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Research Article

A Novel Nonlinear Fault Tolerant Control for Manipulator under Actuator Fault

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A fault tolerant control (FTC) scheme based on adaptive sliding mode control technique is proposed for manipulator with actuator fault. Firstly, the dynamic model of manipulator is introduced and its actuator faulty model is established. Secondly, a fault tolerant controller is designed, in which both the parameters of actuator fault and external disturbance are estimated and updated by online adaptive technology. Finally, taking a two-joint manipulator as example, simulation results show that the proposed fault tolerant control scheme is effective in tolerating actuator fault; meanwhile it has strong robustness for external disturbance.

1. Introduction

With the rapid development of modern science technology, manipulator has emerged as an important area of research, and more manipulators are applied in our life to reduce the burden of work. In [1, 2] two cleaning robots are designed to help people complete household cleaning tasks better. Besides, some tasks cannot be completed by a single manipulator, but two more cooperating manipulators are required. Thus, a control method of dual manipulators is proposed to replace the human workers to assemble and grasp objects [3, 4]. Reference [5] addressed a decentralized controller with constrained error variable and a radial basis function network for space manipulator. Besides the above application, manipulator also plays an important role in dangerous environment where people can not directly participate. In outer space, nobody can stay much long. Therefore, an advanced mechanical arm system is needed to perform some exploratory and experimental tasks, especially extravehicular mission, such as space assembly, spacecraft maintenance, satellite interaction, and outer space exploration. Thus, it greatly reduces the risks of the astronauts going out of the cabin and improves the efficiency and safety of space mission [6]. In recent years, many researchers have done a great deal of products on manipulators. The American space robot remote service system and the lightweight modular space manipulator system developed by German and Canadian giant robotic arm have been successfully launched following the rocket and completed the task. What is more, there are also some other space manipulators serving in International Space Station, including Dextre, SSRMS, and ERA [7–9].

In the current industrial application, manipulator has become increasingly important [10], therefore, the stability and reliability of manipulator system are crucial factors [11–15]. Sophisticated and dangerous work such as welding and space tasks that require high precision are assigned to robots. The robotic manipulator is a typical complex underactuated system with redundancy, multivariate, highly nonlinearities and coupling. On the one hand, as friction coefficient between joints always changes over time, and external disturbance is uncertain [16], fault may occur in manipulator. In particular, in dangerous environment, fault may occur more easily, such as hard environment conditions, particle radiation, electromagnetic interference [17], and low temperature; consequently the performance will greatly decrease, even leading mission to fail. On the other hand, artificial repair is nearly impossible in outer space. In conclusion manipulator is needed to tolerate fault and continue the given operation task. Consequently, fault tolerant control is vital in security assurance for manipulator. FTC technique also applies on UAV team, cooperative control, distributed control, mobile wireless, networks, and communications [18]. The performance of feedback control system depends on actuators, sensors, and data acquisition/interface components. Faulty components will lead to the deterioration of the overall system stability, which has been a safety problem in control system [19]. At present, there have been a large number of FTC schemes. In the FTC literature, different approaches have been reported, such as robust FTC presented in [20], adaptive FTC designed in [21, 22], nonlinear FTC proposed in [23], and sliding mode FTC proposed in [24, 25]. However, there are not many FTC schemes for manipulator. In [26] a novel finite time FTC based on adaptive neural network nonsingular fast terminal sliding mode is addressed for uncertain robot manipulators with actuator faults, and it is verified that the system possesses strong robustness, no singularity, less chattering, and fast finite time convergence by simulation [27–33]. In [34] a robust LQR/LQI FTC method is developed for a 2DOF unmanned bicycle robot with actuator fault. Different from [26], [34, 35] proposed a decentralized FTC for reconfigurable manipulator with sensor fault.

Compared with the above-mentioned methods, sliding mode control (SMC) has attractive advantages of efficient characteristics thanks to its insensibility to matched uncertainties and disturbances [36]. Therefore, SMC is adopted to design the fault tolerant controller in this paper. However, chattering of the sliding mode control signal has become the major issue to its actual applications, which can lead to the deterioration of the overall system. In order to solve such problem, a novel sliding mode control technology, dynamic sliding mode control, is designed [37]. In FTC method, observers are usually designed to estimate disturbances and fault information; for example, in [38] a fault diagnosis via higher order sliding mode observers is proposed for manipulator system; different from [39, 40], observers are not considered in this paper, in which, the proposed controller is much easier to realize in practical application.

The main theoretical contributions of this paper can be briefly outlined as follows.

(1) Compared with the design of traditional sliding surface, the novel dynamic sliding mode controller proposed in this paper can reduce the chattering effectively.

(2) In comparison with traditional method to handle unknown system parameters and disturbances, adaptive algorithm is adopted to update parameters online; it is not necessary to obtain the accurate value of disturbances.

(3) In comparison with the conventional FTC method, there is no need to design a fault diagnosis and detection observer, and the unknown fault can be estimated by the proposed adaptive algorithm.

The rest of this paper is organized as follows. In Section 2, the dynamic model of space manipulator and its actuator fault model are introduced. Then a fault tolerant controller is designed, in which parameters of actuator fault and external disturbance are estimated and updated by online adaptive method in Section 3. Finally, simulation results demonstrate the proposed fault tolerant controller is able to tolerate actuator fault, as well as the strong robustness for external disturbance.

2. Mathematic Model

2.1. Dynamic model. An articular robot system is a typical redundant, multivariable nonlinear complex dynamic coupling system [39]. When dealing with manipulator, we usually adopt its simplified model as follows due to the extreme complexity of its general dynamic model manipulator:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau + \tau_d \tag{1}$$

where $q \in \mathbb{R}^n$, $\dot{q} \in \mathbb{R}^n$, $\ddot{q} \in \mathbb{R}^n$ represent joint position, velocity, and acceleration vector, respectively; here position refers to joint angle. $H(q) \in \mathbb{R}^{n\times n}$ denotes symmetric positive definite inertia matrix. $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents Coriolis force and centrifugal force matrix. $G(q) \in \mathbb{R}^n$ denotes gravity torque vector. $F(\dot{q}) \in \mathbb{R}^n$ is friction torque vector. $\tau_d \in \mathbb{R}^n$ denotes external disturbance and model parameter uncertainties torque vector. $\tau \in \mathbb{R}^n$ denotes control torque vector.

