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Research Article

Mixed H_2/H_∞ Fault Detection Filter for Markovian Jump Linear Systems

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We tackle the mixed H_2/H_∞ fault detection filter problem for a Markov jump linear system (MJLS) in the discrete-time domain. We present three distinct formulations: the first one is to minimize an upper bound on the H_2 subject to a given upper value on the H_∞ norm; the second one is the opposite situation; that is, we minimize an upper bound on H_∞, subject to a given restriction on the H_2 norm; and the third one is the minimization of a weighted combination of the upper bound of both the H_2 and H_∞ norms. We present new conditions in the form of linear matrices inequalities (LMI) that provide the design of the fault detection filter. We also present results for the so-called mode-independent case and the design of robust H_∞ filters in the sense that the system matrices are uncertain. In order to illustrate the feasibility of the proposed approaches, a numerical example is presented.

1. Introduction

Dynamic systems that are subject to sudden changes in their dynamics are present in a multitude of situations, ranging from highly complex industrial processes to day by day routine work, and for that reason this class of systems has been extensively studied in the literature. A particular type of system subject to sudden dynamic behavior is the one in which the changes are caused by the occurrence of faults, so that it is highly desirable to provide solutions able to keep the system operational even in the presence of such issues. A possible approach for this problem is the Fault Detection and Isolation (FDI) algorithms, which have the purpose of detecting unusual behaviors in a wide range of fields in engineering including chemical, nuclear, aerospace, and automotive applications; for instance, see the works in [1–4], respectively. Therefore, whenever a fault happens, the main task is to detect the failure and reorganize the control system with the intent of minimizing the operational loss and the possibility of the occurrence of accidents [5].

One possible way to implement an FDI is to generate a residual signal using a filter device and predetermine a threshold, and whenever the residue surpasses this established limit, consider that a fault occurred [6, 7]. Bearing in mind this general work scheme of an FDI, it is possible to point out three desirable aspects in the residue generator filter. The first one is the sensibility to abrupt changes; that is, the higher the filter sensibility is, the faster the fault detection will take place. The second one is high robustness against noise in order to prevent the occurrence of false alarms. The last aspect is the consideration of communication failure between components that compose the FDI solution, since the occurrence of packet loss may degrade the performance of the fault detection.

The motivation behind the Markov jump linear systems (MJLS) framework usage is the communication between the sensor and the FDI device made via imperfect channels. The communication made through a nonideal network is susceptible to the packet loss, which may be caused, for instance, by collision [8] and channel fading [9]. In the design of a fault detection (FD) filter, it is essential to consider the problems inherent to a communication made through nonideal network. A viable way to model this network characteristic is to use MJLS due to the possibility of modeling the dynamical behavior of systems whose signals are degraded by the possible packet losses in a networked control system; see, for instance, the work in [10].

In relation to the MJLS framework applied to FDI theory, we can mention the work in [11] which tackled the problem of designing H_∞ residual filters for discrete-time MJLS considering that the Markov chain can be measured. More recently, the works in [12, 13] considered also the synthesis of H_∞ residual filters for continuous-time MJLS but with the assumption that the modes of operation of the filter are unmatched in relation to the system being observed. Regarding the aforementioned works, we consider that the fault detection problem is still not completely investigated, since both the works in [11, 12] study a suboptimized H_∞ residual filters, and for that reason some alternative design conditions to the ones given in [11] for H_∞ residual filters would be desirable in order to cope with potential conservative results.

Considering the previous discussions, the main novelty of this paper is the design of mixed H_2/H_∞ filters for FDI devices within the MJLS framework. This is highly motivated, since the filter structure would combine the desirable aspects of signal robustness that is linked to the H_∞ framework, as well as the optimal aspects of the H_2 filtering. We provide design conditions based on the linear matrix inequality (LMI) formulation in order to obtain residual filters that impose bounds on both the H_2 and H_∞ norms of the residual signal. Furthermore, we also investigate the mode-independent residual filter formulation in which the filter matrices do not switch according to the Markov chain and also the design of robust filters with respect to uncertainty on the plant modeling. A numerical example is presented at the end of this paper in order to illustrate our results and compare the proposed approaches.

This work is organized as follows. Section 2 presents the notation used during the entire work, Section 3 shows the basic theoretical concepts in order to understand the following sections, Section 4 presents the fault detection problem, and Section 5 introduces the main results of this work. Section 6 introduces some secondary results and in Section 7 we present the numerical example that portrays the feasibility of the presented results. Section 8 concludes the paper with some final comments.

2. Notation

The notation is standard. The operator $(')$ denotes the matrix or vector transpose; $(\bullet)$ indicates each symmetric block of a symmetric matrix. We consider the convex set

$$\Upsilon = \left\{ Q;\ Q = \sum_{l=1}^{V} \mu_l Q^l,\ \mu_l \geq 0,\ \sum_{l=1}^{V} \mu_l = 1 \right\} \qquad (1)$$

where V is the number of vertices in the polytope. The set of Markov chain states is represented by $\mathbb{K} = \{1, 2, \ldots, N\}$. The convex combinations of the matrix X_j and the weight ρ_{ij} are given by $\varepsilon_i(X) = \sum_{j=1}^{N} \rho_{ij} X_j$ for $i \in \mathbb{K}$. The symbol $\varepsilon(\cdot)$ represents mathematical expectations. Considering the stochastic signal $z(k)$, its norm is defined by $\|z\|_2^2 = \sum_{k=0}^{\infty} \varepsilon\{z(k)' z(k)\}$. On a probabilistic space $(\Omega, \mathcal{F}, \mathcal{F}_k, P)$, the set of signals $z(k) \in \mathbb{R}^p$, such that $z(k)$ is $\mathcal{F}_k$ measurable, for all $k \in \mathbb{N}$ and $\|z\|_2 < \infty$, is indicated by $\mathcal{L}^2$.

3. Theoretical Background

We define in this section the concepts of mean square stability, H_∞ norm and H_2 norm.

3.1. Markovian Jump Linear System. We initially consider the following general discrete-time Markovian jump linear system (MJLS):

$$\mathcal{G}: \begin{cases} x(k+1) = A_{\theta_k} x(k) + J_{\theta_k} w(k) \\ z(k) = C_{\theta_k} x(k) + D_{\theta_k} w(k) \end{cases} \qquad (2)$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^q$ is the measured output vector, $z(k) \in \mathbb{R}^p$ is the estimated output, $w(k) \in \mathbb{R}^m$ is the exogenous input, and $\{\theta_k\}$ is a Markov chain taking values in $\mathbb{K}$. We define the transition probability matrix of $\{\theta_k\}$ by $\mathbb{P} = [\rho_{ij}]$, where $\rho_{ij} \geq 0$ is such that $\rho_{ij} = P(\theta_{k+1} = j \mid \theta_k = i)$ and $\sum_{j=1}^{N} \rho_{ij} = 1$.

3.2. Mean Square Stability. We recall the definition of mean square stability presented in [14].

Definition. System (2) is mean square stable (MSS) if, for any initial condition $x(0) = x_0 \in \mathbb{R}^n$ and initial distribution $\theta(0) = \theta_0 \in \mathbb{K}$,

$$\lim_{k \longrightarrow \infty} \varepsilon\left\{ x(k)' x(k) \mid x_0, \theta_0 \right\} = 0; \qquad (3)$$

see, for instance, [15].

