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Research Article

Finite-Time Boundedness Control for Nonlinear Networked Systems with Randomly Occurring Multi-Distributed Delays and Missing Measurements

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This paper investigates the stochastic finite-time H_∞ boundedness problem for nonlinear discrete time networked systems with randomly occurring multi-distributed delays and missing measurements. The randomly occurring multi-distributed delays and missing measurements are described as Bernoulli distributed white noise sequence. The goal of this paper is to design a full-order output-feedback controller to guarantee that the corresponding closed-loop system is stochastic finite-time H_∞ bounded and with desired H_∞ performance. By constructing a new Lyapunov-Krasovskii functional, sufficient conditions for the existence of output-feedback are established. The desired full-order output-feedback controller is designed in terms of the solution to linear matrix inequalities (LMIs). Finally, a numerical example is provided to show the validity of the designed method.

1. Introduction

Networked control system (NCS) is a new research field originated from control engineering practice, which is composed of controller, actuator, and sensor. Due to low cost, low power consumption, and practical resource sharing, the dynamic behavior of NCSs has been widely concerned in recent years [1–5]. However, network is not a reliable communication medium. Due to the physical limitations of network bandwidth and service capability, time delay and missing measurement are inevitable problems in network transmission. These problems are an important reason for the deterioration of system performance and the instability of NCSs. Therefore, in the past few decades, a large number of new methods have been used to deal with NCSs with various time delays. At present, the following three models are mainly adopted: constant delay model [6], the upper and lower bounds with time-varying delay model [7], and random time delay with a certain probability distribution delay model (such as finite state markov process, Bernoulli distribution, etc. [8, 9]). In [8], the H_∞ filtering problem of the Markov delay network system with a partially unknown distribution is studied. By using Lyapunov theory, the authors analyze the robust hybrid controller, controller design, and synchronous dissipation state estimation of the network control system with Bernoulli distribution in [10–13], respectively. On the other hand, missing measurement is also a major cause of NCSs performance degradation and system instability. Study of missing measurement: one of the most common ways to describe the missing measurement is probability method; this method assumes the missing measurement to satisfy a certain probability distribution, such as Bernoulli distribution. Many control problems are considered based on the Bernoulli distribution, such as [14–17]. In [14], recursive state estimation for time-varying complex networks with probability distribution missing measurements is investigated by using the mean square constraint method. The rapid state estimation of discrete nonlinear systems with random missing measurement is studied in [15]. In [16], the author extends the above research to the network control system with multiple missing measurement and studies the extended Kalman filter problem of the NCSs with random nonlinearity and multiple packet loss. In [17–19], the authors further extended the above results to the study of robust H_∞ control, quantised recursive filtering, and event-based filtering of the NCSs with Bernoulli distribution and concluded that the use of Bernoulli distribution to describe missing measurement is more general.

It should be pointed out that the above-mentioned literature on system stability considers Lyapunov's stability at an infinite interval. In practice, however, we are concerned not only with the stability of the system on an infinite interval, but with the stability on a finite interval. The finite time stability (FTS) is different from the stability in the general sense. It studies the state behavior in the finite time; that is, the system should meet certain transient performance requirements (for example, meeting the requirements of the system orbit for a certain degree of deviation from the equilibrium point). In order to study the transient performance of the system, Dorato proposed the concept of FTS in 1961 and obtained extensive research [20]. In recent years, some scholars have extended FTS to studies on finite time boundedness (FTB) and finite time H_∞ control. In [21–25], the authors studied the finite time H_∞ problem of discrete time system based on linear matrix inequality. In [26–29], the authors extended the FTB and finite time H_∞ control problem to the NCSs. Further, in [30], finite time H_∞ control problem is extened to fuzzy discrete system. In [31, 32], the authors discussed FTB and finite time H_∞ state estimation of neural networks with missing measurement based on the semini-research approach and gave the design method of the state estimator. Recently, the issues of nonfragile finite time state estimators and event-driven finite time state estimators for neural networks have been investigated [33–35].

In addition, it is worth noting that, due to the influence of internal and external environmental factors, nonlinearity always exists in the actual system. At present, the control problem of the network system with nonlinearity is widely concerned [36]. Since the nonlinear perturbation phenomena may vary arbitrarily, Bernoulli distribution is more appropriate to describe the randomly occurring nonlinearity. In recent years, the problem of randomly occurring nonlinearities has been widely discussed. For example, in [37], the authors studied the H_∞ control problem for randomly occurring nonlinear systems with saturation and channel fading. In [38, 39], the authors studied the reliable finite time H_∞ filtering and quantized fault detection filters problem for discrete time systems, respectively.

To the best of the authors' knowledge, the SFTB control problem for a discrete-time NCS with randomly occurring multi-distributed delays and missing measurements has not been fully investigated, which motivates the main purpose of our study. The main contributions of the paper are summarized as follows: (1) The discrete-time NCS under consideration is more general networked control systems that include randomly occurring nonlinearities, multi-distributed delays, and missing measurements, where the randomly occurring nonlinearities, multi-distributed delays, and missing measurements are governed by a set of Bernoulli distributed white noise sequences; (2) definitions of SFTB and SFTH_∞B are extended to discrete-time NCSs; (3) sufficient conditions are provided to ensure that the corresponding closed-loop system is stochastic finite-time bounded and H_∞ performance constraint is met by using the Lyapunov-Krasovskii functional method and the LMIs technique; (4) full-order output-feedback controller, a more general controller than the commonly used state feedback controller, is designed.

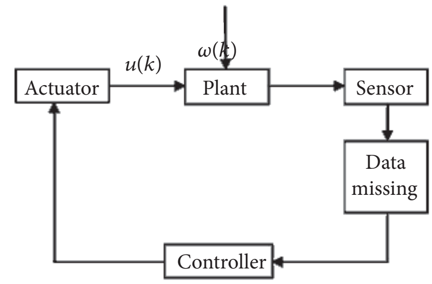

Figure 1: Structure of the considered NCSs.

2. Problem Formulation

Consider a nonlinear discrete-time NCS with randomly multi-distributed delays as shown in Figure 1.

The state-space equation is given as follows:

$$\begin{aligned} x(k+1) &= Ax(k) + A_d\sum_{i=1}^{q} b_i(k)\, x\left(k-d_i(k)\right) + Bu(k) \\ &\quad + \alpha(k)\, f(x(k)) + D_1\omega(k) \\ \tilde{y}(k) &= Cx(k) + D_2\omega(k) \\ z(k) &= Lx(k) \\ x(k) &= \varphi(k), \quad k = -d_M, -d_M+1, \ldots, -1, 0 \end{aligned} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^p$ is the control input, $\tilde{y}(k) \in \mathbb{R}^m$ is the measurement output, $z(k) \in \mathbb{R}^t$ is the controlled output, $f(x(k))$ is a nonlinear function, $\omega(k)$ are external disturbances, $\varphi(k)$ denotes the vector-value initial condition, A, A_d, B, C, D_1, D_2, and L are known real constant matrices, $\sum_{i=1}^{q} b_i(k)x(k-d_i(k))$ denotes the infinite distributed delays, $d_i(k)$ $(i = 1, \ldots, q)$ are time-varying delays satisfying

$$d_m \le d_i(k) \le d_M, \tag{2}$$

where d_m and d_M are nonnegative scalars. $b_i(k)$ $(i = 1, \ldots, q)$ are uncorrelated random variables and satisfy the following probability distribution:

$$\begin{aligned} \operatorname{Prob}\{b_i(k) = 1\} &= \bar{b}_i, \\ \operatorname{Prob}\{b_i(k) = 0\} &= 1 - \bar{b}_i, \end{aligned} \tag{3}$$

where $\bar{b}_i \in [0,1]$ is a known constant and the variance $\tilde{b}_i^2 = \bar{b}_i(1-\bar{b}_i)$.

The stochastic variable $\alpha(k)$ is a Bernoulli distributed white noise sequence with

$$\begin{aligned} \operatorname{Prob}\{\alpha(k) = 1\} &= \bar{\alpha}, \\ \operatorname{Prob}\{\alpha(k) = 0\} &= 1 - \bar{\alpha}, \end{aligned} \tag{4}$$

where $\overline{\alpha} \in [0, 1]$ is a known constant, and the variance $\widetilde{\alpha}^2 = \overline{\alpha}(1 - \overline{\alpha})$. Throughout the paper, we assume that $\alpha(k)$ and $b_i(k)$ $(i = 1, \ldots, q)$ are mutually independent.