The above manipulator system (1) has the following property which is beneficial in subsequent controller design.

Property. $\dot{H}(q) - 2C(q,\dot{q})$ is a skew symmetric matrix [41]; i.e., $\Gamma^T(\dot{H}(q) - 2C(q,\dot{q}))\Gamma = 0, \forall\Gamma \in \mathbb{R}^n$.

2.2. Fault Model. Instead of locked-joint fault, loss of effectiveness considered in this paper means the free-swinging fault; that is, fault joint swings freely without constraints. On the contrary, locked-joint fault means that fault joint is locked at its current position and cannot move any more. The move of manipulator system depends on the rotation of motor, which ranges from 0 degree to 300 degree. The free-swinging fault here that may be caused by a hardware or software fault in a manipulator can lead to the loss of torque (or force) on a joint [42]. Then the free-swinging fault model is established as follows:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau_f + \tau_d \tag{2}$$

where $\tau_f = E\tau$, $E = \mathrm{diag}\{e_i\}$, $i = 1, 2, \ldots, n$, and $e_i \in (0, 1)$ represents actuator loss of effectiveness factor which refers to the free-swinging fault in this paper, and the proposed FTC of manipulator is mainly designed under such case. When $e_i = 1$, the i_{th} joint is normal without fault; when $e_i = 0$, the i_{th} joint is with lock in place that means that the joint is locked at its current position, which is not considered in this paper. Define $\Delta E(t) = I - E = \mathrm{diag}\{1 - e_i(t)\}$, where I is the identity matrix and $\|\Delta E(t)\| = 1 - \min\{e_i(t)\}$; then system (2) can be transformed as

$$H\ddot{q} = \tau - \Delta E(t)\tau + \tau_d + g(q,\dot{q}) \tag{3}$$

where $g(q,\dot{q}) = -C(q,\dot{q})\dot{q} - G(q) - F(\dot{q})$. For convenience, define $H \triangleq H(q)$, $C \triangleq C(q,\dot{q})$, $G \triangleq G(q)$, $F \triangleq F(\dot{q})$, and $g \triangleq g(q,\dot{q})$.

3. Fault Tolerant Controller Design

3.1. Problem Statement. In this paper, we are absorbed in investigating an FTC method for manipulator with actuator fault. The trajectory tracking problem of fault system (2) is considered. The control objective can be described as that for a manipulator control system with actuator fault, an FTC method is proposed to ensure that the closed-loop system is stable, i.e., when $t \longrightarrow \infty$, $q \longrightarrow q_d$, where q_d denotes the desired position signal. For this purpose, an FTC method based on adaptive dynamic sliding mode technology is proposed. Firstly, an assumption is given as follows.

Assumption 1. The external disturbance τ_d is assumed to be norm-bounded.

$$\|\tau_d\| \leq K \tag{4}$$

where K is an unknown positive constant and $\|\cdot\|$ represents L_∞ *norm* in this paper.

3.2. Controller Design. As mentioned in the above section, the desired position signal is defined as q_d, so the tracking error is $z = q - q_d$. Then the conventional sliding mode surface is selected as

$$S = \dot{z} + lz \tag{5}$$

where $l \in \mathbb{R}^{n\times n}$ is a positive matrix; further,

$$\dot{S} = \ddot{z} + l\dot{z} = \ddot{q} - \ddot{q}_d + l\left(\dot{q} - \dot{q}_d\right). \tag{6}$$

Next, the dynamic sliding mode surface is selected as

$$J = S + \chi \tag{7}$$

where χ can be considered as the error between J and S and its derivative of time is $\dot{\chi} = \rho_1\|S\|^{0.5}\operatorname{sgn}(S) - \rho_2\|J\|^{0.5}\operatorname{sgn}(J)$, where $\operatorname{sgn}(S) = [\operatorname{sgn}(S_1), \operatorname{sgn}(S_2), \ldots, \operatorname{sgn}(S_n)]^T$, $\operatorname{sgn}(J) = [\operatorname{sgn}(J_1), \operatorname{sgn}(J_2), \ldots, \operatorname{sgn}(J_n)]^T$, ρ_1 is the sliding mode gain of conventional sliding mode surface S, and ρ_2 is the sliding mode gain of dynamic sliding mode surface J. They are two positive constants satisfying $\rho_1 > 0$, $\rho_2 > 0$, $\rho_1 \neq \rho_2$, and for more details please refer to Remark 6.

Theorem 2. *Considering manipulator system with free-swinging fault and external disturbance (2) under Assumption 1, a fault tolerant control input based on adaptive sliding mode controller (8) and adaptive law (9) can guarantee asymptotic output tracking of manipulator control system in both cases of no fault and fault, which guarantees the boundedness of all the closed-loop signals and asymptotic output tracking.*

$$\tau = -g - Hf - CJ - \phi\operatorname{sgn}(J) - \widehat{K}\operatorname{sgn}(J) \tag{8}$$

$$\dot{\widehat{K}} = \mu\sum_{i=1}^{n}\left|J_i\right| \tag{9}$$

where $\widehat{K}$ is the estimated value of K, ε is a positive constant, $f = -\ddot{q}_d + l(\dot{q} - \dot{q}_d) + \dot{\chi}$, $\phi = (b/(1-b))[\|g + Hf + CJ\| + \widehat{K}\operatorname{sgn}(J) + \varepsilon]$, and $b = \|\Delta E\|$. To satisfy the stability of the system, achieve a good tracking effectiveness, and estimate the value of disturbances, (9) is obtained to put an integral action in the definition of (8).