3.3. H_∞ Norm. Assuming that (2) is MSS with $x_0 = 0$, the H_∞ norm of $\mathcal{G}$ is given by (see [16])

$$\|\mathcal{G}\|_\infty = \sup_{0 \neq w \in \mathcal{L}_2, \theta_0 \in \mathbb{K}} \frac{\|z\|_2}{\|w\|_2}. \qquad (4)$$

Notice that the case $\mathbb{K} = \{1\}$ corresponds to the deterministic case, that is, the case without jumps.

It is possible to calculate the H_∞ norm using the so-called Bounded Real Lemma for Markovian jump linear systems, first presented in [17] and stated below.

Lemma. System (2) is MSS and satisfies the norm constraint $\|\mathcal{G}\|_\infty^2 < \delta$ if and only if there exist matrices $P_i = P_i' > 0$ such that

$$\begin{bmatrix} A_i & J_i \\ C_i & D_i \end{bmatrix}' \begin{bmatrix} \varepsilon_i(P) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_i & J_i \\ C_i & D_i \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \delta I \end{bmatrix} < 0, \qquad (5)$$

$$\forall i \in \mathbb{K}.$$

Applying the Schur complement to (5), we get that

$$\begin{bmatrix} P_i & \bullet & \bullet & \bullet \\ 0 & \delta I & \bullet & \bullet \\ \varepsilon_i(P)A_i & \varepsilon_i(P)J_i & \varepsilon_i(P) & \bullet \\ C_i & D_i & 0 & I \end{bmatrix} > 0 \quad (6)$$

and the LMI constraint (6) can also be described by the following inequality:

$$\begin{bmatrix} P_i & \bullet & \bullet & \bullet \\ 0 & \delta I & \bullet & \bullet \\ A_i & J_i & \varepsilon_{\theta_k}(P)^{-1} & \bullet \\ C_i & D_i & 0 & I \end{bmatrix} > 0. \quad (7)$$

See, for instance, [18].

3.4. H_2 Norm. Assuming that (2) is MSS with $x_0 = 0$, the H_2 norm is given by

$$\|\mathcal{G}\|_2^2 = \sum_{s=1}^{m} \sum_{i=1}^{N} \mu_i \left\| z^{s,i} \right\|_2^2 \quad (8)$$

where $z^{s,i}$ represents the output $z(0), z(1), \ldots$ obtained when

(i) the input is given by $w(k) = e_s \delta(k)$, where $e_s \in \mathbb{R}^m$ is the sth column of the $m \times m$ identity matrix and $\delta(k)$ is the unitary impulse; see [19];

(ii) $\theta_0 = i \in \mathbb{K}$ with probability $\mu_i = P(\theta_0 = i)$.

In [20], it is shown that if the Markov chain is ergodic and taking $\mu_i = \rho_i$, where $\rho_i = \lim_{k \to \infty} P(\theta(k) = i)$, the norm defined in (8) can also be written as

$$\|G\|_2^2 = \lim_{k \to \infty} \varepsilon \left[z(k)' z(k) \right], \quad (9)$$

where $z(k)$ is the controlled output and $w(k)$ represents a wide-sense white-noise with covariance given by the identity matrix that is independent of the initial condition x_0 and the Markov chain $\{\theta_k\}$.

In [15] or [19], we have that if there is a solution $S = (S_1, \ldots, S_N) > 0$, $P = (P_1, \ldots, P_N) > 0$ of the following LMI:

$$\sum_{i=1}^{N} \mu_i tr(S_i) < \gamma, \quad (10)$$

$$\begin{bmatrix} S_i & \bullet & \bullet \\ \varepsilon_i(P)J_i & \varepsilon_i(P) & \bullet \\ D_i & 0 & I \end{bmatrix} > 0, \quad (11)$$

$$\begin{bmatrix} P_i & \bullet & \bullet \\ \varepsilon_i(P)A_i & \varepsilon_i(P) & \bullet \\ C_i & 0 & I \end{bmatrix} > 0, \quad \forall i \in \mathbb{K}, \quad (12)$$

which can be equivalently written as

$$\begin{bmatrix} S_i & \bullet & \bullet \\ J_i & \varepsilon_i(P)^{-1} & \bullet \\ D_i & 0 & I \end{bmatrix} > 0, \quad (13)$$

$$\begin{bmatrix} P_i & \bullet & \bullet \\ A_i & \varepsilon_i(P)^{-1} & \bullet \\ C_i & 0 & I \end{bmatrix} > 0, \quad (14)$$

and then $\|\mathcal{G}\|_2^2 < \gamma$.

4. The Fault Detection Problem Formulation

The MJLS subject to faults we consider in this work is represented by

$$\mathcal{G}_a: \begin{cases} x(k+1) = A_{\theta_k} x(k) + B_{\theta_k} u(k) + B_{d\theta_k} d(k) + B_{f\theta_k} f(k) \\ y(k) = C_{\theta_k} x(k) + D_{d\theta_k} d(k) + D_{f\theta_k} f(k) \\ x(0) = x_0, \end{cases} \quad (15)$$

where $x(k) \in \mathbb{R}^n$ is the state variable, $y(k) \in \mathbb{R}^q$ is the measured output, $u(k) \in \mathbb{R}^m$ is the known input, $d(k) \in \mathbb{R}^p$ is the exogenous input, and $f(k) \in \mathbb{R}^t$ is the fault vector which is considered as an unknown time function. We also consider that $f(k), d(k) \in \mathcal{L}^2$.

Usually the fault detection system is divided into two distinct stages, a residual generator and a residual evaluation.

4.1. Residual Generator. For the purpose of generating the residual signal $r(k)$, a Markovian filter is considered with the following definition:

$$\mathcal{F}: \begin{cases} \eta(k+1) = A_{\eta\theta_k} \eta(k) + M_{\eta\theta_k} u(k) + B_{\eta\theta_k} y(k) \\ r(k) = C_{\eta\theta_k} \eta(k) + D_{\eta\theta_k} y(k) \\ \eta(0) = \eta_0 \end{cases} \quad (16)$$

where $\eta(k) \in \mathbb{R}^n$ represents the filter states and $r(k) \in \mathbb{R}^o$ is the filter residue. We point out that this filter structure also depends on the Markov mode θ_k.

Similar to the continuous-time case presented in [7] and the discrete-time case in [11], a weighting matrix $W_f(z)$ is used with the intention to increase the fault detection performance (see, e.g., [21]), where $\hat{f}(z) = W_f(z) f(z)$. A minimal realization of $\hat{f}(z) = W_f(z) f(z)$ is

$$\mathcal{W}_f: \begin{cases} x_f(k+1) = A_{wf} x_f(k) + B_{wf} f(k) \\ \hat{f}(k) = C_{wf} x_f(k) + D_{wf} f(k) \\ x_f(0) = 0 \end{cases} \quad (17)$$

where $x_f(k) \in \mathbb{R}^t$ is the weighting matrix state vector, $\hat{f}(k) \in \mathbb{R}^o$ is the weighted fault signal, and $f(k)$ is the same fault as in (15).

