

## Research Article

# Venture Capital Contracting with Double-Sided Moral Hazard and Fairness Concerns

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The development of new venture enterprise is the result of joint efforts of entrepreneurs and venture capitalists who collaborate based on complementary resources. In this paper, we analyze a venture capital incentive contracting model in which a venture capitalist interacts with an entrepreneur who is risk neutral and fairness concerned, offering him an equity contract. We solve the venture capitalist's maximization problem in the presence of double-sided moral hazard. Our results show that fairness concerns change the structure of the optimal contract. More importantly, we show that the solution to the contract regarding the optimal share given to the entrepreneur is nonlinear and is a fixed point between 0 and 1. Further, we simulate the model under the assumption that venture project's revenue is a Constant Elasticity of Substitution (CES) function and obtain the following results. (1) When the two efforts are complementary, the venture capitalist's effort does not monotonically decrease in the share allocated to the entrepreneur, while the entrepreneur's effort does not monotonically increase in his share. (2) Relative to the benchmark case where the entrepreneur is fairness neutral, the optimal equity share allocated to the fair-minded entrepreneur is larger than 1/2, and as the degree of efforts complementarity increases, the optimal equity share tends to 60%. In this scenario, for a given efforts substitution parameter, the fair-minded entrepreneur provides a higher effort level than the venture capitalist.

## 1. Introduction

Venture capital is the primary means through which innovative ideas are financed, nurtured, and brought to fruition and therefore plays a crucial role in economic growth. Over the past 30 years, venture capital has been an important source of financing for innovative firms. Many highly successful companies, such as Facebook, Google, Amazon, Apple, FedEx, Starbucks, and Alibaba, have all been backed by venture capitalist (VC). Gornall and Strebulaev [1] estimate that public companies previously received VC backing account for one-fifth of the market capitalization and 44% of the research and development spending of US public companies. According to a report by KPMG in January 2018, global venture capital investment reached US \$155 billion in 2017, setting the highest level in 10 years. Meanwhile, a report jointly published by PricewaterhouseCoopers and CB Ventures in January 2018 shows that Asia is approaching the United States and is

about to become the region that receives the most venture capital investment. In 2017, the total amount of venture capital investment in Asia reached \$708 billion, while that in the United States was \$719 billion. The report also said China is becoming a "magnet" for capital attraction, with money flowing from taxi services to shared bicycles, to artificial intelligence (AI) and vehicle charging.

This paper studies the optimal contracting problem of an early start-up seeking venture capital finance within the context of two noncontractible efforts, entrepreneurial (denoted by EN) effort and VC effort, which are crucial for the success of start-up. An EN possesses an innovative idea for a new venture but may lack the personal funds and business expertise needed to get the business started. In addition to financing, VC provides advice and expertise in management, commercialization, and development which enhance the value, success, and marketability of high-risk, high return projects. Further, VC often places members of

the venture capital team on the firm's board. VC and EN both play an important and irreplaceable role in VC-backed innovative startups. In the case of incomplete information, the EN and the VC exert unobservable efforts to improve the profitability of a venture project, and the asymmetry of efforts and the arrangement of the residual claims make both parties have the motive to seek private benefits. Therefore, the start-up firms are faced with double-sided moral hazard. In this case, the incentive problem is particularly prominent, and double-sided moral hazard has become the research direction of venture capital financing contract [2].

A unique feature in the VC-EN relationship results from the role of VC as active investors, which ideally goes far beyond the traditional principal-agent context. Casamatta [3] divides the role of VC into two types: financing and consulting and analyzes a double-sided moral hazard problem whereby both EN and VC must exert effort to improve the profitability of a venture. With moral hazard, Casamatta argues that if the EN's effort is less costly than the VC's effort, the EN will not contract the VC unless the latter contributes capital to fund the project. Repullo and Suarez [4] point out that VC and EN face bilateral moral hazard due to the bounded rationality of individuals and the uncertainty of future returns of investment projects, and both of the two partners need an appropriate incentive mechanisms in order to increase their willingness to exert more effort into the project. The authors suggest that the optimal contract arrangement should give VC a warrant-like claim. Dang [5] studies financial contracting in a two-period financing model with double moral hazard, as EN effort choices and profits are unobservable and nonverifiable. He finds that under well-defined conditions, an incentive mechanism that ensures truthful reporting of profits can effectively elicit the high level of effort. Fairchild [6] develops a behavioral game-theoretic model in which an EN chooses between a VC and an angel to provide start-up finance for his venture. After the EN has chosen his financier, the dyad faces a double-sided moral hazard problem in the form of effort-shirking. The model shows that the EN's choice of financier is affected by the level of EN/angel empathy relative to the VC's value-adding abilities.

In the study of how to guard against the moral hazard caused by information asymmetry, most scholars choose the perspective of contract instruments to stimulate the efforts of both parties. More recently, Vergara et al. [7] study an optimal contract design problem in the context of double-sided moral hazard but in an economy in which efforts are complements. The advisory services, networking, and the experience of VC are complemented by the technological and innovation skills of EN. The synergy induced by the complementarity between the EN and VC has a considerable effect on the enterprise's market value. Additionally, they simulate the model and show that the EN's effort does not monotonically decrease by the share allocated to the VC, while the VC's effort does not monotonically increase with respect to his share, and as the efforts of the EN and the VC are highly complementary, the project cash flows are distributed nearly equally. Using a Nash bargaining approach, Fu, Yang, and An [8] analyze the

financing contract between the EN and the VC with double-sided moral hazard in a start-up enterprise.

The existing financial contracting models assume that decision makers are self-interested in that their utility function depends only on their own material payoffs [3–5, 7, 8]. However, a large number of empirical evidence has pointed out that many individuals are also motivated by other psychological considerations, such as inequity aversion and overconfidence [9]. In particular, Fehr and Schmidt [10] propose a class of preferences with fairness concerns (in which players are concerned about fairness of outcomes) and investigate their implications in economic models. According to [10], fairness concern refers to that decision makers care not only about their own payoffs, but also about whether their payoffs are higher or lower than others. This type of preference explains many experimental observations. Furthermore, Fehr, Klein, and Schmidt [11] show experimentally that concerns for fairness have an important impact on the actual and the optimal design of contracts. In addition, numerous studies have confirmed that venture-backed performance may also be affected by behavioral factors. It is argued that VC/EN cooperative value-creating efforts may be influenced by reciprocal feelings of fairness, trust, and empathy [6, 12–16].

It is observed that other-regarding preferences may have a positive role in moral hazard situations [17–19]. The observation has stimulated research into a theoretical analysis of the contract theory. By using a continuous-effort model of Holmstrom [20], Englmaier and Wambach [21] set up a model where an agent has an inequality-averse preference, which is a variant of the Fehr-Schmidt preference, and provide a comprehensive treatment of the moral hazard problem under inequality aversion. Bartling [22] analyzes a principal-multiagent model and assumes that the agents have inequity aversion or status preferences. Luo, Wang, and Li [23] apply the agency model in the social network setting, study the impact of an inequity-averse agent's deserved concerns on her behaviors, and derive the optimal output sharing. Banerjee and Sarkar [24] analyze optimal contracts when an other-regarding principal interacts separately with a self-regarding and other-regarding agent.

If the agent's preference indeed does exhibit fairness concern, then the optimal contract design must take this into account. How is the structure of the optimal contract changed if the agent is no longer selfish but concerns about fairness? Furthermore, what are the implications for incentive provisions? To solve these questions, we consider a double-sided moral hazard model of venture financing with a fairness concern agent. In the model, a VC who is self-interested and profit maximizing hires an EN to implement a potentially profitable project. The EN is a utility-maximizing agent who is fairness concerned towards his principal. A contract specifies a share of the total output for each of the contracting parties. Hence, we investigate an equity contract according to the corporate finance literature.

