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Research Article

Adaptive Tracking Control for a Class of Manipulator Systems with State Constraints and Stochastic Disturbances

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An adaptive controller is constructed for a class of stochastic manipulator nonlinear systems in this paper. The states are constrained in the compact set. A tan-type Barrier Lyapunov Function (BLF) is employed to deal with state constraints. The proposed control scheme guarantees the output error convergence to a small neighbourhood of zero. All the signals in the closed-loop system are bounded. The simulation results illustrate the validity of the proposed method.

1. Introduction

Adaptive control of strict-feedback nonlinear systems has received a lot of attention since the appearance of recursive backstepping design [1] and feedforward design [2, 3]; a great deal of work has been done for this class of systems in the past decades; for example, see [4–9]. Constraints are widespread in many real systems such as robotic manipulators and physical engineering systems. Therefore robotic manipulators have received increasing attention over the last few years. For this reason, many methods have been used to handle the issue of constraints, for example, [10–16]. In detail, in [10], the author studies the neural network adaptive control design for robotic systems with the velocity constraints and input saturation. An adaptive finite-time controller is considered in [11] for a class of strict-feedback nonlinear systems with parametric uncertainties and full state constraints. In [12], the problem of position control of manipulators operating in the task space under state constraints has been addressed. Literatures [15, 16] discussed the trajectory and tracking control problems of the mobile manipulator with constrained end-effector and adaptive controllers are proposed.

The practical systems are inevitable to contain the stochastic disturbance and it can cause instability of system. So the stability of stochastic nonlinear systems has attracted great attention [17–19]. In [20], the author has proposed the state equations of the stochastic dynamics of an open-chain manipulator in a fluid environment. This paper has given an algorithm for the discretization of the state equations and explained how the interaction between a fluid and a manipulator can be taken into account in the control of manipulators. The work [21] studies the problem of output feedback stabilization for a class of stochastic feedforward nonlinear systems with state and input delays and the unknown output function. In [22], the stochastic response of a mobile robotic manipulator has been investigated. This paper has studied the sensitivities of the joints responses to base velocity, the surface roughness coefficients, manipulator configuration, and damping in detail.

In this paper, an adaptive tracking controller will be designed for stochastic manipulator nonlinear systems with full state constraints. A backstepping technique with a tan-type Barrier Lyapunov Function (BLF) will be constructed to address the state constraints problem and all the states in stochastic nonlinear systems are not violated. The error signals have converged to an arbitrarily small neighbourhood of zero and all the signals in the closed-loop system are bounded.

2. System Description

2.1. Problem Statement and Preliminary Results. Consider the one-link manipulator which contains motor dynamics and stochastic disturbances. The model is described as [9]

$$D\ddot{q} + \theta\dot{q} - N\sin(q) = \tau_r + \tau_d,$$
$$M\dot{\tau}_r + H\tau_r = u - k_m\dot{q}, \tag{1}$$

where $q, \dot{q}, \ddot{q}$ are the link position, velocity, and acceleration, respectively; τ_r denotes the torque produced by the electrical system; τ_d is the known torque stochastic disturbance; u is the electromechanical torque control input; D is a known mechanical inertia; θ is unknown coefficient of viscous friction at the joint; N is a known constant related to the mass of the load and the coefficient of gravity; M is the known armature inductance; H is the known armature resistance; k_m is the known back electromotive force coefficient; m is the link mass; and l is the length of the link which is known. Then, following change of coordinates $\xi_1 = q$, $\xi_2 = \dot{q}$, $\xi_3 = \tau_r$, we can get

$$\begin{aligned} d\xi_1 &= \xi_2 dt, \\ d\xi_2 &= \frac{1}{D}\left(\xi_3 - \theta\xi_2 + 10\sin(\xi_1)\right)dt + \frac{1}{D}\sin(\xi_1)\,\xi_1 d\omega, \\ d\xi_3 &= \frac{1}{M}\left(u - k_m\xi_2 - H\xi_3\right)dt, \\ y &= \xi_1. \end{aligned} \tag{2}$$

All the states are constrained in the compact set as $\Omega_\xi := \{\xi_i(t) \in R, |\xi_i(t)| \le k_{c_i}\}$, where $i = 1, 2, 3$, k_{c_i} are positive constants.

