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Research Article

A Mathematical Method for Determining Optimal Quantity of Backfill Materials Used for Grounding Resistance Reduction

Jovan Trifunovic

Faculty of Electrical Engineering, University of Belgrade, Bulevar Kralja Aleksandra 73, 11000 Belgrade, Serbia

Correspondence should be addressed to Jovan Trifunovic; jovan.trifunovic@etf.rs

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During installation of grounding system, which represents a significant part of any electrical power system, various backfill materials are used for grounding resistance reduction. The general mathematical method for determining an optimal quantity of backfill materials used for grounding resistance reduction, based on the mathematical tools, 3D FEM modeling, numerical analysis of the obtained results, and the "knee" of the curve concept, as well as on the engineering analysis based on the designer's experience, is developed and offered in this paper. The proposed method has been tested by applying it to a square loop enveloped by a backfill material and buried in a 2-layer soil. The results obtained by the presented method showed a good correlation with the experimentally obtained data from literature. The proposed method can help the designers to avoid the saturation areas in order to maximize efficiency of backfill material usage.

1. Introduction

A proper design of grounding systems is essential to assure the safety of the persons and avoid interruptions of power supply, as well as to protect the electrical and electronic equipment [1]. In order to meet the electrical safety standards, grounding resistance of a grounding system must be lower than the demanded values (in further text denoted as R_{dem}), which can vary from $10\,\Omega$ for lightning protection [2] to below $0.1\,\Omega$ for sites where protective devices must operate very quickly [3]. This is not always easy to obtain, especially in troubled environments (high soil resistivity and/or soil which forms a poor contact with grounding system electrodes) [4]. In such cases grounding system resistance can be decreased by increasing the number of rods or the electrode length or by using appropriate backfill materials. The latter solution, although generally not suitable for large grounding systems, in some cases can be efficient for electrodes covering small areas. Various backfill materials are used in practice to eliminate the contact resistance component of grounding resistance, as well as to reduce the grounding resistance to R_{dem} (usage of bentonite was analyzed in [5, 6]; usage of coconut coir peat, planting-clay soil, and paddy dust was analyzed in [7, 8]; usage of granulated blast furnace slag and fly ash was analyzed in [9] and [10], respectively; usage of waste drilling mud was analyzed in [11]; general analysis of backfill material usage was performed in [12, 13]).

The analysis conducted in [14] indicated that in soils which form a poor contact with the grounding system electrodes the value of grounding resistance could be significantly decreased using a backfill material which is characterized either by ability to provide excellent contact with grounding electrodes or by low resistivity or by proper combination of those 2 features. It means that in such terrains the whole contact resistance component (which is not taken into account by standard engineering methods and formulas, e.g., given in [15, 16]) can be eliminated, and the values of grounding resistance R could be decreased to those which are computed by standard engineering methods and formulas (in further text, for convenience, denoted as the baseline values R_0) using relatively small quantities of a backfill material characterized by ability to achieve perfect contact with both the grounding electrodes and the surrounding soil. In that case the whole contact resistance component would be eliminated even if the backfill material is characterized by the same high resistivity as the surrounding soil [14]. According to findings presented in [17] (based on experimental investigations), the sufficient amount of a backfill material characterized by ability to achieve perfect contact with both the grounding electrodes and the surrounding soil, such as bentonite, which should provide the successful elimination of the whole contact resistance component, is $0.02\,\mathrm{m}^3$ per 1 m of grounding strip ($V'_{\min}$).

However, if the used backfill material is characterized by a lower resistivity than the surrounding soil, the additional decrease of grounding resistance can be achieved. The grounding resistance R of a grounding system decreases with increasing quantity of the used backfill material. Nevertheless, this grounding resistance reduction effect will display the saturation phenomenon when the quantity of the used backfill material increases to a certain level [18]. The best understanding of this saturation phenomenon can be obtained analyzing the effect of the volume V of used backfill material, as well as the effect of its resistivity ρ_{bf}, on the grounding resistance reduction rate δR (%), defined by the following expression:

$$\delta R\ \ (\%) = \frac{R_0 - R}{R_0} \cdot 100. \tag{1}$$

Optimization methodologies, based on mathematical tools and computer-aided design, are required for minimizing investment costs of power systems, parts of which are power transmission lines [19] and their grounding systems [20]. Hence, such optimization methodologies for determining theoretical maximum efficiency of backfill material usage for grounding rods are offered in [13, 18, 21]. Being based on grounding resistance calculation techniques which are suitable only for solving 2D problems, the application of those optimization methodologies is limited only to grounding systems with geometries characterized by rotational symmetries (e.g., grounding rods surrounded by a cylinder of backfill material), buried in uniform soil, which can be reduced to 2D problems. They cannot be applied to grounding systems characterized by more complex geometries, buried in nonuniform soils, for example, square loop buried in a 2-layer soil (Figure 1) and enveloped by a backfill material (Figure 2).