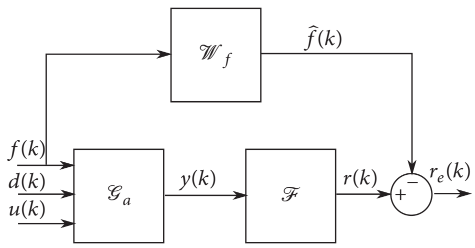


Figure 1: Block diagram.

Remark. The matrices that compose system (17) are supposed to be known. The block diagram presented in Figure 1 represents the equivalent system.

Considering $r_e(k) = r(k) - \hat{f}(k)$, the equivalent system can be written in the augmented form as

$$\mathscr{G}_{aug}: \begin{cases} \overline{x}(k+1) = \widetilde{A}_{\theta_k}\overline{x}(k) + \widetilde{B}_{\theta_k}\overline{w}(k) \\ r_e(k) = \widetilde{C}_{\theta_k}\overline{x}(k) + \widetilde{D}_{\theta_k}\overline{w}(k) \end{cases} \quad (18)$$

where the augmented state and the input signal are $\overline{x}(k) = \left[x(k)' \ \ \eta(k)' \ \ x_f(k)'\right]'$ and $\overline{w} = \left[u(k)' \ \ d(k)' \ \ \hat{f}(k)'\right]'$ with

$$\left[\begin{array}{c|c} \widetilde{A}_{\theta_k} & \widetilde{B}_{\theta_k} \\ \hline \widetilde{C}_{\theta_k} & \widetilde{D}_{\theta_k} \end{array}\right] = \left[\begin{array}{ccc|ccc} A_{\theta_k} & 0 & 0 & B_{\theta_k} & B_{d\theta_k} & B_{f\theta_k} \\ B_{\eta\theta_k}C_{\theta_k} & A_{\eta\theta_k} & 0 & M_{\eta\theta_k} & B_{\eta\theta_k}D_{d\theta_k} & B_{\eta\theta_k}D_{f\theta_k} \\ 0 & 0 & A_{wf} & 0 & 0 & B_{wf} \\ \hline D_{\eta\theta_k}C_{\theta_k} & C_{\eta\theta_k} & -C_{wf} & 0 & D_{\eta\theta_k}D_{d\theta_k} & D_{\eta\theta_k}D_{f\theta_k} - D_{wf} \end{array}\right]. \quad (19)$$

In this paper, we tackle three distinct problems: the H_∞ case, the H_2 case, and the mixed H_2/H_∞ case. For the H_∞ case, the fault detection (FD) filter problem corresponds to obtaining matrices that compose observer (16) in such a way that system (18) is MSS when $u = 0$, $d = 0$ $f = 0$, and δ is as small as possible in the feasibility of (20), meaning that

$$\left\|\mathscr{G}_{aug}\right\|_\infty = \sup_{\|\overline{w}\|_2 \neq 0, \overline{w} \in \mathscr{L}_2} \frac{\|r_e\|_2}{\|\overline{w}\|_2} < \delta^{1/2}. \quad (20)$$

For the H_2 case, the FD filter problem has the goal of obtaining matrices that compose the filter as in (16) in such a way that system (18) is MSS, minimizing γ in the equation.

$$\left\|\mathscr{G}_{aug}\right\|_2^2 = \sum_{s=1}^{m}\sum_{i=1}^{N}\mu_i \left\|r_e^{s,i}\right\|_2^2 < \gamma. \quad (21)$$

For the mixed H_2/H_∞ case, a way to describe the mixed problem is by setting the objective function as

$$\inf\left\{g(\gamma,\delta), \ \text{such that} \ \left\|G_{aug}\right\|_2^2 < \gamma \ \text{and} \ \left\|G_{aug}\right\|_\infty^2 < \delta\right\}, \quad (22)$$

which considers the restrictions as defined in (20) and (21). By inspection, it is possible to note that there are three possible ways to define the objective function in (22), as described below.

First Case. Find a minimum guaranteed cost γ for the H_2 norm of system (18), subject to a given upper bound $\delta > 0$ on the H_∞ norm. In this case, we have

$$g(\gamma,\delta) = \gamma. \quad (23)$$

Second Case. Find a minimum guaranteed cost δ for the H_∞ norm of system (18), subject to a given upper bound $\gamma > 0$ on H_2. In this case, we have

$$g(\gamma,\delta) = \delta. \quad (24)$$

Third Case. Find a minimum for a weighted combination of the guaranteed cost for both H_2 and H_∞ norms of system (18). Thus, for given scalars $\beta^{(\infty)} \geq 0$ and $\beta^{(2)} \geq 0$, we set

$$g(\gamma,\delta) = \delta\beta^{(\infty)} + \gamma\beta^{(2)} \quad (25)$$

where $\beta^{(\cdot)}$ represents the weight for each upper bound. A similar approach is presented in [22].

4.2. Residual Evaluation. In the evaluation stage, it is necessary to set an evaluation function $J(\overline{r}(k))$ and also a threshold $J_{th}(k)$, both as defined in [11]. We consider L as the evaluation time, and with that we are able to separate the evaluation process into two distinct cases: the first one is defined by $k - L \geq 0$ and the second one by $k - L < 0$. Thus, we define the auxiliary vectors for each case as

$$\text{for } k - L \geq 0, \quad \overline{r}(k) = \left[r(k)' \ \ r(k-1)' \ \cdots \ r(k-L)'\right] \quad (26)$$

$$\text{for } k - L < 0, \quad \overline{r}(k) = \left[r(k)' \ \ r(k-1)' \ \cdots \ r(0)'\right]$$

and, given the discrepancy between the intervals, the evaluation functions for each case are set as

$$\text{for } k - L \geq 0, \quad J(\overline{r}(k)) = \left\{\sum_{\sigma=k}^{\sigma=k-L}\overline{r}'(\sigma)\,\overline{r}(\sigma)\right\}^{1/2},$$
$$\text{for } k - L < 0, \quad J(\overline{r}(k)) = \left\{\sum_{\sigma=k}^{\sigma=0}\overline{r}'(\sigma)\,\overline{r}(\sigma)\right\}^{1/2}. \quad (27)$$

The threshold is defined as

$$J_{th}(k) = \sup_{d \in \mathscr{L}^2, f=0} \varepsilon\left(J(\overline{r}(k))\right) \quad (28)$$

and the decision rule for the fault detection is taken by analyzing the value of $J(\overline{r}(k))$ as follows:

$$J(\overline{r}(k)) < J_{th}(k), \quad \text{means that the system is in the nominal mode,}$$
$$J(\overline{r}(k)) \geq J_{th}(k), \quad \text{means that a fault occurred at the instant } k. \quad (29)$$

5. Main Results

In this section, we introduce the three main results of this paper: the H_∞ FD filter, the H_2 FD filter, and the mixed H_2/H_∞ FD filter. A theorem is presented for each case.