Given a reference trajectory q_d, the control objective is to design an adaptive control algorithm such that q tracks the desired trajectory q_d as much as possible; all the signals in closed-loop systems are bounded; the state constraint requirements are not violated. To facilitate control system design, the following assumption and lemma are proposed.

Assumption 1. The reference trajectory $y_d(t)$ and its derivatives up to the n-th ones are continuous and bounded. That is, for any constant $k_{c_i} > 0$, there exist positive constants $Y_i, i = 0, \ldots, n$, such that $|y_d(t)| \le Y_0 < k_{c_1}$, $|y_d^{(i)}(t)| \le Y_i$, $i = 1, \ldots, n$.

Remark 2. This assumption is reasonable. Assumption 1 is the worst case one. The requirement on derivatives is widely fixed in backstepping control [23]. This is because the standard backstepping technique requires the reference signal to be continuous and derivable to design the desired controller. $y_0 < k_{c_i}$ in assumption is always true in practice for the requirement of output tracking control. A similar assumption is also considered in [24, 25].

Next, consider the following stochastic nonlinear system:

$$dx = f(x)\,dt + g(x)\,d\omega, \tag{3}$$

where $x \in R^n$ is the system state vector; $f(x) \in R^n$ and $g(x) \in R^{n\times r}$ satisfy the locally Lipschitz functions and the linear growth condition and $f(0) = 0, g(0) = 0$; ω is an r-dimensional standard Wiener process.

Definition 3 (see [2]). For any Lyapunov function $V(x,t) \in C^{2,1}$, we define the differential operator L as follows:

$$L[V(x,t)] = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}f + \frac{1}{2}\mathrm{Tr}\left\{g^T\frac{\partial^2 V}{\partial x^2}g\right\}, \tag{4}$$

where $Tr(\cdot)$ is the matrix trace.

Lemma 4 (see [26]). *There exist a $C^{2,1}$ function V, two constants $\gamma > 0, \rho > 0$, and K_∞ class functions α_1, α_2 satisfying $\alpha_1(\|x\|) \le V(x) \le \alpha_2(\|x\|)$ and $LV(x) \le -\gamma V(x) + \rho$; then, there is a unique strong solution which satisfies*

$$E[V(x)] \le V(x_0)e^{-\gamma t} + \frac{\rho}{\gamma}. \tag{5}$$

3. Control Design and Stability Analysis

Step 1. Define the tracking error $e_1 = \xi_1 - y_d, e_2 = \xi_2 - \alpha_1$, where α_i are the virtual controllers and $|\alpha_i| < \overline{\alpha}_i$ with $\overline{\alpha}_i$ are positive constants, and $e_i \in \Omega_e := \{e_i \in R, |e_i| < k_{b_i}, i = 1, \ldots, n\}$ with $k_{b_1} = k_{c_1} - Y_0 > 0, k_{b_i} = k_{c_i} - \overline{\alpha}_{i-1} > 0$. We can get

$$de_1 = d\xi_1 - dy_d = (e_2 + \alpha_1 - \dot{y}_d)\,dt. \tag{6}$$

Consider a candidate BLF as follows:

$$V_1 = \frac{k_{b_1}^4}{\pi}\tan\left(\frac{\pi e_1^4}{4k_{b_1}^4}\right). \tag{7}$$

Based on Definition 3, one has

$$LV_1 = \frac{e_1^3}{\cos^2\left(\pi e_1^4/4k_{b_1}^4\right)}\left(e_2 + \alpha_1 - \dot{y}_d\right). \tag{8}$$

Design the virtual controller α_1 as

$$\alpha_1 = -\frac{K_1\sin\left(\pi e_1^4/4k_{b_1}^4\right)\cos\left(\pi e_1^4/4k_{b_1}^4\right)}{e_1^3} + \dot{y}_d, \tag{9}$$

where $K_1 > 0$ is a design parameter. Substituting (9) into (8), we can obtain

$$LV_1 \le -K_1\tan\left(\frac{\pi e_1^4}{4k_{b_1}^4}\right) + \frac{e_1^3}{\cos^2\left(\pi e_1^4/4k_{b_1}^4\right)}e_2. \tag{10}$$

Step 2. Define the tracking error $e_3 = \xi_3 - \alpha_2$; the following can be obtained:

$$\begin{aligned} de_2 &= d\xi_2 - d\alpha_1 \\ &= \frac{1}{D}\left(e_3 + \alpha_2 - \theta\xi_2 + 10\sin(\xi_1)\right)dt \\ &\quad + \frac{1}{D}\sin(\xi_1)\,\xi_1 d\omega \\ &\quad - \left(\frac{\partial\alpha_1}{\partial\xi_1}\xi_2 + \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial y_d^{(i-1)}}y_d^{(i)}\right)dt. \end{aligned} \tag{11}$$