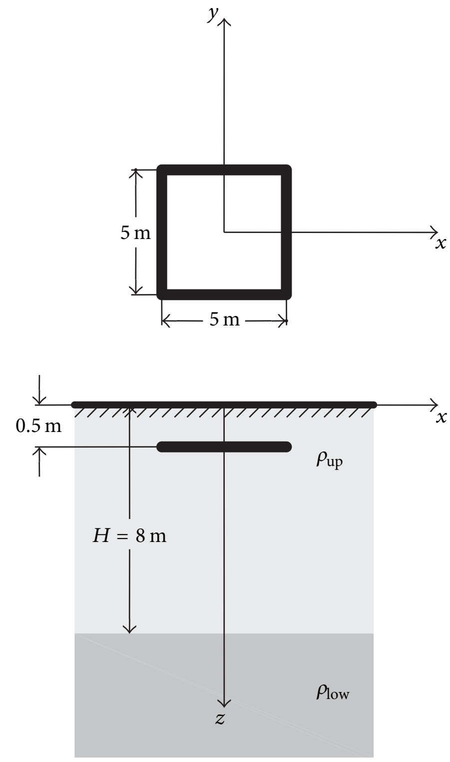

Figure 1: The considered grounding loop installed in a 2-layer soil.

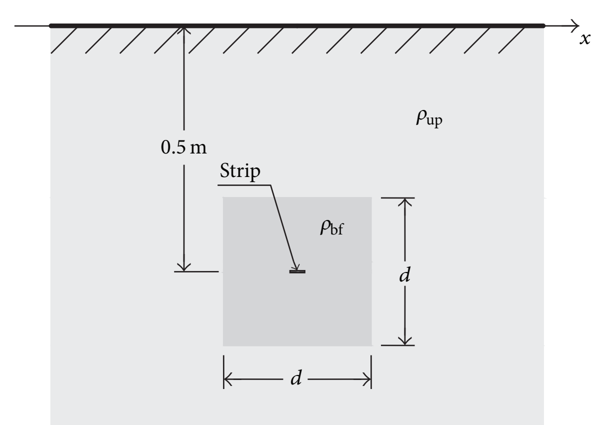

Figure 2: Cross-section showing grounding strip of loop surrounded by backfill material.

For such complex geometries, the general mathematical method for determining a backfill material optimal volume, based on 3D FEM modeling, numerical analysis of the obtained results, and the "knee" of the curve concept, as well as on the engineering analysis based on the designer's experience, is developed and offered in this paper. The proposed method is suitable only for cases in which using backfill materials has advantages (in terms of efficiency and cost) in relation to simply increasing the number of rods or the electrode length.

The "knee" of the curve concept was adopted following examples from engineers working in different areas of system design which use the "knee" of the curve (i.e., of the graph of a continuous function which is relevant for the system behavior), representing the border between the saturated and unsaturated area of a curve, in their optimization methodologies. General system design analyses with "knee" concept were given in [22, 23], and usage of "knee" concept in information technologies was explained in [24, 25], in chemical engineering in [26], and in projectile design in [27].

The experimental setup and the results of measurements of the 2 identical square grounding loops buried in a 2-layer soil (the one backfilled with bentonite suspension and the other conventional), presented in [11], were analyzed using the proposed method. Being based on 3D FEM modeling, the proposed method is suitable for any kind of multilayer soil, as well.