Proof. Define a Lyapunov function as follows:

$$V = \frac{1}{2}J^T HJ + \frac{1}{2\mu}\widetilde{K}^2 \tag{10}$$

where $\widetilde{K}$ denotes the estimated error of disturbance; that is, $\widetilde{K} = K - \widehat{K}$ and μ is a positive constant. The time derivative of V is obtained.

$$\begin{aligned}\dot{V} &= \frac{1}{2}J^T\dot{H}J + J^T H\dot{J} - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \\ &= \frac{1}{2}J^T\left(\dot{H} - 2C\right)J + J^T CJ + J^T H\dot{J} - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \\ &= J^T\left(CJ + H\dot{J}\right) - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \\ &= J^T\left(CJ + H\ddot{q} + Hf\right) - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}}\end{aligned} \tag{11}$$

Substituting (3) into the above equation, one can obtain the following.

$$\dot{V} = J^T\left(CJ + \tau - \triangle E(t)\tau + \tau_d + g + Hf\right) - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \tag{12}$$

Applying controller (8) and adaptive law (9) into the above equation, one can obtain

$$\begin{aligned}\dot{V} &= J^T\left[-\triangle E(t)\tau - \phi\operatorname{sgn}(J) - \widehat{K}\operatorname{sgn}(J) + \tau_d\right] \\ &\quad - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \\ &= J^T\left[-\triangle E(t)\tau - \phi\operatorname{sgn}(J)\right] - \widehat{K}J^T\operatorname{sgn}(J) + J^T\tau_d \\ &\quad - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \\ &\leq J^T\left[-\triangle E(t)\tau - \phi\operatorname{sgn}(J)\right] - \widehat{K}\sum_{i=1}^{n}\left|J_i\right| + K\sum_{i=1}^{n}\left|J_i\right| \\ &\quad - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \\ &= J^T\left[-\triangle E(t)\tau - \phi\operatorname{sgn}(J)\right] + \widetilde{K}\left(\sum_{i=1}^{n}\left|J_i\right| - \frac{1}{\mu}\dot{\widehat{K}}\right)\end{aligned} \tag{13}$$

Further simplify

$$\begin{aligned}\dot{V} &= J^T\left[-\triangle E(t)\tau - \phi\operatorname{sgn}(J)\right] \\ &= J^T \triangle E(t)\left[g + Hf + CJ + \phi\operatorname{sgn}(J) + \widehat{K}\operatorname{sgn}(J)\right] \\ &\quad - J^T\phi\operatorname{sgn}(J) \\ &\le b\|J\|\left[\|g + Hf + CJ\| + \widehat{K}\operatorname{sgn}(J)\right] + b\phi\sum_{i=1}^{n}|J_i| \\ &\quad - \phi\sum_{i=1}^{n}|J_i| \\ &\le b\sum_{i=1}^{n}|J_i|\left[\|g + Hf + CJ\| + \widehat{K}\operatorname{sgn}(J)\right] \\ &\quad - \phi(1-b)\sum_{i=1}^{n}|J_i| = -b\varepsilon\sum_{i=1}^{n}|J_i| \le 0\end{aligned} \tag{14}$$

and, thus, the stability of the closed-loop system is verified. □

Remark 3. In practical application, the smallest valuable of actuator fault $\min\{e_i(t)\}$ is usually unknown; therefore it is necessary to design an adaptive law to estimate fault information. To solve this problem, an adaptive fault tolerant controller will be designed in the following, in which adaptive scheme is adopted to estimate both the actuator fault and external disturbance.

When dealing with the faulty term, define $b = \|\Delta E(t)\|$, $\xi = 1/(1-b)$, and then adaptive algorithm is adopted to estimate ξ, which can compensate for the existent fault. Regarding the disturbance, based on Assumption 1, $\widehat{K}$ is used to compensate for the existent disturbance by adaptive technique.

Theorem 4. *Based on Theorem 2, considering manipulator system with free-swinging fault and external disturbance (2) under Assumption 1, the minimum boundary value of actuator fault $\min\{e_i(t)\}$ is unknown, and an FTC input based on adaptive sliding mode controller (15) and adaptive laws (16)-(17) can guarantee asymptotic output tracking of manipulator control system in both cases, i.e., no fault and fault, which guarantees the boundedness of all the closed-loop signals and asymptotic output tracking.*

$$\tau = -g - Hf - CJ - \gamma(t)\operatorname{sgn}(J) - \widehat{K}\operatorname{sgn}(J) \tag{15}$$

$$\dot{\widehat{K}} = \mu\sum_{i=1}^{n}|J_i| \tag{16}$$

$$\dot{\widehat{\xi}} = \beta\delta\sum_{i=1}^{n}|J_i| \tag{17}$$

where $\widehat{K}$ is the estimated value of K, $\delta = \|g + Hf + CJ\| + \widehat{K} + \varepsilon$, and ε is a positive constant. $\gamma(t) = -\delta + \widehat{\xi}\delta$, and $\widehat{\xi}$ is the estimated value of ξ.

Proof. In order to prove the stability of the overall system, the Lyapunov function is selected as

$$V = V_1 + V_2 + V_3 \tag{18}$$

where V_1, V_2 V_3 represent three Lyapunov functions which will be elaborated in the following 3 steps; further,

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3. \tag{19}$$

The process of proof is divided into three steps.