In the present study, we discuss how incorporating behavioral biases in the analysis of incentives may affect the predictions of the classical moral hazard model. The VC maximizes his own profit whereas the EN maximizes

his utility depending on the profits of both members. The optimal contract under moral hazard takes into account the acceptance condition for the EN and his choice of effort. Moreover, it is often the case that arbitrarily low or high payments are not feasible, which would introduce additional constraints into the VC's optimization program.

This paper thus focuses on how the complementarity of efforts between a VC and an EN affects the equity share that the VC is willing to allocate to the EN. We investigate the combined impact of double-sided moral hazard and the EN's fairness concerns on VC/EN contracting. Following the approach of [3–8], in our model, the VC's investment in the project is also endogenous, which is equivalent to assuming that the VC buys a share in the project and pays the price that covers start-up costs, including an upfront payment to the EN [7, 25, 26]. We simulate the model under the assumption that project revenue is a Constant Elasticity of Substitution (CES) function, whereby we analyze the combined effects of complementarity and fairness concern on effort dynamics, and the function of the optimal equity distribution.

Our model is similar to that in [7] but we depart from it in two ways. First, we assume the VC possesses the bargaining power and then the optimization problems are solved from the VC's perspective. Second, we introduce the EN's social preferences, captured by inequity aversion in the spirit of Fehr and Schmidt [10]; i.e., the EN cares about his own payoff and does not like absolute difference between his own payoff and the payoff of VC.

Another paper closely related to our work is that of Fairchild [12], who also analyzes the effects of fairness norms and VC's value-adding abilities on financial contracting and the venture's performance. In Fairchild's model, the VC makes an equity proposal to the EN, and the EN compares it with the social norm, where the social norm is the equity proposal that equates the EN's and VC's expected payoff. A deviation from the social norm provides a disutility for the EN. Our approach, by applying distribution-based inequity aversion, is more in the spirit of Fehr and Schmidt [10]. In addition, as in [3, 26], Fairchild [12] assumes that the VC's and EN's efforts are perfect substitutes, meaning that in this scenario the synergy of efforts is irrelevant. In contrast, we investigate how the complementarity of efforts between an EN and a VC affects the equity share that the VC is willing to allocate to the EN. As a special case, we consider a scenario in which the efforts of VC and EN are perfect substitutes. The main findings in Fairchild [12] are that, for a given level of VC-ability, a greater feeling of fairness induces the VC to offer more equity to the EN, which in turn induces the EN to exert more effort. However, we simulate the model and show that when the two efforts are complementary, the VC's effort does not monotonically decrease in the share allocated to the EN, while the EN's effort does not monotonically increase in his share. Furthermore, as the degree of efforts complementarity increases, the fair-minded EN provides a higher effort level than the VC. When the two efforts are perfect substitutes, we obtain similar results to Fairchild [12].

The remainder of the paper is structured as follows: in Section 2, we describe the key elements of the model and introduce notation. In Section 3, we set up the model, and we

simulate the model in Section 4. Section 5 numerically examines the influence of fairness concerns on the equilibrium result and the optimal effort levels. Section 6 will conclude our research and point out the future directions.

## 2. Problem Statement and Basic Assumptions

We consider a setting in which an EN has initially formed a firm to develop a potentially profitable project, requiring start-up funds  $I > 0$ ; however, EN lacks personal finance and must raise outside finance. Therefore, EN approaches a VC to obtain that additional funding, and the amount to be financed from VC is  $I_{vc}$ . EN's own technology is quantified as  $I - I_{vc}$ . Following [3, 7], we make the VC's investment endogenous. Although the EN has the technology of the developing project and innovation skills, but he lacks the necessary operation and management experience and also the important commercial channels to organize production and sales. So in fact, VC is not only providing funds for the firm, but, more importantly, providing intellectual support in management consulting services to improve the probability of project success.

Due to the high levels of uncertainty involved, it is difficult for startups to seek support from traditional financial intermediaries (such as banks); EN may not have sufficient bargaining power in the process of seeking financial support from VC. Different from the hypothesis of [7], we assume that the VC has all of the bargaining power and so makes a take-it-or-leave-it equity offer to the EN. Once the financial contract has been agreed, the EN and the VC exert respective, unobservable effort level  $e$  and  $a$  in developing the business. Specifically, the project is risky and may either succeed with probability  $p$  or fail with probability  $1 - p$ , where  $0 < p < 1$ . In the case of success, the project provides income  $R(e, a) > 0$ . In the case of failure, the project delivers zero income.

When the project is successful, the VC offers the EN an equity share  $s$  and he gets the other part  $1 - s$ . The output of the project is an increasing function of  $e$  and  $a$  and is a concave function; that is,  $R_e$  and  $R_a$  are positive, and  $R_{ee}$  and  $R_{aa}$  are negative. In addition, the cross-derivative  $R_{ea} = R_{ae}$  is positive [7]. The efforts are costly. Let  $C(e)$  denote the EN's disutility of effort and  $B(a)$  the VC's disutility of effort. It is assumed that both functions are increasing and also convex, i.e.,  $C_e, B_a > 0$ ,  $C_{ee}, B_{aa} > 0$ . It is further assumed that  $C(0) = B(0) = C_e(0) = B_a(0) = 0$ . The optimal sharing  $s^*$  makes a trade-off between providing incentives to the EN and providing incentives to the VC.

As a benchmark, let us first determine the optimal level of effort for both EN and VC when their efforts are observable and verifiable. According to the classic principal-agent theory, this corresponds to the first-best solution that maximizes the social value of the venture project.

The social value of the project is expressed as  $R(e, a) - C(e) - B(a) - I$ , then the first-best solution is expressed as  $R_e/R_a = C_e/B_a$ .

This means that the financing strategy is irrelevant; in other words, it does not matter who funds the project. However, when EN and VC cannot observe each other's level

of effort, the form of funding and how the project cash flows are distributed affecting the way in which efforts are made, resulting in what is called double-sided moral hazard in the literature.

In this paper, our objective is to catalyze the research agenda by developing the formal game-theoretic model to incorporate fairness preference and double-sided moral hazard into VC/EN financial contracting.

### 3. The Model

We model the interaction between a profit-maximizing VC (principal) and a utility-maximizing EN (agent). In our model, both EN and VC make efforts that are not observable to each other. Therefore, this is a double-sided moral hazard problem. The sequence of events in the development of the business is as follows. At date 0, a wealth-constrained EN has in mind a positive net present value project, which requires an initial capital outlay  $I > 0$ . Then, the EN approaches a self-interested VC in an attempt to obtain financing. Both the EN and the VC are risk neutral, but the EN has fairness concerns and the VC is fairness neutral.

At date 1, the VC offers an ultimatum proposal regarding the equity allocation  $s \in [0, 1]$  and  $1-s$  for the EN and the VC, respectively (this reflects the idea that the VC possesses all of the bargaining power). The EN chooses whether to accept or reject the proposal. If the EN rejects, both partners get payoffs of zero. If the EN accepts, the game continues to date 2.

In date 2, the EN and the VC make simultaneous, nonobservable efforts to develop the business. After the success or failure of the project is disclosed at date 3, both partners are paid according to the financial contract that was signed at date 1, and the game ends.