For the aforementioned system, we make the following assumptions.

Assumption 1. $f(\cdot)$ is a nonlinear function, and we assume that $f(\cdot)$ satisfies $f(0) = 0$ and the sector-bounded condition

$$\begin{aligned} &\left[f(x) - f(y) - U_1(x - y)\right]^T \\ &\quad \cdot \left[f(x) - f(y) - U_2(x - y)\right] \leq 0, \quad \forall x, y \in \mathbb{R}^n, \end{aligned} \tag{5}$$

where $U_1, U_2 \in \mathbb{R}^{n \times n}$ are known real matrices and $U_1 - U_2$ is a known positive definite matrix.

Assumption 2. For any given positive number δ, external disturbances input $\omega(k)$ is time varying and satisfies

$$\sum_{k=0}^{N} \omega^T(k)\,\omega(k) \leq \delta, \quad \delta \geq 0. \tag{6}$$

In this paper, the actual system output is expressed as

$$y(k) = \beta(k)\,\widetilde{y}(k), \tag{7}$$

where the stochastic variable $\beta(k)$ takes values on 0 and 1 with the following probabilities:

$$\begin{aligned} &\operatorname{Prob}\{\beta(k) = 1\} = \overline{\beta}, \\ &\operatorname{Prob}\{\beta(k) = 0\} = 1 - \overline{\beta}, \end{aligned} \tag{8}$$

where $\overline{\beta} \in [0, 1]$ is a known constant and the variance $\widetilde{\beta}^2 = \overline{\beta}(1 - \overline{\beta})$.

Equation (7) can be written in the light of system (1)

$$y(k) = \beta(k)\left[Cx(k) + D_2\omega(k)\right]. \tag{9}$$

In this paper, the full-order dynamic output feedback controller for system (1) is described as

$$\begin{aligned} \widehat{x}(k+1) &= A_K\widehat{x}(k) + B_K y(k) \\ u(k) &= C_K\widehat{x}(k). \end{aligned} \tag{10}$$

where $\widehat{x}(k) \in \mathbb{R}^n$ is the state estimation of system (1) and A_K, B_K, and C_K are the controller parameters to be determined.

By combining the controller (10) with (1), we can obtain the following closed-loop system:

$$\begin{aligned} \eta(k+1) = {} & \overline{A}\eta(k) + \left(\beta(k) - \overline{\beta}\right)\widetilde{A}\eta(k) + \overline{C}f(x(k)) \\ & + (\alpha(k) - \overline{\alpha})\widetilde{C}f(x(k)) + \overline{D}\omega(k) \\ & + \left(\beta(k) - \overline{\beta}\right)\widetilde{D}\omega(k) \\ & + \sum_{i=1}^{q}\overline{A}_{di}H\eta(k - d_i(k)) \\ & + \sum_{i=1}^{q}\left(b_i(k) - \overline{b}_i\right)\widetilde{A}_{di}H\eta(k - d_i(k)) \\ z(k) = {} & \overline{L}\eta(k), \end{aligned} \tag{11}$$

where

$$\begin{aligned} \eta(k) &= \begin{bmatrix} x(k) \\ \widehat{x}(k) \end{bmatrix}, \\ \overline{A} &= \begin{bmatrix} A & BC_K \\ \overline{\beta}B_K C & A_K \end{bmatrix}, \\ \widetilde{A} &= \begin{bmatrix} 0 & 0 \\ B_K C & 0 \end{bmatrix}, \\ \overline{A}_{di} &= \begin{bmatrix} \overline{b}_i A_d \\ 0 \end{bmatrix}, \\ \widetilde{A}_{di} &= \begin{bmatrix} A_d \\ 0 \end{bmatrix}, \\ \overline{C} &= \begin{bmatrix} \overline{\alpha}I \\ 0 \end{bmatrix}, \\ \widetilde{C} &= \begin{bmatrix} I \\ 0 \end{bmatrix}, \\ \overline{D} &= \begin{bmatrix} D_1 \\ \overline{\beta}B_K D_2 \end{bmatrix}, \\ \widetilde{D} &= \begin{bmatrix} 0 \\ B_K D_2 \end{bmatrix}, \\ H &= \begin{bmatrix} I \\ 0 \end{bmatrix}^T, \\ \overline{L} &= \begin{bmatrix} L \\ 0 \end{bmatrix}^T, \end{aligned} \tag{12}$$

In this paper, our goal is to design a full-order output feedback controller to ensure that the corresponding closed-loop dynamic system is SFTB and with a desired H_∞ perfoamance. The definitions of SFTB and SFTH_∞B and a lemma are introduced before sequel.

Definition 3 (SFTB). Given positive numbers $c_1, c_2 (c_1 < c_2), \delta$, N, and a symmetric positive definite matrix R, the closed-loop system (11) is stochastic finite-time boundedness w.r.t. (c_1, c_2, δ, R, N) if

$$\begin{aligned} &\sup_{i \in \{-d_M, -d_M+1, \cdots -1, 0\}} \mathbb{E}\left\{\eta(i)^T R\eta(i)\right\} \leq c_1 \Longrightarrow \\ &\qquad \mathbb{E}\left\{\eta(k)^T R\eta(k)\right\} \leq c_2, \\ &\qquad\qquad k \in \{1, \ldots, N\}. \end{aligned} \tag{13}$$

Definition 4 (SFTH_∞B). Given positive numbers $\gamma, c_1, c_2 (c_1 < c_2), \delta, N$, and a symmetric positive definite matrix R, the closed-loop system (11) is stochastic finite-time H_∞ boundedness w.r.t. $(c_1, c_2, \delta, \gamma, R, N)$ if the closed-loop system (11) is

SFTB w.r.t. (c_1, c_2, δ, R, N) and under the zero-initial condition the output $z(k)$ satisfies

$$\mathbb{E}\left\{\sum_{s=0}^{N} z^T(s)\, z(s)\right\} \le \gamma^2 \sum_{s=0}^{N} \omega^T(s)\,\omega(s) \tag{14}$$

for all $\omega(k)$ satisfying Assumption 2.

Lemma 5 ([23] (Schur complement)). *Given constant matrices $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$, then $\mathcal{S}_1 + \mathcal{S}_3^T \mathcal{S}_2^{-1} \mathcal{S}_3 < 0$ if and only if*

$$\begin{aligned} &\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^T \\ * & -\mathcal{S}_2 \end{bmatrix} < 0 \\ \text{or} \quad &\begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ * & \mathcal{S}_1, \end{bmatrix} < 0, \end{aligned} \tag{15}$$

where $\mathcal{S}_1 = \mathcal{S}_1^T$ and $\mathcal{S}_2 = \mathcal{S}_2^T > 0$.

3. Main Result

In this section, we first give sufficient conditions for the SFTB of closed-loop system (11) and then discuss the design of dynamic output feedback controller (10).

3.1. Stochastic Finite-Time Boundedness Analysis. The following theorem provides sufficient conditions for the SFTB of closed-loop system (11) by employing Lyapunov-Krasovskii functional approach.

