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Research Article

The Adaptive Neural Control for a Class of High-Order Uncertain Stochastic Nonlinear Systems

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This paper studies the problem of the adaptive neural control for a class of high-order uncertain stochastic nonlinear systems. By using some techniques such as the backstepping recursive technique, Young's inequality, and approximation capability, a novel adaptive neural control scheme is constructed. The proposed control method can guarantee that the signals of the closed-loop system are bounded in probability, and only one parameter needs to be updated online. One example is given to show the effectiveness of the proposed control method.

1. Introduction

Ever since the stochastic stability theory was established by [1–3], the design and analysis of backstepping controller for stochastic nonlinear systems have achieved remarkable development in recent years; see [4–20] and the references therein. Based on the backstepping technique, Pan and Basar [8] firstly studied a class of stochastic nonlinear systems under a risk-sensitive cost criterion. Then, by combining backstepping technique with different nonlinear control methods, [9–13] obtained the state-feedback stabilization results of stochastic nonlinear systems in various structures. In the case of system states being unmeasurable, [14–19] further studied the problem of the output-feedback stabilization for stochastic nonlinear systems with the help of observer design. In addition, by applying the backstepping design and Lyapunov stability analysis, the finite-time control with fast convergence rate has been achieved for stochastic nonlinear systems in [10, 20, 21].

Note that when stochastic nonlinear system is of high-order, it may be nonsmooth and in general not stabilizable. How to deal with this problem is difficult. To handle this case, [9–13, 17, 20] have done remarkable work on stochastic high-order nonlinear systems and obtained different control results. Particularly, the homogeneous domination approach was extended to stochastic nonlinear system in [17], which provides an effective design methods for high-order stochastic nonlinear systems. However, the above-mentioned results were subjected to the nonlinear dynamics models which are known exactly or unknown parameters existing linearly. Thus, these results cannot be used for the stochastic systems with structured uncertainties. Naturally, one raises the problem of how to design the controller for the high-order stochastic nonlinear systems with structured uncertainties.

To handle the structured uncertainties for the stochastic nonlinear systems, the radial basis function neural network (RBF NN) or the fuzzy logic is used to approximate the uncertain functions, which ensures the growth assumptions can be weakened or removed. Based on these methods and some useful adaptive backstepping control approaches, fruitful results have been introduced and obtained in [22–30] and the references therein. Reference [22] studied the problem of fuzzy backstepping control for a class of stochastic nonlinear strict-feedback systems. Reference [23] considered the adaptive neural network output-feedback control for nonlinear systems with dynamical uncertainties. Reference [25] considered NN output-feedback control for stochastic nonlinear systems with unknown control coefficients. Reference [24] studied NN output-feedback control for stochastic time-delay nonlinear systems with unknown control coefficients. Furthermore, more problems of stochastic nonlinear systems with unmodeled dynamics were studied in [26–30].

Motivated by the aforementioned literatures, one raises the following meaningful problem: *how to relax or remove the* *matching conditions on drift and diffusion terms by RBF NN? And how to design the adaptive NN state-feedback controller for a class of high-order stochastic nonlinear systems with unknown control directions*?

In this paper, we will discuss the problem of adaptive neural control for a class of high-order stochastic nonlinear systems with structured uncertainties. By using backstepping recursive approach, Young's inequality, etc., the restrictions on systems nonlinearities are removed and the procedure of the design is simpler. A novel adaptive neural controller is constructed, which assures that the closed-loop system is bounded in probability. In addition, in the design progress, there is only one parameter that needs to be updated online.

The rest of this paper is organized as follows. The notations and some preliminaries are provided in Section 2. In Section 3, we present the main results. In Section 4, we give the simulation example to illustrate the effectiveness of the proposed results, and the conclusion is drawn in Section 5.

2. Preliminaries and Problem Formulation

Notations. R denotes the set of all real numbers, R^+ denotes the set of all nonnegative real numbers, and R^n denotes the real n-dimensional space. C^i stands for the family of the functions with ith continuous partial derivations. For a given vector or matrix X, X^T denotes its transpose, $Tr\{X\}$ denotes its trance when X is square, and $|X|$ is the Euclidean norm of a vector X. For simplicity, the smooth function $\varrho(\cdot)$ is sometimes denoted by ϱ.

Consider a class of high-order stochastic nonlinear systems as follows:

$$dx_i = h_i(t)\, x_{i+1}^{P_i} dt + \varphi_i(t, \overline{x}_i)\, dt + g_i(\overline{x}_i) \sum(t)\, d\omega, \quad i = 1, \ldots, n-1, \quad (1)$$

$$dx_n = h_n(t)\, u^{P_n} dt + \varphi_n(t, \overline{x}_n)\, dt + g_n(\overline{x}_n) \sum(t)\, d\omega$$

where $x = [x_1, \ldots, x_n]^T \in R^n$ and $u \in R$ are the system states and the control input, respectively. $\overline{x}_i = [x_1, \ldots, x_i]^T \in R^i$, $i = 1, 2, \ldots, n$ and $\overline{x}_n = x$. $\omega \in R^r$ is an r-dimensional standard Wiener process defined on the complete probability space (Ω, F, P) with Ω being a sample space, F being a filtration, and P being measure. $\varphi_i(\cdot)$ and $g_i(\cdot)$ are unknown smooth functions, with $\varphi_i(0,0) = 0$, $g_i(0) = 0$ for $i = 1, 2, \ldots, n$. The disturbed virtual control coefficients $h_i(t) : R \longrightarrow R$ $(i = 1, 2, \ldots, n)$ are unknown and continuous functions, respectively; $\sum(t) : R^+ \longrightarrow R^{r\times r}$ is the Borel bounded measurable functions.

2.1. Preliminary Results. Next we introduce several technical lemmas which will play an important role in our later control design.

Consider the following stochastic nonlinear system:

$$dx = f(t, x)\, dt + h(t, x)\, d\omega \quad (2)$$

where f and g are the Borel measurable functions. $f : R^+ \times R^{n+1} \longrightarrow R^n$ and $g : R^+ \times R^{n+1} \longrightarrow R^{n\times r}$ are assumed to be C^1 in their arguments.

Definition 1 (see [2]). Given $V(x,t) \in C^{1,2}$ for stochastic nonlinear system (2), the differential operator L is defined as follows:

$$LV(x,t) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\left(h^T \frac{\partial^2 V}{\partial x^2} h\right) \quad (3)$$

where $C^{1,2}(R^n \times R^+; R^+)$ denotes all nonnegative functions $V(x,t)$ on $R^n \times R^+$; i.e., $V(x,t)$ satisfies C^1 in t and C^2 in x. Simply, the smooth function $f(\cdot)$ is denoted by f.

Definition 2 (see [15]). The solution of stochastic system (2) is said to be bounded in probability if it satisfies

$$\lim_{c\longrightarrow\infty} \sup_{0\le t\le\infty} P\{|x(t)| > c\} = 0. \quad (4)$$

Definition 3 (see [15]). Consider system (2) with $f(t,0) = 0$ and $h(t,0) = 0$; the equilibrium $x = 0$ is globally stable in probability if, for any $\varepsilon > 0$, there exists a class K_∞ function $\gamma(\cdot)$ such that

$$P\{|x(t)| < \gamma(|x_0|)\} \ge 1 - \varepsilon, \quad \forall t \ge 0,\ x_0 \in R^n \setminus \{0\}. \quad (5)$$

Lemma 4 (Young's inequality [4]). *For any $(x, y) \in R^2$, the following inequality holds:*

$$xy \le \frac{\varepsilon^p}{p} |x|^p + \frac{1}{q\varepsilon^q} |y|^q. \quad (6)$$

Lemma 5 (see [15]). *Consider the stochastic system (2). Assume that $f(x,t)$ and $h(x,t)$ are C^1 in their arguments, and $f(0,t)$ and $h(0,t)$ are bounded uniformly in t if there exist functions $V(x,t) \in C^{1,2}(R^n \times R^+, R^+)$, $u_1(\cdot), u_2(\cdot) \in K_\infty$ and constants $a_0 > 0$, $b_0 > 0$ such that*

$$\begin{gathered} u_1|x| \le V(x,t) \le u_2|x|, \\ LV \le -a_0 V(x,t) + b_0. \end{gathered} \quad (7)$$

Then the solution of (2) is bounded in probability.