The payoff function of the VC and the EN can be expressed respectively as

$$\begin{aligned} x_1 &= (1-s)pR(e, a) - I_{vc} \\ x_2 &= spR(e, a) - (I - I_{vc}) \end{aligned} \quad (1)$$

It is assumed that the VC is a self-interested profit maximizer and his expected utility is given by

$$\Pi_{vc} = (1-s)pR(e, a) - I_{vc} - B(a) \quad (2)$$

In this study, we assume that the EN is risk neutral but inequity averse towards the VC in the sense of Fehr and Schmidt [10]. According to [10], the EN cares about his own payoff and dislikes absolute payoff differences between his own payoff and the payoff of VC. In other words, the EN maximizes his utility accounting for his monetary payoff and his concerns for fairness. The reference point is the VC's material payoff, and the EN thus compares his payoff with the VC's payoff. Specifically, the EN's fairness utility function can be denoted as

$$\begin{aligned} U_{en}(x_1, x_2) \\ = x_2 - [\alpha \max(x_1 - x_2, 0) + \beta \max(x_2 - x_1, 0)] \end{aligned} \quad (3)$$

where  $\alpha > \beta$  and  $0 \leq \beta < 1$ . The terms in the square bracket are the payoff effects of compassion (advantageous

inequality)  $\beta$  and envy (disadvantageous inequality)  $\alpha$ . We see that if the EN's payoff is greater than the payoff of VC, then EN feels compassion towards VC. However, if the EN's payoff is smaller than the payoff of VC, then EN feels envy towards VC. The EN's inequity aversion towards VC is characterized by the pair of parameters  $(\alpha, \beta)$ .

At the same time, many scholars proved that the decision maker is more caring about the envy than the compassion [27–29]; in other words, a preference for advantageous inequality is much less prominent. Moreover, as pointed by Scheer et al. [30], people in some culture do not care about advantageous inequality. Scheer et al. surveyed 417 US automobile dealers and 289 Dutch automobile dealers and found that while Dutch firms react to both disadvantageous and advantageous inequality, US firms react only to disadvantageous inequality. In order to simplify mathematical analysis and avoid tedious mathematical calculations, we intend not to incorporate the advantageous inequality in the EN's utility function. This allows us to achieve relatively concise results, focusing on the most important aspects related to inequality aversion. The similar utility function including only disadvantageous inequality is also used by [29, 31–33]. Hence, the expected utility function of EN is expressed as follows:

$$\begin{aligned} \Pi_{en} &= x_2 - \alpha(x_1 - x_2) - C(e) \\ &= (1 + \alpha)x_2 - \alpha x_1 - C(e) \end{aligned} \quad (4)$$

where  $\alpha$  is the distributional fairness parameter; the greater it is, the more EN is concerned about the fairness of the distribution. In this study, we assume that the parameter  $\alpha$  is common knowledge for both parties.

We solve the game-theoretic model using backward induction. That is, we firstly solve for VC's and EN's optimal data 2 effort levels, given the agreed equity stakes, and then move back to solve for VC's optimal data 1 equity offer. It is important to highlight that we do not impose any specific functional form for the project revenue function or the cost of the two players' efforts. In the next section, we simulate the model under the assumption that venture project's revenue is a CES function and assume that the disutility of the players' efforts is quadratic function.

Given that the VC has proposed equity allocation  $s$ , the level of effort is chosen by the VC deriving from his incentive-compatibility constraint:

$$a = \arg \max (1-s)pR(e, a) - I_{vc} - B(a) \quad (5)$$

That is, the VC maximizes his expected profit based on his share of the revenues as stipulated in the contract  $1-s$ , his rational expectation of the EN's effort  $e$ , the cost of his effort  $B(a)$ , and also his financial contribution  $I_{vc}$ .

In the similar way, the EN also chooses his level of effort based on his incentive-compatibility constraint:

$$\begin{aligned} e &= \arg \max [(1 + 2\alpha)s - \alpha] pR - (1 + \alpha)I \\ &\quad + (1 + 2\alpha)I_{vc} - C(e) \end{aligned} \quad (6)$$

Given the assumptions upon, the principal-agent problem faced by the VC is

$$\max_{s,e,a,I_{vc}} (1-s) pR(e,a) - B(a) - I_{vc} \quad (7a)$$

subject to

$$e = \arg \max \Pi_{en} \quad (7b)$$

$$a = \arg \max \Pi_{vc} \quad (7c)$$

$$\Pi_{en} \geq 0 \quad (7d)$$

The main difficulty in solving the above principal-agent problem is related to the fact that the incentive-compatibility constraint is itself a maximization problem. That is to say, the VC's maximization problem includes two additional optimization problems expressed in (7b) and (7c). To overcome this obstacle, for the cases where the effort levels are continuous, a theoretical shortcut widely used is to replace the EN and the VC's incentive-compatibility with the first order condition of their individual rationality constraint respectively [34]. Following the approach of Holmstrom [20], (7b) and (7c) are replaced by their first order conditions, and then the VC's maximization problem can be described as follows:

$$\max_{s,e,a,I_{vc}} (1-s) pR(e,a) - B(a) - I_{vc} \quad (8a)$$

subject to

$$((1+2\alpha)s - \alpha) pR_e = C_e \quad (8b)$$

$$(1-s) pR_a = B_a \quad (8c)$$

$$\begin{aligned} & ((1+2\alpha)s - \alpha) pR + \alpha(2I_{vc} - I) - C(e) \\ & \geq (I - I_{vc}) \end{aligned} \quad (8d)$$

The incentive-compatibility equations (8b) and (8c) reflect the double-sided moral hazard problem. Immediately, by these equations, we can easily obtain that the equity share awarded to the EN cannot be  $s = 0$  or  $s = 1$ . Thus, the project cash flows must be shared. The equity contract  $s$  is given by

$$s = \frac{\alpha + (C_e/R_e)(R_a/B_a)}{1 + 2\alpha + (C_e/R_e)(R_a/B_a)} \in \left( \frac{\alpha}{1 + 2\alpha}, 1 \right) \quad (9)$$

When  $s = \alpha/(1+2\alpha)$ , then  $C_e = 0$  from (8b), which means that the EN makes no effort ( $e = 0$ ). Similarly, by (8c), if  $s = 1$ , then  $B_a = 0$ , in which case the VC makes no effort ( $a = 0$ ). Hence, if both the VC and the EN each provide a positive effort level, the project cash flows must be shared. Equation (9) shows that the lower bound of the equity share level is related to the EN's fairness preference, an increase in the inequality-aversion parameter  $\alpha$  causes the lower bound of the share level awarded to the EN monotonically increasing, and the upper bound of  $s$  is 0.5. Moreover, if  $\alpha = 0$ , then  $s \in (0, 1)$ , which is similar to the case that [7] considered.

According to (9), the admissible range for  $s$  is  $(\alpha/(1+2\alpha), 1)$ , but this equation does not identify the optimal value

of  $s$ . In order to find the optimal share level  $s^*$ , problem (8a) must be solved. Let  $\lambda_1$  and  $\lambda_2$  denote the shadow price of the incentive-compatibility constraint of the EN and the VC, respectively, and  $\lambda_3$  denote the shadow price of the EN's participation constraint. According to [7],  $\lambda_3$  measures the sensitivity of the VC's objective function to a change in the amount of funding provided by the EN. The following Theorem 1 shows that EN's participation constraint (8d) is binding.

**Theorem 1.** (a) The EN's participation constraint is binding because  $\lambda_3 > 0$ . (b) The level of equity share  $s^*$  that solves the VC's optimization problem is obtained when the marginal value of the VC's effort is equal to the marginal value of the EN's effort multiplied by  $(1 + 2\alpha)$ , i.e.,  $\lambda_1(1 + 2\alpha)R_e = \lambda_2R_a$ .