Step 1 (adaptive law analysis). To obtain the adaptive laws of fault and disturbance, the Lyapunov function is chosen as

$$V_1 = \frac{1}{2}J^T HJ + \frac{1}{2\mu}\widetilde{K}^2 + \frac{1-b}{2\beta}\widetilde{\xi}^2 \tag{20}$$

where $\widetilde{K}$ denotes the estimated error of disturbance, $\widetilde{\xi}$ denotes the estimated error of fault, $\widetilde{K} = K - \widehat{K}$, $\widetilde{\xi} = \xi - \widehat{\xi}$, and μ and β are both positive constants. The time derivative of V_1 is obtained.

$$\begin{aligned}\dot{V}_1 &= \frac{1}{2}J^T\dot{H}J + J^T H\dot{J} - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} - \frac{1-b}{\beta}\widetilde{\xi}\dot{\widehat{\xi}} \\ &= \frac{1}{2}J^T\left(\dot{H} - 2C\right)J + J^T CJ + J^T H\dot{J} - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \\ &\quad - \frac{1-b}{\beta}\widetilde{\xi}\dot{\widehat{\xi}} \\ &= J^T\left(CJ + H\ddot{q} + Hf\right) - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} - \frac{1-b}{\beta}\widetilde{\xi}\dot{\widehat{\xi}}\end{aligned} \tag{21}$$

Substituting (3) into the above equation, one obtains the following.

$$\begin{aligned}\dot{V}_1 &= J^T\left(CJ + \tau - \triangle E(t)\tau + \tau_d + g + Hf\right) - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} \\ &\quad - \frac{1-b}{\beta}\widetilde{\xi}\dot{\widehat{\xi}}\end{aligned} \tag{22}$$

Applying controller (15) into the above equation yields the following.

$$\begin{aligned}\dot{V}_1 &= J^T\left[-\triangle E(t)\tau - \gamma(t)\operatorname{sgn}(J) - \widehat{K}\operatorname{sgn}(J) + \tau_d\right] \\ &\quad - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} - \frac{1-b}{\beta}\widetilde{\xi}\dot{\widehat{\xi}} \\ &= J^T\left[-\triangle E(t)\tau - \gamma(t)\operatorname{sgn}(J)\right] + \widetilde{K}\sum_{i=1}^{n}|J_i| \\ &\quad - \frac{1}{\mu}\widetilde{K}\dot{\widehat{K}} - \frac{1-b}{\beta}\widetilde{\xi}\dot{\widehat{\xi}} \\ &= J^T\left[-\triangle E(t)\tau - \gamma(t)\operatorname{sgn}(J)\right] \\ &\quad + \widetilde{K}\left(\sum_{i=1}^{n}|J_i| - \frac{1}{\mu}\dot{\widehat{K}}\right) - \frac{1-b}{\beta}\widetilde{\xi}\dot{\widehat{\xi}}\end{aligned} \tag{23}$$

Substituting adaptive law (16) into the above equation yields the following.

$$\begin{aligned}
\dot{V}_1 &\le J^T\left[-\triangle E(t)\tau - \gamma(t)\operatorname{sgn}(J)\right] - \frac{1-b}{\beta}\tilde{\xi}\dot{\tilde{\xi}} \\
&= -\gamma(t)\sum_{i=1}^{n}|J_i| - \frac{1-b}{\beta}\tilde{\xi}\dot{\tilde{\xi}} + J^T \\
&\quad \triangle E(t)\left[g + Hf + CJ + \gamma(t)\operatorname{sgn}(J) + \widehat{K}\operatorname{sgn}(J)\right] \\
&\le \left(1-\widehat{\xi}\right)\delta\sum_{i=1}^{n}|J_i| - \frac{1-b}{\beta}\tilde{\xi}\dot{\tilde{\xi}} \\
&\quad + b\left\|J^T\right\|\left[\|g + Hf + CJ\| + \gamma(t) + \widehat{K}\right] \\
&\le \left(1-\widehat{\xi}\right)\delta\sum_{i=1}^{n}|J_i| - \frac{1-b}{\beta}\tilde{\xi}\dot{\tilde{\xi}} + b\sum_{i=1}^{n}|J_i|\left(-\varepsilon + \widehat{\xi}\delta\right) \\
&= \left(1-\widehat{\xi}+b\widehat{\xi}\right)\delta\sum_{i=1}^{n}|J_i| - \frac{1-b}{\beta}\tilde{\xi}\dot{\tilde{\xi}} - \varepsilon b\sum_{i=1}^{n}|J_i| \\
&= (1-b)\left(\xi-\widehat{\xi}\right)\delta\sum_{i=1}^{n}|J_i| - \frac{1-b}{\beta}\tilde{\xi}\dot{\tilde{\xi}} - \varepsilon b\sum_{i=1}^{n}|J_i| \\
&= (1-b)\tilde{\xi}\left(\delta\sum_{i=1}^{n}|J_i| - \frac{1}{\beta}\dot{\tilde{\xi}}\right) - \varepsilon b\sum_{i=1}^{n}|J_i|
\end{aligned} \tag{24}$$

As a result, applying adaptive law (17) into the above equation, one can obtain the following.

$$\dot{V}_1 = -\varepsilon b\sum_{i=1}^{n}|J_i| \le 0 \tag{25}$$

Step 2 (reach time analysis). Before analysis, a lemma is proposed as follows to obtain the convergence time.

Lemma 5. *The dynamic sliding mode function J exists and can converge within finite time t_J; please see [43] for more details.*

After time t_J, the surface $J = 0$; from (7), one can obtain the following.

$$\dot{S} = -\dot{\chi} = -\rho_1\|S\|^{0.5}\operatorname{sgn}(S) \tag{26}$$

Further, a Lyapunov function is selected as follows to obtain the convergence time of system.

$$V_2 = \frac{1}{2}S^TS \tag{27}$$

Further,

$$\dot{V}_2 = S^T\dot{S} = S^T\left(-\rho_1\|S\|^{0.5}\operatorname{sgn}(S)\right) \le -\rho_1\|S\|^{1.5} \le 0. \tag{28}$$

To solve the above differential equation, the following equation can be obtained by (27).

$$V_2 = \frac{1}{2}\|S\|^2 \tag{29}$$

Further,

$$\|S\| = \left|2V_2\right|^{1/2} \tag{30}$$

Substituting (30) to (28) yields the following.