2. The Experimental Setup and the Results of Measurements

As reported in [11], the 2 identical square loops were installed in a former stone bed. The site was described by a 2-layer soil ($\rho_{up} = 170\,\Omega$m, $\rho_{low} = 75\,\Omega$m, and $H = 8$ m (Figure 1)). The upper soil layer was made of stones (karst terrain). The dimensions of the loops (5 m × 5 m) belong to the range of the dimensions of grounding loops which are frequently used as parts of grounding systems of 35 kV transmission line towers and 10 kV/0.4 kV transformer stations. The loops made of rectangular cross-section (30 mm × 4 mm) zinc protected steel strips were installed at a depth of 0.5 m. It was proven in [14] (by 3D FEM modeling of both loops) that the input data and the results of measurements from this experimental setup, reported in [11], were obtained with a reasonable accuracy.

The backfill material of the first loop channel was the excavated material. The grounding resistance of the loop was measured to be $R = 50.2\,\Omega$, and the calculated resistance amounted to $R = 14.6\,\Omega$ [11]. It was suggested in [11] and shown by 3D FEM modeling in [28, 29] that the huge difference between the measured and calculated grounding resistances in this particular case was caused by reduced contact surface between the grounding electrodes and the surrounding soil (i.e., very high contact resistance component), which was not taken into consideration by the applied calculation formula in [11]. Due to the type of soil (stones, karst terrain), it was impossible to achieve good contact between electrodes and soil by compacting the soil above the electrode, which is the common practice for avoiding such very high contact resistance component. It is also possible that when electrodes are subjected to impressed currents by faults or other occurrences (for instance, lightning-related currents), the high values of the associate electric field at the electrode surface naturally would promote a good contact of this surface with the surrounding soil.

The second loop channel was backfilled with 1.2 m^3 of bentonite suspension (0.06 m^3 of bentonite per 1 m of the grounding strip). Resistivity of this backfill material was $\rho_{bf} = \rho_{bentonite} = 2.5\,\Omega$m. The grounding resistance of this loop was measured to be $R = 12.5\,\Omega$ [11]. It is obvious that by the use of bentonite not only is the whole contact resistance component eliminated, but the additional decrease of grounding resistance is achieved.

3. The "Knee" of the Curve Concept

In cases of grounding systems with backfill materials, if the volume of used backfill material corresponds to the values from the saturated area of the relevant curve $\delta R(V)$, it is likely that this volume is oversized, and therefore, material and human or machine efforts may be wasted and investment increased without a justified reason. Therefore, the values of the "knee" point coordinates, V_{knee} and δR_{knee}, practically can be considered as the maximum volume of backfill material, V_{max}, that should be used and the maximum grounding resistance reduction rate, δR_{max}, which realistically can be achieved with the use of the considered backfill material for the considered grounding system at the considered installation site.

On the contrary, if the volume of used backfill material corresponds to the values from the unsaturated area of the relevant curve, it is likely that this volume is undersized, and hence, the opportunity to additionally reduce the grounding resistance of a grounding system with relatively small additional investment, using volume V_{dem} of backfill material ($V_{dem} \leq V_{max}$), sufficient to achieve $R \leq R_{dem}$, could unjustifiably be wasted.

However, it is not always easy to determine the "knee" point of a curve. It should not be read from the graphic because an "optical illusion," produced by using different aspect ratios in the V and δR axes, can occur, misleading an engineer and giving him a false "knee" point data. Therefore, a mathematical approach for finding coordinates of a "knee" point is adopted. It is based on differential calculus and the mathematical definition of curvature for continuous functions, considering the formal definition of a "knee" for continuous functions given in [22], where it is defined as a point of maximum curvature of a curve. The point of the maximum curvature of a curve corresponds to the point of the minimum radius of curvature. The radius of curvature, r, of a graph of a continuous function (curve) at a point is the length of the radius of the circular arc which represents the best approximation of the curve at that point. For any continuous function $y(x)$, if given in Cartesian coordinates and assuming that it is differentiable up to the second order, the radius of curvature at an arbitrary point of its graph can be determined using the following expression [30]:

$$r = \frac{\left(1 + (dy/dx)^2\right)^{3/2}}{\left|d^2 y/dx^2\right|}. \tag{2}$$

The "knee" point of a curve, that is, the point characterized by the minimum radius of curvature, can be determined using the equation

$$\frac{dr}{dx} = 0, \tag{3}$$

and satisfying the condition

$$\frac{d^2 r}{dx^2} > 0. \tag{4}$$

While the radius of curvature is well defined for continuous functions, it is not well defined for discrete data sets. Note that in the considered case only the discrete data sets, several pairs of values $(V, \delta R)$ for each value of ρ_{bf}, can be obtained by 3D FEM calculations. In the discrete case, the radius of curvature and the "knee" point of a curve can be determined by fitting a suitable continuous function to the available data, followed by the application of (2)–(4) to that function.