*Proof.* The Lagrangian of the VC's maximization problem is defined as follows:

$$\begin{aligned} L = & [(1-s) pR(e,a) - B(a) - I_{vc}] \\ & + \lambda_1 [((1+2\alpha)s - \alpha) pR_e - C_e] + \lambda_2 [(1-s) pR_a \\ & - B_a] + \lambda_3 \{[(1+2\alpha)s - \alpha] pR(e,a) \\ & + (1+2\alpha)I_{vc} - (1+\alpha)I - C(e)\} \end{aligned} \quad (10)$$

(1) The First Order Condition for the VC's investment level  $I_{vc}$  is

$$\frac{\partial L}{\partial I_{vc}} = -1 + \lambda_3(1 + 2\alpha) = 0 \quad (11)$$

(2) The First Order Condition for the EN's effort  $e$  is

$$\begin{aligned} \frac{\partial L}{\partial e} = & (1-s) pR_e + \lambda_1 [((1+2\alpha)s - \alpha) pR_{ee} - C_{ee}] \\ & + \lambda_2(1-s) pR_{ae} \\ & + \lambda_3 [((1+2\alpha)s - \alpha) pR_e - C_e] = 0 \end{aligned} \quad (12)$$

where the last term is zero because of the EN's incentive-compatibility constraint in (8b). Thus, the above expression can be equivalently restated as follows:

$$\begin{aligned} & \lambda_1 [((1+2\alpha)s - \alpha) pR_{ee} - C_{ee}] + \lambda_2(1-s) pR_{ae} \\ & = -(1-s) pR_e \end{aligned} \quad (13)$$

(3) The First Order Condition for the VC's effort  $a$  is

$$\begin{aligned} \frac{\partial L}{\partial a} = & (1-s) pR_a - B_a + \lambda_1((1+2\alpha)s - \alpha) pR_{ae} \\ & + \lambda_2((1-s) pR_{aa} - B_{aa}) \\ & + \lambda_3((1+2\alpha)s - \alpha) pR_a = 0 \end{aligned} \quad (14)$$

where the first term is zero because of the VC's incentive-compatibility constraint in (8c). In combination with (11), the above expression would be

$$\begin{aligned} & \lambda_1((1+2\alpha)s - \alpha) pR_{ea} + \lambda_2((1-s) pR_{aa} - B_{aa}) \\ & = -\frac{((1+2\alpha)s - \alpha)}{1 + 2\alpha} pR_a \end{aligned} \quad (15)$$

(4) At a given equity participation level  $s$  allocated to the EN:

$$-pR + \lambda_1 (1 + 2\alpha) pR_e - \lambda_2 pR_a + \lambda_3 (1 + 2\alpha) pR = 0 \quad (16)$$

Given (11), the above expression can be written as follows:

$$\lambda_1 (1 + 2\alpha) R_e = \lambda_2 R_a \quad (17)$$

Equation (11) shows that  $\lambda_3 = 1/(1 + 2\alpha) > 0$  and, therefore, that the EN's equity participation constrain (8d) is binding. Furthermore, it is deduced from (13) and (15) that  $\lambda_1 > 0$ ,  $\lambda_2 > 0$ . If this does not occur, (13) and (15) reduce to  $(1-s)pR_e = 0$  and  $pR_a = 0$ , which cannot be the case because  $s \in (\alpha/(1+2\alpha), 1)$ ,  $p > 0$ ,  $R_e > 0$ ,  $R_a > 0$ .  $\square$

The next theorem provides the equity share given to the EN that maximizes the VC's problem.

**Theorem 2.** *When the EN is concerned with fairness, the optimal equity participation level given to the EN that solves the VC's problem is nonlinear and at a fixed point takes the form of  $s^* = (\varphi(s^*) + \alpha\psi(s^*)) / (\varphi(s^*) + (1 + 2\alpha)\psi(s^*))$ , where  $\varphi(s^*) = (1 + 2\alpha)^2 R_e^2 [(1 - s^*) pR_{aa} - B_{aa}]$ ,  $\psi(s^*) = R_a^2 [(1 + 2\alpha)s^* - \alpha] pR_{ee} - C_{ee}$ .*

*Proof.* Using (13) and (15), we obtain the following system of equations for  $\lambda_1$  and  $\lambda_2$ :

$$\begin{aligned} & \begin{bmatrix} \gamma pR_{ee} - C_{ee} & (1-s) pR_{ae} \\ \gamma pR_{ae} & (1-s) pR_{aa} - B_{aa} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \\ & = \begin{bmatrix} -(1-s) pR_e \\ -\frac{\gamma}{1+2\alpha} pR_a \end{bmatrix} \end{aligned} \quad (18)$$

then

$$\begin{aligned} \lambda_1 &= \frac{|A_1|}{|A|}, \\ \lambda_2 &= \frac{|A_2|}{|A|} \end{aligned} \quad (19)$$

where

$$\gamma = (1 + 2\alpha) s - \alpha$$

$$\begin{aligned} |A| &= (\gamma pR_{ee} - C_{ee}) [(1-s) pR_{aa} - B_{aa}] \\ &\quad - \gamma (1-s) p^2 R_{ae}^2 \\ |A_1| &= \frac{\gamma}{1+2\alpha} (1-s) p^2 R_a R_{ae} \end{aligned} \quad (20)$$

$$-(1-s) pR_e [(1-s) pR_{aa} - B_{aa}]$$

$$|A_2| = \gamma (1-s) p^2 R_e R_{ae} - \frac{\gamma}{1+2\alpha} pR_a (\gamma pR_{ee} - C_{ee})$$

Substituting  $\lambda_1$  and  $\lambda_2$  into (17), we obtain

$$\begin{aligned} & (1-s) [(1-s) pR_{aa} - B_{aa}] \\ & = \frac{\gamma}{(1+2\alpha)^2} \frac{R_a^2}{R_e^2} (\gamma pR_{ee} - C_{ee}) \end{aligned} \quad (21)$$

and reordering, we solve for the optimal equity participation level allocated to the EN, and that solves the VC's optimization problem:

$$s^* = \frac{\varphi(s^*) + \alpha\psi(s^*)}{\varphi(s^*) + (1 + 2\alpha)\psi(s^*)} \quad (22)$$

Equation (22) generalises the model of [7].  $\square$

Note that under the assumptions of [35] concerning the revenue and effort functions, namely,  $R(e, a) = \theta_1 e + \theta_2 a$ ,  $C(e) = e^2/2$ ,  $B(a) = a^2/2$ , (22) is expressed as follows:

$$s^* = \frac{(1 + 2\alpha)^2 \theta_1^2 + \alpha\theta_2^2}{(1 + 2\alpha)^2 \theta_1^2 + (1 + 2\alpha)\theta_2^2} \quad (23)$$

According to (23), when the two efforts are perfect substitutes and the parameter  $\alpha$  is given, the optimal equity participation level  $s^*$  increases by the elasticity of the EN's effort while decreases by the elasticity of the VC's effort. Furthermore, as  $\alpha = 0$ , then  $s^* = \theta_1^2 / (\theta_1^2 + \theta_2^2) \in (0, 1)$ , which identify with the argument by [7]. However, when  $\alpha > 0$ , we can show that  $s^* \in (1/2, 1)$  by (23); that is, if the EN is concerned with fairness, the equity share  $s^*$  assigned to the EN will be larger than 1/2.

The expression of the optimal equity share is complex, so the analysis of Theorem 2 is supported by the numerical experiments given in the next section.

#### 4. Model Analysis

In this section, we discuss the effects that the degree of complementarity of efforts, the efficiency of the EN's efforts and the VC's efforts, and the fairness preference of the EN all have on the dynamics of the effort best-response functions and on the optimal equity participation level expressed in (22). Inspired by [7], we simulate (22) by assuming a Constant Elasticity of Substitution (CES) project revenue function, which has the following specification:

$$R(e, a) = A [\theta_1 e^\rho + \theta_2 a^\rho]^{1/\rho} \quad (24)$$

where  $\theta_1$  and  $\theta_2$  correspond to the elasticity of the partners' efforts,  $0 < \theta_i < 1$ ,  $i = 1, 2$ ,  $A$  determines the productivity, and  $\rho$  determines the (constant) elasticity of substitution. Here, the two inputs of production,  $e$  and  $a$ , can contribute to the revenues of the project.