$$\dot{V}_2 \le -\rho_1\left|2V_2\right|^{3/4}. \tag{31}$$

Thus

$$\begin{aligned}
V_2^{-3/4}dV_2 \le -2^{3/4}\rho_1 dt \int_{t_J}^{t} V_2^{-3/4}dV_2 \\
\le \int_{t_J}^{t} -2^{3/4}\rho_1 dt4\left[V_2(t)^{1/4} - V_2(t_J)^{1/4}\right] \\
\le -2^{3/4}\rho_1(t-t_J).
\end{aligned} \tag{32}$$

As time t reaches t_S, the conventional sliding mode surface S will converge to zero; i.e., when $t = t_S$, $S = 0$; that is, $V_2(t) = V_2(t_S)$; thus

$$-4V_2(t_J)^{1/4} \le -2^{3/4}\rho_1(t_S - t_J) \tag{33}$$

Thus, one can obtain the following.

$$t_S \le \frac{4V_2(t_J)^{1/4} + 2^{3/4}\rho_1 t_J}{2^{3/4}\rho_1} \tag{34}$$

Consequently, sliding mode surface S will converge to zero within finite time.

$$t_S \le \frac{4V_2(t_J)^{1/4} + 2^{3/4}\rho_1 t_J}{2^{3/4}\rho_1} \tag{35}$$

Now, it is verified that conventional sliding mode surface S and dynamic sliding mode surface J can both converge to zero within finite time, and dynamic surface converges faster than conventional surface; i.e., $\lim_{t\ge t_J}(J/S) = 0$. As $t \longrightarrow t_S$ is reached, $S = 0$; substituting this into (5), one can obtain the following.

$$\dot{z} = -lz \tag{36}$$

Step 3 (tracking error analysis). To prove the convergence of the tracking error z, the Lyapunov function is defined as follows.

$$V_3 = \frac{1}{2}z^Tz \tag{37}$$

Thus

$$\dot{V}_3 = z^T\dot{z} \le -l\|z\|^2 \le 0 \tag{38}$$

and, therefore, tracking error z is convergent, which means that when $t \longrightarrow \infty$, $q \longrightarrow q_d$, $z \longrightarrow 0$. Therefore, according to the above three steps, it is easy to be seen that $\dot{V} < 0$ in (19), which means that the overall system can be stable with the proposed controller. The proof is completed. □

Remark 5. Practically, due to the hysteresis of nonlinear and switching, $\|J\|$ cannot converge to zero accurately within a finite time; therefore, the adaptive parameters $\widehat{K}$ and $\widehat{\xi}$ of the estimated values for K and ξ may increase boundlessly. In the other words, $\widehat{K}$ and $\widehat{\xi}$ obtained by the proposed adaptive algorithm will be not accurate, which may increase towards infinity. In practical engineering, it is difficult to apply (16)-(17) directly. Consequently, to solve such a problem, dead zone technique is used [44] and adaptive laws (16) and (17) are modified as

$$\dot{\widehat{K}} = \begin{cases} \mu \sum_{i=1}^{n} |J_i|, & |J_i| \geq \lambda_1 \\ 0, & |J_i| < \lambda_1 \end{cases} \tag{39}$$

$$\dot{\widehat{\xi}} = \begin{cases} \beta\delta \sum_{i=1}^{n} |J_i|, & |J_i| \geq \lambda_2 \\ 0, & |J_i| < \lambda_2 \end{cases} \tag{40}$$

where λ_1 and λ_1 are both small positive constants.

Remark 6. The dynamic sliding function J reaches and remains on the sliding surface $J = 0$ before the conventional sliding function S gets to the sliding surface $S = 0$ if and only if ρ_1, ρ_2 satisfy [45]

$$|J(0)| \leq \left(\frac{\rho_2}{\rho_1}\right)^2 |S(0)|. \tag{41}$$

4. Simulation

In order to verify the validity of control method proposed in this paper, we applied it to two-joint manipulator model. In this section, four cases with considering signal fault and double joints faults, respectively, will be simulated to present the tracking effectiveness of manipulator.

4.1. Simulation Cases

Case 1. Link 1 and link 2 are both healthy without fault; i.e., $E = I$, where I is the identity matrix.

Case 2. Link 1 is healthy, and actuator fault occurs in link 2 at 10s; i.e.,

$$e_1 = 1$$
$$e_2 = \begin{cases} 1, & t < 10s \\ 0.2, & t \geq 10s. \end{cases} \tag{42}$$

Case 3. Actuator fault occurs in link 1 at 10s, and link 2 is healthy; i.e.,

$$e_1 = \begin{cases} 1, & t < 10s \\ 0.6, & t \geq 10s. \end{cases}$$
$$e_2 = 1 \tag{43}$$

Case 4. Actuator fault occurs in link 1 and link 2 at 10s; i.e.,

$$e_1 = \begin{cases} 1, & t < 10s \\ 0.6, & t \geq 10s \end{cases}$$
$$e_2 = \begin{cases} 1, & t < 10s \\ 0.2, & t \geq 10s. \end{cases} \tag{44}$$

Simulation Parameters. The parameters of simulation are designed as follows.

$$H(q) = \begin{bmatrix} 3.66 + 1.74\cos q_2 & 0.76 + 0.87\cos q_2 \\ 0.76 + 0.87\cos q_2 & 0.76 \end{bmatrix}$$
$$G(q) = \begin{bmatrix} 3.04g\cos q_1 + 0.87g\cos(q_1 + q_2) \\ 0.87g\cos(q_1 + q_2) \end{bmatrix} \tag{45}$$
$$\tau_d = \begin{bmatrix} 0.2\sin \dot{q}_1 \\ 0.2\sin \dot{q}_2 \end{bmatrix}$$

The initial state of manipulator system is selected as $q_0 = (0,0)^T rad/s$, $\dot{q}_0 = (0,0)^T rad/s$.