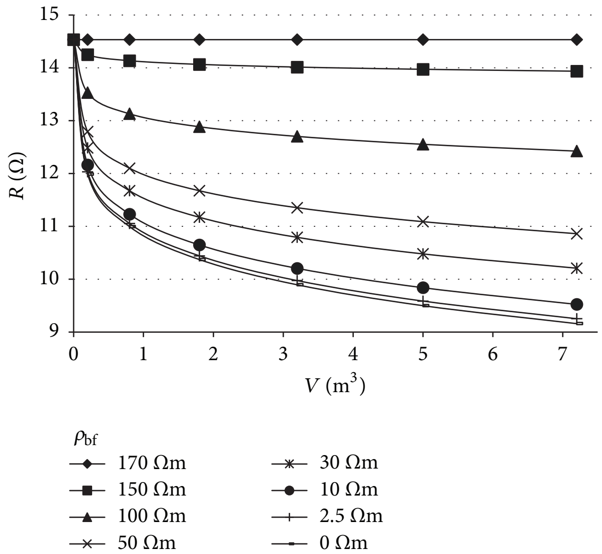

Figure 3: The effect of the volume of a backfill material and its resistivity on the grounding resistance of the considered loop.

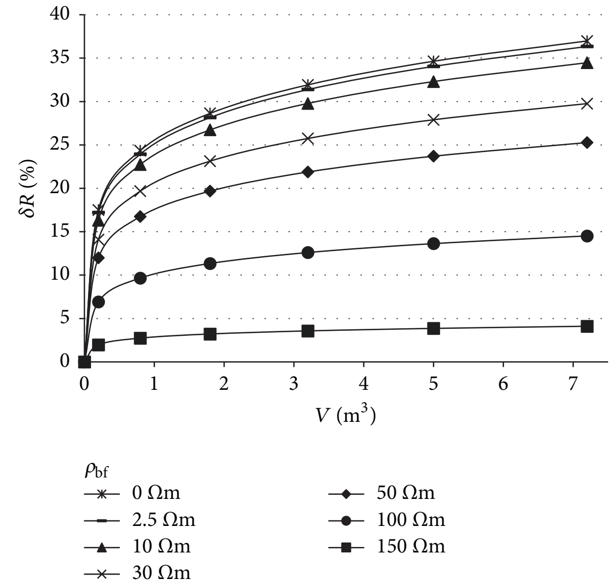

Figure 4: The effect of the volume of a backfill material and its resistivity on the grounding resistance reduction rate.

4. Determining the Curve $\delta R(V)$ and the Values of the "Knee" Point Coordinates for the Considered Grounding Loops

The described experimental setup (Figure 1) was 3D modeled applying FEM. The used model is described in detail in [14, 28, 31]. Note that any kind of multilayer soil could also easily be modeled using 3D FEM, in the same manner as it was done in case of 2-layer soil for the described experimental setup. Backfill material was modeled as subdomain which surrounds the grounding loop, cross-section of which is shown in Figure 2.

Values of resistivity of backfill material were varied from $0\,\Omega$m (ideal conducting material) to $170\,\Omega$m (ρ_{up}), as well as the subdomain's dimension d (Figure 2) from 0.1 m to 0.6 m, and the grounding resistance of the considered loop was calculated for each case using FEM. The variation of the grounding resistance of considered loop R as a function of volume V of backfill material, for its various resistivities ρ_{bf}, is presented in Figure 3. The volume V of backfill material is calculated as

$$V = d^2 \cdot P \tag{5}$$

(P is the grounding loop perimeter, 20 m in the considered case).

It is obvious from curves shown in Figure 3 that the values of the grounding resistance lower than the baseline value ($R_0 = 14.53\,\Omega$, calculated for the considered case using 3D FEM) can be obtained with backfill materials characterized by different values of their resistivities, as long as their resistivities are lower than the resistivity of surrounding soil ($\rho_{up} = 170\,\Omega$m).