Choose a candidate BLF as follows:

$$V_2 = V_1 + \frac{k_{b_2}^4}{\pi} \tan\left(\frac{\pi e_2^4}{4k_{b_2}^4}\right) + \frac{1}{2}\tilde{\theta}^2. \tag{12}$$

Here $\tilde{\theta} = \hat{\theta} - \theta$, $\hat{\theta}$ denotes the estimation of θ. Based on Definition 3, we can obtain

$$\begin{aligned}
LV_2 &= LV_1 + \frac{e_2^3}{\cos^2\left(\pi e_2^4/4k_{b_2}^4\right)}\left(\frac{1}{D}\left(e_3 + \alpha_2 - \theta\xi_2 + 10\right.\right. \\
&\quad \left.\cdot \sin\left(\xi_1\right)\right) - \frac{\partial\alpha_1}{\partial\xi_1}\xi_2 - \left.\sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial y_d^{(i-1)}} y_d^{(i)}\right) \\
&\quad + \frac{3k_{b_2}^4 e_2^4 \cos\left(\pi e_2^4/4k_{b_2}^4\right) + 2\pi e_2^6 \sin\left(\pi e_2^4/4k_{b_2}^4\right)}{2k_{b_2}^4 \cos^3\left(\pi e_2^4/4k_{b_2}^4\right)}\left(\frac{1}{D}\right. \\
&\quad \left.\cdot \sin\left(\xi_1\right)\xi_1\right)^2 + \tilde{\theta}\dot{\hat{\theta}} \le LV_1 + \tilde{\theta}\dot{\hat{\theta}} \\
&\quad + \frac{e_2^3}{\cos^2\left(\pi e_2^4/4k_{b_2}^4\right)}\left(\frac{1}{D}\left(\alpha_2 - \hat{\theta}\xi_2 + 10\sin\left(\xi_1\right)\right)\right. \\
&\quad - \frac{\partial\alpha_1}{\partial\xi_1}\xi_2 - \sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial y_d^{(i-1)}} y_d^{(i)} \\
&\quad + \frac{3k_{b_2}^4 e_2 \cos\left(\pi e_2^4/4k_{b_2}^4\right) + 2\pi e_2^3 \sin\left(\pi e_2^3/4k_{b_2}^4\right)}{2k_{b_2}^4 \cos\left(\pi e_2^4/4k_{b_2}^4\right)}\left(\frac{\xi_1}{D}\right. \\
&\quad \left.\left.\cdot \sin\left(\xi_1\right)\right)^2\right) + \frac{e_2^3 e_3}{D\cos^2\left(\pi e_2^4/4k_{b_2}^4\right)} \\
&\quad - \frac{\tilde{\theta} e_2^3 \xi_2}{D\cos^2\left(\pi e_2^4/4k_{b_2}^4\right)}.
\end{aligned} \tag{13}$$

Design the virtual controller α_2 as

$$\begin{aligned}
\alpha_2 &= -\frac{K_2 D \sin\left(\pi e_2^4/4k_{b_2}^4\right)\cos\left(\pi e_2^4/4k_{b_2}^4\right)}{e_2^3} + \hat{\theta}\xi_2 - 10 \\
&\quad \cdot \sin\left(\xi_1\right) + D\frac{\partial\alpha_1}{\partial\xi_1}\xi_2 + D\sum_{i=1}^{2}\frac{\partial\alpha_1}{\partial y_d^{(i-1)}} y_d^{(i)} \\
&\quad - \frac{3k_{b_2}^4 e_2 \cos\left(\pi e_2^4/4k_{b_2}^4\right) + 2\pi e_2^3 \sin\left(\pi e_2^3/4k_{b_2}^4\right)}{2Dk_{b_2}^4 \cos\left(\pi e_2^4/4k_{b_2}^4\right)} \\
&\quad \cdot \xi_1^2 \sin^2\left(\xi_1\right),
\end{aligned} \tag{14}$$

where $K_2 > 0$ is a design parameter. An tuning function is chosen as $\tau_1 = e_2^3\xi_2/D\cos^2(\pi e_2^4/4k_{b_2}^4)$. Substituting (14) into (13) yields

$$\begin{aligned}
LV_2 &\le -\sum_{i=1}^{2} K_i \tan\left(\frac{\pi e_i^4}{4k_{b_i}^4}\right) + \tilde{\theta}\left(\dot{\hat{\theta}} - \tau_1\right) \\
&\quad + \frac{e_2^3 e_3}{D\cos^2\left(\pi e_2^4/4k_{b_2}^4\right)}.
\end{aligned} \tag{15}$$