5.1. FD Filter for the H_∞ Case

Theorem 1. *There exists a mode-dependent FD filter as in (16) satisfying $\|\mathcal{G}_{aug}\|_\infty^2 < \delta$ if there exist symmetric matrices Z_i, X_i, and W_i and matrices H_i, Δ_i, O_i, F_i, and G_i with compatible dimensions that satisfy the following LMI constraint:*

$$\begin{bmatrix} Z_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ Z_i & X_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & W_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & \delta I & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & \delta I & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 0 & 0 & 0 & \delta I & \bullet & \bullet & \bullet & \bullet \\ \varepsilon_i(Z)A_i & \varepsilon_i(Z)A_i & 0 & \varepsilon_i(Z)B_i & \varepsilon_i(Z)B_{di} & \varepsilon_i(Z)B_{fi} & \varepsilon_i(Z) & \bullet & \bullet & \bullet \\ \varepsilon_i(X)A_i+\Delta_iC_i+O_i & \varepsilon_i(X)A_i+\Delta_iC_i & 0 & \varepsilon_i(X)B_i+H_i & \varepsilon_i(X)B_{di}+\Delta_iD_{di} & \varepsilon_i(X)B_{fi}+\Delta_iD_{fi} & \varepsilon_i(Z) & \varepsilon_i(X) & \bullet & \bullet \\ 0 & 0 & \varepsilon_i(W)A_{wf} & 0 & 0 & \varepsilon_i(W)B_{wf} & 0 & 0 & \varepsilon_i(W) & \bullet \\ G_iC_i+F_i & G_iC_i & -C_{wf} & 0 & G_iD_{di} & G_iD_{fi}-D_{wf} & 0 & 0 & 0 & I \end{bmatrix} > 0 \quad (30)$$

for all $i \in \mathbb{K}$. If a feasible solution for (30) is obtained, then a suitable FD filter is given by $A_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}O_i$, $B_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}\Delta_i$, $C_{\eta i} = F_i$, $D_{\eta i} = G_i$, and $M_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H_i$, for all $i \in \mathbb{K}$.

Proof. The first step to derive the result is to impose the following structure, similar to the structure in [23], for the matrices P_i and P_i^{-1}:

$$P_i = \begin{bmatrix} X_i & U_i & 0 \\ U_i' & \widehat{X}_i & 0 \\ 0 & 0 & W_i \end{bmatrix}, \qquad P_i^{-1} = \begin{bmatrix} Y_i & V_i & 0 \\ V_i' & \widehat{Y}_i & 0 \\ 0 & 0 & H_i \end{bmatrix}, \quad (31)$$

and also consider the following structure for the matrices $\varepsilon_i(P)$ and $\varepsilon_i(P)^{-1}$:

$$\varepsilon_i(P) = \begin{bmatrix} \varepsilon_i(X) & \varepsilon_i(U) & 0 \\ \varepsilon_i(U)' & \varepsilon_i(X) & 0 \\ 35pt]0 & 0 & \varepsilon_i(W) \end{bmatrix},$$

$$\varepsilon_i(P)^{-1} = \begin{bmatrix} R_{1i} & R_{2i} & 0 \\ R_{2i}' & R_{3i} & 0 \\ 0 & 0 & \varepsilon_i(W)^{-1} \end{bmatrix}. \quad (32)$$

We define the matrices π and ζ as follows:

$$\pi = \begin{bmatrix} I & I & 0 \\ V_i'Y_i^{-1} & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, \qquad \zeta = \begin{bmatrix} R_{1i}^{-1} & \varepsilon_i(X) & 0 \\ 0 & \varepsilon_i(U)' & 0 \\ 0 & 0 & \varepsilon_i(G) \end{bmatrix}. \quad (33)$$

Considering $U_i = Z_i - X_i$ in (31), we get from (31) and (33) that $Y_i = V_i'$ and $Y_i = Z_i^{-1}$. Also considering $U_i = -\widehat{X}_i$, we get $R_{1i}^{-1} = \varepsilon_i(X+U) = \varepsilon_(Z)$. Moreover, we have that $R_{1i}^{-1} = \varepsilon_i(Z)$, and so we have that

$$\pi' P_i \pi = \begin{bmatrix} Y_i^{-1} & Y_i^{-1} & 0 \\ Y_i^{-1} & X_i & 0 \\ 0 & 0 & W_i \end{bmatrix},$$

$$\zeta' \widetilde{A}_i \pi = \begin{bmatrix} R_{1i}^{-1}A_i & R_{1i}^{-1}A_i & 0 \\ \varepsilon_i(X)A_i + \varepsilon_i(U)B_{\eta i}C_i + \varepsilon_i(U)A_{\eta i}V_i'Y_i^{-1} & \varepsilon_i(X)A_i + \varepsilon_i(U)B_{\eta i}C_i & 0 \\ 0 & 0 & \varepsilon_i(W)A_{wf} \end{bmatrix},$$

$$\zeta'\widetilde{B}_i = \begin{bmatrix} R_{1i}^{-1}B_i & R_{1i}^{-1}B_{di} & R_{1i}^{-1}B_{fi} \\ \varepsilon_i(X)B_i + \varepsilon_i(U)M_{\eta i} & \varepsilon_i(X)B_{di} + \varepsilon_i(U)B_{\eta i}D_{di} & \varepsilon_i(X)B_{fi} + \varepsilon_i(U)B_{\eta i}D_{fi} \\ 0 & 0 & \varepsilon_i(W)B_{wf} \end{bmatrix},$$

$$\zeta'\varepsilon_i(P)^{-1}\zeta = \begin{bmatrix} R_{1i}^{-1} & \varepsilon_i(Z) & 0 \\ \varepsilon_i(Z) & \varepsilon_i(X) & 0 \\ 0 & 0 & \varepsilon_i(W) \end{bmatrix},$$

$$\widetilde{C}_i\pi = \left[D_{\eta i}C_i + C_{\eta i}V_i'Z_i \quad D_{\eta i}C_i \quad -C_{wf}\right],$$

$$\widetilde{D}_i = \left[0 \quad D_{\eta i}D_{di} \quad D_{\eta i}D_i - D_{wf}\right]. \tag{34}$$

Applying the change of variables $\varepsilon_i(U)B_{\eta i} = \Delta_i$, $\varepsilon_i(U)A_{\eta i}V_i'Z_i = O_i$, $\varepsilon_i(U)M_{\eta i} = H_i$, $C_{\eta i}V_i'Z_i = F_i$, and $D_{\eta i} = G_i$ and also substituting $\varepsilon_i(Z) = R_{1i}^{-1}$ in (30), we get the following inequality:

$$\begin{bmatrix} \pi'\widetilde{P}_i\pi & \bullet & \bullet & \bullet \\ 0 & \delta I & \bullet & \bullet \\ \zeta'\widetilde{A}_i\pi & \zeta'\widetilde{B}_i & \zeta'\varepsilon_i(P)^{-1}\zeta & \bullet \\ \widetilde{C}_i\pi & \widetilde{D}_i & 0 & I \end{bmatrix} > 0, \tag{35}$$

and it is easy to see that inequality (35) is equivalent to inequality (30). Multiplying to the right by diag$[\pi^{-1}, I, \zeta^{-1}, I]$ and to the left by its transpose, we get inequality (7) and with that we can guarantee that $\|\mathscr{G}\|_\infty^2 < \delta$. □