Theorem 6. *Given positive constants $c_1, c_2 (c_1 < c_2), \delta, N$, a symmetric positive-definite matrix R, closed-loop system (11) is SFTB w.r.t. (c_1, c_2, δ, R, N) if, for a scalar $\mu \ge 1$, there exist symmetric positive definite matrices P, Q_i $(i = 1, \ldots, q)$ and positive scalars λ_j $(j = 1, 2, 3)$ such that the following inequalities hold:*

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ * & \Sigma_{22} & \Sigma_{23} \\ * & * & \Sigma_{33} \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} r\delta - c_2\mu\lambda_1 & c_1\mu^{N+1}\lambda_2 & \rho\lambda_3 \\ * & -c_1\mu^{N+1}\lambda_2 & 0 \\ * & * & -\rho\lambda_3 \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{aligned}
\Sigma_{11} &= \operatorname{diag}\left\{-\mu P, -\mu^{d_m} Q_1, \ldots, -\mu^{d_m} Q_q\right\}, \\
\Sigma_{22} &= \operatorname{diag}\left\{-I, -\frac{\gamma}{\mu^N} I\right\}, \\
\Sigma_{33} &= \operatorname{diag}\left\{-P, -P, -\widehat{Q}\right\}, \\
\Sigma_{12} &= \begin{bmatrix} -H^T \widetilde{R}_2 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \\
\Sigma_{13} &= \begin{bmatrix} \overline{A}^T P & \widetilde{\beta} \widetilde{A}^T P & H^T \widehat{Q} \\ \overline{A}_{d1}^T P & \widetilde{b}_1 \widetilde{A}_{d1}^T P & 0 \\ \vdots & \vdots & \vdots \\ \overline{A}_{dq}^T P & \widetilde{b}_q \widetilde{A}_{dq}^T P & 0 \end{bmatrix}, \\
\Sigma_{23} &= \begin{bmatrix} \overline{C}^T P & \widetilde{\alpha} \widetilde{C}^T P & 0 \\ \overline{D}^T P & \widetilde{\beta} \widetilde{D}^T P & 0 \end{bmatrix}, \\
\widehat{Q} &= (d_M - d_m + 1) \sum_{i=1}^{q} Q_i - \widetilde{R}_1, \\
\rho &:= c_1 q \mu^{N+d_M-1} \left[d_M \mu \right. \\
&\qquad \left. + \frac{d_M(d_M - 1) - d_m(d_m - 1)}{2} \right], \\
\overline{P} &= R^{-1/2} P R^{-1/2}, \\
\lambda_1 &= \lambda_{\min}\left(\overline{P}\right), \\
\lambda_2 &= \lambda_{\max}\left(\overline{P}\right), \\
\overline{Q}_i &= R^{-1/2} H^T Q_i H R^{-1/2}, \\
\lambda_3 &= \max_{i \in \{1, \ldots, q\}} \left\{\lambda_{\max}\left(\overline{Q}_i\right)\right\}, \\
\widetilde{R}_1 &= \frac{U_1^T U_2 + U_2^T U_1}{2}, \\
\widetilde{R}_2 &= -\frac{U_1^T + U_2^T}{2}.
\end{aligned} \tag{18}$$

Proof. Choose the following Lyapunov-Krasovskii functional candidate:

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{19}$$

where

$$\begin{aligned}
V_1(k) &= \eta^T(k) P \eta(k), \\
V_2(k) &= \sum_{j=1}^{q} \sum_{s=k-d_j(k)}^{k-1} \mu^{k-1-s} \eta^T(s) H^T Q_j H \eta(s), \\
V_3(k) &= \sum_{j=1}^{q} \sum_{s=-d_{M+2}}^{-d_m+1} \sum_{t=k+s-1}^{k-1} \mu^{k-1-t} \eta^T(t) H^T Q_j H \eta(t).
\end{aligned} \tag{20}$$

Let us assume that

$$\mathbb{E}\left\{\eta(k)^T R\eta(k)\right\} \le c_1, \quad \forall k=-d_M,-d_M+1,\ldots,0. \quad (21)$$

The following goal is to show that if conditions (16) and (17) hold, then

$$\mathbb{E}\left\{\eta(k)^T R\eta(k)\right\} \le c_2, \quad \forall k=1,\ldots,N. \quad (22)$$

First, calculating the difference variation of $V_i(k)$ $(i = 1,2,3)$, and taking the mathematical expectation, one has

$$\begin{aligned}\mathbb{E}\left\{V_1(k+1)-\mu V_1(k)\right\} &= \mathbb{E}\left\{\xi^T(k)\Phi^T P\Phi\xi(k)\right.\\ &\left.+\xi^T(k)\Pi^T P\Pi\xi(k)-\mu\eta^T(k)P\eta(k)\right\},\end{aligned} \quad (23)$$

where

$$\begin{aligned}&\xi^T(k)\\ &=\left[\eta(k)\;\; x(k-d_1(k))\;\;\cdots\;\; x\left(k-d_q(k)\right)\;\; f(x(k))\;\;\omega(k)\right],\\ &\Phi=\left[\overline{A}\;\;\overline{A}_{d1}\;\;\cdots\;\;\overline{A}_{dq}\;\;\overline{C}\;\;\overline{D}\right],\\ &\Pi=\left[\widetilde{\beta}\widetilde{A}\;\;\widetilde{b}_1\widetilde{A}_{d1}\;\;\cdots\;\;\widetilde{b}_q\widetilde{A}_{dq}\;\;\widetilde{\alpha}\widetilde{C}\;\;\widetilde{\beta}\widetilde{D}\right].\end{aligned} \quad (24)$$

Similarly, it can be obtained that

$$\begin{aligned}&\mathbb{E}\left\{V_2(k+1)-\mu V_2(k)\right\}\\ &=\mathbb{E}\left\{\sum_{j=1}^{q}\sum_{s=k+1-d_j(k+1)}^{k}\mu^{k-s}\eta^T(s)H^TQ_jH\eta(s)\right.\\ &\left.-\sum_{j=1}^{q}\sum_{s=k-d_j(k)}^{k-1}\mu^{k-s}\eta^T(s)H^TQ_jH\eta(s)\right\}\\ &=\sum_{j=1}^{q}\mathbb{E}\left\{\eta^T(k)H^TQ_jH\eta(k)\right.\\ &+\sum_{s=k+1-d_m}^{k-1}\mu^{k-s}\eta^T(s)Q_j\eta(s)\\ &+\sum_{s=k+1-d_j(k+1)}^{k-d_m}\mu^{k-s}\eta^T(s)Q_j\eta(s)\\ &-\sum_{s=k+1-d_j(k)}^{k-1}\mu^{k-s}\eta^T(s)Q_j\eta(s)\\ &\left.-\mu^{d_m}x^T\left(k-d_j(k)\right)Q_jx\left(k-d_j(k)\right)\right\}.\end{aligned} \quad (25)$$

Considering $d_m \le d_j(k) \le d_M$,

$$\begin{aligned}&\mathbb{E}\left\{\sum_{s=k+1-d_m}^{k-1}\mu^{k-s}\eta^T(s)Q_j\eta(s)\right.\\ &\left.-\sum_{s=k-d_j(k)+1}^{k-1}\mu^{k-s}\eta^T(s)Q_j\eta(s)\right\}\le 0.\end{aligned} \quad (26)$$

It follows from (25) that

$$\begin{aligned}&\mathbb{E}\left\{V_2(k+1)-\mu V_2(k)\right\}\\ &\le\mathbb{E}\left\{\sum_{j=1}^{q}\left[\eta^T(k)H^TQ_jH\eta(k)\right.\right.\\ &+\sum_{s=k+1-d_j(k+1)}^{k-d_m}\mu^{k-s}\eta^T(s)Q_j\eta(s)\\ &\left.\left.-\mu^{d_m}x^T\left(k-d_j(k)\right)Q_jx\left(k-d_j(k)\right)\right]\right\}.\end{aligned} \quad (27)$$

Similarly, we have

$$\begin{aligned}&\mathbb{E}\left\{V_3(k+1)-\mu V_3(k)\right\}\\ &\le\mathbb{E}\left\{\sum_{j=1}^{q}\left[\left(d_M-d_m\right)\eta^T(k)H^TQ_jH\eta(k)\right.\right.\\ &\left.\left.-\sum_{s=k-d_M+1}^{k-d_m}\mu^{k-s}\eta^T(s)H^TQ_jH\eta(s)\right]\right\}.\end{aligned} \quad (28)$$

It is obtained from (23), (27), and (28) that

$$\begin{aligned}\mathbb{E}\left\{V(k+1)-\mu V(k)\right\}&\le\mathbb{E}\left\{\xi^T(k)\Phi^TP\Phi\xi(k)\right.\\ &+\xi^T(k)\Pi^TP\Pi\xi(k)-\mu\eta^T(k)P\eta(k)\\ &+\left(d_M-d_m+1\right)\eta^T(k)\sum_{j=1}^{q}Q_j\eta(k)\\ &-\mu^{d_m}\sum_{j=1}^{q}x^T\left(k-d_j(k)\right)Q_jx\left(k-d_j(k)\right)\\ &+z^T(k)z(k)-\frac{\gamma}{\mu^N}\omega^T(k)\omega(k)-z^T(k)z(k)\\ &\left.+\frac{\gamma}{\mu^N}\omega^T(k)\omega(k)\right\}\\ &\le\mathbb{E}\left\{\xi^T(k)\left[\Phi^TP\Phi+\Pi^TP\Pi+\Delta\right]\xi(k)\right\},\end{aligned} \quad (29)$$

where $\Delta = \mathrm{diag}\{-\mu P + (d_M - d_m + 1)\sum_{i=1}^{q} H^TQ_iH, -\mu^{d_m}Q_1,\ldots,-\mu^{d_m}Q_q, 0, -(\gamma/\mu^N)I\}$.