The pairs of values $(V, \delta R)$ for various values of ρ_{bf}, calculated for the considered case using 3D FEM modeling, are presented at the diagram shown in Figure 4 as points marked by different items for each value of ρ_{bf}. As expected, for lower resistivities of the backfill material, the lower grounding resistances of the considered loop were obtained. Nevertheless, the saturation phenomenon in grounding resistance reduction effect obviously appears even if backfill material is an ideal conducting material ($\rho_{bf} = 0\,\Omega$m). According to the diagram shown in Figure 4, it would be very difficult to achieve the value of $\delta R = 40\%$ in the considered case, even if large volume of an ideal conducting material was used as a backfill material. For backfill materials characterized by higher resistivities, δR gets saturated at its smaller values, even for very small used volumes of a backfill material.

Analyzing values presented at the diagram shown in Figure 4, it was perceived that the grounding resistance reduction rate δR (%), as a function of the used volume V (m^3) of backfill material, can be approximated by the following expression:

$$\delta R = a \cdot \ln (b \cdot V + 1), \tag{6}$$

where a and b are positive parameters, different for each value of ρ_{bf}, which describe the shape of the $\delta R(V)$ curve in the considered case. This function was chosen among other candidate functions because it was the most suitable one for the determination of the radius of curvature. It is differentiable up to the second order and its first and second derivatives were easily determined:

$$\frac{d\delta R}{dV} = \frac{a \cdot b}{b \cdot V + 1}, \qquad \frac{d^2\delta R}{dV^2} = -\frac{a \cdot b^2}{(b \cdot V + 1)^2}. \tag{7}$$

The parameters a and b, which describe the shape of the $\delta R(V)$ curve in the considered case, were determined by fitting the continuous function expressed by (6) through the points presented at the diagram shown in Figure 4, using the method of least squares and the iterative calculation method. The values of the determined a and b parameters are given in Table 1 for several values of ρ_{bf}.

The radius of curvature $r(V)$ at arbitrary point of a curve $\delta R(V)$ can be determined by applying (2) on (6), which gives

$$r = \frac{\left(1 + (d\delta R/dV)^2\right)^{3/2}}{\left|d^2\delta R/dV^2\right|}, \quad (8)$$

and then incorporating (7) into (8), which after rearranging becomes

$$r = \frac{\left(\left(a^2 \cdot b^2\right) / \left((b \cdot V + 1)^2 + 1\right)\right)^{3/2} \cdot (b \cdot V + 1)^2}{a \cdot b^2}. \quad (9)$$

The first and the second derivative of $r(V)$ function, respectively, are

$$\frac{dr}{dV} = \frac{\left(\left(a^2 \cdot b^2\right) / (b \cdot V + 1)^2 + 1\right)^{1/2} \cdot \left(2 \cdot b^2 \cdot V^2 - a^2 \cdot b^2 + 4 \cdot b \cdot V + 2\right)}{a \cdot b \cdot (b \cdot V + 1)},$$
$$\frac{d^2 r}{dV^2} = \frac{2 \cdot a^4 \cdot b^4 + a^2 \cdot b^4 \cdot V^2 + 2 \cdot a^2 \cdot b^3 \cdot V + a^2 \cdot b^2 + 2 \cdot b^4 \cdot V^4 + 8 \cdot b^3 \cdot V^3 + 12 \cdot b^2 \cdot V^2 + 8 \cdot b \cdot V + 2}{a \cdot \left(\left(a^2 \cdot b^2\right) / (b \cdot V + 1)^2 + 1\right)^{1/2} \cdot (b \cdot V + 1)^4}. \quad (10)$$

The "knee" point of $\delta R(V)$ function can be determined by using the equation

$$\frac{dr}{dV} = 0 \quad (11)$$

and satisfying the condition

$$\frac{d^2 r}{dV^2} > 0. \quad (12)$$

The solution of (11) which is realistic and which satisfies the condition expressed by (12) is

$$V_{\text{knee}} = \frac{\sqrt{2} \cdot a \cdot b - 2}{2 \cdot b}. \quad (13)$$

This solution represents the V coordinate of the "knee" point of $\delta R(V)$ function. The δR coordinate of the "knee" point of $\delta R(V)$ function can be determined introducing the solution expressed by (13) into (6), which after rearranging becomes

$$\delta R_{\text{knee}} = a \cdot \ln\left(\frac{\sqrt{2} \cdot a \cdot b}{2}\right). \quad (14)$$

The values of the "knee" point coordinates, V_{knee} and δR_{knee}, are given in Table 1 for several values of ρ_{bf}. They are calculated incorporating the corresponding a and b parameters into (13) and (14).