Tracked objective trajectory is $q_{d_1} = 0.7\sin(\pi t) rad$, $q_{d_2} = \sin(\pi t) rad$, respectively.

To make the overall system stable, equipped with strong robustness and fault tolerance, the parameters of the proposed controller are selected as $\varepsilon = 1$, and the learning gains of adaptive laws are adopted as $\mu = 0.5$, $\beta = 0.1$. To realize fast convergence of the sliding mode surface, sliding mode parameters are designed as $l = \mathrm{diag}\{0.001, 0.001\}$, $\rho_1 = 0.3$, $\rho_2 = 5$ $\lambda_1 = \lambda_2 = 0.05$.

Simulation Results and Analysis. In the simulation, according to the designed control law (15) and adaptive laws (39), (40), the time of fault tolerance in each case is shown in Table 1, and the corresponding simulation results are depicted in Figures 1–8, which show time responses of link position tracking, velocity tracking.

Figure 1 shows the tracking responses in Case 1. From Figure 1, we can see that the position and speed signal of link 1 can track the corresponding desired signal within 5s. Meanwhile, Figure 1 shows that tracking trajectories of position and speed of link 2 between actual signal and desired signal can converge to zero in 5s and then reach stability. From Figure 5, it is easy to be seen that the control torque can converge within 4 seconds without actuator fault in this case.

Figure 2 depicts the time responses of trajectory tracking in Case 2. From Figure 2, it can be easily seen that the position and speed tracking errors of link 1 between actual signal and desired signal can converge to zero in 5s without actuator fault. Also, Figure 2 shows that tracking error of two links can converge to zero in 5s; after actuator fault occurs in link 2 at 10s, system can rapidly deal with the fault and realize trajectory tracking within 5 seconds. From Figure 6, we can see that when actuator fault occurs in link 2, the control

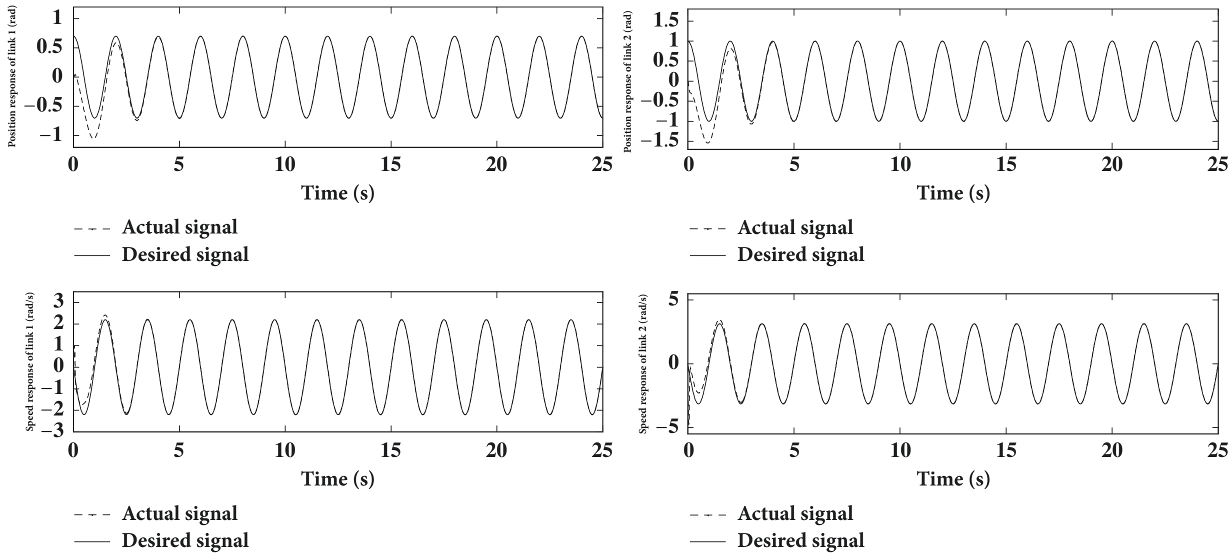

Figure 1: Tracking responses of links 1 and 2 in Case 1.

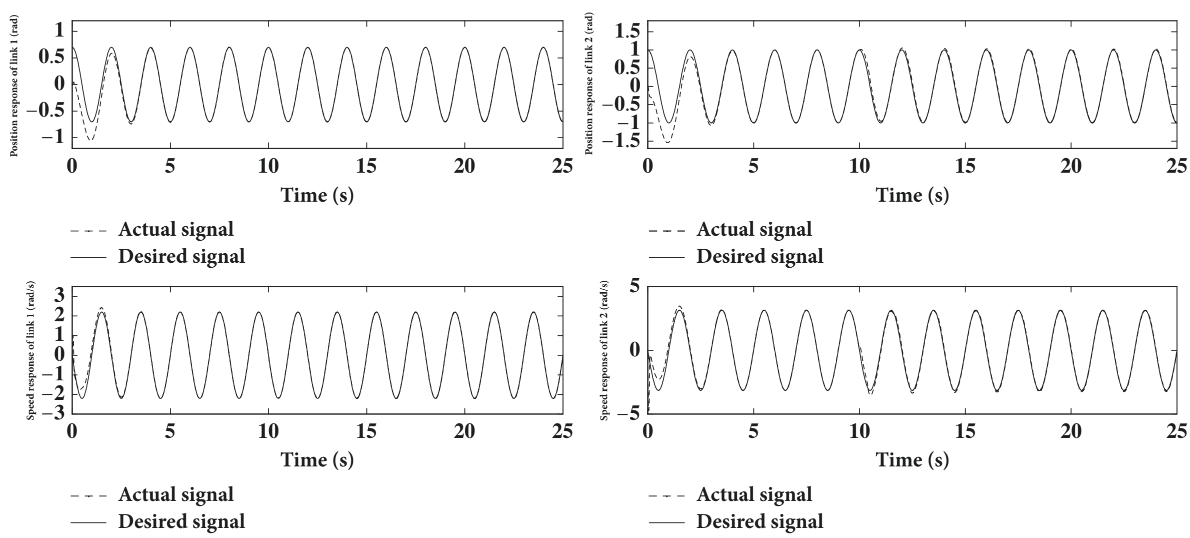

Figure 2: Tracking responses of links 1 and 2 in Case 2.