Notice that (5) implies the following inequality:

$$\begin{bmatrix}\eta(k)\\ f(x(k))\end{bmatrix}^T\begin{bmatrix}H^T\widetilde{R}_1H & \widetilde{R}_2^TH\\ * & I\end{bmatrix}\begin{bmatrix}\eta(k)\\ f(x(k))\end{bmatrix}\le 0. \quad (30)$$

Combining (29) with (30), we have

$$\begin{aligned}&\mathbb{E}\left\{V(k+1)-\mu V(k)\right\}\\ &\le\mathbb{E}\left\{\xi^T(k)\left[\Phi^TP\Phi+\Pi^TP\Pi+\Theta\right]\xi(k)\right\}\\ &+\frac{\gamma}{\mu^N}\omega^T(k)\omega(k),\end{aligned} \quad (31)$$

where

$$\Theta = \begin{bmatrix} -\mu P + (d_M - d_m + 1)\sum_{i=1}^{q} H^T Q_i H - H^T \tilde{R}_1 H & 0 & \cdots & 0 & -H^T \tilde{R}_2 & 0 \\ * & -\mu^{d_m} Q_1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ * & * & \cdots & -\mu^{d_m} Q_q & 0 & 0 \\ * & * & \cdots & * & -I & 0 \\ * & * & \cdots & * & * & -\frac{\gamma}{\mu^N} \end{bmatrix}. \quad (32)$$

By applying Lemma 5 to (16), we deduce that $\Phi^T P\Phi + \Pi^T P\Pi + \Theta < 0$. Thus, we obtain from (31) that

$$\mathbb{E}\{V(k+1) - \mu V(k)\} \le \frac{\gamma}{\mu^N}\omega^T(k)\,\omega(k), \quad \forall k \in Z_+. \quad (33)$$

That is,

$$\mathbb{E}\{V(k+1)\} \le \mu\mathbb{E}\{V(k)\} + \frac{\gamma}{\mu^N}\omega^T(k)\,\omega(k), \quad \forall k \in Z_+. \quad (34)$$

It follows from (34) that

$$\mathbb{E}\{V(k)\} \le \mu\mathbb{E}\{V(k-1)\} + \frac{\gamma}{\mu^N}\omega^T(k-1)\,\omega(k-1), \quad \forall k \in Z_+. \quad (35)$$

By iteration and taking Assumption 2 into account, it can be obtained that

$$\begin{aligned} &\mathbb{E}\{V(k+1) - \mu V(k)\} \\ &\le \mu^k\mathbb{E}\{V(0)\} + \frac{\gamma}{\mu^N}\sum_{s=0}^{k-1}\mu^{k-1-s}\omega^T(s)\,\omega(s) \\ &\le \mu^N\mathbb{E}\{V(0)\} + \frac{\gamma}{\mu^N}\mu^{N-1}\sum_{s=0}^{N-1}\omega^T(s)\,\omega(s) \\ &\le \mu^N\mathbb{E}\{V(0)\} + \frac{\gamma}{\mu}\delta. \end{aligned} \quad (36)$$

Noting definition of λ_i $(i = 2, 3)$, we have

$$\begin{aligned} \mathbb{E}\{V(0)\} = \mathbb{E}\Bigg\{&\eta^T(0)\,P\eta(0) \\ &+ \sum_{j=1}^{q}\sum_{s=-d_j(0)}^{-1}\mu^{-1-s}\eta^T(s)\,H^T Q_j H\eta(s) \\ &+ \sum_{j=1}^{q}\sum_{s=-d_M+2}^{-d_m+1}\sum_{t=s-1}^{-1}\mu^{-1-t}\eta^T(t)\,H^T Q_j H\eta(t)\Bigg\} \\ &< \lambda_2\mathbb{E}\left\{\eta^T(0)\,R\eta(0)\right\} \\ &+ q\lambda_3\mu^{d_M-1}\sum_{s=-d_M}^{-1}\mathbb{E}\left\{\eta^T(s)\,R\eta(s)\right\} \\ &+ q\lambda_3\mu^{d_M+2}\sum_{s=-d_M-2}^{-d_m+1}\sum_{t=-1+s}^{-1}\mathbb{E}\left\{\eta^T(t)\,R\eta(t)\right\} < \Bigg[\lambda_2 \\ &+ q\lambda_3 d_M\mu^{d_M-1} \\ &+ q\lambda_3\frac{d_M(d_M-1) - d_m(d_m-1)}{2}\mu^{d_M-2}\Bigg]c_1. \end{aligned} \quad (37)$$

By taking (36) and (37) into account, we have

$$\mathbb{E}\{V(k+1) - \mu V(k)\} \le \mu^N\sigma + \frac{\gamma}{\mu}\delta, \quad \forall k \in Z_+, \quad (38)$$

where $\sigma := [\lambda_2 + q\lambda_3 d_M\mu^{d_M-1} + q\lambda_3((d_M(d_M-1) - d_m(d_m-1))/2)\mu^{d_M-2}]c_1$.

On the other hand, also from the definition of λ_i $(i = 1, 2, 3)$, we have

$$\mathbb{E}\{V(k+1) - \mu V(k)\} \ge \lambda_1\mathbb{E}\left\{\eta^T(k)\,R\eta(k)\right\}, \quad \forall k \in Z_+. \quad (39)$$

Consequently, we get from (38) and (39) that

$$\mathbb{E}\left\{\eta^T(k)\,R\eta(k)\right\} < \frac{1}{\lambda_1\mu}\left(\mu^{N+1}\sigma + \gamma\delta\right). \quad (40)$$

Note that

$$\begin{aligned} &\mu^{N+1}\sigma + \gamma\delta - c_2\mu\lambda_1 < 0 \Longleftrightarrow \\ &\gamma\delta - c_2\mu\lambda_1 + c_1\mu^{N+1}\lambda_2 \\ &\quad + c_1 q\mu^{N+d_M-1}\left[d_M\mu + \frac{d_M(d_M-1) - d_m(d_m-1)}{2}\right]\lambda_3 \\ &\quad < 0 \Longleftrightarrow \\ &\gamma\delta - c_2\mu\lambda_1 + c_1\mu^{N+1}\lambda_2 + \rho\lambda_3 < 0 \Longleftrightarrow \\ &\gamma\delta - c_2\mu\lambda_1 \end{aligned}$$

$$
\begin{aligned}
&+ \begin{bmatrix} c_1\mu^{N+1}\lambda_2 & \rho\lambda_3 \end{bmatrix} \begin{bmatrix} \left(c_1\mu^{N+1}\lambda_2\right)^{-1} & 0 \\ 0 & \left(\rho\lambda_3\right)^{-1} \end{bmatrix} \begin{bmatrix} c_1\mu^{N+1}\lambda_2 \\ \rho\lambda_3 \end{bmatrix} \\
&< 0 \Longleftrightarrow \\
&\gamma\delta - c_2\mu\lambda_1 \\
&+ \begin{bmatrix} c_1\mu^{N+1}\lambda_2 & \rho\lambda_3 \end{bmatrix} \begin{bmatrix} c_1\mu^{N+1}\lambda_2 & 0 \\ 0 & \rho\lambda_3 \end{bmatrix}^{-1} \begin{bmatrix} c_1\mu^{N+1}\lambda_2 \\ \rho\lambda_3 \end{bmatrix} < 0.
\end{aligned} \tag{41}
$$