Step 3. Defining the tracking error $e_3 = \xi_3 - \alpha_2$, we have

$$\begin{aligned}
de_3 &= d\xi_3 - d\alpha_2 = \frac{1}{M}\left(u - k_m\xi_2 - H\xi_3\right)dt \\
&\quad - \left(\frac{\partial\alpha_2}{\partial\xi_1}\xi_2 + \frac{\partial\alpha_2}{\partial\xi_2}\left(\frac{1}{D}\left(\xi_3 - \theta\xi_2 + 10\sin\left(\xi_1\right)\right)\right)\right. \\
&\quad + \sum_{i=1}^{3}\frac{\partial\alpha_2}{\partial y_d^{(i-1)}} y_d^{(i)} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} \\
&\quad \left. + \frac{1}{2}\frac{\partial^2\alpha_2}{\partial\xi_2^2}\left(\frac{1}{D}\sin\left(\xi_1\right)\xi_1\right)^2\right)dt.
\end{aligned} \tag{16}$$

Construct a candidate BLF as follows:

$$V_3 = V_2 + \frac{k_{b_3}^4}{\pi}\tan\left(\frac{\pi e_3^4}{4k_{b_3}^4}\right). \tag{17}$$

Based on Definition 3, computing LV_3, we can figure out

$$\begin{aligned}
LV_3 &= LV_2 + \frac{e_3^3}{\cos^2\left(\pi e_3^4/4k_{b_3}^4\right)}\left(\frac{1}{M}\left(u - k_m\xi_2 - H\xi_3\right)\right. \\
&\quad - \frac{\partial\alpha_2}{\partial\xi_1}\xi_2 + \frac{\partial\alpha_2}{\partial\xi_2}\left(\frac{1}{D}\left(\xi_3 - \theta\xi_2 + 10\sin\left(\xi_1\right)\right)\right) \\
&\quad + \sum_{i=1}^{3}\frac{\partial\alpha_2}{\partial y_d^{(i-1)}} y_d^{(i)} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} \\
&\quad \left. + \frac{1}{2}\frac{\partial^2\alpha_2}{\partial e_2^2}\left(\frac{1}{D}\sin\left(\xi_1\right)\xi_1\right)^2\right) \le \frac{e_2^3 e_3}{D\cos^2\left(\pi e_2^4/4k_{b_2}^4\right)} \\
&\quad + \frac{e_3^3}{\cos^2\left(\pi e_3^4/4k_{b_3}^4\right)}\left(\frac{1}{M}\left(u - \xi_2 - 0.5\xi_3\right) - \frac{\partial\alpha_2}{\partial\xi_1}\xi_2\right. \\
&\quad + \frac{\partial\alpha_2}{\partial\xi_2}\left(\frac{1}{D}\left(\xi_3 - \theta\xi_2 + 10\sin\left(\xi_1\right)\right)\right) \\
&\quad + \sum_{i=1}^{3}\frac{\partial\alpha_2}{\partial y_d^{(i-1)}} y_d^{(i)} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} \\
&\quad \left. + \frac{1}{2}\frac{\partial^2\alpha_2}{\partial e_2^2}\left(\frac{1}{g_m}\sin\left(\xi_1\right)\xi_1\right)^2\right) + \sum_{i=1}^{2} K_i \tan\left(\frac{\pi e_i^4}{4k_{b_i}^4}\right) \\
&\quad + \tilde{\theta}\left(\dot{\hat{\theta}} - \tau_1\right).
\end{aligned} \tag{18}$$

