}	0.2250	0.0418
A_{11}	0.2264	0.0365
A_{12}	0.2008	0.1148
A_{13}	0.2211	0.0445
A_{14}	0.1645	0.1532
A_{15}	0.2391	0.0107

Table 4: Distance from the ideal and the negative ideal solutions.

	Jahanshahloo et al. [6]	Our results	Difference
A_1	1	2	−1
A_2	6	4	+2
A_3	4	7	−3
A_4	14	11	+3
A_5	9	10	−1
A_6	2	1	+1
A_7	5	5	0
A_8	15	8	+7
A_9	8	9	−1
A_{10}	13	13	0
A_{11}	11	14	−3
A_{12}	7	6	+1
A_{13}	12	12	0
A_{14}	3	3	0
A_{15}	10	15	−5

On the strength the concept of n-dimensional Euclidean distance, the separation of each alternative from the ideal and the negative ideal solutions is given in Table 3.

Consequently, the ranking of alternatives can be obtained based on the closeness to positive ideal solutions, which is demonstrated and compared with that of Jahanshahloo et al. [6] in Table 4. Jahanshahloo et al. [6] subjectively assigned equal weights to each criterion. As shown in Table 4, 11 out of 15 alternatives are ranked differently. We observe that the ranking of Alternative 8 is increased from 15 to 8, while that of Alternative 15 drops from 10 to 15. This is more reasonable because the upper bound performance rating of Alternative 8 with respect to C1 is ranked at the first, while the upper bound performance ratings of Alternative 15 with respect to C1 and C3 are placed at the bottom.

5. Conclusions

In this paper we develop a mathematical programming model to determine objective weights for the interval extension of TOPSIS. The contribution of this study is to provide a solution to MCDM problems that not only incorporate interval-valued criteria into TOPSIS method, but also provide a set of objective weights to reducing decision bias from subjectivity and arbitrariness. An illustrative example is presented to compare our results with that of Jahanshahloo et al. [6].

A new and fresh practical application would deeply improve the novelty of the proposed method, for instance, online rating [27] and online review [28]. Future research should consider the novel application as a publication trial.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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