The CES function is a well-known function widely used in the production part of the microeconomics and operational research literature. Leontief, linear and Cobb-Douglas production functions are special cases of the CES production function [36]. That is, if the substitution parameter  $\rho$  equals one, efforts are perfect substitutes, and we have a linear or perfect substitutes production function; that is, the firm can obtain revenues from using either EN's effort or VC's effort independently; if  $\rho$  approaches zero in the limit, we get the Cobb-Douglas production function; as  $\rho$  approaches negative infinity, we get the Leontief or perfect complements production function; that is, they must be used in fixed

TABLE 1: Simulation parameters. Table 1 is reproduced from Marcos Vergara et al. (2016).

Parameter	Case 1	Case 2	Case 3	Case 4
$\theta_1$	0.5	0.5	0.5	0.5
$\theta_2$	0.5	0.5	0.5	0.5
$\delta_{en}$	30	30	30	30
$\delta_{vc}$	30	30	30	30
$A$	1	1	1	1
$p$	0.7	0.7	0.7	0.7
$\alpha$	(0,0.8,1.8)	(0,0.8,1.8)	(0,0.8,1.8)	(0,0.8,1.8)
$\rho$	1	0.5	-1	-10

proportions to produce revenues. If  $-\infty < \rho < 1$ , we have different degrees of complementarity between the inputs; that is, they are mixed together in different proportions in order to produce revenues.

Both efforts are costly. We will also assume that the cost of the EN's effort is given by  $C(e) = \delta_{en}e^2/2$ , while the cost of the VC's effort is  $B(a) = \delta_{vc}a^2/2$ , where  $\delta_{en} > 0$  and  $\delta_{vc} > 0$  are efficiency parameters of the EN and VC, respectively. The quadratic cost function is widely used in the venture capital and contract theory literatures (e.g., [3–8, 23–27]).

According to the incentive-compatibility constraints, we obtain the best-response functions concerning the equity share assigned to the EN at date 1 of the game. As the production function is CES, and given the assumptions of the cost of efforts, the effort dynamics of the EN and the VC are expressed as follows (see Appendix A for the proof):

$$e = \frac{\gamma\theta_1 pA}{\delta_{en}} \left[ \theta_1 + \theta_2 \left[ \frac{(1-s)\theta_2/\delta_{vc}}{\gamma\theta_1/\delta_{en}} \right]^{\rho/(2-\rho)} \right]^{(1-\rho)/\rho} \quad (25)$$

$$a = \frac{(1-s)\theta_2 pA}{\delta_{vc}} \left[ \theta_2 + \theta_1 \left[ \frac{\gamma\theta_1/\delta_{en}}{(1-s)\theta_2/\delta_{vc}} \right]^{\rho/(2-\rho)} \right]^{(1-\rho)/\rho} \quad (26)$$

where  $\gamma = (1 + 2\alpha)s - \alpha$ .

**Theorem 3.** *Under the condition that the two levels of efforts are perfect substitutes: the equity share  $s$  awarded to the EN is inversely proportional to the VC's effort, whereas it is positively related to the EN's effort.*

*Proof.* If the two efforts are perfect substitutes, i.e.,  $\rho = 1$ , according to (25) and (26), the efforts can be written as  $e = \gamma p\theta_1 A/\delta_{en}$ ,  $a = (1-s)p\theta_2 A/\delta_{vc}$ . Then

$$\begin{aligned} \frac{\partial e}{\partial s} &= \frac{(1+2\alpha)p\theta_1 A}{\delta_{en}} > 0, \\ \frac{\partial a}{\partial s} &= -\frac{\theta_2 pA}{\delta_{vc}} < 0. \end{aligned} \quad (27)$$

□

Theorem 3 shows that if the two levels of efforts are perfect substitutes, an increase in the equity share allocated to the EN

causes the VC's effort to monotonically decrease and the EN's effort to monotonically increase. This occurs because neither EN nor VC has incorporated the characteristics of the other partner or equity participation into their effort functions.

In order to investigate the effect that the substitution parameter  $\rho$  has on the dynamics of the efforts supplied by the EN and the VC, Figure 1 presents the effort dynamics of each partner at different levels of equity share  $s$  using the parameters in Table 1. The (0,0.8,1.8) in Table 1 indicates that for a given value of  $\rho$ , the fairness concerns parameter  $\alpha$  takes 0, 0.8, and 1.8 three different values respectively. Moreover, from (26), when  $\rho = 1$ , the VC's effort level is not related to the EN's fairness preference, which is the case that [7] considered.

It can be found from Figure 1 that (1) when the two players' effort levels are perfect substitutes, as shown in case 1, and as established in Theorem 3, the EN's effort level monotonically increases by  $s$ , particularly, reaching a maximum when all the equity is awarded to him,  $s = 1$ . On the contrary, if the VC retains all the property of the effort,  $s = 0$ , he exerts maximum effort, and his effort level monotonically decreases by  $s$ , reaching no effort when  $s = 1$ . (2) When the two effort levels have different degrees of complementarity, as shown in case 2-4, the EN and the VC effort level both become concave function of the equity participation share. The graphs show that there must be a level of equity participation awarded to EN which maximizes the VC's effort. Similarly, there must be a level of equity participation that maximizes EN's effort. This observation is established in Theorem 4. (3) When  $s > 1/2$ , both the two players' effort levels increase with EN's fairness concern, and vice versa if  $s < 1/2$ . (4) When the two partners share the equity in equal parts,  $s = 0.5$ , they exert the same effort levels. (5) For a given degree of effort complementarity and fairness concerns, when  $s > 1/2$ , EN provides a higher level of effort than VC, while when  $s < 1/2$ , the opposite is true.

It is worth noting that when the EN does not have fairness concerns, as the degree of efforts complementarity increases, for instance case 4:  $\rho = -10$ , both the EN and the VC exert no effort if  $s = 0$  or  $s = 1$ . A large degree of complementarity means that in order for the endeavour to succeed, both partners must put their efforts into work at the same time. If one of them is not part of the project,  $s = 0$  or  $s = 1$ , then the other partner does not exert effort at all because it is worthless.