By Lemma 5, (41) is equivalent to LMI (23); it can be verified that

$$
\mathbb{E}\left\{\eta^T(k) R\eta(k)\right\} < \frac{1}{\lambda_1\mu}\left(\mu^{N+1}\sigma + \gamma\delta\right) < c_2. \tag{42}
$$

Therefore, according to Definition 3, closed-loop system (11) is SFTB. The above discussion completes the proof of Theorem 6. □

Remark 7. If nonlinearities and perturbation are not considered in system (11), that is, $f(k) = 0$ and $\omega(k) = 0$, the SFTB problem of system (11) is deduced by analyzing the SFTB of the following system:

$$
\begin{aligned}
\eta(k+1) &= \overline{A}\eta(k) + \left(\beta(k) - \overline{\beta}\right)\widetilde{A}\eta(k) \\
&\quad + \sum_{i=1}^{q} \overline{A}_{di} H\eta\left(k - d_i(k)\right) \\
&\quad + \sum_{i=1}^{q} \widetilde{A}_{di} H\eta\left(k - d_i(k)\right) \\
z(k) &= \overline{L}\eta(k),
\end{aligned} \tag{43}
$$

where system parameters are the same as before.

The following criterion for system (43) is given based on Theorem 6.

Corollary 8. *Given positive constants $c_1, c_2, (c_1 < c_2), \delta, N$, a symmetric positive-definite matrix R, closed-loop system (43) is SFTB w.r.t.(c_1, c_2, δ, R, N) if, for a scalar $\mu \geq 1$, there exist symmetric positive definite matrices P, Q_i and positive scalars λ_i $(i = 1, 2, 3)$ such that the following inequalities hold:*

$$
\Theta = \begin{bmatrix}
-\mu P & 0 & 0 & 0 & \overline{A}^T P & \widetilde{\beta}\widetilde{A}^T P & H^T\widehat{Q} \\
* & -\mu^{d_m} Q_1 & 0 & 0 & \overline{A}_{d1}^T P & \widetilde{b}_1\widetilde{A}_{d1}^T P & 0 \\
* & * & \ddots & 0 & \vdots & \vdots & 0 \\
* & * & * & -\mu^{d_m} Q_q & \overline{A}_{dq}^T P & \widetilde{b}_q\widetilde{A}_{dq}^T P & 0 \\
* & * & * & * & -P & 0 & 0 \\
* & * & * & * & * & -P & 0 \\
* & * & * & * & * & * & -\widehat{Q}
\end{bmatrix} < 0,
$$

$$
\begin{bmatrix}
-c_2\mu\lambda_1 & c_1\mu^{N+1}\lambda_2 & \rho\lambda_3 \\
* & -c_1\mu^{N+1}\lambda_2 & 0 \\
* & * & -\rho\lambda_3
\end{bmatrix} < 0. \tag{44}
$$

3.2. Stochastic Finite-Time H_∞ Boundedness Analysis. In this subsection, we will provide sufficient condition to ensure SFTH_∞B of closed-loop system (11) based on Theorem 6.

Theorem 9. *Given positive constants c_1, c_2 $(c_1 < c_2)$, δ, γ, N, a symmetric positive-definite matrix R, closed-loop system (11) is SFTH_∞B w.r.t. $(c_1, c_2, \delta, \gamma, R, N)$ if, for a scalar $\mu \geq 1$, there exist symmetric positive definite matrices P and Q_i $(i = 1, \ldots, q)$, and positive scalars λ_j $(j = 1, 2, 3)$ such that the following inequalities hold:*

$$
\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} \\
* & \Omega_{22} & \Omega_{23} \\
* & * & \Omega_{33}
\end{bmatrix} < 0, \tag{45}
$$

$$
\begin{bmatrix}
r\delta - c_2\mu\lambda_1 & c_1\mu^{N+1}\lambda_2 & \rho\lambda_3 \\
* & -c_1\mu^{N+1}\lambda_2 & 0 \\
* & * & -\rho\lambda_3
\end{bmatrix} < 0, \tag{46}
$$

where

$$
\begin{aligned}
\Omega_{11} &= \operatorname{diag}\left\{-\mu P, -\mu^{d_m} Q_1, \ldots, -\mu^{d_m} Q_q\right\}, \\
\Omega_{22} &= \operatorname{diag}\left\{-I, -\frac{\gamma}{\mu^N} I\right\}, \\
\Omega_{33} &= \operatorname{diag}\left\{-P, -P, -\widehat{Q}, -I\right\} \\
\Omega_{12} &= \begin{bmatrix} -H^T\widetilde{R}_2 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}, \\
\Omega_{13} &= \begin{bmatrix}
\overline{A}^T P & \widetilde{\beta}\widetilde{A}^T P & H^T\widehat{Q} & L^T \\
\overline{A}_{d1}^T P & \widetilde{b}_1\widetilde{A}_{d1}^T P & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots \\
\overline{A}_{dq}^T P & \widetilde{b}_q\widetilde{A}_{dq}^T P & 0 & 0
\end{bmatrix}, \\
\Omega_{23} &= \begin{bmatrix}
\overline{C}^T P & \widetilde{\alpha}\widetilde{C}^T P & 0 & 0 \\
\overline{D}^T P & \widetilde{\beta}\widetilde{D}^T P & 0 & 0
\end{bmatrix}.
\end{aligned} \tag{47}
$$

Other parameters are shown in Theorem 6.

Proof. Obviously LMI (45) implies LMI (16); according to Theorem 6, we can easily conclude that the closed-loop system (11) is SFTB w.r.t. (c_1, c_2, δ, R, N).

In the following, we shall prove that inequality (14) holds. By introducing the same Lyapunov-Krasovskii functional, we can get from (33)

$$\mathbb{E}\{V(k+1)\} \leq \mu\mathbb{E}\{V(k)\} + \frac{\gamma}{\mu^N}\omega^T(k)\omega(k), \quad \forall k \in Z_+. \tag{48}$$

And by iteration, we obtain that

$$\begin{aligned}\mathbb{E}\{V(k)\} &\leq \mu^{k-1}\mathbb{E}\{V(0)\} \\ &+ \sum_{s=0}^{k-1}\mu^{k-1-s}\left[\frac{\gamma}{\mu^N}\omega^T(s)\omega(s) - \mathbb{E}\left\{z^T(s)z(s)\right\}\right].\end{aligned} \tag{49}$$

Under the zero initial condition $\mathbb{E}\{V(0)\} = 0$, and noting that $\mathbb{E}\{V(k)\} \geq 0$, $\forall k \in \{1, 2, \ldots, N\}$, we get

$$\sum_{s=0}^{k-1}\mu^{k-1-s}\mathbb{E}\left\{z^T(s)z(s)\right\} \leq \sum_{s=0}^{k-1}\mu^{k-1-s}\left[\frac{\gamma}{\mu^N}\omega^T(s)\omega(s)\right]. \tag{50}$$

Letting $k = N + 1$, we then have

$$\sum_{s=0}^{N}\mu^{N-s}\mathbb{E}\left\{z^T(s)z(s)\right\} \leq \sum_{s=0}^{N}\mu^{N-s}\left[\frac{\gamma}{\mu^N}\omega^T(s)\omega(s)\right]. \tag{51}$$

Notice that $1 \leq \mu^{N-s} \leq \mu^N$, $\forall s \in \{0, 1, \ldots, N\}$; (51) immediately yields

$$\mathbb{E}\left\{\sum_{s=0}^{N}z^T(s)z(s)\right\} \leq \gamma\sum_{s=0}^{N}\omega^T(s)\omega(s). \tag{52}$$

which implies that system (11) is SFTH_∞B. This completes proof of Theorem 9. □

3.3. Stochastic Finite-Time H_∞ Output Feedback Control. In this subsection, we will deal with the problem of the design of output-feedback controller for system (1).