5.2. FD Filter for the H_2 Case

Theorem 2. *There exists a mode-dependent FD filter in the form of (16) satisfying $\|\mathscr{G}_{aug}\|_2^2 < \gamma$ if there exist symmetric matrices Z_i, X_i, S_i, and T_i and matrices H_i, Δ_i, O_i, F_i, and G_i with compatible dimensions that satisfy the LMI constraints (36), (37), and (38):*

$$\sum_{i=1}^{N}\mu_i tr(S_i) < \gamma \tag{36}$$

$$\begin{bmatrix} & [S_i] & & \bullet & \bullet & \bullet & \bullet \\ & & & \bullet & \bullet & \bullet & \bullet \\ & & & \bullet & \bullet & \bullet & \bullet \\ \varepsilon_i(Z)B_i & \varepsilon_i(Z)B_{di} & \varepsilon_i(Z)B_{fi} & \varepsilon_i(Z) & \bullet & \bullet & \bullet \\ \varepsilon_i(X)B_i + H_i & \varepsilon_i(X)B_{di} + \Delta_i D_{di} & \varepsilon_i(X)B_{fi} + \Delta_i D_{fi} & \varepsilon_i(Z) & \varepsilon_i(X) & \bullet & \bullet \\ 0 & 0 & \varepsilon_i(T)B_{wf} & 0 & 0 & \varepsilon_i(T) & \bullet \\ 0 & G_iD_{di} & G_iD_{fi} - D_{wf} & 0 & 0 & 0 & I \end{bmatrix} > 0 \tag{37}$$

$$\begin{bmatrix} Z_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ Z_i & X_i & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & T_i & \bullet & \bullet & \bullet & \bullet \\ \varepsilon_i(Z)A_i & \varepsilon_i(Z)A_i & 0 & \varepsilon_i(Z) & \bullet & \bullet & \bullet \\ \varepsilon_i(X)A_i + \Delta_iC_i + O_i & \varepsilon_i(X)A_i + \Delta_iC_i & 0 & \varepsilon_i(Z) & \varepsilon_i(X) & \bullet & \bullet \\ 0 & 0 & \varepsilon_i(T)A_{wf} & 0 & 0 & \varepsilon_i(T) & \bullet \\ G_iC_i + F_i & G_iC_i & -C_{wf} & 0 & 0 & 0 & I \end{bmatrix} > 0 \tag{38}$$

for all $i \in \mathbb{K}$. If a feasible solution for (36), (37), and (38) is obtained, then a suitable FD filter is given by $A_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}O_i$, $B_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}\Delta_i$, $C_{\eta i} = F_i$, $D_{\eta i} = G_i$, and $M_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H_i$, for all $i \in \mathbb{K}$.

Proof. In the same way as presented for the H_∞ case, the structures for the matrices T_i and T_i^{-1} are as shown in (31) for, respectively, P_i and P_i^{-1}. For the matrices $\varepsilon_i(T)$ and $\varepsilon_i(T)^{-1}$, the structures are equal to the one in (32) for, respectively,

$\varepsilon_i(P)$ and $\varepsilon_i(P)^{-1}$. Furthermore, the matrices π and ζ are as shown in (33). Applying the change of variables $\varepsilon_i(U)B_{\eta i} = \Delta_i$, $\varepsilon_i(U)A_{\eta i}V_i'Z_i = O_i$, $\varepsilon_i(U)M_{\eta i} = H_i$, $C_{\eta i}V_i'Z_i = F_i$, and $D_{\eta i} = G_i$ and also substituting $\varepsilon_i(Z) = R_{1l}^{-1}$ in (37) and (38), we get the following inequalities:

$$\sum_{i=1}^{N} \mu_i tr(S_i) < \gamma, \quad (39)$$

$$\begin{bmatrix} S_i & \bullet & \bullet \\ \zeta'\widetilde{B}_i & \zeta'\varepsilon_i(P)^{-1}\zeta & \bullet \\ \widetilde{D}_i & 0 & I \end{bmatrix} > 0, \quad (40)$$

$$\begin{bmatrix} \pi' P_i \pi & \bullet & \bullet \\ \zeta'\widetilde{A}_i\pi & \zeta'\varepsilon_i(P)^{-1}\zeta & \bullet \\ \widetilde{C}_i\pi & 0 & I \end{bmatrix} > 0. \quad (41)$$

Multiplying (40) to the right by $\operatorname{diag}[I, \zeta^{-1}, I]$ (resp. (41) by $\operatorname{diag}[\pi^{-1}, \zeta^{-1}, I]$) and to the left by its transpose, we get inequalities (13) and (14) that, combined with (39), yield that $\|\mathcal{G}_{aug}\|_2^2 < \gamma$. □

5.3. FD Filter for the Mixed H_2/H_∞ Case. In this subsection, we consider the mixed H_2/H_∞ case. The set of variables is defined as

$$\psi = \{Z_i > 0,\ X_i > 0,\ W_i > 0,\ T_i > 0,\ S_i, H_i, \Delta_i, O_i, F_i, G_i\} \cup \zeta \quad (42)$$

where ζ represents a set that contains γ, δ, or both, depending on whether these parameters γ and δ are assumed to be given or a variable of the problem. See Section 4.1 and (23), (24), and (25) for the possible cases to design the FD filter. We also define

$$\Psi = \{\psi \text{ as in (42) such that the LMIs (30), (36), (37), (38) are simultaneously feasible}\}. \quad (43)$$

The next theorem provides a sufficient condition for the FD filter design for the mixed H_2/H_∞ case.

Theorem 3. *There exists a mode-dependent FD filter as in (16) such that $\|G_{aug}\|_2^2 < \gamma$ and $\|G_{aug}\|_\infty^2 < \delta$ if there exists $\psi \in \Psi$, where ψ is defined as in (43). If a feasible solution is obtained, then a suitable FD filter is given by $A_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}O_i$, $B_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}\Delta_i$, $C_{\eta i} = F_i$, $D_{\eta i} = G_i$, and $M_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H_i$, for all $i \in \mathbb{K}$.*

Proof. The proof follows directly from the proofs for Theorems 1 and 2. □

6. Secondary Results

In this section, we derive the mode-independent case and the case with parametric uncertainties.

6.1. Mode-Independent Case. We tackle in this subsection the mode-independent case, in which the FD filter does not depend on the Markov chain parameter $\theta(k)$. In this particular case, we design a single FD filter suitable for all system modes.

For the sake of obtaining a single FD filter for the N modes, it is necessary to fix the variables, $\Delta_i = \Delta$, $O_i = O$, $F_i = F$, $G_i = G$, and $H_i = H$, in the LMI constraints (30) or (36), (37), and (38) or all the set of LMI constraints, depending on whether the constraints are related to the problems analyzed in Theorem 1, Theorem 2, or Theorem 3. Furthermore, it is also necessary to assume that

$$\rho_{ij} = \rho_j, \quad \forall (i,j) \in \mathbb{K} \quad (44)$$

which corresponds to the Bernoulli case. Notice that the addition of this assumption is necessary due to the terms in Theorems 1 and 2 ($\varepsilon_i(Z) - \varepsilon_i(X)$), which in the Bernoulli case become ($\varepsilon(Z) - \varepsilon(X)$), where $\varepsilon(X) = \sum_{j=1}^{N} \rho_j X_j$, similarly for $\varepsilon(Z)$. Theorem 4 presents this result.