The purpose of this paper is to construct a smooth adaptive neural state-feedback controller such that the solution process of system (1) is bounded in probability.

To design the controller for system (1), the following assumptions are needed:

- (A_1) $p_1 = p_2 = \cdots p_n = p \ge 1$ are odd integers.
- (A_2) For any $Y \in \Omega_Y$, there exists an ideal constant weight vector W^* such that $0 < \|W^*\|_\infty \le W_{max}$ and $0 < |\delta| \le \delta_{max}$.
- (A_3) For $i = 1, \ldots, n$, there are positive constants γ_{i1} and γ_{i2}, such that $\gamma_{i1} \le |h_i(t)| \le \gamma_{i2}$.
- (A_4) There exists constant $M > 0$ such that $\| \sum^T(t) \sum(t)\|_F^2 \le M$.

Remark 6. If $p_1 = p_2 = \cdots p_n = 1$, system (1) becomes the strict-feedback form. The problem of the feedback control has been studied in [22–24, 26, 27]. However, they did not consider $p_1 = p_2 = \cdots p_n > 1$. In this paper, we will consider the problem of the feedback control under the case $p_1 = p_2 = \cdots p_n > 1$.

The following radial basis function neural network (RBFNN) will be considered and used to approximate unknown continuous functions:

$$\Psi(Y) : R^n \longrightarrow R,$$
$$\Psi_{nn}(Y) = W^T S(Y) \tag{8}$$

where $Y \in \Omega_Y \subset R^q$ is the input vector with q being the neural networks input dimension. $W = [w_1, w_2, \ldots, w_l]^T \in R^l$ denotes the weight vector. $l > 1$ is the neural network mode number. $S(Y) = [s_1(Y), s_2(Y), \ldots, s_l(Y)]^T$ and $s_i(Y) = \exp[-\|Y - u_i\|^2/\delta_i]$, $i = 1, 2, \ldots, l$ are the basis function vectors. Here $u_i = [u_{i1}, u_{i2}, \ldots, u_{iq}]^T$ is the center of the receptive field, and δ_i is the width of the Gaussian function. Equation (8) can approximate any unknown continuous function over the compact set $\Omega_Y \subset R^q$ with arbitrary accuracy. Namely,

$$\Psi(Y) = W^{*T} S(Y) + \delta(Y), \quad \forall Y \in \Omega_Y. \tag{9}$$

The ideal constant weight vector W^* is defined as $W^* := \arg\min_{W \in R^l}\{\sup_{Y \in \Omega_Y} |\Psi(Y) - W^T S(Y)|\}$, and $\delta(Y)$ is the approximation error.

From (9), we can easily get

$$\begin{aligned} W^{*T} S(Y) + \delta &\leq \left|W^{*T} S(Y)\right| + |\delta| \\ &\leq \sum_{i=1}^{l} \left|S_i(Y)\right| W_{max} + \delta_{max} \leq \Theta \pi(Y) \end{aligned} \tag{10}$$

where $\pi(Y) = \sqrt{(l+1)(\sum_{i=1}^{l} S_i^2(Y) + 1)}$, $\Theta = \max(W_{max}, \delta_{max})$.

To design a state-feedback controller, we first introduce the following transformation:

$$\begin{aligned} \eta_i &= x_i - \alpha_i\left(\overline{x}_i, \widehat{\Theta}\right), \\ \alpha_i &= -\eta_{i-1}\beta_{i-1}(\cdot). \quad i = 1, 2, \ldots, n \end{aligned} \tag{11}$$

where $\alpha_1 = 0$, $\alpha_i(\cdot)$ is the virtual control law and $\beta_i(\cdot) > 0$ can be designed in the following form.

Using (11), we have

$$\begin{aligned} d\eta_1 &= dx_1 = h_1 x_2^p dt + \overline{\varphi}_1 dt + G_1 \sum(t)\, d\omega \\ d\eta_i &= d\left(x_i - \alpha_i\right) \\ &= h_i x_{i+1}^p dt + \overline{\varphi}_i dt - \frac{\partial \alpha_i}{\partial \widehat{\Theta}} \dot{\widehat{\Theta}} dt + G_i \sum(t)\, d\omega \\ &\qquad i = 2, \ldots, n, \end{aligned} \tag{12}$$

where $x_{n+1} = u$, $\overline{\varphi}_i = \varphi_i - \sum_{l=1}^{i-1}(\partial\alpha_i/\partial x_l)\varphi_l - \sum_{l=1}^{i-1}(\partial\alpha_i/\partial x_l)x_{l+1}^p - (1/2)\sum_{k,m=1}^{i-1}(\partial^2\alpha_i/\partial x_k \partial x_m) g_k \sum(t) \sum^T(t) g_m^T$, and $G_i = g_i - \sum_{l=1}^{i-1}(\partial\alpha_i/\partial x_l) g_l$.

3. Controller Design and Stability Analysis

3.1. Controller Design. In this section, by using the backstepping method and the BRFNN, we construct the adaptive neural controller and approximate the unknown nonlinear functions, respectively.

Step 1. Consider the Lyapunov function $V_1 = (1/4)\eta_1^4 + (1/2)\widetilde{\Theta}^2$, where $\widetilde{\Theta} = \Theta - \widehat{\Theta}$ is the parameter error. By (3) and (12), we have

$$\begin{aligned} LV_1 = {} & \eta_1^3\left(h_1 x_2^p + \overline{\varphi}_1\right) + \frac{3}{2}\eta_1^2 Tr\left(G_1^T \sum^T(t) \sum(t)\, G_1\right) \\ & - \widetilde{\Theta}\dot{\widehat{\Theta}}. \end{aligned} \tag{13}$$

From Lemma 4 and assumptions $(A_2) - (A_4)$, there exists constant $a_{1j} > 0$, $j = 1, 2$, such that

$$\eta_1^3 \overline{\varphi}_1 \leq \eta_1^{p+3} \frac{3}{p+3} a_{11}^{3/(p+3)} \overline{\varphi}_1^{(p+3)/3} + \frac{1}{p+3} a_{11}^{p+3} \tag{14}$$

and

$$\begin{aligned} & \frac{3}{2}\eta_1^2 Tr\left(G_1^T \sum^T(t) \sum(t)\, G_1\right) \\ & \leq \frac{3}{2}\eta_1^2 \left\|G_1\right\|^2 \left\|\sum^T(t) \sum(t)\right\|_F^2 \\ & \leq \frac{2}{p+3} a_{12}^{2/(p+3)} \eta_1^{p+3} \left\|G_1\right\|^{p+3} M \\ & \quad + \frac{1}{p+3} a_{12}^{p+3} M. \end{aligned} \tag{15}$$

Substituting (14) and (15) into (13), we obtain

$$LV_1 \leq \eta_1^3 h_1 x_2^p + a_1 - \widetilde{\Theta}\dot{\widehat{\Theta}} + \eta_1^{p+3}\Psi_1 \tag{16}$$

where $\Psi_1 = (3/(p+3)) a_{11}^{3/(p+3)} \overline{\varphi}_1^{(p+3)/3} + (2/(p+3)) a_{12}^{2/(p+3)} \|G_1\|^{p+3} M$, $a_1 = (1/(p+3)) a_{11}^{p+3} + (2/(p+3)) a_{12}^{p+3} M$. Obviously, Ψ_1 is an unknown function since it has unknown function φ_1 and g_1. In practice, it cannot be used directly. Moreover, there exists a neural network $W_1^{*T} S_1(Y_1)$, $Y_1 = x_1 \in \Omega_{Y_1} \subset R^1$, such that

$$\Psi_1 = W_1^{*T} S_1\left(Y_1\right) + \delta_1\left(Y_1\right) \leq \Theta \pi_1(\cdot) \tag{17}$$

where $\pi_1(\cdot) = \sqrt{(l+1)\sum_{k=1}^{l} S_{1k}^2 + 1}$. In the view of (16) and (17), we can get

$$\begin{aligned} LV_1 \leq {} & -\widetilde{\Theta}\dot{\widehat{\Theta}} + \eta_1^3 h_1\left(x_2^p - \alpha_2^p\right) + \eta_1^{p+3}\Theta\pi_1 + \eta_1^3 h_1 \alpha_2^p \\ & + a_1. \end{aligned} \tag{18}$$