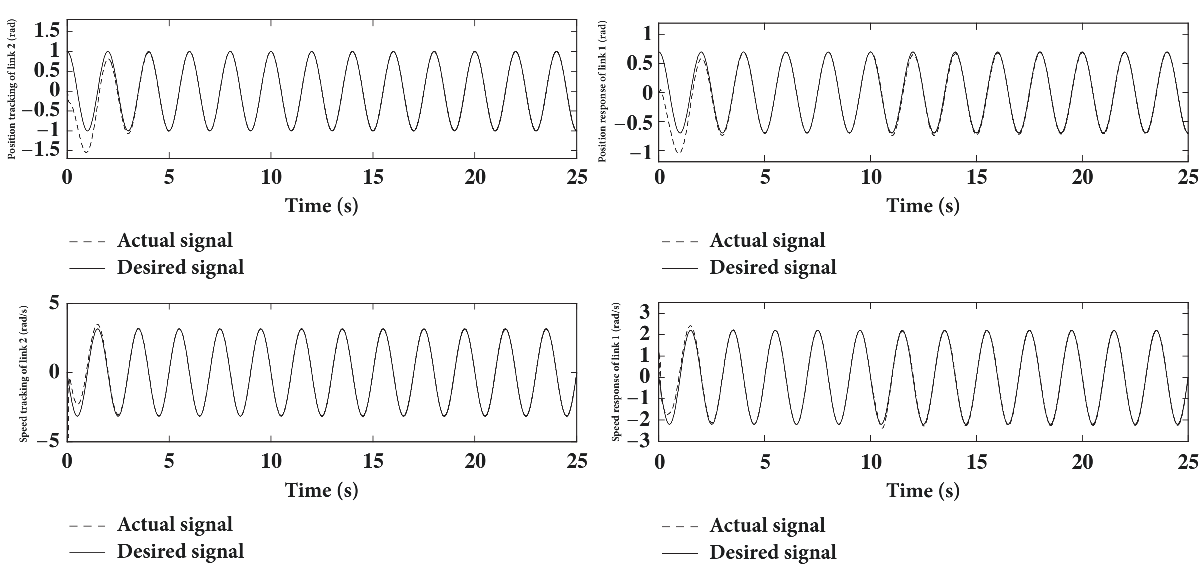

Figure 3: Tracking responses of links 1 and 2 in Case 3.

Table 1: Time of fault tolerance.

CASES	Position tracking of link1(s)	Speed tracking of link1(s)	Control torque of link1(s)	Position tracking of link2(s)	Speed tracking of link2(s)	Control torque of link2(s)
1	/	/	/	/	/	/
2	/	/	/	5	5	3
3	7	7	3	/	/	/
4	7	7	3	5	5	3

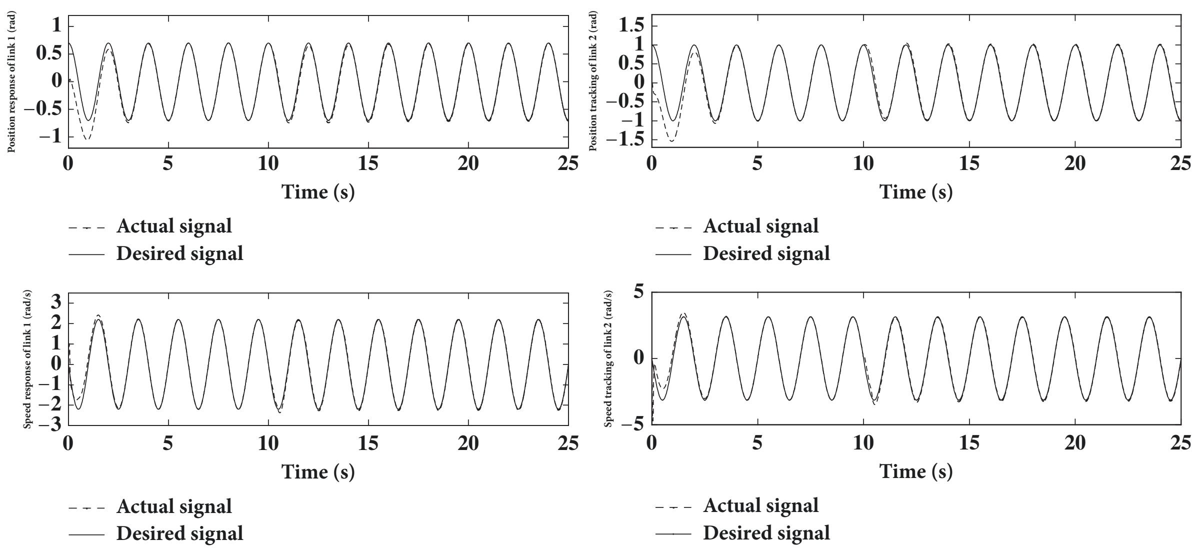

Figure 4: Tracking responses of links 1 and 2 in Case 4.

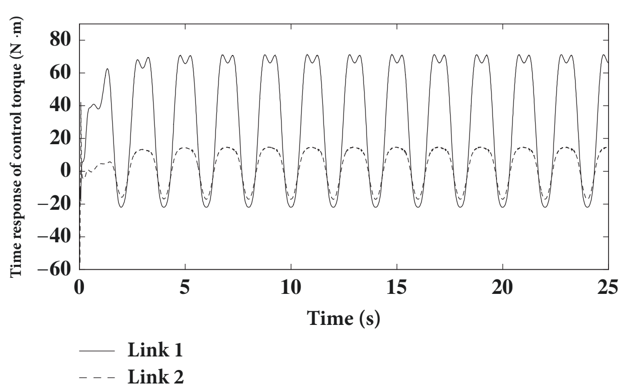

Figure 5: Time responses of control torque in Case 1.

torque of link 2 can handle the fault within 3 seconds, but a little chattering phenomenon appears.