Theorem 4. *There exists a mode-independent FD filter as in (16) satisfying the constraint presented in Theorem 1 or Theorem 2 or Theorem 3, if there are Z_i, X_i, and W_i (and S_i and T_i for the H_2 and mixed cases) and matrices H, Δ, O, F, and G (independent of i) satisfying (30) for the H_∞ case, (36)-(38) for the H_2 case, and simultaneously (30), (36), (37), and (38) for the mixed case. If a feasible solution is obtained, a suitable FD filter is given by $A_\eta = (\varepsilon(Z)-\varepsilon(X))^{-1}O$, $B_\eta = (\varepsilon(Z)-\varepsilon(X))^{-1}\Delta$, $C_\eta = F$, $D_\eta = G$, and $M = (\varepsilon(Z) - \varepsilon(X))^{-1}H$.*

Proof. The proof can be derived directly from the proofs for Theorems 1 and 2. □

6.2. Parametric Uncertain Case. The last special case we work on is the procedure to add parametric uncertainties in Theorems 1, 2, and 3. In order to describe system (15) with polytopic uncertainties, we consider that for vertex matrices

$$\left[\begin{array}{c|c} A_i^l & B_i^l \\ \hline C_i^l & 0 \end{array}\right], \quad i \in \mathbb{K} \quad (45)$$

we have that

$$\left[\begin{array}{c|c} A_{\theta_k} & B_{\theta_k} \\ \hline C_{\theta_k} & 0 \end{array}\right] \in \Upsilon \quad (46)$$

where Υ is the polytope as described in (1) and $l \in \{1, \ldots, V\}$ represents the uncertain polytopic vertex. We replace in (30) and (37) and (38) the matrices A_i, B_i, and C_i by, respectively, A_i^l, B_i^l, and C_i^l, so that adding this new index implies adding V new constraints in (30) for Theorem 1 or (37) and (38) for Theorem 2 or (30), (37), and (38) for Theorem 3. We have the following result.

Theorem 5. *There exists a mode-dependent FD filter as in (16) satisfying the constraints in Theorem 1 or Theorem 2 or Theorem 3 if there exist symmetric matrices Z_i, X_i, and W_i, (S_i and T_i for the H_2 and mixed cases) and matrices H_i, Δ_i, O_i, F_i, and $D_{\eta i}$ satisfying (30) for the H_∞ case, (36)-(38) for the H_2 case, and simultaneously (30), (36), (37), and (38) for the mixed case. If a feasible solution is obtained, a suitable FD filter is given by $A_{\eta i} = (\varepsilon_i(Z)-\varepsilon_i(X))^{-1}O_i$, $B_{\eta i} = (\varepsilon_i(Z)-\varepsilon_i(X))^{-1}\Delta_i$, $C_{\eta i} = F_i$, $D_{\eta i} = G_i$ =, and $M_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H_i$, for all $i \in \mathbb{K}$.*

Proof. The proof derives directly from the proofs for Theorems 1 and 2. □

7. Numerical Example

In this section, some numerical examples are presented. The first analysis is on the H_∞ and H_2 norm behavior when we consider variation on some of the parameters in the transition matrix $\mathbb{P}$.

The second analysis is related to the mixed H_2/H_∞ case, and we consider the three cases presented in Section 4.1. For the first case, defined by (23), the upper bound for the H_∞ norm δ is fixed and the upper bound for the H_2 norm γ is minimized. For the second case, defined by (24), the opposite situation is considered; that is, the upper bound γ is fixed and the upper bound δ is minimized. For the third case, defined by (25), we vary the scalars β^∞ and β^2 and consider the behavior of both upper bounds γ and δ.

The third analysis is a temporal simulation comparing the H_∞, H_2, and all three mixed H_2/H_∞ cases when two kinds of failures occur: an abrupt failure and a smooth failure. This analysis is divided into two stages: the first one with a single sample and the second one with a Monte Carlo simulation with 2000 random samples. The system used for this example was extracted from [11] and its matrices are given by

$$A_1 = \begin{bmatrix} 0.1 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.5 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.3 & 0 & -1 & 0 \\ -0.1 & 0.2 & 0 & -0.5 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0.8 \\ -2.4 \\ 1.6 \\ 0.8 \end{bmatrix},$$

$$B_f = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix},$$

$$D_d = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$$

$$D_f = \begin{bmatrix} 2 \\ -1 \end{bmatrix},$$

$$A_{wf} = 0.5,$$

$$B_{wf} = 0.25,$$

$$C_{wf} = 1,$$

$$D_{wf} = 0.5. \tag{47}$$

The matrix $\mathbb{P}$ is given by

$$\mathbb{P} = \begin{bmatrix} \rho_1 & 1-\rho_1 \\ 1-\rho_2 & \rho_2 \end{bmatrix} \tag{48}$$

where $0 \le \rho_i \le 1$, $i \in \mathbb{K}$.

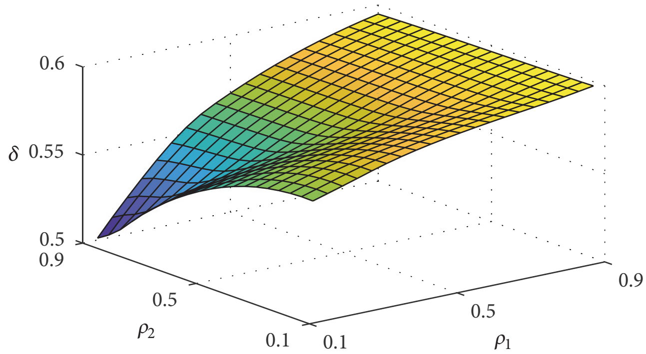
7.1. Norm Behavior. The norm behavior is important to be analyzed in order to identify the worst case scenario and the system sensibility to the parameters variation in the transition matrix $\mathbb{P}$.

7.1.1. FD Filter for the H_∞ Case and the H_2 Case. Considering that $\rho_1 = \rho_2 \in [0.1, 0.9]$ and $\mathbb{P}$ as in (48), the obtained curves are presented in Figure 2. It is possible to observe that in both cases the variation range is small, showing that the system sensibility is small in terms of the variation of the probability of jumps. Another important information that could be extracted from the graphics is that the system has a good level of robustness, given that, from Figure 2(a), the H_∞ norm's maximum value is less than 0.6. The variation of the values for the H_2 norm, which represents the total energy dissipated by the impulsive inputs, is shown in Figure 2(b), which indicates that the system is sensible to this class of signals.