Now we choose the virtual control laws

$$\alpha_2 = \left(\frac{-c_1 - \widehat{\Theta}\pi_1}{\gamma_{11}}\right)^{1/p} \eta_1 = -\beta_1\eta_1,$$
$$\beta_1 = \left(\frac{c_1 + \widehat{\Theta}\pi_1}{\gamma_{11}}\right)^{1/p} > 0 \quad (19)$$

where $c_1 > 0$ is a constant to be chosen.

Substituting α_2 into (18), it can be rewritten as

$$LV_1 \leq \widetilde{\Theta}\left(\eta_1^{p+3}\pi_1 - \dot{\widehat{\Theta}}\right) + \eta_1^3 h_1\left(x_2^p - \alpha_2^p\right) - c_1\eta_1^{p+3} + a_1. \quad (20)$$

By (11), Lemma 4, assumptions (A_2), (A_3), and $(a+b)^n = \sum_{i=0}^{n} C_n^k a^k b^{n-k}$, we obtain

$$\begin{aligned}\eta_1^3 h_1\left(x_2^p - \alpha_2^p\right) &= \eta_1^3 h_1\left\{(\eta_2 + \alpha_2)^p - \alpha_2^p\right\} \\ &= \eta_1^3 h_1 \sum_{k=0}^{p-1} C_p^k \alpha_2^k \eta_2^{p-k} = \eta_1^3 h_1 \sum_{k=0}^{p-1} C_p^k \left(-\eta_1\beta_1\right)^k \eta_2^{p-k} \\ &\leq \gamma_{12} \sum_{k=0}^{(p-1)/2} C_p^{2k} \left|\eta_1^{3+2k}\right| \left|\eta_2^{p-2k}\right| \beta_1^{2k} \\ &\leq \sum_{k=0}^{(p-1)/2} \frac{3+2k}{p+3} \epsilon_{1k}^{(p+3)/(3+2k)} \eta_1^{3+p} \\ &\quad + \sum_{k=0}^{(p-1)/2} \frac{p-2k}{p+3} \epsilon_{1k}^{(p-2k)/(p+3)} \left(C_p^{2k}\beta_1^{2k}\gamma_{12}\right)^{(p+3)/(p-2k)} \\ &\quad \cdot \eta_2^{3+p} \leq \epsilon_1 \eta_1^{p+3} + \gamma_{20}\eta_2^{p+3}\end{aligned} \quad (21)$$

where $\epsilon_1 \geq \sum_{k=0}^{(p-1)/2}((3+2k)/(p+3))\epsilon_{1k}^{(p+3)/(3+2k)}$, $\gamma_{20} \geq \sum_{k=0}^{(p-1)/2}((p-2k)/(p+3))\epsilon_{1k}^{(p-2k)/(p+3)}(C_p^{2k}\beta_1^{2k}\gamma_{12})^{(p+3)/(p-2k)}$, and $\epsilon_{1k} > 0 (k = 0, 1, \cdots (p-1)/2)$ are constants.

Substituting (21) into (20), we can get

$$LV_1 \leq \widetilde{\Theta}\left(\eta_1^{p+3}\pi_1 - \dot{\widehat{\Theta}}\right) - \left(c_1 - \epsilon_1\right)\eta_1^{p+3} + \gamma_{20}\eta_2^{p+3} + a_1. \quad (22)$$

Step 2. We choose the Lyapunov function $V_2 = V_1 + (1/4)\eta_2^4$ to design the control law α_3. From (11), (12), and (22), we have

$$\begin{aligned}LV_2 &= LV_1 + \eta_2^3\left(h_2 x_3^p + \overline{\varphi}_2 - \frac{\partial\alpha_2}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) \\ &\quad + \frac{3}{2}\eta_2^2 Tr\left(G_2^T \sum^T(t) \sum(t) G_2\right) \\ &= \eta_2^3\left\{h_2 x_3^p + \overline{\varphi}_2 - \frac{\partial\alpha_2}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right\} \\ &\quad + \frac{3}{2}\eta_2^2 Tr\left(G_2^T \sum^T(t) \sum(t) G_2\right) \\ &\quad + \widetilde{\Theta}\left(\eta_1^{p+3}\pi_1 - \dot{\widehat{\Theta}}\right) - \left(c_1 - \epsilon_1\right)\eta_1^{p+3} + \gamma_{20}\eta_2^{p+3} \\ &\quad + a_1.\end{aligned} \quad (23)$$

From the definition of $\overline{\varphi}_2$ and Lemma 4, there exist constants $a_{2j} > 0$, $j = 1, 2$, such that

$$\eta_2^3\overline{\varphi}_2 \leq \eta_2^{p+3}\frac{3}{p+3}a_{21}^{3/(p+3)}\left|\overline{\varphi}_2\right|^{(p+3)/3} + \frac{1}{p+3}a_{21}^{p+3} \quad (24)$$

and

$$\begin{aligned}&\frac{3}{2}\eta_2^2 Tr\left(G_2^T \sum^T(t) \sum(t) G_2\right) \\ &= \frac{3}{2}\eta_2^2\left|G_2\right|^2\left\|\sum^T(t)\sum(t)\right\|_F^2 \\ &\leq \eta_2^{p+3}\frac{2}{p+3}{a_{22}}^{2/(p+3)}\left\|G_2\right\|^{p+3} M + \frac{1}{p+3}a_{22}^{p+3} M.\end{aligned} \quad (25)$$

By (23), (24), and (25), we can obtain

$$\begin{aligned}LV_2 &\leq \eta_2^3\left(h_2 x_3^p - \frac{\partial\alpha_2}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) + \widetilde{\Theta}\left(\eta_1^{p+3}\pi_1 - \dot{\widehat{\Theta}}\right) - (c_1 \\ &\quad - \epsilon_1)\eta_1^{p+3} + \gamma_{20}\eta_2^{p+3} + a_1 \\ &\quad + \eta_2^{p+3}\left\{\frac{2}{p+3}{a_{22}}^{2/(p+3)}\left\|G_2\right\|^{p+3} M \right. \\ &\quad \left. + \frac{3}{p+3}a_{21}^{3/(p+3)}\left|\overline{\varphi}_2\right|^{(p+3)/3}\right\} + \frac{1}{p+3}a_{21}^{p+3} + \frac{1}{p+3} \\ &\quad \cdot a_{22}^{p+3} M.\end{aligned} \quad (26)$$

Further, adding and subtracting $\eta_2^3 f_2$ in the first bracket in (26) and using Lemma 4, there exists a constant $a_{23} > 0$, such that

$$-\eta_2^3 f_2 \leq \eta_2^{p+3}\frac{3}{p+3}a_{23}^{3/(p+3)}\left|f_2\right|^{(p+3)/3} + \frac{1}{p+3}a_{23}^{p+3}. \quad (27)$$