Design the controller u as follows:

$$\begin{aligned} u = M\Bigg(&-\frac{K_3 \sin\left(\pi e_3^4/4k_{b_3}^4\right)\cos\left(\pi e_3^4/4k_{b_3}^4\right)}{e_3^3} \\ &+ \sum_{i=1}^{3}\frac{\partial\alpha_2}{\partial y_d^{(i-1)}} y_d^{(i)} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{1}{2}\frac{\partial^2\alpha_2}{\partial e_2^2}\left(\frac{1}{D}\sin\left(\xi_1\right)\xi_1\right)^2 \\ &- \hat{\theta}^T\frac{1}{D}\frac{\partial\alpha_2}{\partial\xi_2}\xi_2 + \frac{\partial\alpha_2}{\partial\xi_1}\xi_2 + \frac{\partial\alpha_2}{\partial\xi_2}\frac{1}{D}\xi_3 - \frac{\vartheta_{e_2} g_2 e_3}{\vartheta_{e_3}}\Bigg), \end{aligned} \tag{19}$$

where $K_3 > 0$ is a design parameter. Choosing $\tau_2 = \tau_1 + (1/D)(\partial\alpha_2/\partial\xi_2)\xi_2$ and substituting (18) and (19) into (15) result in

$$LV_3 \le -\sum_{i=1}^{3} K_i \tan\left(\frac{\pi e_i^4}{4k_{b_i}^4}\right) + \tilde{\theta}\left(\dot{\hat{\theta}} - \tau_2\right). \tag{20}$$

The adaptive law is given as $\dot{\hat{\theta}} = \tau_2 - \sigma\hat{\theta}$ with a design parameter σ. Then, we can obtain

$$\begin{aligned} LV_3 &\le -\sum_{i=1}^{3} K_i \tan\left(\frac{\pi e_i^4}{4k_{b_i}^4}\right) - \sigma\tilde{\theta}\hat{\theta} \\ &\le -\sum_{i=1}^{3} K_i \tan\left(\frac{\pi e_i^4}{4k_{b_i}^4}\right) + \frac{\sigma\theta^2}{2} - \frac{\sigma\tilde{\theta}^2}{2}. \end{aligned} \tag{21}$$

Choose $\eta_1 = \min\{K_1\pi/k_{b_1}^2, K_2\pi/k_{b_2}^2, K_3\pi/k_{b_3}^2, \sigma\}$; then (21) can be rewritten as

$$LV_3 \le -\eta_1\left(\sum_{i=1}^{3}\frac{k_{b_i}^2}{\pi}\tan\left(\frac{\pi e_i^4}{4k_{b_i}^4}\right) + \frac{1}{2}\tilde{\theta}^2\right) + C_1, \tag{22}$$

where $C_1 = \sigma\theta^2/2$; that is,

$$LV_3 \le -\eta_1 V_3 + C_1. \tag{23}$$

Theorem 5. *Consider system (1) under Assumption 1; the controller u is given in (19), and the adaption law $\dot{\hat{\theta}} = \tau_2 - \sigma\hat{\theta}$. Then, the following are guaranteed:*

(1) *The full state constraints are not violated.*

(2) *All the signals in the closed-loop system are bounded.*

(3) *The error signals $e_i(t)$ will converge to $\Xi = \{e_i : E(|e_i|^4) \le (4k_{b_i}^4/\pi)\tan^{-1}((V_3(0) + C_1/\eta_1)(\pi/k_{b_i}^4))\}$.*

Proof. Define a candidate BLF as follows:

$$V_3 = \sum_{i=1}^{3}\frac{k_{b_i}^4}{\pi}\tan\left(\frac{\pi e_i^4}{4k_{b_i}^4}\right) + \frac{1}{2}\tilde{\theta}^2. \tag{24}$$

From (23), we can get that

$$LV_3 \le -\eta_1 V_3 + C_1. \tag{25}$$

Based on Lemma 4, we know that

$$0 \le \left(E\left[V_3(t)\right]\right) \le V_3(0)e^{-\eta_1 t} + \frac{C_1}{\eta_1}. \tag{26}$$

Then, the following inequality holds:

$$\begin{aligned} E\left(\frac{k_{b_i}^4}{\pi}\tan\left(\frac{\pi e_i^4}{4k_{b_i}^4}\right)\right) &\le \left(E\left[V_3(t)\right]\right) \\ &\le V_3(0)e^{-\eta_1 t} + \frac{C_1}{\eta_1}. \end{aligned} \tag{27}$$