5. Determining a Backfill Material Optimal Volume for the Considered Grounding Loops by Engineering Analysis

For determining an optimal volume, V_{opt}, of used backfill material, a designer must be able to estimate the influence of use of arbitrary volume, V, of a backfill material on grounding resistance, R, of a grounding system. For the considered grounding loops it can be done by incorporating value of δR (obtained by (6)), along with the baseline value of R_0, into the following expression obtained rearranging (1):

$$R = R_0 \cdot \left(1 - \frac{\delta R}{100}\right). \quad (15)$$

By incorporating (6) into (15), which after rearranging becomes

$$R = R_0 \cdot \ln\left(\frac{e}{(b \cdot V + 1)^{a/100}}\right), \quad (16)$$

direct $R(V)$ dependence is obtained, which can be used for the same purpose (e is the base of the natural logarithm).

For the considered loops, if it is assumed that $R_{\text{dem}} = 10\,\Omega$, $\rho_{bf} = \rho_{\text{bentonite}} = 2.5\,\Omega\text{m}$ and that the sufficient amount of bentonite, which should provide the successful elimination of the whole contact resistance component, is $V'_{\min} = 0.02\,\text{m}^3$ per 1 m of grounding strip (according to findings presented in [17], based on experimental investigations), characteristic backfill material volumes are $V_{\min} = P \cdot V'_{\min} = 0.4\,\text{m}^3$, $V_{\max} = V_{\text{knee}} = 3.8\,\text{m}^3$ (obtained using (13), $V'_{\max} = 0.19\,\text{m}^3$ of bentonite per 1 m of strip), and $V_{\text{dem}} = 2.97\,\text{m}^3$ (obtained using (16), $V'_{\text{dem}} \approx 0.15\,\text{m}^3$ of bentonite per 1 m of strip).

Comparing the experimentally obtained values of the grounding resistance ($R = 50.2\,\Omega$ for the conventional loop and $R = 12.5\,\Omega$ for the loop backfilled with $1.2\,\text{m}^3$ of bentonite suspension ($0.06\,\text{m}^3$ per 1 m of strip)) with the baseline value of $R_0 = 14.53\,\Omega$ calculated using FEM and the values of $R = 11.55\,\Omega$, $R = 10.71\,\Omega$, and $R = 9.81\,\Omega$, calculated using (16) for loop backfilled with $V_{\min} = 0.4\,\text{m}^3$, $V = 1.2\,\text{m}^3$, and $V_{\max} = 3.8\,\text{m}^3$ of bentonite suspension, respectively, it can be concluded that the amount of $0.06\,\text{m}^3$ of bentonite suspension per 1 m of the grounding strip used in experimental setup [11] was sufficient to eliminate the whole contact resistance component, but if that was the only goal, it could be achieved using 3 times smaller volume ($0.02\,\text{m}^3$ per 1 m of strip [17]).

Table 1: The values of the determined a and b parameters and the “knee” point coordinates for various resistivities of backfill material, for the considered case.

ρ_{bf} (Ωm)	a	b	V_{knee} (m^3)	δR_{knee} (%)
0	5.48	109.43	3.87	33.17
2.5	5.39	109.43	3.80	32.51
10	5.11	109.69	3.61	30.58
30	4.41	110.96	3.11	25.77
50	3.74	112.39	2.63	21.28
100	2.14	115.67	1.51	11.06
150	0.60	118.42	0.42	2.37

If the goal was to achieve $R_{dem} = 10\,\Omega$, nearly 2.5 times larger volume (0.15 m^3 of bentonite per 1 m of strip) should be used. However, it must be taken into consideration that there is some difference between the experimentally obtained value of grounding resistance of the loop backfilled with $V = 1.2\,m^3$ bentonite and the one calculated using (16) ($R = 12.5\,\Omega$ and $R = 10.71\,\Omega$, respectively), because the input data and the results of measurements from this experimental setup, reported in [11], were obtained only with a reasonable, not perfect, accuracy. Therefore, during the design process, a designer should definitely foresee usage of somewhat larger volumes of backfill material than those estimated by (16), in order to compensate potential errors caused by inaccuracy of certain input parameters and their seasonal variations, but certainly not larger than V_{max} (= 3.8 m^3 in the considered case), because the increase of volume over this value could not provide significant positive effect but would only needlessly increase the installation costs. All things considered, in the end, it is again on the designer to estimate the optimal volume, V_{opt}, of a backfill material, on the basis of the existing situation on the site, the impact of considered grounding system on the safety of people and equipment, and the available budget, as well as on the basis of personal experience and engineering sense.