Then, substituting (27) into (26), we get

$$\begin{aligned}LV_2 &\leq \eta_2^3\left(h_2 x_3^p - \frac{\partial\alpha_2}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}} + f_2\right) + \widetilde{\Theta}\left(\eta_1^{p+3}\pi_1 - \dot{\widehat{\Theta}}\right) \\ &\quad - \left(c_1 - \epsilon_1\right)\eta_1^{p+3}\eta_2^{p+3}\frac{3}{p+3}a_{23}^{3/(p+3)}\left|f_2\right|^{(p+3)/3} \\ &\quad + \frac{1}{p+3}a_{23}^{p+3} + \gamma_{20}\eta_2^{p+3} + \eta_2^{p+3}\Psi_2 + \sum_{i=1}^{2}a_i\end{aligned} \quad (28)$$

where $a_2 = (1/(p+3))a_{21}^{p+3} + (1/(p+3))a_{23}^{p+3} + (1/(p+3))a_{22}^{p+3}M$, $\Psi_2 = (2/(p+3))a_{22}{}^{2/(p+3)}\|G_2\|^{p+3}M + (3/(p+3))a_{21}^{-3/(p+3)}|\overline{\varphi}_2|^{(p+3)/3} + (3/(p+3))a_{23}^{3/(p+3)}|f_2|^{(p+3)/3}$, and $f_2 = -(\partial\alpha_2/\partial\widehat{\Theta})c_0\widehat{\Theta} + (\partial\alpha_2/\partial\widehat{\Theta})\eta_1^{p+3}\pi_1 - |\partial\alpha_2/\partial\widehat{\Theta}|\eta_2^{p+3}\pi_2$.

Similar to Step 1, obviously Ψ_2 is an unknown function. Hence there exist the neural networks $W_2^{*T}S_2(Y_2)$ and $Y_2 = [\overline{x_2}, \widehat{\Theta}] \in \Omega_{Y_2} \subset R^3$ such that

$$\Psi_2 = W_2^{*T}S_2(Y_2) + \delta_2(Y_2) \leq \Theta\pi_2(\cdot) \tag{29}$$

where $\pi_2(\cdot) = \sqrt{(l+1)\sum_{k=1}^{l} S_{2k}^2 + 1}$.

We add and subtract $h_2\eta_2^3\alpha_3^p$ in the formula (26), respectively, and, substituting (29) into (26), we can get

$$\begin{aligned} LV_2 \leq{}& \eta_2^3 h_2\left(x_3^p - \alpha_3^p\right) + \eta_2^3\left(f_2 - \frac{\partial\alpha_2}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) \\ &+ \widetilde{\Theta}\left(\eta_1^{p+3}\pi_1 - \dot{\widehat{\Theta}}\right) + h_2\eta_2^p\alpha_3^p - (c_1 - \epsilon_1)\eta_1^{p+3} \\ &+ \gamma_{20}\eta_2^{p+3} + \sum_{i=1}^{2} a_i + \eta_2^{p+3}\Theta\pi_2. \end{aligned} \tag{30}$$

Now we choose the virtual control law $\alpha_3 = ((-c_2 - \widehat{\Theta}\pi_2 - \gamma_{20})/\gamma_{11})^{1/p}\eta_2 = -\beta_2\eta_2$ and $\beta_2 = ((c_2 + \widehat{\Theta}\pi_2 + \gamma_{20})/\gamma_{11})^{1/p} > 0$, where $c_2 > 0$ is a constant to be chosen. Substitution α_3 into (30), (30) can be calculated as

$$\begin{aligned} LV_2 \leq{}& \eta_2^3 h_2\left(x_3^p - \alpha_3^p\right) + \eta_2^3\left(f_2 - \frac{\partial\alpha_2}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) \\ &+ \widetilde{\Theta}\left(\sum_{k=1}^{2}\eta_k^{p+3}\pi_k - \dot{\widehat{\Theta}}\right) - (c_1 - \epsilon_1)\eta_1^{p+3} + \sum_{i=1}^{2} a_i \\ &- c_2\eta_2^{p+3}. \end{aligned} \tag{31}$$

By (11), Lemma 4, assumptions $(A_2) - (A_4)$, and $(a+b)^n = \sum_{i=0}^{n} C_n^k a^k b^{n-k}$, we have

$$\begin{aligned} \eta_2^3 h_2\left(x_3^p - \alpha_3^p\right) &= \eta_2^3 h_2\left\{(\eta_3 + \alpha_3)^p - \alpha_3^p\right\} \\ &= \eta_2^3 h_2 \sum_{k=0}^{p-1} C_p^k \alpha_3^k \eta_3^{p-k} = \eta_2^3 h_2 \sum_{k=0}^{p-1} C_p^k\left(-\eta_2\beta_2\right)^k \eta_3^{p-k} \\ &\leq \sum_{k=0}^{(p-1)/2} C_p^{2k}\left|\eta_2^{3+2k}\right|\left|\eta_3^{p-2k}\right|\beta_2^{2k} \\ &\leq \sum_{k=0}^{(p-1)/2}\frac{3+2k}{p+3}\epsilon_{2k}^{(p+3)/(3+2k)}\eta_2^{3+p} \\ &+ \sum_{k=0}^{(p-1)/2}\frac{p-2k}{p+3}\epsilon_{2k}^{(p-2k)/(p+3)}\left(C_p^{2k}\beta_2^{2k}\gamma_{22}\right)^{(p+3)/(p-2k)} \\ &\cdot \eta_3^{3+p} \leq \epsilon_2\eta_2^{p+3} + \gamma_{30}\eta_2^{p+3} \end{aligned} \tag{32}$$

where $\epsilon_2 \geq \sum_{k=0}^{(p-1)/2}((3+2k)/(p+3))\epsilon_{2k}^{(p+3)/(3+2k)}$, $\gamma_{30} \geq \sum_{k=0}^{(p-1)/2}((p-2k)/(p+3))\epsilon_{2k}^{(p-2k)/(p+3)}(C_p^{2k}\beta_2^{2k}\gamma_{22})^{(p+3)/(p-2k)}$, $\epsilon_{2k} > 0, k = 0, 1, \ldots, (p-1)/2$.

Substituting (32) into (31), we get

$$\begin{aligned} LV_2 \leq{}& \widetilde{\Theta}\left(\sum_{i=1}^{2}\eta_i^{p+3}\pi_i - \dot{\widehat{\Theta}}\right) - \sum_{i=1}^{2}(c_i - \epsilon_i)\eta_i^{p+3} \\ &+ \gamma_{30}\eta_3^{p+3} + \eta_2^3\left(f_2 - \frac{\partial\alpha_2}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) + \sum_{i=1}^{2} a_i. \end{aligned} \tag{33}$$

Remark 7. Since $\dot{\widehat{\Theta}}$ contains η_1, η_2, the term $(\partial\alpha_2/\partial\Theta)\dot{\widehat{\Theta}}$ cannot be used directly to design the virtual control law α_2. And the function f_2 will be used to compensate for $(\partial\alpha_2/\partial\Theta)\dot{\widehat{\Theta}}$. As a result, the term $\eta_2^3(f_2 - (\partial\alpha_2/\partial\Theta)\dot{\widehat{\Theta}})$ will be considered in the later section.