Theorem 10. *Consider the discrete-time NCS (1) with randomly occuring multi-distrbuted delays and missing measurement. For given positive constants c_1, $c_2 (c_1 < c_2)$, δ and N, and a symmetric positive-definite matrix $R = \operatorname{diag}\{R_1, R_2\}$, closed-loop system (11) is SFTH_∞B w.r.t. $(c_1, c_2, \delta, \gamma, R, N)$ if, for a scalar $\mu \geq 1$, there exist symmetric positive definite matrices U, Y, Q_i $(i = 1, \ldots, q)$, matrices Ω_j $(j = 1, 2, 3)$, and positive scalars λ'_t $(t = 1, 2, 3,)$, such that the following inequalities hold:*

$$\begin{bmatrix} J_1 & 0 & H_1^T & H_2^T & H_5^T \\ * & J_2 & 0 & H_3^T & 0 \\ * & * & J_3 & H_4^T & 0 \\ * & * & * & J_4 & 0 \\ * & * & * & * & J_5 \end{bmatrix} < 0, \tag{53}$$

$$\begin{bmatrix} r\delta - c_2\mu\lambda'_1 & c_1\mu^{N+1}\lambda'_2 & \rho\lambda'_3 \\ * & -c_1\mu^{N+1}\lambda'_2 & 0 \\ * & * & -\rho\lambda'_3 \end{bmatrix} < 0, \tag{54}$$

$$\begin{gathered}\lambda'_1 R_1 < Y < \lambda'_2 R_1, \\ 0 < Q_i < \lambda'_3 R_1, \\ (i = 1, \ldots, q),\end{gathered} \tag{55}$$

where

$$\begin{aligned}
J_1 &= \begin{bmatrix} -\mu U & -\mu I \\ -\mu I & -\mu Y \end{bmatrix}, \\
J_2 &= \operatorname{diag}\left\{-\mu^{d_m}Q_1, \ldots, -\mu^{d_m}Q_q\right\}, \\
J_3 &= \operatorname{diag}\left\{-I, -\frac{\gamma}{\mu^N}I\right\}, \\
J_4 &= \begin{bmatrix} -U & -I & 0 & 0 \\ -I & -Y & 0 & 0 \\ 0 & 0 & -U & -I \\ 0 & 0 & -I & -Y \end{bmatrix}, \\
J_5 &= \operatorname{diag}\left\{-I, -\widehat{Q}\right\}, \\
H_1^T &= \begin{bmatrix} U\widetilde{R}_2^T & 0 \\ \widetilde{R}_2^T & 0 \end{bmatrix}, \\
H_2^T &= \begin{bmatrix} UA^T + \Omega_3 B^T & \Omega_1 & 0 & \widetilde{\beta}UC^T\Omega_2^T \\ A^T & A^T R + \overline{\beta}C^T\Omega_2 & 0 & C^T\Omega_2^T \end{bmatrix}, \\
H_3^T &= \begin{bmatrix} \overline{b}_1 A_d^T & \overline{b}_1 A_d^T Y & \widetilde{b}_1 A_d^T & \widetilde{b}_1 A_d^T Y \\ \vdots & \vdots & \vdots & \vdots \\ \overline{b}_q A_d^T & \overline{b}_q A_d^T Y & \widetilde{b}_q A_d^T & \widetilde{b}_q A_d^T Y \end{bmatrix}, \\
H_4^T &= \begin{bmatrix} \overline{\alpha}I & \overline{\alpha}Y & \widetilde{\alpha}I & \widetilde{\alpha}Y \\ D_1^T & D_1^T W + \overline{\beta}D_2^T\Omega_2 & 0 & D_2^T\Omega_2^T \end{bmatrix}, \\
H_5^T &= \begin{bmatrix} UL^T & U\widehat{Q} \\ L^T & I \end{bmatrix}.
\end{aligned} \tag{56}$$

R_1 and R_2 are symmetric positive matrices.

In this case, the parameters of the output feedback controller can be designed as

$$\begin{aligned}
A_K &= X_{12}^{-1}\left(\Omega_1 - YAU - \overline{\beta}\Omega_2 CU - YB\Omega_3\right)Y_{12}^{-T}, \\
B_K &= X_{12}^{-1}\Omega_2, \\
C_K &= Y_{12}^{-T}\Omega_3,
\end{aligned} \tag{57}$$

where X_{12} and Y_{12} are any nonsingular matrices satisfying $X_{12}Y_{12}^T = I - UY$.

Proof. Evidently, from Theorem 9, system (11) is SFTH_∞B if, for a scalar $\mu \geq 1$, LMIs (45) and (46) hold. To do this, let P and P^{-1} be as follows:

$$P = \begin{bmatrix} Y & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} S^{-1} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}, \tag{58}$$

where the partitioning of P and P^{-1} is compatible with that of $\overline{A}$, $\widetilde{A}$, $\overline{A}_{di}$, $\widetilde{A}_{di}$ and X_{22} and Y_{22} are symmetric positive matrices.

Define

$$T_1 = \begin{bmatrix} S^{-1} & I \\ Y_{12}^T & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} I & Y \\ 0 & X_{12}^T \end{bmatrix}, \tag{59}$$

and this implies that

$$PT_1 = T_2, \quad T_1^T P T_1 = T_1^T T_2. \tag{60}$$

Thus

$$P = T_2 T_1^{-1}, \quad X_{22} = Y_{12}^{-1} S^{-1} (Y - S) S^{-1} Y_{12}^{-T} > 0. \tag{61}$$

It is clear that, by Lemma 5, if LMI (53) is feasible, then we have $J_1 < 0$; thus $Y - S < 0$ and therefore $I - YS^{-1}$ is nonsingular, which implies that $P > 0$.

First, it can be verified that the LMI (53) can be rewritten as

$$\Sigma = \begin{bmatrix} \Sigma_{11} & 0 & \Sigma_{13} & 0 & \Sigma_{15} & \Sigma_{16} & \Sigma_{17} \\ * & \Sigma_{22} & 0 & 0 & \Sigma_{25} & \Sigma_{26} & 0 \\ * & * & \Sigma_{33} & 0 & \Sigma_{35} & \Sigma_{36} & 0 \\ * & * & * & \Sigma_{44} & \Sigma_{45} & \Sigma_{46} & 0 \\ * & * & * & * & \Sigma_{55} & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & 0 \\ * & * & * & * & * & * & \Sigma_{77} \end{bmatrix} < 0, \tag{62}$$

where A_K, B_K, C_K are defined in (57),

$$\begin{aligned}
\Sigma_{11} &= \begin{bmatrix} -\mu S^{-1} & -\mu I \\ * & -\mu Y \end{bmatrix}, \\
\Sigma_{22} &= \operatorname{diag}\left\{-\mu^{d_m} Q_1, \ldots, -\mu^{d_m} Q_q\right\}, \\
\Sigma_{33} &= -I, \\
\Sigma_{44} &= -\frac{\gamma}{\mu^N}, \\
\Sigma_{55} = \Sigma_{66} &= \begin{bmatrix} -S^{-1} & -I \\ * & -Y \end{bmatrix}, \\
\Sigma_{77} &= \begin{bmatrix} -I & 0 \\ * & -\widehat{Q} \end{bmatrix}, \\
\Sigma_{13} &= \begin{bmatrix} S^{-1}\widetilde{R}_2 \\ \widetilde{R}_2 \end{bmatrix}, \\
\Sigma_{15} &= \begin{bmatrix} S^{-1}A^T + \Omega_2^T B^T & \Omega_1 \\ A^T & A^T Y + \overline{\beta} C^T \Omega_2^T \end{bmatrix}, \\
\Sigma_{16} &= \begin{bmatrix} 0 & \widetilde{\beta} S^{-1} C^T \Omega_2^T \\ 0 & C^T \Omega_2^T \end{bmatrix}, \\
\Sigma_{17} &= \begin{bmatrix} S^{-1} L^T & S^{-1}\widehat{Q} \\ L^T & I \end{bmatrix}, \\
\Sigma_{25} &= \begin{bmatrix} \overline{b}_1 A_d^T & \overline{b}_1 A_d^T Y \\ \vdots & \vdots \\ \overline{b}_q A_d^T & \overline{b}_q A_d^T Y \end{bmatrix}, \\
\Sigma_{26} &= \begin{bmatrix} \widetilde{b}_1 A_d^T & \widetilde{b}_1 A_d^T Y \\ \vdots & \vdots \\ \widetilde{b}_q A_d^T & \widetilde{b}_q A_d^T Y \end{bmatrix}, \\
\Sigma_{35} &= \left[\overline{\alpha} I \;\; \overline{\alpha} Y\right], \\
\Sigma_{36} &= \left[\widetilde{\alpha} I \;\; \widetilde{\alpha} Y\right], \\
\Sigma_{45} &= \left[D_1^T \;\; D_1^T Y + \overline{\beta} D_2^T \Omega_2^T\right], \\
\Sigma_{46} &= \left[0 \;\; \widetilde{\alpha} D_2^T \Omega_2^T\right].
\end{aligned} \tag{63}$$