The tracking responses in Case 3 are shown in Figure 3, from which, it can be easily seen that link position can deal with fault by itself and basically track the desired position signal in 7s when actuator fault appears in link 1 at 10s. Compared with other cases, the error between actual signal and desired signal is a little high, but is still in an acceptable range. Figure 3 also provides the actual speed of link 1 with actuator fault being able to track expected signal within 7s. From Figure 3 we can, respectively, see position and speed of link 2 with no fault being able to easily realize the trajectory tracking in 5s. Figure 7 shows the time response of control torque, from which we can see that the amplitude of link 1 increases and control torque can reconverge within 3 seconds.

Figure 4 provides the trajectory tracking responses of two links in this case. The tracking response of link 1 is shown in Figure 4, from which, it is easy to see that when actuator of link 1 failed at 10s, position signal and speed signal of link 1 can deal with fault and basically track the desired signals in 7 seconds. Although there exists a certain error in both position and speed tracking, the error is acceptable in programming. Figure 4 also depicts the tracking response of link 2, from which, we can see that it takes 5 seconds for position and

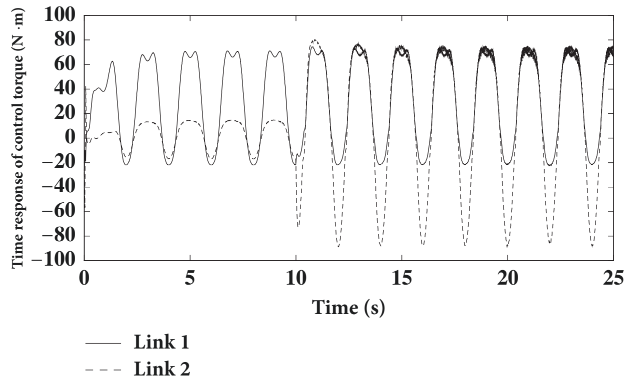

Figure 6: Time responses of control torque in Case 2.

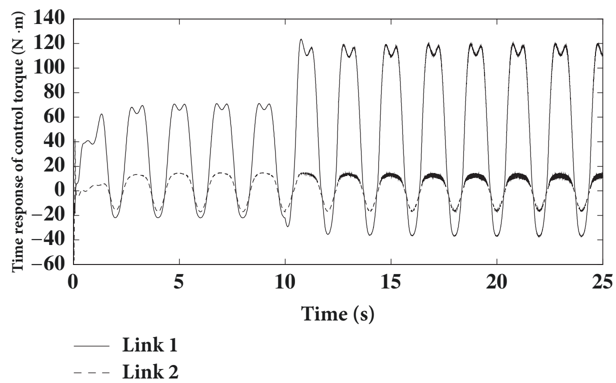

Figure 7: Time responses of control torque in Case 3.

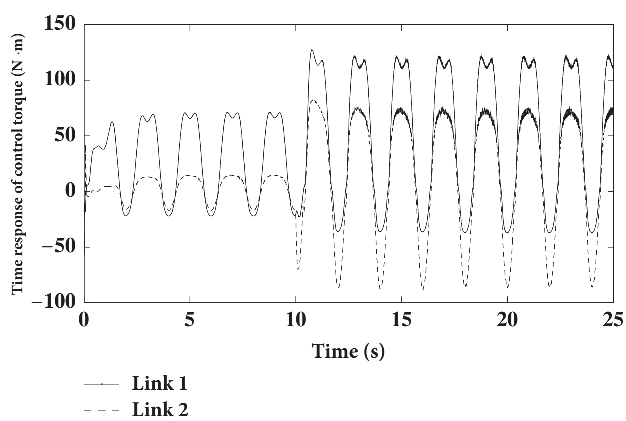

Figure 8: Time responses of control torque in Case 4.

speed signal to track the corresponding desired signals when actuator fault occurs in link 2 at 10s. Figure 8 depicts the time response of control torque in this case, from which it can be easily seen that the amplitudes of 2 links increase, and control torque can reconverge within 3 seconds but a little chattering phenomenon appears.

From Table 1 and Figures 1–8, we can see that when both actuators are healthy without fault, the overall system can realize trajectory tracking within 5 seconds; when actuator fault occurs in link 2, tracking responses of link 2 can reconverge to desired signal within 5 seconds; when actuator fault occurs in link 1, tracking responses of link 1 can reconverge to desired signal within 7 seconds; when actuator fault occurs in two links, tracking responses of links 1 and 2 can reconverge to desired signal within 7 and 5 seconds, respectively. Therefore, the trajectory tracking effectiveness of the FTC method proposed in this paper is verified.

5. Conclusions

In this paper, a novel FTC scheme based on adaptive sliding mode method is investigated for manipulator with actuator fault and external disturbance. Firstly the general dynamic model of space manipulator is introduced and further its actuator faulty model is established. Secondly, an adaptive fault tolerant controller is designed for manipulator with actuator fault. Parameters of fault and disturbance are estimated and updated by online adaptive method. Finally the proposed controller is applied to two-joint manipulator; from the simulation results it is shown that the controller proposed in this paper can not only realize a good trajectory tracking, but also tolerate actuator fault and present strong robustness for external disturbance, while effectively reducing the chattering phenomenon of sliding mode control.

Data Availability

The data supporting the conclusions of our manuscript are some open access articles that have been properly cited, and the readers can easily obtain these articles to verify the conclusions, replicate the analysis, and conduct secondary analyses. Therefore, we did not create a publicly available data repository.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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