Step i $(3 \leq i \leq n)$. We choose the following Lyapunov function $V_i = V_{i-1} + (1/4)\eta_i^4$. From (3) and (12), we have

$$\begin{aligned} LV_i ={}& LV_{i-1} + \eta_i^3 h_i x_{i+1}^p + \eta_i^3\overline{\varphi}_i \\ &+ \frac{3}{2}\eta_i^2 Tr\left(G_i^T \sum^T(t)\sum(t)G_i\right) + \eta_i^3\frac{\partial\alpha_i}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}} \end{aligned} \tag{34}$$

where $V_{i-1} = (1/4)\sum_{j=1}^{i-1}\eta_j^4 + (1/2)\widetilde{\Theta}^2$, and

$$\begin{aligned} LV_{i-1} \leq{}& -\sum_{j=1}^{i-1}(c_j - \epsilon_j)\eta_j^{p+3} + \widetilde{\Theta}\left(\sum_{j=1}^{i-1}\eta_j^{p+3}\pi_j - \dot{\widehat{\Theta}}\right) \\ &+ \sum_{j=2}^{i-1}\eta_j^3\left(f_j - \frac{\partial\alpha_j}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) + \gamma_{io}\eta_i^{p+3} + \sum_{j=1}^{i-1} a_j \end{aligned} \tag{35}$$

where

$$f_j = -\frac{\partial\alpha_j}{\partial\widehat{\Theta}}c_0\widehat{\Theta} + \frac{\partial\alpha_j}{\partial\widehat{\Theta}}\sum_{k=1}^{i-1}\eta_k^{p+3}\pi_k - \eta_j^p\pi_j\sum_{l=2}^{j}\left|\frac{\partial\alpha_l}{\partial\widehat{\Theta}}\eta_l^3\right|. \tag{36}$$

Similar to the Steps 1 and 2, from the definition of $\overline{\varphi}_i$, Lemma 4, and assumptions $(A_2) - (A_4)$, there exist constants $a_{ij} > 0$, $j = 1, 2$, such that

$$\eta_i^3\overline{\varphi}_i \leq \eta_i^{p+3}\frac{3}{p+3}a_{i1}^{3/(p+3)}\left|\overline{\varphi}_i\right|^{(p+3)/3} + \frac{1}{p+3}a_{i1}^{p+3} \tag{37}$$

and

$$\begin{aligned} &\frac{3}{2}\eta_i^2 Tr\left(G_i{}^T\sum^T(t)\sum(t)G_i\right) \\ &= \frac{3}{2}\eta_i^2\left|G_i\right|^2\left\|\sum^T(t)\sum(t)\right\|_F^2 \\ &\leq \eta_i^{p+3}\frac{2}{p+3}a_{i2}{}^{2/(p+3)}\left\|G_i\right\|^{p+3}M + \frac{1}{p+3}a_{i2}^{p+3}M. \end{aligned} \tag{38}$$

Substituting LV_{i-1}, (37), and (38) into (34), we can get

$$\begin{aligned} LV_i \le{}& \eta_i^3\left(h_i x_{i+1}^p - \frac{\partial\alpha_i}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) + \widetilde{\Theta}\left(\sum_{j=2}^{i-1}\eta_j^{p+3}\pi_j - \dot{\widehat{\Theta}}\right) \\ &- \sum_{j=1}^{i-1}\left(c_j - \epsilon_j\right)\eta_j^{p+3} + \gamma_{i0}\eta_i^{p+3} + \sum_{j=1}^{i-1}a_j + \eta_i^{p+3} \\ &\cdot\left\{\frac{2}{p+3}a_{i2}^{\ 2/(p+3)}\left\|G_i\right\|^{p+3}M\right. \\ &\left.+\frac{3}{(p+3)}a_{i1}^{3/(p+3)}\left|\overline{\varphi}_i\right|^{(p+3)/3}\right\} + a_i \\ &+ \sum_{j=2}^{i-1}\eta_j^3\left(f_j - \frac{\partial\alpha_j}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) + \frac{1}{p+3}a_{i1}^{p+3} + \frac{1}{p+3} \\ &\cdot a_{i2}^{p+3}M. \end{aligned} \tag{39}$$

Further, add and subtract $\eta_i^3 f_i$ in the first bracket in (39), and, using the Lemma 4, there exists a constant $a_{i3} > 0$, such that

$$-\eta_i^3 f_i \le \eta_i^{p+3}\frac{3}{p+3}a_{i3}^{3/(p+3)}\left|f_i\right|^{(p+3)/3} + \frac{1}{p+3}a_{i3}^{p+3} \tag{40}$$

Then, substituting (40) into (39), we get

$$\begin{aligned} LV_i \le{}& \eta_i^3\left(h_i x_{i+1}^p - \frac{\partial\alpha_2}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}} + f_i\right) + \widetilde{\Theta}\left(\sum_{j=2}^{i-1}\eta_j^{p+3}\pi_j\right. \\ &\left.-\dot{\widehat{\Theta}}\right) - \sum_{j=1}^{i-1}\left(c_j - \epsilon_j\right)\eta_j^{p+3} + \gamma_{i0}\eta_i^{p+3} + \sum_{j=1}^{i-1}a_j \\ &+ \eta_i^{p+3}\left\{\frac{2}{p+3}a_{i2}^{\ 2/(p+3)}\left\|G_i\right\|^{p+3}M\right. \\ &\left.+\frac{3}{(p+3)}a_{i1}^{3/(p+3)}\left|\overline{\varphi}_i\right|^{(p+3)/3}\right\} + a_i \\ &+ \sum_{j=2}^{i-1}\eta_j^3\left(f_j - \frac{\partial\alpha_j}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) + \frac{1}{p+3}a_{i1}^{p+3} + \frac{1}{p+3}a_{i2}^{p+3}M \\ &+ \eta_i^{p+3}\frac{3}{p+3}a_{i3}^{3/(p+3)}\left|f_i\right|^{(p+3)/3} + \frac{1}{p+3}a_{i3}^{p+3} \\ ={}& \eta_i^3 h_i x_{i+1}^p + \widetilde{\Theta}\left(\sum_{j=2}^{i-1}\eta_j^{p+3}\pi_j - \dot{\widehat{\Theta}}\right) \\ &- \sum_{j=1}^{i-1}\left(c_j - \epsilon_j\right)\eta_j^{p+3} + \gamma_{i0}\eta_i^{p+3} + \sum_{j=1}^{i}a_j + \eta_i^{p+3}\Psi_i \\ &+ \sum_{j=2}^{i}\eta_j^3\left(f_j - \frac{\partial\alpha_j}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right) + \gamma_{i0}\eta_i^{p+3}. \end{aligned} \tag{41}$$

where

$$\begin{aligned} a_i &= \frac{1}{p+3}a_{i1}^{p+3} + \frac{1}{p+3}a_{i2}^{p+3}M + \frac{1}{p+3}a_{i3}^{p+3}, \\ \Psi_i &= \frac{2}{p+3}a_{i2}^{\ 2/(p+3)}\left\|G_i\right\|^{p+3}M \\ &\quad + \frac{3}{(p+3)}a_{i1}^{3/(p+3)}\left|\overline{\varphi}_i\right|^{(p+3)/3} \\ &\quad + \frac{3}{p+3}a_{i3}^{3/(p+3)}\left|f_i\right|^{(p+3)/3} \end{aligned} \tag{42}$$

and

$$f_i = -\frac{\partial\alpha_i}{\partial\widehat{\Theta}}c_0\widehat{\Theta} + \frac{\partial\alpha_i}{\partial\widehat{\Theta}}\sum_{j=1}^{i-1}\eta_j^{p+3}\pi_j - \eta_i^p\pi_i\sum_{j=2}^{i}\left|\frac{\partial\alpha_j}{\partial\widehat{\Theta}}\eta_j^3\right|. \tag{43}$$

Since the unknown functions φ_i and g_i can be derived by Ψ_i, it cannot be used to design the control law directly. Thus there exist the neural networks $W_i^{*T}S_i(Y_i)$ and $Y_i = [\overline{x_i}, \widehat{\Theta}] \in \Omega_{Y_i} \subset R^{i+1}$ such that

$$\Psi_i = W_i^{*T}S_i\left(Y_i\right) + \delta_i\left(Y_i\right) \le \Theta\pi_i\left(\cdot\right) \tag{44}$$

where $\pi_i(\cdot) = \sqrt{(l+1)\sum_{k=1}^{l}S_{ik}^2 + 1}$, $f_i = -(\partial\alpha_i/\partial\widehat{\Theta})c_0\widehat{\Theta} + (\partial\alpha_i/\partial\widehat{\Theta})\sum_{j=1}^{i-1}\eta_j^{p+3}\pi_j - \eta_i^p\pi_i\sum_{j=2}^{i}|(\partial\alpha_j/\partial\widehat{\Theta})\eta_j^3|$.