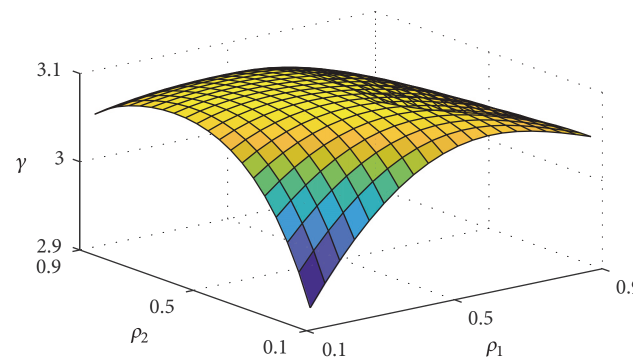
7.2. FD Filter for the Mixed H_2/H_∞ Case. In this subsection, we present the norm behavior for each case below:

(i) *First case:* the behavior of γ when δ varies between (3.6, 10)

(ii) *Second case:* the behavior of δ when γ varies between (3.6, 10)

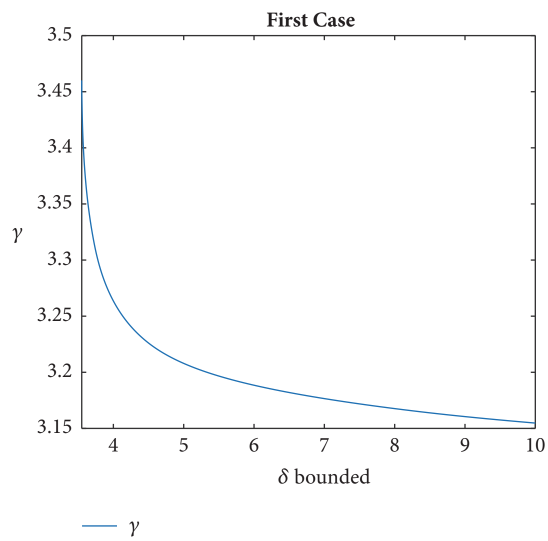


(a) H_∞ norm

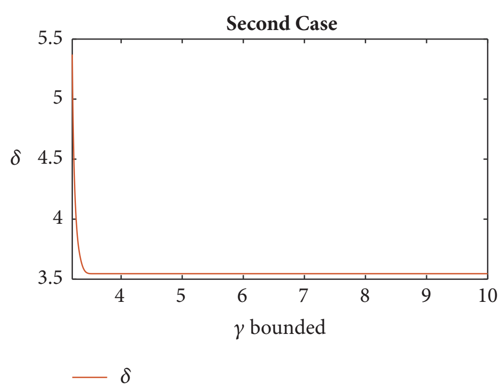


(b) H_2 norm

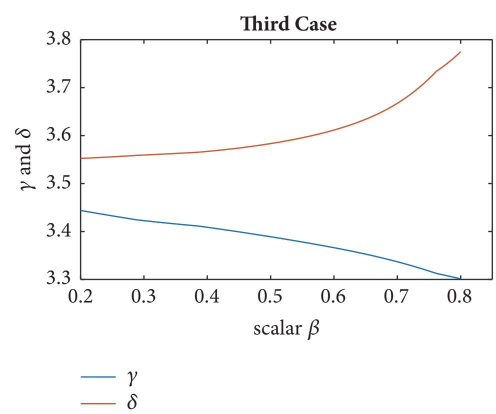
FIGURE 2: Norm behavior for H_∞ and H_2 cases.



(a) First case



(b) Second case

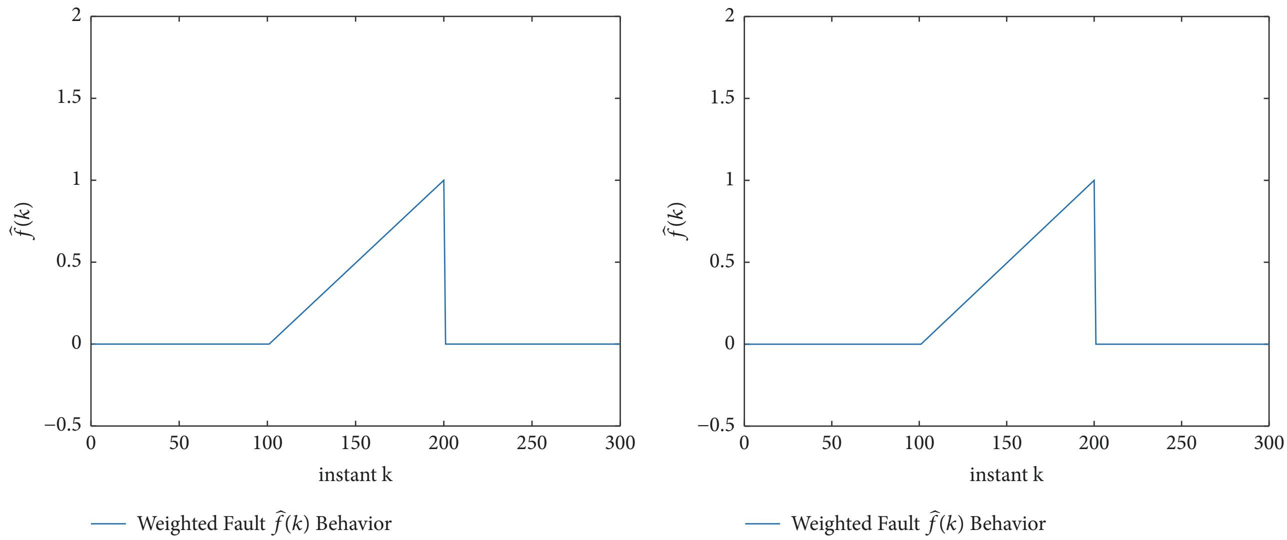


(c) Third case

FIGURE 3: Norm behavior for the mixed cases.

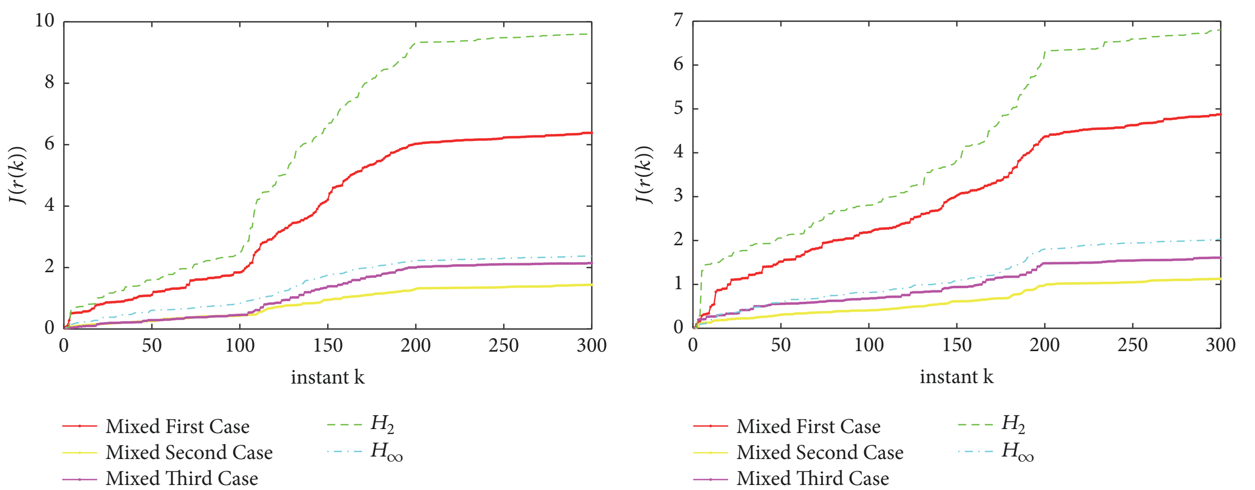
(iii) *Third case:* the behavior of γ and δ when β^2 varies between $(0.2, 0.8)$, considering that $\beta^\infty = 1 - \beta^2$

It is possible to observe that, in the first case, shown in Figure 3(a), as the value of δ increases, the upper bound γ decreases, which is expected, since the conservatism in the optimization problem is reduced. A similar situation occurs in Figure 3(b), since that, as the value of γ increases, the value of δ decreases. For the third case (Figure 3(c)), it is important to observe that when $\beta^2 = 0$ the optimization problem falls in



(a) Abrupt failure (b) Smooth failure

FIGURE 4: Failure behavior.