That is,

$$\Sigma' = \begin{bmatrix} \Sigma'_{11} & \Sigma'_{12} & \Sigma'_{13} & \Sigma'_{14} \\ * & \Sigma'_{22} & \Sigma'_{23} & 0 \\ * & * & \Sigma'_{33} & 0 \\ * & * & * & \Sigma'_{44} \end{bmatrix} < 0, \tag{64}$$

where

$$\begin{aligned}
\Sigma'_{11} &= \operatorname{diag}\left\{-\mu T_1^T P T_1, -\mu^{d_m} Q_1, \ldots, -\mu^{d_m} Q_q\right\}, \\
\Sigma'_{22} &= \operatorname{diag}\left\{-I, -\frac{\gamma}{\mu^N}\right\}, \\
\Sigma'_{33} &= \operatorname{diag}\left\{-T_1^T P^{-1} T_1, -T_1^T P^{-1} T_1\right\}, \\
\Sigma'_{44} &= \operatorname{diag}\left\{-I, -\widehat{Q}\right\},
\end{aligned}$$

$$\Sigma_{12}' = \begin{bmatrix} -\widetilde{R}_2 H T_1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}^T,$$

$$\Sigma_{13}' = \begin{bmatrix} T_1^T \overline{A}^T T_2 & \widetilde{\beta} T_1^T \widetilde{A}^T T_2 \\ \overline{A}_{d1}^T T_2 & \widetilde{A}_{d1}^T T_2 \\ \vdots & \vdots \\ \overline{A}_{dq}^T T_2 & \widetilde{A}_{dq}^T T_2 \end{bmatrix},$$

$$\Sigma_{14}' = \begin{bmatrix} T_1^T L^T & T_1^T H^T \widehat{Q} \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix},$$

$$\Sigma_{23}' = \begin{bmatrix} \overline{C}^T T_2 & \widetilde{C}^T Y \\ \overline{D}^T T_2 & \widetilde{D}^T Y \end{bmatrix}. \tag{65}$$

Now letting $U = S^{-1}$ and applying the congruence transformations $\operatorname{diag}\{T_1^{-T}, I, \ldots, I, I, T_2^{-T}, T_2^{-T}, I, I\}$ to (64), we can obtain LMI (45).

In addition, since Y and Q_i are fringed matrices of P and $\widehat{Q}_i = H^T Q_i H$, by the property of fringed matrices, we have

$$\begin{aligned} &\lambda_{\max}(Y) \geq \lambda_{\max}(P), \\ &\lambda_{\min}(Y) \leq \lambda_{\min}(P), \\ &\lambda_{\max}(Q_i) \geq \lambda_{\max}(\widehat{Q}_i), \\ &\lambda_{\min}(Q_i) \leq \lambda_{\min}(\widehat{Q}_i). \end{aligned} \tag{66}$$

Thus, (46) follows from inequalities (54) and (55). This completes the proof □

Remark 11. Being different from literature [25], on the one hand, we consider NCSs with the stochastic and nonlinearity. On the other hand, state feedback controller is designed in [25]. However when not all the state variables are available, the results in [25] cannot be applicable. In this paper, output feedback controllers are designed in the stochastic context.

Remark 12. In order to design a controller independent of the matrix X_{12}, we can make $X_{12} = Y$; then A_K, B_K, and C_K become

$$\begin{aligned} &A_K \\ &\quad = Y^{-1}\left(\Omega_1 - YAU - \overline{\beta}\Omega_2 CU - YB\Omega_3\right)(I - UY)^{-1} Y, \\ &B_K = Y^{-1}\Omega_2, \\ &C_K = (I - UY)^{-1} Y\Omega_3. \end{aligned} \tag{67}$$

Remark 13. The feasibility of Theorem 10 can be represented by the following basic LMIs with a parameter μ:

$$\begin{aligned} &\min \quad (c_2 + \gamma) \\ &\text{s.t.} \quad (53), (54) \text{ and } (55). \end{aligned} \tag{68}$$

Remark 14. So far, we have discussed problem of SFTH_∞B analysis for a class of nonlinear discrete networked systems with randomly occurring multi-distributed delays and missing measurements. Also, some sufficient conditions are given to reflect the impacts from the involved phenomena. When the error analysis of the proposed theoretical result becomes a concern, additional efforts will be made with some criterion which will appear in the near future.

4. Numerical Example

In this section, a numerical example is used to verify the validity of the controller design approach proposed. Consider a discrete-time NCS described by system (1) and its parameters are selected as

$$\begin{aligned} A &= \begin{bmatrix} 0.4 & -0.1 \\ -0.1 & 0.035 \end{bmatrix}, \\ A_d &= \begin{bmatrix} -0.125 & 0.1 \\ 0.05 & -0.275 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, \\ C &= \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix}^T, \\ D_1 &= \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \\ D_2 &= 0.2, \\ L &= \begin{bmatrix} -0.1 \\ 0.5 \end{bmatrix}^T, \\ U_1 &= \begin{bmatrix} -0.3 & 0 \\ 0 & 0 \end{bmatrix}, \\ U_2 &= \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}, \\ R_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned} \tag{69}$$

We let $q = 2$ and the stochastic parameters are chosen as

$$\begin{aligned} \mathbb{E}\{\alpha(k)\} &= \overline{\alpha} = 0.8, \\ \mathbb{E}\{\beta(k)\} &= \overline{\beta} = 0.8, \\ \mathbb{E}\{b_1(k)\} &= \overline{b}_1 = 0.6, \\ \mathbb{E}\{b_2(k)\} &= \overline{b}_2 = 0.8. \end{aligned} \tag{70}$$

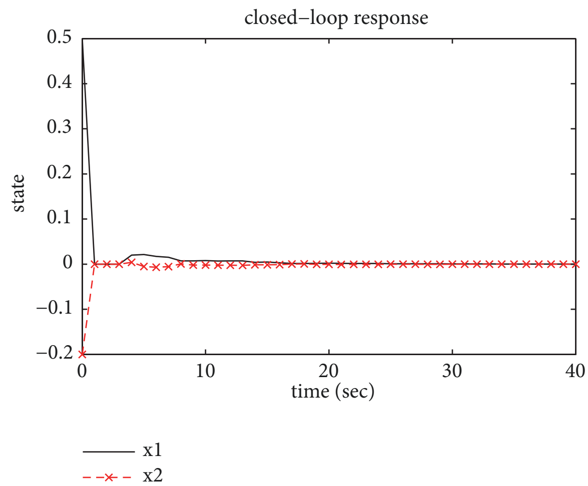

Figure 2: The trajectory of state $x(k)$ when $\mathbb{E}\{\alpha(k)\} = 0.8$, $\mathbb{E}\{\beta(k)\} = 0.8$, $\mathbb{E}\{b_1(k)\} = 0.6$, $\mathbb{E}\{b_2(k)\} = 0.8$.

Moreover, the time-varying delays satisfy $1 \leq d_i(k) \leq 4 (i = 1, 2)$, the exogenous disturbance $\omega(k)$ is chosen as $\omega(k) = 1/(0.1 + k^2)$, and the nonlinear function $f(x(k))$ is selected as $f(x(k)) = \left[\begin{smallmatrix} \tanh(-0.3x_1(k)) \\ \tanh(0.5x_2(k)) \end{smallmatrix} \right]$.