We add and subtract $h_i\eta_i^3\alpha_{i+1}^p$ in the formula (39), respectively, and, substituting (44) into (39), it has

$$\begin{aligned} LV_i \le{}& \eta_i^3 h_i\left(x_{i+1}^p - \alpha_{i+1}^p\right) + \eta_i^3 h_i\alpha_{i+1}^p \\ &+ \widetilde{\Theta}\left(\sum_{j=2}^{i-1}\eta_j^{p+3}\pi_j - \dot{\widehat{\Theta}}\right) - \sum_{j=1}^{i-1}\left(c_j - \epsilon_j\right)\eta_j^{p+3} \\ &+ \gamma_{i0}\eta_i^{p+3} + \sum_{i=1}^{i}a_j + \eta_i^{p+3}\pi_i\Theta \\ &+ \sum_{j=2}^{i}\eta_j^3\left(f_j - \frac{\partial\alpha_{j-1}}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right). \end{aligned} \tag{45}$$

Choose the virtual control law

$$\begin{aligned} \alpha_{i+1} &= \left(\frac{-c_i - \widehat{\Theta}\pi_i - \gamma_{i0}}{\gamma_{i1}}\right)^{1/p}\eta_i = -\beta_i\eta_i, \\ \beta_i &= \left(\frac{c_i + \widehat{\Theta}\pi_i + \gamma_{i0}}{\gamma_{i1}}\right)^{1/p} > 0, \end{aligned} \tag{46}$$

where $c_i > 0$ is a constant to be chosen. Then, by substituting α_{i+1} into (45) and using (11), Lemma 5, and $(a+b)^n = \sum_{i=0}^{n}C_n^k a^k b^{n-k}$, one yields

$$\begin{aligned} \eta_i^3 h_i\left(x_{i+1}^p - \alpha_{i+1}^p\right) &= \eta_i^3 h_i\left\{\left(\eta_{i+1} + \alpha_{i+1}\right)^p - \alpha_{i+1}^p\right\} \\ &= \eta_i^3 h_i\sum_{k=0}^{p-1}C_p^k\alpha_{i+1}^k\eta_{i+1}^{p-k} = \eta_i^3 h_i\sum_{k=0}^{p-1}C_p^k\left(-\eta_i\beta_i\right)^k\eta_{i+1}^{p-k} \end{aligned}$$

$$\le \sum_{k=0}^{(p-1)/2} C_p^{2k} \left|\eta_i^{3+2k}\right| \left|\eta_{i+1}^{p-2k}\right| \beta_i^{2k}$$
$$\le \sum_{k=0}^{(p-1)/2} \frac{3+2k}{p+3} \epsilon_{ik}^{(p+3)/(3+2k)} \eta_i^{3+p}$$
$$+ \sum_{k=0}^{(p-1)/2} \frac{p-2k}{p+3} \epsilon_{ik}^{(p-2k)/(p+3)} \left(C_p^{2k} \beta_i^{2k} \gamma_{i2}\right)^{(p+3)/(p-2k)}$$
$$\cdot \eta_{i+1}^{3+p} \le \epsilon_i \eta_i^{p+3} + \gamma_{i+10} \eta_{i+1}^{p+3} \tag{47}$$

where $\epsilon_i \ge \sum_{k=0}^{(p-1)/2}((3+2k)/(p+3))\epsilon_{ik}^{(p+3)/(3+2k)}$, $\gamma_{i+10} \ge \sum_{k=0}^{(p-1)/2}((p-2k)/(p+3))\epsilon_{ik}^{(p-2k)/(p+3)}(C_p^{2k}\beta_i^{2k}\gamma_{i2})^{(p+3)/(p-2k)}$, $\epsilon_{ik} > 0, k = 0, 1, \ldots, (p-1)/2$.

In the view of (47), (45) can be calculated as

$$LV_i \le \widetilde{\Theta}\left(\sum_{j=2}^{i} \eta_j^{p+3} \pi_j - \dot{\widehat{\Theta}}\right) - \sum_{j=1}^{i} \left(c_j - \epsilon_j\right) \eta_j^{p+3}$$
$$+ \gamma_{i+10} \eta_{i+1}^{p+3} + \sum_{j=1}^{i} a_j + \sum_{j=2}^{i} \eta_j^3 \left(f_j - \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \dot{\widehat{\Theta}}\right). \tag{48}$$

Remark 8. Since $\dot{\widehat{\Theta}}$ contains η_j, $j = 1, 2, \ldots, i$, the term $(\partial \alpha_j/\partial \Theta)\dot{\widehat{\Theta}}$ cannot be used directly to design the virtual control law α_i. And the function f_j will be used to compensate for $(\partial \alpha_j/\partial \Theta)\dot{\widehat{\Theta}}$. As a result, the term $\eta_j^3(f_j - (\partial \alpha_j/\partial \Theta)\dot{\widehat{\Theta}})$ will be considered in the later section.

Finally, when $i = n$, $\eta_{n+1} = u$ is the actual control. Choose the actual controller

$$u = -\eta_n \beta_n,$$
$$\beta_n = \left(\frac{c_n + \gamma_{n0} + \pi_n \widehat{\Theta}}{\gamma_{n1}}\right)^{1/p} > 0 \tag{49}$$

where $c_n > 0$ is a design parameter to be chosen. It can be deduced that

$$LV_n \le -\sum_{j=1}^{n} \left(c_j - \varepsilon_j\right) \eta_j^{p+3} + \sum_{j=1}^{n} a_j$$
$$+ \widetilde{\Theta}\left(\sum_{j=2}^{n} \eta_j^{p+3} \pi_j - \dot{\widehat{\Theta}}\right) + \sum_{j=2}^{n} \eta_j^3 \left(f_j - \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \dot{\widehat{\Theta}}\right) \tag{50}$$

where $V_n(\eta, \widehat{\Theta}) = \sum_{k=1}^{n}(1/4)\eta_k^4 + (1/2)(\widehat{\Theta} - \Theta)^2$, $\eta = (\eta_1, \cdots \eta_n)$. We finish the controller design procedure.

3.2. Analysis of Stability. We have obtained the main result in the following theorem.

Theorem 9. *Suppose that assumptions $(A_1) - (A_4)$ hold for the stochastic nonlinear system (1). Furthermore, suppose that the unknown functions Ψ_i $(1 \le i \le n)$ can be approximated by the RBF neural networks. Given a control law with the virtual control signals α_i, it is constructed in (49), and the adaptive law satisfies*

$$\dot{\widehat{\Theta}} = \sum_{k=1}^{n} \eta_k^{p+3} \pi_k - c_0 \widehat{\Theta} \tag{51}$$

where the design parameter $c_0 > 0$. Then the signals of the closed-loop system are bounded in probability.

Proof. Choose the Lyapunov function $V = V_n$ such that

$$LV \le -\sum_{j=1}^{n} \left(c_j - \varepsilon_j\right) \eta_j^{p+3} + \sum_{j=1}^{n} a_j$$
$$+ \widetilde{\Theta}\left(\sum_{j=2}^{n} \eta_j^{p+3} \pi_j - \dot{\widehat{\Theta}}\right) + \sum_{j=2}^{n} \eta_j^3 \left(f_j - \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \dot{\widehat{\Theta}}\right). \tag{52}$$

Substituting the adaptive law (51) into the penultimate term in (52) results in

$$LV \le -\sum_{j=1}^{n} \left(c_j - \varepsilon_j\right) \eta_j^{p+3} + \sum_{j=1}^{n} a_j + c_0 \widetilde{\Theta} \widehat{\Theta}$$
$$+ \sum_{j=2}^{n} \eta_j^3 \left(f_j - \frac{\partial \alpha_j}{\partial \Theta} \dot{\widehat{\Theta}}\right). \tag{53}$$