(a) For the abrupt failure (b) For the smooth failure

FIGURE 5: Temporal simulation.

the first case, and for $\beta^\infty = 0$ the optimization problem falls in the second case. Bearing this information in mind, the third graphic (Figure 3(c)) represents exactly what it was expected; that is, when β is closer to zero the δ value is closer to the value obtained in the second case; otherwise, the value of γ gets closer to the value obtained in the first case.

7.3. Temporal Simulation. Hereafter, the matrix $\mathbb{P}$ is given by

$$\mathbb{P} = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}. \quad (49)$$

Two distinct faults were selected: the first one an abrupt fault and the second one a smooth signal, and both are presented in Figure 4.

TABLE 1: Amount of time until detection in a single simulation.

	Abrupt failure	Smooth failure
H_∞ case	104	125
H_2 case	105	122
Mixed first case	107	123
Mixed second case	110	132
Mixed third case	112	123

In order to analyze the applicability of our approaches, we present in Figure 5 the simulation results considering a single sample for the cases H_2, H_∞, and mixed H_∞/H_2. Observing the values in Table 1, we notice that the H_∞ approach has a small advantage when compared to the other approaches.

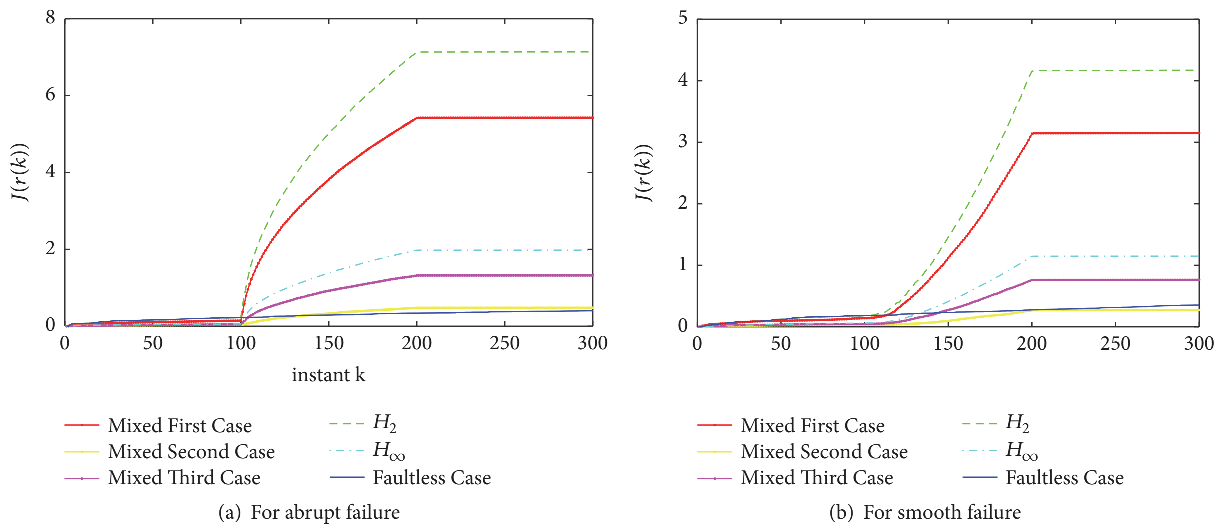


FIGURE 6: Monte Carlo simulation.

TABLE 2: Average amount of time until detection and the standard deviation.

	Abrupt failure	Smooth failure
H_∞ case	105.2750 ± 2.1336	127.4950 ± 7.0018
H_2 case	104.7400 ± 2.2489	127.0150 ± 7.2773
Mixed first case	104.6000 ± 2.0100	127.9650 ± 6.5688
Mixed second case	106.4350 ± 3.0795	128.0850 ± 7.2893
Mixed third case	105.1900 ± 3.0922	127.9400 ± 9.3075

TABLE 3: The sensibility against the failure and resilience against exogenous input.

	r/f	r/d
H_∞ case	0.1979	0.0145
H_2 case	0.7130	0.2163
Mixed first case	0.5357	0.1648
Mixed second case	0.0450	0.0567
Mixed third case	0.1288	0.0389

Other information obtained is that the smooth failure, as expected, demands more time to be detected.

Using the same parameter as in the previous simulation and performing a Monte Carlo simulation with 2000 samples, we obtain the curves as in Figure 6.

Observing the graphics presented in Figure 6, it is possible to see that the H_2 and second mixed H_2/H_∞ cases, explained in Section 4.1 and represented by (24), are the most sensible approaches, as can also be observed from Table 2.

As explained in [7], the main goal is to provide a FD filter with a maximum ratio between fault sensitivity (r/f) and disturbance sensitivity (r/d), and, as observed, the solution that fits better in this condition is the first mixed H_2/H_∞ case, explained in the Section 4.1 and represented by (23), where the robustness is provided by the H_∞ norm and the fault sensibility is provided by the H_2 norm.

According to the graphic presented in Figure 6, the approach with higher level of fault sensibility is the H_2 case, but this approach has the lowest level of exogenous input resilience. Thus, the H_2 approach does not provide the best option when considering the desirable aspect of a FD filter of high sensibility to the fault and low sensibility to the disturbance. For this reason, analyzing Table 3, the best performance is provided by the first mixed H_2/H_∞ case, explained in Section 4.1 and represented by (23), due to the high level of sensibility against the fault and low sensibility to the disturbance. The H_∞ case provides the lowest sensibility to disturbance, but the level of sensibility against the fault does not get close to the levels presented by the H_2 norm and the first mixed H_2/H_∞ case.

8. Conclusion

In this paper, the fault detection problem under the discrete-time Markovian jump linear systems framework is studied. We propose a convex formulation in terms of LMI for the design of mixed H_2/H_∞ FD filters such that the H_2 and H_∞ norms with respect to the residual signal are bounded. Three possible design formulations are presented: (1) an upper bound of the H_2 norm is minimized while guaranteeing that the H_∞ norm is bounded; (2) the inverse scheme (minimizing an upper bound of the H_∞ norm by restricting its H_2 norm); (3) a weighed combination of both bounds on H_2 and H_∞ norms is minimized. Design conditions for mode-independent filters in the scope of the Bernoulli case, as well as robust filters (in the sense of uncertain system matrices), are also given. Regarding the simulation results, it is important to point out that all approaches provided a plausible solution to the fault detection problem. However, following the comparison criterion presented in [7], the approach with the best ratio between fault sensitivity and disturbance sensitivity was the first mixed H_2/H_∞ case as in (23), since this solution presented the best results as can be seen in Table 3.

Along the lines of the present paper, a possible next step would be to consider that the Markov chain mode is not accessible, as considered in [22], which is an assumption that brings new challenges to the fault detection problem.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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