Hence,

$$\begin{aligned} E\left(|e_i|^4\right) &\le \frac{4k_{b_i}^4}{\pi}\tan^{-1}\left(\left(V_3(0)e^{-\eta_1 t} + \frac{C_1}{\eta_1}\right)\frac{\pi}{k_{b_i}^4}\right) \\ &\le \frac{4k_{b_i}^4}{\pi}\tan^{-1}\left(\left(V_3(0) + \frac{C_1}{\eta_1}\right)\frac{\pi}{k_{b_i}^4}\right). \end{aligned} \tag{28}$$

Hence, the size of $(4k_{b_i}^4/\pi)\tan^{-1}((V_3(0) + C_1/\eta_1)(\pi/k_{b_i}^4))$ can be made small enough by choosing appropriate parameters. On the other hand, from the above inequality, we can obtain that $E[V_3(t)]$ is limited by C/η_1. Then, we know that $V_3(t)$ is bounded. So $\tilde{\theta}$ is bounded and $\hat{\theta} = \tilde{\theta} + \theta$ is bounded. Then we know that e_i, y_d, α_i, and u are bounded. From $e_1 = \xi_1 - y_d$ and $|y_d| \le Y_0$, we know that $|\xi_1| \le |e_1| + |y_d| < k_{b_1} + Y_0 = k_{c_1}$. From $e_2 = \xi_2 - \alpha_1$ and $|\alpha_1| \le \overline{\alpha}_1$, we can obtain that $|\xi_2| \le |e_2| + |\alpha_1| < k_{b_2} + \overline{\alpha}_1 = k_{c_2}$. In a similar way, we can obtain $|\xi_3| < k_{c_3}$. Thus, the full state constraints are not violated. □

4. Simulation

In this section, simulation is introduced to demonstrate the effectiveness of the proposed scheme.

$$\begin{aligned} d\xi_1 &= \xi_2 dt, \\ d\xi_2 &= \frac{1}{D}\left(\xi_3 - \theta\xi_2 + 10\sin\left(\xi_1\right)\right)dt \\ &\quad + \frac{1}{D}\sin\left(\xi_1\right)\xi_1 d\omega, \\ d\xi_3 &= \frac{1}{M}\left(u - K_m\xi_2 - H\xi_3\right)dt, \\ y &= \xi_1, \end{aligned} \tag{29}$$

where $D = 1$, $M = 1$, $N = 10$, $B = 1$, $H = 0.5$, and $k_m = 1$. All the states are constrained in the compact set as $\Omega_\xi := \{\xi_i(t) \in R, |\xi_i(t)| \le 1.5,\}$. The reference signal is chosen as $y_d = 0.5\sin t$.

In the simulation, the design parameters are chosen as $\xi_i(0) = 0.1$, $K_i = 1$, $i = 1, 2, 3$, $\hat{\theta}(0) = 0.1$, $k_{b_1} = k_{b_2} = 1$, $k_{b_3} = 0.9$, and $k_{b_4} = 1.1$. The results of the simulation are shown in Figures 1–4. The output tracking q and q_d is illustrated in Figure 1. It can be seen that the position state q can primely

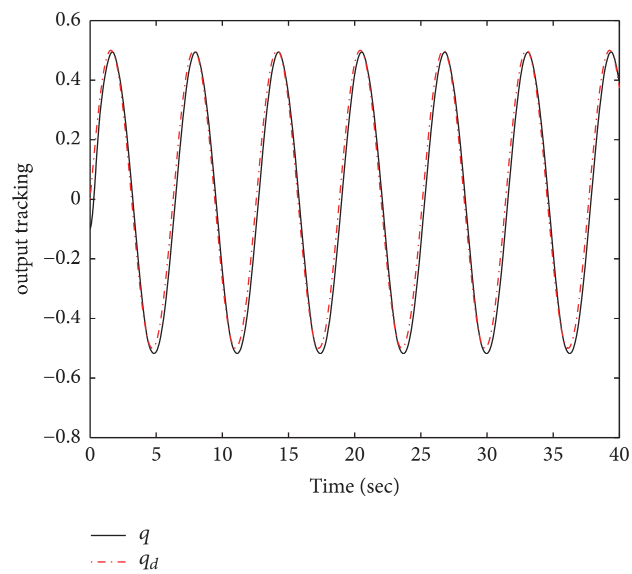

FIGURE 1: The trajectories of q and q_d.