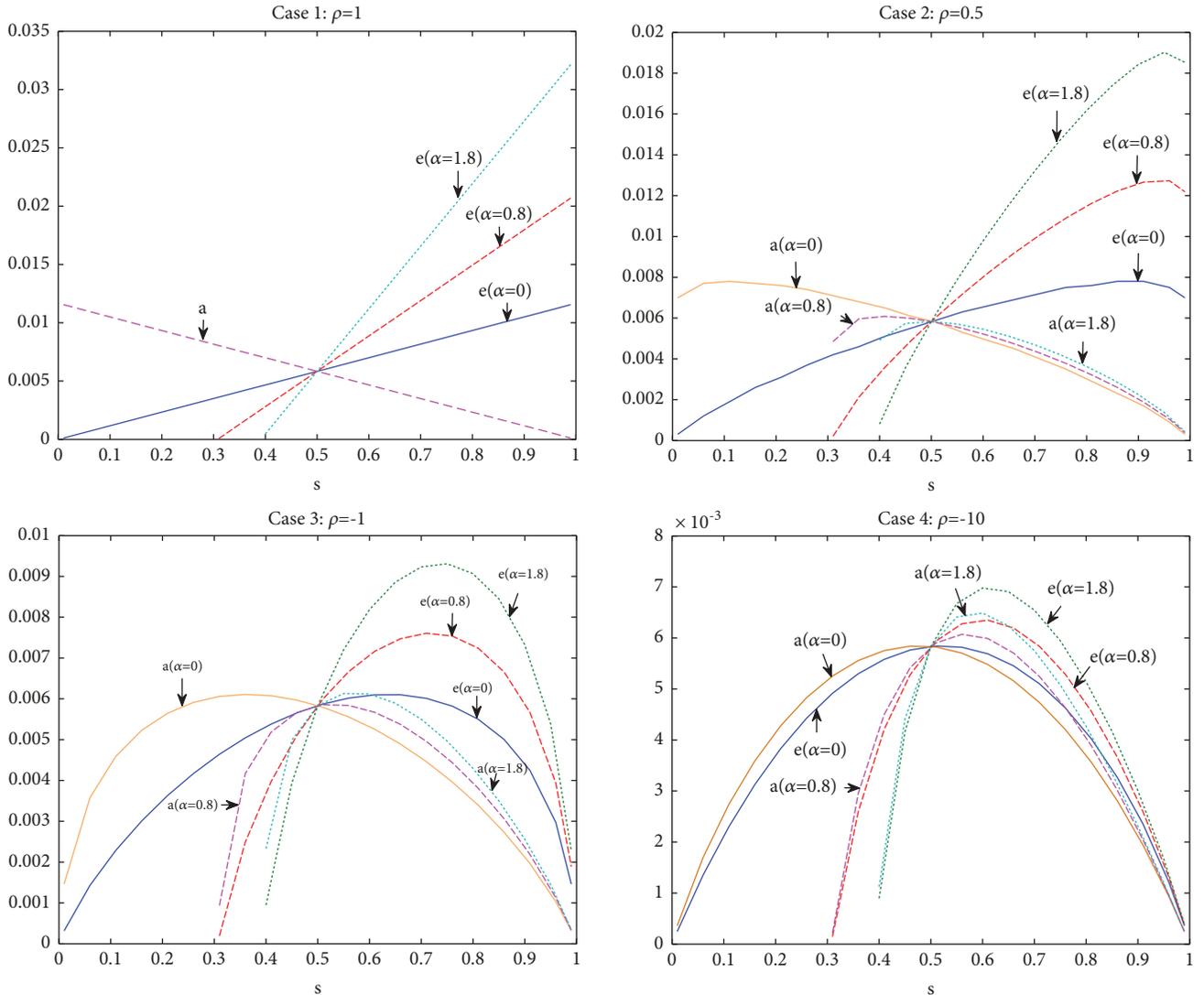


FIGURE 1: Effort dynamics for different degrees of effort complementarity  $\rho$ .

**Theorem 4.** *If efforts of the EN and the VC are complementary, then we have the following:*

(a) *There is an equity participation level  $s_{en}^*$  assigned to the EN which maximizes the effort of EN. This equity level is nonlinear and at a fixed point takes the form of  $s_{en}^* = g(s_{en}^*)$ .*

(b) *There is an equity participation level  $s_{vc}^*$  assigned to the EN which maximizes the effort of VC. This equity level is nonlinear and at a fixed point takes the form of  $s_{vc}^* = h(s_{vc}^*)$ .*

*Proof.* See Appendix B.  $\square$

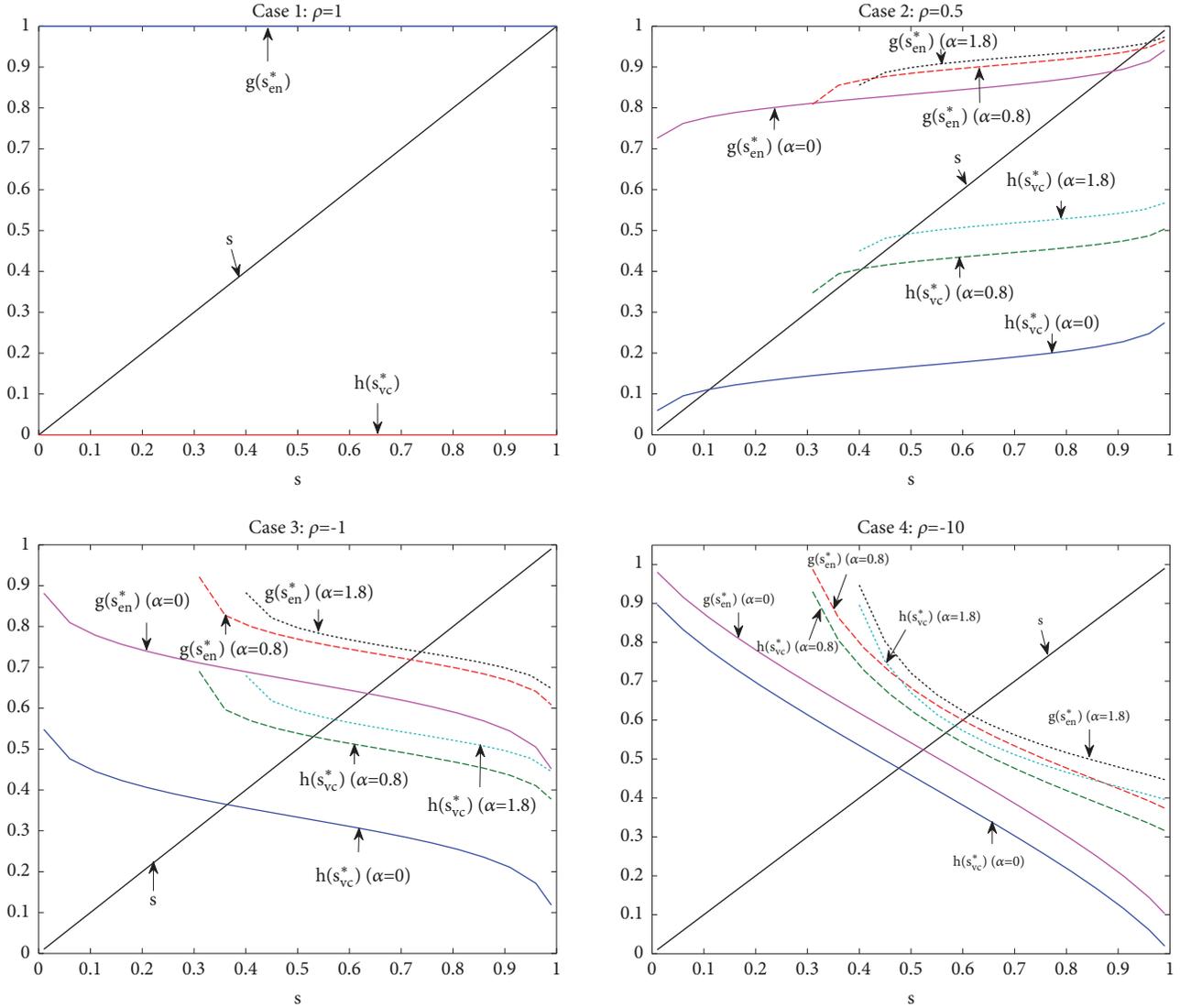
Figure 2 depicts the results of Theorem 4. The graphs demonstrate the simulation results obtained using the parameters in Table 1. The figure shows the equilibrium point of equity sharing that maximizes the efforts of both the EN and the VC. The equilibrium point appears when the functions  $g(s_{en}^*)$  and  $h(s_{vc}^*)$  intersect the 45° line.

According to Theorem 4, when  $\rho = 1$ , the EN's effort increases by the equity participation level, while the VC's

effort decreases by the equity participation level. It follows that the equity share that maximizes the EN's effort is  $s_{en}^* = 1$ , and the equity share that maximizes the VC's effort is  $s_{vc}^* = 0$ .

When  $\rho < 1$ , that is, when the two efforts are complementary, the equity share that maximizes the EN's effort is  $s_{en}^* < 1$  and the equity share that maximizes the VC's effort is  $s_{vc}^* > 0$ . The graphs show that  $s_{en}^* > s_{vc}^*$ . However, the distance between the two equity participation levels decreases with the complementarity of efforts, which means that the equity levels that maximize the two efforts tend to be similar.