For given $N = 40$, $\delta = 1$, $c_1 = 1$, and $\mu = 1.002$, it can be easily checked that $\mathbb{E}\{x^T(k)R_1x(k)\} \leq c_1$ is satisfied under the initial condition $x(0) = [0.5 \ \ -0.2]^T$ and $\hat{x}(0) = [0 \ \ 0]^T$.

By solving the minimization problem involving LMIs conditions (53) and (54) and using LMI toolbox in Matlab, we can obtain the optimal value $c_{2\min} = 39.5855$ and the optimal H_∞ performance $\gamma_{\min} = 0.0779$. By Theorem 10, the resulting controller gain matrices are expressed as

$$A_K = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$
$$B_K = \begin{bmatrix} 0.0163 \\ 00350 \end{bmatrix}, \qquad (71)$$
$$C_K = \begin{bmatrix} 2.1100 \\ 2.1324 \end{bmatrix}^T.$$

Figures 2 and 3 show the system state trajectories and the curve of $\mathbb{E}\{x^T(k)R_1x(k)\}$. From Figures 2 and 3, we can get that all the state variables converge to zero quickly in the finite-time, which means that system (1) is stochastic finite-time stable. Moreover, for all $x(0)$ that satisfy the initial condition $\mathbb{E}\{x^T(0)R_1x(0)\} \leq c_1$, we can get that the system state trajectory cannot exceed the given optimal value $c_{2\min} = 39.5855$, which means the obtained output-feedback controller can guarantee that system (11) is SFTB with prescribed properties. This proves the validity of the theoretical results.

In the following, for further assessment of the control method, we consider $\mathbb{E}\{\alpha(k)\} = \overline{\alpha} = 0.3$, $\mathbb{E}\{\beta(k)\} = \overline{\beta} = 0.6$, $\mathbb{E}\{b_1(k)\} = \overline{b}_1 = 0.5$, $\mathbb{E}\{b_2(k)\} = \overline{b}_2 = 0.5$. Other parameters are chosen based on the values represented above.

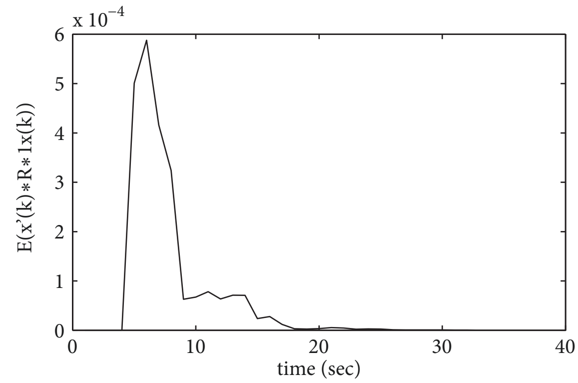

Figure 3: The curve of $\mathbb{E}\{x^T(k)R_1x(k)\}$.

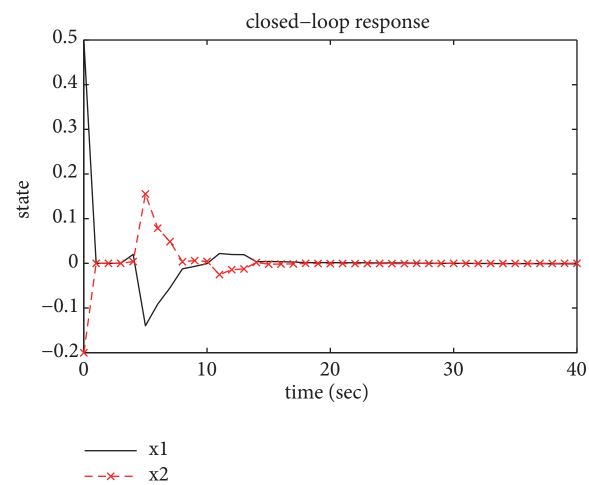

Figure 4: The trajectory of state $x(k)$ when $\mathbb{E}\{\alpha(k)\} = 0.3$, $\mathbb{E}\{\beta(k)\} = 0.6$, $\mathbb{E}\{b_1(k)\} = \mathbb{E}\{b_2(k)\} = 0.5$.

By solving the minimization problem, we can get the optimal values $c_{2\min} = 39.6743$, $\gamma_{\min} = 0.1313$, and the resulting controller gain matrices are

$$A_K = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$
$$B_K = \begin{bmatrix} -0.1120 \\ 0.2499 \end{bmatrix}, \qquad (72)$$
$$C_K = \begin{bmatrix} 1.3480 \\ 2.1717 \end{bmatrix}^T.$$

Simulation results are illustrated in Figures 4 and 5. From Figures 4 and 5, it is apparent that the state responses satisfy $\mathbb{E}\{x^T(k)R_1x(k)\} \leq 39.6743$, $k \in \{1, \ldots, N\}$, which means that the system is also SFTB in this case. Compared with the first case, we can get that when randomly occurring multi-distributed delays and missing measurements are more serious, the state trajectory of the system converges to zero for a long time. In addition, the maximum value of $\mathbb{E}\{x^T(k)R_1x(k)\}$ obtained in the second case is much greater

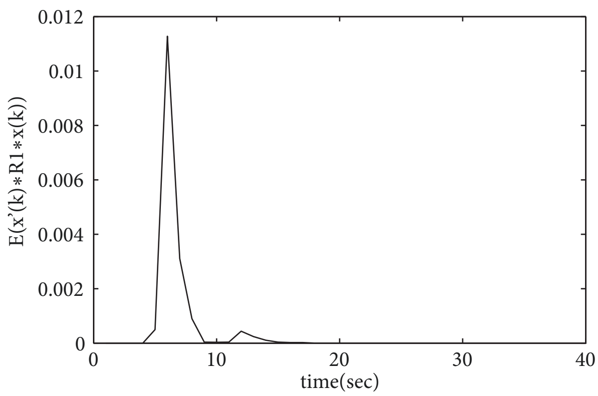

Figure 5: The curve of $\mathbb{E}\{x^T(k)R_1x(k)\}$.

than that in the first case. In short, from the above results, randomly occurring multi-distributed delays and missing measurements have an important effect on the stability of the system. In addition, the adequate performance of NCS (1) is achieved by using the proposed controller, which confirms that the STFH_∞B of the NCS (1) is secured in this paper.

5. Conclusions

This paper has investigated the SFTH_∞B control problem for a class of discrete-time NCS. In this addressed problem, the effects of randomly occurring multi-distributed delays and missing measurement have been taken into account simultaneously. Firstly, the concept of FTB has been extended to SFTB in the discrete-time NCSs. Then, sufficient conditions with less conservative have been established to ensure the existence of the output feedback controller by constructing a set of improved Lyapunov-Krasovskii functional with power function μ^{k-1-s}. And the obtained conditions are expressed based on LMI. The gain of controller has been designed by solving LMIs. Lastly, a numerical example is given to show the validity of the verified results. It is worth mentioning that the main results obtained in this paper can be extended to the stochastic finite-time passive or stochastic finite-time dissipative control problems. The results will appear in the near future. In addition, the methods proposed in this paper can be further used to deal with SFTH_∞B control problems for discrete-time NCS with more complicated network-induced phenomena such as randomly occurring faults [6] and randomly occurring quantized effect [13, 22, 35].

Nomenclature

$\mathbb{R}^n$: Space of n-dimension real vector
$\mathbb{R}^{m\times n}$: $m \times n$ real matrices
Prob$\{\cdot\}$: Occurrence probability of the event
$\mathbb{E}\{\cdot\}$: Mathematical expectation operator with respect to the given probability measure
diag$\{\cdot\}$: Block diagonal matrix
$\lambda_{\max}$: Maximum eigenvalues of matrix
$\lambda_{\min}$: Minimum eigenvalues of matrix
$*$: Matrix element induced by symmetry
M^T: Transposition of matrix M
$X < Y$: $X - Y$ is negative definite
w.r.t.: With respect to
SFTB: Stochastic finite-time boundedness
SFTH_∞B: Stochastic finite-time H_∞ boundedness.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors have no conflicts of interest regarding the manuscript.

Authors' Contributions

Ling Hou contributed significantly to analysis and manuscript preparation, and she performed the data analysis and wrote the manuscript; Dongyan Chen helped perform the analysis with constructive discussions.

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