In the following, we will prove that the last term $\sum_{j=2}^{n} \eta_j^3(f_j - (\partial \alpha_j/\partial \widehat{\Theta})\dot{\widehat{\Theta}})$ in (53) is negative. It is clear that

$$-\sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \Theta} \dot{\widehat{\Theta}} = \sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \Theta} c_0 \widehat{\Theta} - \sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \sum_{i=1}^{n} \eta_i^{p+3} \pi_i$$
$$= \sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \widehat{\Theta}} c_0 \widehat{\Theta} - \sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \sum_{i=1}^{n} \eta_i^{p+3} \pi_i \tag{54}$$

and

$$-\sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \sum_{i=1}^{n} \eta_i^{p+3} \pi_i = -\sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \sum_{i=1}^{j-1} \eta_i^{p+3} \pi_i$$
$$- \sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \sum_{i=j}^{n} \eta_i^{p+3} \pi_i$$
$$\le -\sum_{j=2}^{n} \eta_j^3 \frac{\partial \alpha_j}{\partial \widehat{\Theta}} \sum_{i=1}^{j-1} \eta_i^{p+3} \pi_i$$
$$+ \sum_{j=2}^{n} \eta_j^{p+3} \pi_j \sum_{i=2}^{j} \left|\eta_i^3 \frac{\partial \alpha_i}{\partial \widehat{\Theta}}\right|. \tag{55}$$

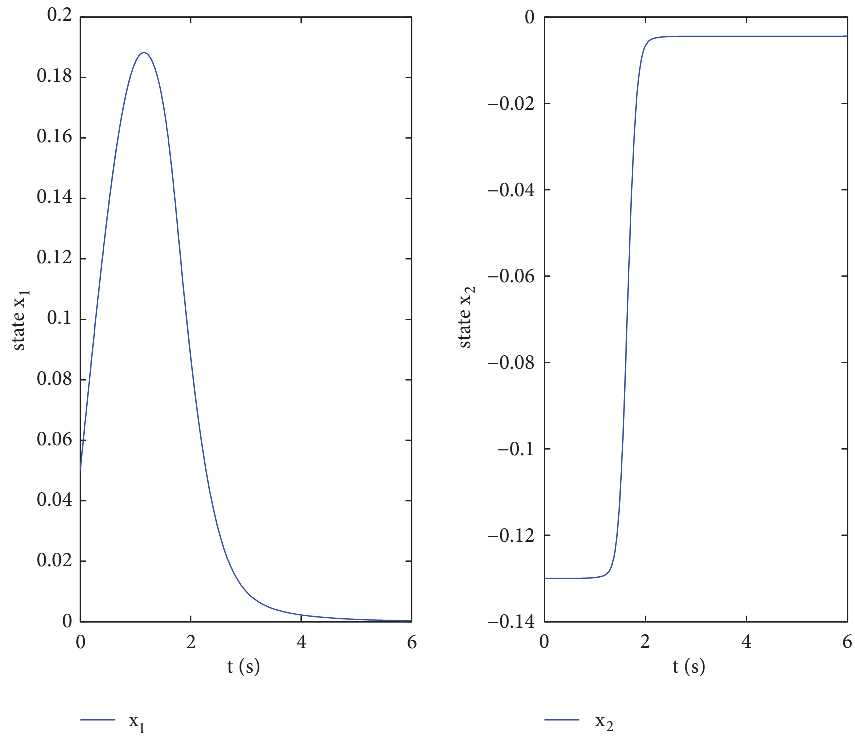

FIGURE 1: The responses of states x_1 and x_2 relative to time.

Substituting (55) into (54), one arrives at

$$\begin{aligned}
-\sum_{j=2}^{n}\eta_j^3\frac{\partial\alpha_j}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}} &= \sum_{j=2}^{n}\eta_j^3\frac{\partial\alpha_j}{\partial\widehat{\Theta}}\left(c_0\widehat{\Theta}-\sum_{i=1}^{n}\eta_i^{p+3}\pi_i\right) \\
&\le \sum_{j=2}^{n}\eta_j^3\frac{\partial\alpha_j}{\partial\widehat{\Theta}}c_0\widehat{\Theta}-\sum_{j=2}^{n}\eta_j^3\frac{\partial\alpha_j}{\partial\widehat{\Theta}}\sum_{i=1}^{j-1}\eta_i^{p+3}\pi_i+\sum_{j=2}^{n}\eta_j^{p+3}\pi_j \\
&\cdot\sum_{i=2}^{j}\left|\eta_i^3\frac{\partial\alpha_i}{\partial\widehat{\Theta}}\right| = -\sum_{j=2}^{n}\eta_j^3 \\
&\cdot\left\{-\frac{\partial\alpha_j}{\partial\widehat{\Theta}}c_0\widehat{\Theta}+\frac{\partial\alpha_j}{\partial\widehat{\Theta}}\sum_{i=1}^{j-1}\eta_i^{p+3}\pi_i-\eta_j^{p}\pi_j\sum_{i=2}^{j}\left|\eta_i^3\frac{\partial\alpha_i}{\partial\widehat{\Theta}}\right|\right\} \\
&= -\sum_{j=2}^{n}\eta_j^3 f_j.
\end{aligned} \tag{56}$$

By the definition of f_i, which implies that

$$\sum_{j=2}^{n}\eta_j^3\left(f_j-\frac{\partial\alpha_j}{\partial\widehat{\Theta}}\dot{\widehat{\Theta}}\right)\le 0 \tag{57}$$

and

$$c_0\widetilde{\Theta}\widehat{\Theta} = c_0\widetilde{\Theta}\left(\Theta-\widetilde{\Theta}\right) = c_0\widetilde{\Theta}\Theta - c_0\widetilde{\Theta}^2 \le \frac{c_0}{2}\Theta^2-\frac{c_0}{2}\widetilde{\Theta}^2 \tag{58}$$

substituting (57) and (58) into (53) yields

$$LV \le -\sum_{j=1}^{n}\left(c_j-\varepsilon_j\right)\eta_j^{p+3}+\sum_{j=1}^{n}a_j+\frac{c_0}{2}\Theta^2-\frac{c_0}{2}\widetilde{\Theta}^2. \tag{59}$$

Furthermore, let $a_0 = \min\{4(c_j-\epsilon_j), c_0\}$, $j = 1,2,\cdots n$, and $b_0 = \sum_{j=1}^{n} a_j + (c_0/2)\Theta^2$, and it follows that

$$LV \le -a_0V+b_0, \quad t\ge 0. \tag{60}$$

Therefore, according to Lemma 5, η_i, $i = 1,2,\ldots,n$, and $\widetilde{\Theta}$ are bounded in probability. Since Θ is a constant, $\widehat{\Theta}$ is bounded in probability. It can be obtained that the control law α_j, $j = 1,2,\ldots,n$, is also bounded in probability because α_j is the function of η_j and $\widehat{\Theta}$. So far we get that all the states of the closed-loop system (1) are bounded in probability. □

4. Simulation Example

In this section, we will give an example to show the effectiveness of the proposed control method in this paper.