Notice that the equilibrium points of the functions  $g(s_{en}^*)$  and  $h(s_{vc}^*)$  do not solve the VC's problem because we have only discussed the problem at the level of incentive-compatibility constraints of the two partners. According to Theorem 2, the equilibrium point  $s^*$  that solves the VC's maximization problem is expressed in (22). The equity share allocated to the EN that solves the VC's problem is at a fixed point  $s^* = l(s^*)$ , where  $s^* \in (s_{vc}^*, s_{en}^*)$ .


 FIGURE 2: Equity share that maximizes effort, for different degrees of complementarity  $\rho$ .

The results of Theorems 2 and 4 are further illustrated in Figure 3. The graphs demonstrate the results of simulations using the parameters in Table 1. It can be observed from Figure 3 that (1) for given fairness concern parameter  $\alpha$  and different degrees of complementarity, the equity participation that solves the VC's maximization problem (8a) is found between the share that maximizes the VC's effort and the share that maximizes the EN's effort, i.e.,  $s^* \in (s_{vc}^*, s_{en}^*)$ ; (2) an increase in the complementarity of efforts makes the equilibrium points in Theorem 4 approximate to the level that would solve the VC's maximization problem; (3) when the EN does not care about fairness, the project cash flows are distributed in similar proportions, at approximately 50% for each partner. However, when the EN is concerned with fairness, it is optimal for the VC to offer  $s > 1/2$  to the EN, and as the degree of efforts complementarity increases, the optimal equity share that awarded to the EN tends to 60%.

## 5. Influence of Fairness Concerns

In this section, we investigate the influence of fairness concerns on the equilibrium result and the optimal effort levels through numerical analysis.

In case the two effort levels are perfect substitutes, i.e.,  $\rho = 1$ , we have got the optimal equity share  $s^*$  assigned to the EN in (23). According to (23), the partial derivative of the optimal equity share with respect to the fairness concerns parameter  $\alpha$  can be expressed as follows:

$$\frac{\partial s^*}{\partial \alpha} = \frac{\theta_2^2 [(\alpha^2 + 8\alpha + 3)\theta_1^2 + (1 + \alpha)\theta_2^2]}{[(1 + 2\alpha)^2\theta_1^2 + (1 + 2\alpha)\theta_2^2]^2} \quad (28)$$

It is easy to find  $\partial s^* / \partial \alpha > 0$ , that means the optimal equity share  $s^*$  increases with EN's fairness concerns degree  $\alpha$ .

Table 2 presents the optimal equity share level  $s^*$  allocated to the EN that solves the VC's maximization problem. When

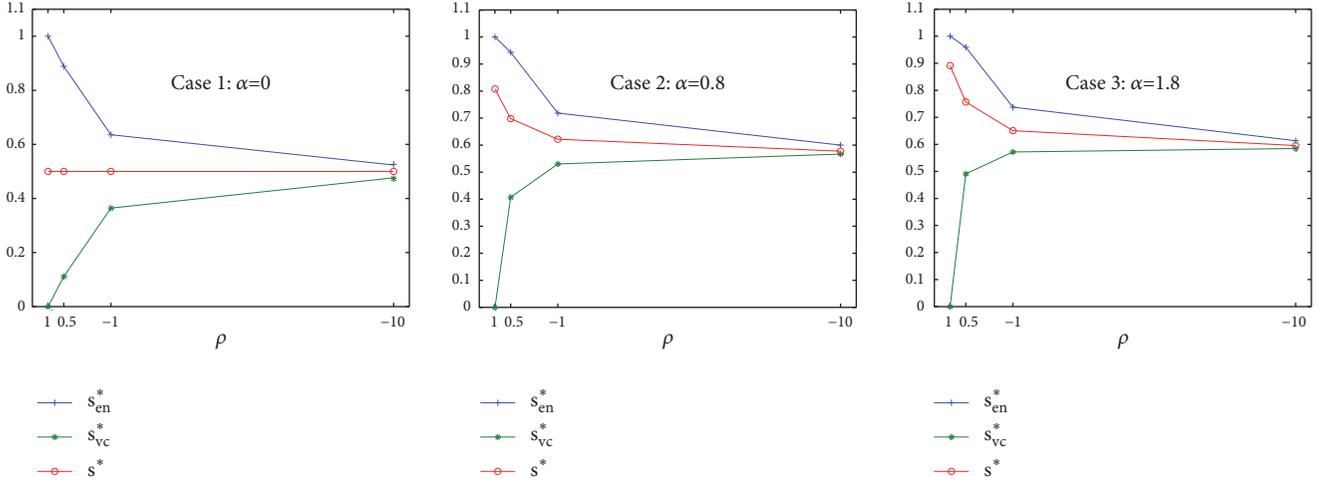


FIGURE 3: Optimal equity share for different degrees of complementarity  $\rho$ .

TABLE 2: Optimal equity share.

Parameter	$\alpha = 0$	$\alpha = 0.8$	$\alpha = 1.8$
$s^*(\rho = 1)$	0.5	0.8077	0.8913
$s^*(\rho = 0.5)$	0.5	0.6980	0.7566
$s^*(\rho = -1)$	0.5	0.6128	0.6507
$s^*(\rho = -10)$	0.5	0.5777	0.5955

TABLE 3: Optimal effort levels.

Parameter	$\alpha = 0$	$\alpha = 0.8$	$\alpha = 1.8$
$e^*(\rho = 1)$	0.0058	0.0152	0.0268
$e^*(\rho = 0.5)$	0.0058	0.0099	0.1490
$e^*(\rho = -1)$	0.0058	0.0073	0.0089
$e^*(\rho = -10)$	0.0058	0.0063	0.0070
$a^*(\rho = 1)$	0.0058	0.0022	0.0013
$a^*(\rho = 0.5)$	0.0058	0.0044	0.0041
$a^*(\rho = -1)$	0.0058	0.0056	0.0059
$a^*(\rho = -10)$	0.0058	0.0061	0.0065

the EN is a self-interested profit maximizer, i.e.,  $\alpha = 0$ , Table 2 shows that at different degrees of complementarity, the optimal equity share is  $s^* = 1/2$ , which is similar to the result of [7]. However, if the EN cares not only about his own profit but also about the fairness of the distribution, i.e.,  $\alpha > 0$ , the optimal equity share  $s^*$  is larger than  $1/2$ , and  $s^*$  increases with the fairness concern parameter  $\alpha$ , meaning the VC will pay additional expense to the EN owing to his inequality aversion. Hence, we extend the results of [7] by addressing EN's fairness concern.

Substituting the optimal equity share level  $s^*$  into (25) and (26), we can obtain the optimal effort levels  $e^*$  and  $a^*$  which should be provided by the EN and the VC in Table 3.

Table 3 shows that when  $\alpha = 0$ , the optimal efforts supplied by two partners are equal,  $e^* = a^* = 0.58$ , this result occurs because the project cash flows are distributed equally, and we assume the elasticity parameters of the partners and

their efficiency are the same. Nevertheless, when the EN is concerned with fairness, i.e.,  $\alpha > 0$ , the EN exerts more effort than when he is fairness neutral. In addition, for a given substitution parameter  $\rho$ , the fair-minded EN provides a higher effort level than the VC. Notice that this result is similar to the findings reported in the literature. Kaplan and Strömberg [37] studied actual contracts between EN and VC and found that “while VCs regularly play a monitoring and advisory role, they do not intend to become too involved in the company”. A plausible interpretation is that VC is more willing to play a nurturing role in the company, rather than to replace the EN and lead the project development.

Providing incentives to an inequality-averse EN is often more costly than to a classical agent. The intuition is that as the project is more profitable, more inequality is created and it is more expensive to satisfy the incentive-compatibility constraint. The significance of our research is that if VC

knows that the EN has inequality-aversion preference and that EN will compare the benefits between them, so VC will increase the incentive level to transfer some of the benefits to the EN. As EN sees VC's kindness to him, he will also exert more effort to reward the VC.