In the considered case, if the position of the grounding system required $R_{dem} = 10\,\Omega$, the logical designer’s choice would be to adopt, for example, $V_{opt} = 3.3\,m^3$ ($\approx 1.1 \cdot V_{dem} < V_{max} = 3.8\,m^3$; additional 10% of backfill material to compensate, to some extent, potential errors caused by inaccuracy of certain input parameters and their seasonal variations). If at the position of the grounding system $R_{dem} = 15\,\Omega$ was acceptable (which is often the case in the power system in Serbia), the logical designer’s choice would be to adopt, for example, $V_{opt} = 0.6\,m^3$ ($= 1.5 \cdot V_{min}$, enough to eliminate the entire contact resistance and additional 50% of backfill material to compensate potential errors). Either way, it can be concluded that the used volume $V = 1.2\,m^3$ of bentonite was wrong choice (undersized if $R_{dem} = 10\,\Omega$, meaning that the additional work would be required after installation; oversized if $R_{dem} = 15\,\Omega$, meaning that the installation costs were increased without a justified reason).

6. Method for Determining a Backfill Material Optimal Volume

The procedure for obtaining (6), (13), (14), and (16) and the determination of necessary coefficients, as well as engineering analysis of the results, which are presented in Sections 4 and 5 for considered grounding loop, represent a new mathematical method for determining optimal quantity of backfill materials used for grounding resistance reduction, which can be applied to any type of grounding system, with various dimensions, placed in any soil structure, with various quantities and characteristics of backfill materials. To summarize, the method consists of the following 6 steps:

(1) 3D FEM modeling of the considered soil structure, grounding system, and backfill material,

(2) calculation (using 3D FEM) of several pairs of values $(V, \delta R)$ for backfill materials (characterized by ρ_{bf}) which are available for grounding system construction on the desired location (Figures 3 and 4 in the considered case),

(3) finding and fitting a suitable $\delta R(V)$ continuous function to a set of the obtained $(V, \delta R)$ points for each backfill material ((6) in the considered case),

(4) determining the “knee” point $(V_{knee}, \delta R_{knee})$ applying mathematical approach based on differential calculus and the mathematical definition of curvature to the obtained continuous function $\delta R(V)$ ((13) and (14) and Table 1 in the considered case),

(5) determining $R(V)$ dependence ((16) in the considered case) and using it to calculate characteristic backfill material volumes (V_{min}, V_{max}, and V_{dem}), as well as values of grounding resistances that correspond to them,

(6) conducting engineering analysis, on the basis of the existing situation on the site, the impact of considered grounding system on the safety of people and equipment, and the available budget, as well as on the basis of designer’s personal experience and engineering sense, in order to estimate the optimal volume, V_{opt}, of a backfill material.

The proposed method can help grounding system designers to avoid the saturation phenomenon when using a backfill material for grounding resistance reduction and maximize the efficiency of its use. Although it does not calculate the exact optimum quantity of a backfill material (which is impossible in a practical sense), it represents a new tool (a kind of which was not available neither in standards nor in scientific and professional literature) for conducting technical and techno-economic analysis, the results of which

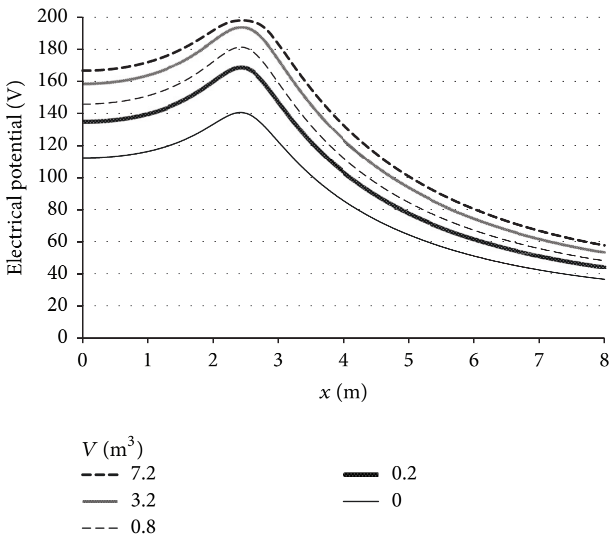

Figure 5: Potential distributions along a line situated at the ground surface, presented for various volumes of backfill material (bentonite).

can assist in assessing the optimum quantity of a backfill material that should be used. However, the practical problems could occur related to implementing the optimized results in real conditions. In some cases it could be difficult, or even impossible, to construct, in real conditions of soil, holes with the dimensions determined with the proposed method for the perfect disposal of the optimized volume of backfill material.