Example 1. Consider the following stochastic nonlinear system:

$$\begin{aligned}
dx_1 &= \left(1+\frac{1}{2}\sin t\right)x_2^3dt + x_1^3\sin x_1tdt \\
&\quad + 3x_1\sin x_1\left(1+\frac{1}{4}\sin t\right)d\omega \\
dx_2 &= \left(1-\frac{1}{4}\sin t\right)u^3dt.
\end{aligned} \tag{61}$$

Obviously, the system satisfies $(A_1)-(A_4)$ and $p_1 = p_2 = 3 > 1$. Now according to Theorem 9, the virtual control

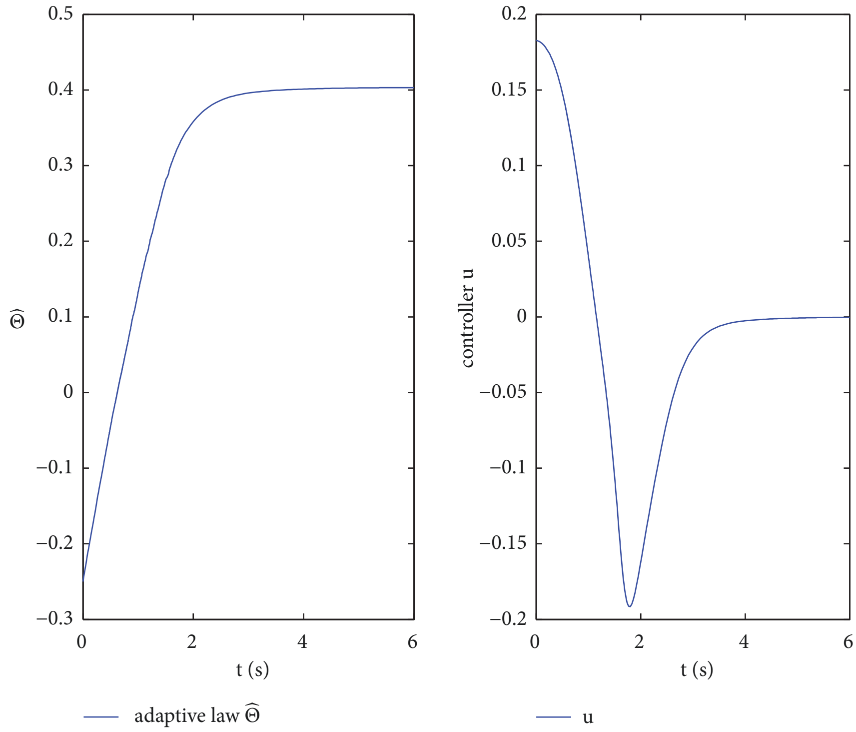

Figure 2: The responses of the adaptive law $\widehat{\Theta}$ and the controller u relative to time.

function α_2, the control law u, and the adaptive law $\dot{\widehat{\Theta}}$ are chosen, respectively, as

$$\begin{aligned} \alpha_2 &= -\eta_1\beta_1 = -\eta_1\left(c_1 + \widehat{\Theta}\pi_1\right)^{1/3}, \\ u &= -\eta_2\beta_2 = -\eta_2\left(c_2 + \widehat{\Theta}\pi_2 + \gamma_{20}\right)^{1/3}, \\ \dot{\widehat{\Theta}} &= \sum_{i=1}^{2}\eta_i^6\pi_i - c_0\widehat{\Theta}, \end{aligned} \tag{62}$$

where $\eta_1 = x_1$, $\eta_2 = x_2 - \alpha_2$, $Y_1 = \eta_1$, $Y_2 = [\eta_1, \eta_2, \widehat{\Theta}]^T$. In the simulation, neural network $W_1^{*T}S_1(Y_1)$ includes 7 nodes with centers spaced evenly in $[-3, -3]$, neural network $W_2^{*T}S_2(Y_2)$ includes 81 nodes with centers spaced evenly in $[-3, -3] \times [-3, -3] \times [0, 3]$, and all the widths are chosen as 2. The design parameters are chosen as $c_1 = 2$, $c_2 = 0.5$, $c_0 = 0.1$. $\gamma_{20} = \sum_{k=0}^{1}((3-2k)/4)\epsilon_{1k}^{(3-2k)/4}(c_3^{2k}\beta_1^{2k})^{4/(3-2k)}$, and $\epsilon_{10} = \epsilon_{11} = 1/4$. The initial condition $[x_1(0), x_2(0), \widehat{\Theta}(0)] = [0.05, -0.13, -0.25]$. Figures 1 and 2 show the simulation results. From the figures, we can see that the proposed adaptive control method can guarantee that all the variables for the closed-loop system are bounded.

5. Conclusions

This paper has investigated the adaptive neural control for a class of high-order uncertain stochastic nonlinear systems. With the help of backstepping technique and separation technique, a smooth adaptive controller is constructed, and it ensures the closed-loop system is the global bounded in probability. Only one adaptive learning parameter needed to be updated online. One example has been given to show the effectiveness of the proposed analytical results. A further work is how to design the output-feedback tracking control for more high-order stochastic systems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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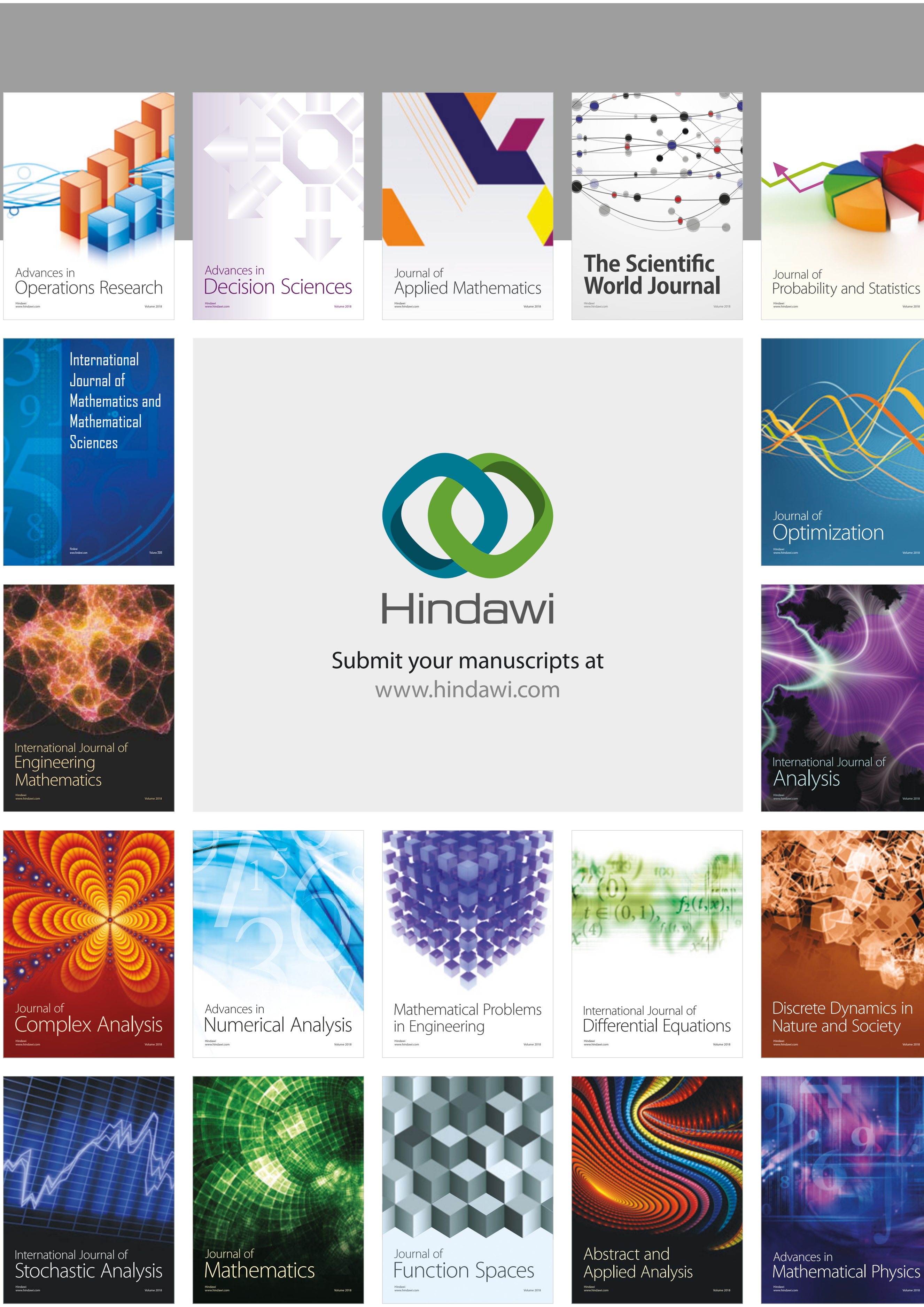
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