## 6. Conclusions and Future Directions

Equity sharing is a common practice in joint ventures, and the VC often provides value-added services by influencing firm-level innovation and commercialization beyond mere capital infusion. Taking a global perspective, the most widespread incentive contracts are sharecropping contracts. We analyze in this paper a contracting problem within a double-sided moral hazard setting where the EN has limited wealth, which constrains his feasible range of actions as the EN's effort induces both monetary and nonmonetary costs. We consider the case that the EN has fairness concerns, but the VC does not. Our analysis shows that incorporating fairness concerns into the analysis of optimal financing contract can improve our understanding of real world incentive schemes. If the EN exhibits an aversion towards inequitable distributions, the optimal contract has to balance the EN's concern for fairness and the VC's desire to provide adequate incentives.

The value-added services of VC and fairness concerns of EN have been studied in the literature independently, but these two factors are seldom studied simultaneously. In this study, we consider the VC's value-added services and behavior regarding fairness concerns using the principal-agent theory. The decision problems are much more complex, but they are closer to reality. We examine the influence of the EN's fairness concerns on the decisions of the two members, thereby demonstrating that their decisions differ from those under perfect rationality.

There are several directions deserving future research. First, we implicitly assume the information to be complete in the current paper. The VC knows the exact values of the EN's fairness related factor  $\alpha$ . This assumption may seem to be strict, because the fairness concern is private information, and the VC only can identify the accurate information about fairness concern by some efforts. It would be interesting to consider a setting wherein the fairness concern information is asymmetric and analyze the value of fairness preference information. Second, we do not consider the case where the VC also has an other-regarding preference. It is reasonable to believe that the VC might also have the same fairness concerns; therefore, the model should be developed to include inequality-averse venture capitalist. This will obviously complicate the analysis. Third, other behavioral factors, such as loss aversion and risk aversion, can be further considered in the venture capital financing contract problem. Finally, numerous studies have confirmed that trust between VC and EN plays an important role in improving the success rate of startups [13–16]. However, there is little research on the dynamic nature of trust between VC and EN. Hence, this provides an exciting future research agenda.

## Appendix

### A. Effort Best-Response Functions

If  $R(e, a) = [\theta_1 e^\rho + \theta_2 a^\rho]^{1/\rho}$ ,  $B(a) = \delta_{vc} a^2/2$ ,  $C(e) = \delta_{en} e^2/2$ , then  $R_e = A[\theta_1 e^\rho + \theta_2 a^\rho]^{1/\rho-1} \theta_1 e^{\rho-1}$ ,  $R_a = A[\theta_1 e^\rho + \theta_2 a^\rho]^{1/\rho-1} \theta_2 a^{\rho-1}$ ,  $C_e = \delta_{en} e$ ,  $B_a = \delta_{vc} a$ . Substituting the expressions above into the partners' incentive-compatibility constraints, we can obtain

$$A\gamma p [\theta_1 e^\rho + \theta_2 a^\rho]^{1/\rho-1} \theta_1 e^{\rho-1} = \delta_{en} e \quad (A.1)$$

$$A(1-s)p [\theta_1 e^\rho + \theta_2 a^\rho]^{1/\rho-1} \theta_2 a^{\rho-1} = \delta_{vc} a \quad (A.2)$$

If we divide (A.1) by (A.2) and reorder, then

$$e = a \left( \frac{\gamma \theta_1 \delta_{vc}}{1-s \theta_2 \delta_{en}} \right)^{1/(2-\rho)} \quad (A.3)$$

Substitute (A.3) into (A.2) to obtain the VC's best-response function expressed as

$$a = \frac{(1-s)\theta_2 p A}{\delta_{vc}} (\theta_2 + \theta_1 \xi^{\rho/(2-\rho)})^{(1-\rho)/\rho} \quad (A.4)$$

where  $\xi = (\gamma\theta_1/\delta_{en})/((1-s)\theta_2/\delta_{vc})$ . Then, substituting  $a$  into (A.1), we also obtain the EN's best-response function:

$$e = \frac{\gamma\theta_1 p A}{\delta_{en}} (\theta_1 + \theta_2 \xi^{-\rho/(2-\rho)})^{(1-\rho)/\rho} \quad (A.5)$$

### B. The Proof of Theorem 4

The derivative of (25) with respect to the equity share  $s$  allocated to the VC is

$$\begin{aligned} \frac{de}{ds} &= \frac{(1+2\alpha)\theta_1 p A}{\delta_{en}} (\theta_1 + \theta_2 \xi^{-\rho/(2-\rho)})^{(1-\rho)/\rho} \\ &\quad - \frac{\gamma\theta_1 p A}{\delta_{EN}} \frac{1-\rho}{\rho} (\theta_1 + \theta_2 \xi^{-\rho/(2-\rho)})^{(1-2\rho)/\rho} \\ &\quad \times \frac{\theta_2 \rho}{2-\rho} \xi^{(2-2\rho)/(2-\rho)} \left( \frac{\theta_2 \delta_{en}}{\theta_1 \delta_{vc}} \frac{1+\alpha}{\gamma^2} \right) \end{aligned} \quad (B.1)$$

Setting the expression above to zero and reordering, then

$$s_{en}^* = g(s_{en}^*) = 1 - \frac{1 + \alpha}{1 + 2\alpha} \frac{((1 - \rho) / (2 - \rho)) \theta_2 [((1 - s_{en}^*) / \gamma(s_{en}^*)) (\theta_2 / \theta_1) (\delta_{en} / \delta_{vc})]^{\rho / (2 - \rho)}}{[\theta_1 + \theta_2 [((1 - s_{en}^*) / \gamma(s_{en}^*)) (\theta_2 / \theta_1) (\delta_{en} / \delta_{vc})]^{\rho / (2 - \rho)}]} \quad (B.2)$$

where  $\gamma(s_{en}^*) = (1 + 2\alpha)s_{en}^* - \alpha$ .

Similarly, the derivative of (26) with respect to the equity shares can be expressed as

$$\frac{\partial a}{\partial s} = -\frac{\theta_2 p A}{\delta_{vc}} [\theta_2 + \theta_1 \xi^{\rho / (2 - \rho)}]^{(1 - \rho) / \rho}$$

$$\begin{aligned} & + \frac{(1 - s) \theta_2 p A}{\delta_{vc}} \frac{1 - \rho}{\rho} [\theta_2 + \theta_1 \xi^{\rho / (2 - \rho)}]^{(1 - 2\rho) / \rho} \\ & \times \frac{\theta_1 \rho}{2 - \rho} \xi^{(2\rho - 2) / (2 - \rho)} \left[ \frac{1 + \alpha}{(1 - s)^2} \frac{\theta_1 \delta_{vc}}{\theta_2 \delta_{en}} \right] \end{aligned} \quad (B.3)$$

If we set the expression above to zero and reorder, then

$$s_{vc}^* = h(s_{vc}^*) = \frac{1 + \alpha}{1 + 2\alpha} \frac{((1 - \rho) / (2 - \rho)) \theta_1 [(\gamma(s_{vc}^*) / (1 - s_{vc}^*)) (\theta_1 / \theta_2) (\delta_{vc} / \delta_{en})]^{\rho / (2 - \rho)}}{[\theta_2 + \theta_1 [(\gamma(s_{vc}^*) / (1 - s_{vc}^*)) (\theta_1 / \theta_2) (\delta_{vc} / \delta_{en})]^{\rho / (2 - \rho)}]} + \frac{\alpha}{1 + 2\alpha} \quad (B.4)$$

where  $\gamma(s_{vc}^*) = (1 + 2\alpha)s_{vc}^* - \alpha$ .

## Data Availability

All data generated or analyzed during this study are included in this paper. The authors are willing to share the implementation scripts in the form of some MATLAB m-files with the interested reader.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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