The author's present attempts are focused on simplifying the presented method. The room for possible simplification may lie in the fact that the "knee" value of the volume of backfill material is almost perfectly linearly dependent from the resistivity of backfill material, according to the data presented in Table 1. Hence, it is possible that only the "knee" value of the volume of backfill material for $\rho_{bf} = 0$ ($V_{knee}(\rho_{bf} = 0)$) has to be calculated using the proposed method, in order to draw a straight line between the points $(V_{knee}, \rho_{bf}) = (V_{knee}(\rho_{bf} = 0), 0)$ and $(V_{knee}, \rho_{bf}) = (0, \rho_{up})$ on diagram with V_{knee} and ρ_{bf} axis, and read from that line the "knee" value of the volume for any ρ_{bf}. However, before the mentioned simplification could be applied, the linear dependence of the "knee" value of the volume from the resistivity of backfill material has to be checked for other types of grounding systems and soil structures.

7. Distribution of Electrical Potential at the Ground Surface

The influence of the used volume of backfill material on the distribution of electrical potential at the ground surface above the buried grounding system (during the earth fault) was also investigated. The diagram shown in Figure 5 contains curves representing the potential distributions along the line between the points $(x, y) = (0\text{ m}, 0\text{ m})$ and $(x, y) = (8\text{ m}, 0\text{ m})$ belonging to the ground surface ($z = 0$ m), calculated using 3D FEM for various volumes of bentonite ($\rho_{bf} = \rho_{bentonite} = 2.5\ \Omega\text{m}$) and for electrical potential of the grounding system electrodes that equals 200 V.

The diagram shown in Figure 5 illustrates how the volume of used backfill material (bentonite) affects the potential distribution on the ground surface during earth fault. It is obvious that the touch voltage, which represents the potential difference between the ground potential rise (electrical potential of the grounding system electrodes) and the ground surface potential at a point where a person is standing while at the same time having a hand in contact with a grounded structure, is decreasing with the increase of backfill material volume (for the same electrical potential of the grounding system electrodes). The step voltage, representing the difference in ground surface potentials experienced by a person bridging a distance of 1 m, is obviously increasing with the increase of backfill material volume (for the same electrical potential of the grounding system electrodes). However, to precisely determine the touch and step voltages—the parameters that indicate the quality of the grounding system—as well as electrical potential of grounding system electrodes in a specific case, conditions in the corresponding electrical circuit must be taken into consideration and deeper analyses have to be performed, which will also be the subject of future author's work.

8. Conclusions

A new mathematical method for determining an optimal quantity of backfill materials used for grounding resistance reduction, based on 3D FEM modeling, numerical analysis of the obtained results, and the "knee" of the curve concept, as well as on the engineering analysis based on the designer's experience, is developed and offered in this paper. Being based on 3D FEM, to the best of the author's knowledge, it is the first such method offered in literature which is general and can be applied to any geometry of electrodes and soil composition.

The proposed method has been tested by applying it to the experimental setup presented in literature [11], containing square loop enveloped by a backfill material and buried in a 2-layer soil. The results obtained by the presented method provide the designer with a range of the optimal volume for any backfill material which is available. It helps a designer to make decision whether to use the volume of backfill material closer to the lower limit of range and try to eliminate only contact resistance component with a smaller investment or to use the volume closer to the upper limit of range and achieve the additional decrease of the grounding resistance with a somewhat higher investment.

However, the proposed method is suitable only for cases in which using backfill materials has advantages (in terms of efficiency and cost) in relation to simply increasing the number of rods or the electrode length. Also, in some cases it could be difficult, or even impossible, to construct, in real conditions of soil, holes with the dimensions determined with the proposed method for the perfect disposal of the optimized volume of backfill material.

Conflicts of Interest

The author declares that there are no conflicts of interest related to this paper.

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