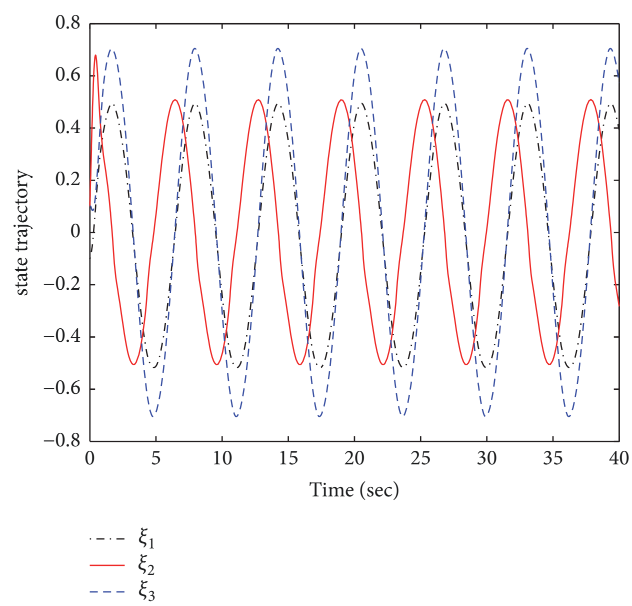

FIGURE 2: The trajectories of ξ_1, ξ_2, and ξ_3.

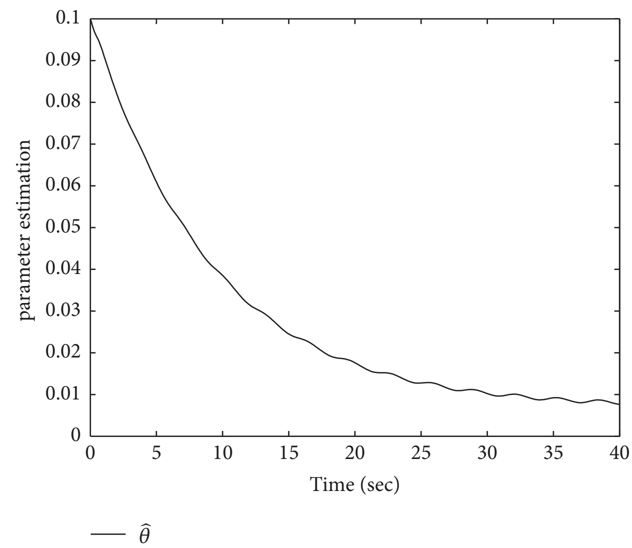

FIGURE 3: The estimation of $\widehat{\theta}$.

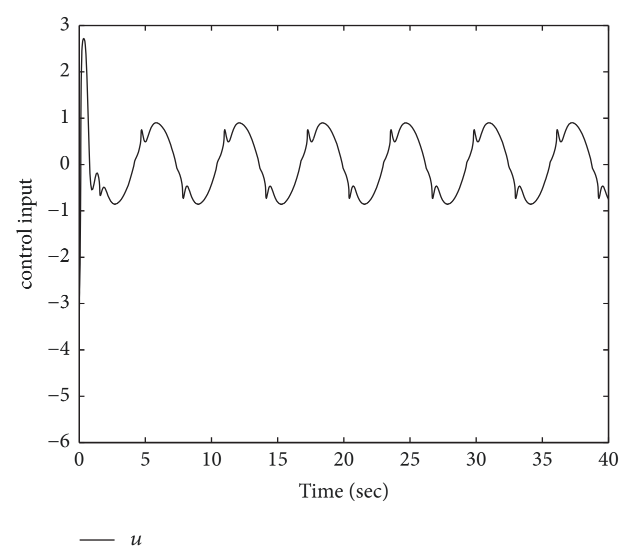

FIGURE 4: The trajectory of control input u.

track the desired trajectory q_d. It is shown in Figure 2 that all the states are strictly constrained in $\{\xi_i \mid -1.5 \leq \xi_i(t) \leq 1.5\}, i = 1, 2, 3$. The parameter updating law $\widehat{\theta}$ and input u are all bounded as shown in Figures 3 and 4. The simulation results demonstrate the effectiveness of the proposed adaptive control scheme.

5. Conclusions

This study carries out the adaptive tracking control for a class of stochastic manipulator nonlinear systems. An adaptive controller is proposed to ensure that the mean square of the tracking error can be made arbitrarily small. Simulation results are presented to illustrate the effectiveness of the proposed control strategy.

Data Availability

In this paper, only numerical simulation is given; all the data are produced by Matlab program. No other data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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