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Research Article

Robust Optimization for the Newsvendor Problem with Discrete Demand

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A robust newsvendor model with discrete demand is initiatively studied, and the steps to obtain the optimal ordering decision are provided. The study shows that the optimal ordering decision with discrete demand is very different from that with continuous demand. Besides, the total number of demand points has almost no effect on the performance of both ordering decisions. Furthermore, for an ultralow-profit product, the ordering decision with discrete demand performs significantly better than that with continuous demand.

1. Introduction

The newsvendor problem has been investigated as a basic problem in stochastic inventory management since the eighteenth century in the economic literature, and it has been universally employed to analyze supply chains with fashionable and perishable products. Since the 50s of the last century, newsvendor problem has been extensively studied in operations research and extended to model various problems in real life. The simplest and most elegant version of the newsvendor problem is an optimal inventory problem in which a newsvendor needs to decide how much newspaper to order for the future demand, where the future demand is uncertain and follows a stationary distribution. This classical newsvendor problem has been extended in many different ways. See Khouja [1] and Qin et al. [2] for a review of various newsvendor-related models. See Choi [3] for the state-of-the-art findings on both theoretical and applied research on the newsvendor problem.

The studies on the newsvendor problem mentioned above focus mainly on scenarios in which the underlying distribution of the random demand is precisely known. However, it is often very hard or impossible to figure out the demand distribution in practice, especially in fast-changing markets. Since the decision is made on the basis of the assumed distribution, it might not hold for other distributions. Therefore, instead of assuming a stationary distribution, it is more reasonable to assume that the distribution belongs to the set of all distributions satisfying the known parameters. These parameters may come from estimates based on past realizations or some prediction by industry experts. The robust approach optimizes the worst-case objective (e.g., expected profit or regret or risk) over the parametric family. Most research that uses the robust approach in newsvendor models describes the distribution set by some known mean and variance. In these researches, Scarf [4] can be regarded as a pioneer who derived a closed-form formula for the optimal ordering rule that maximizes the expected profit against the worst possible distribution of the demand with the given mean and variance. He also pointed out that the worst distribution of the demand has positive mass at two points. Gallego and Moon [5] provided a simpler proof of Scarf's formula and extended his ideas. In the last 25 years, since the appearance of Gallego and Moon [5], there have been numerous results in the literature related to extensions of Scarf's ordering rule to different settings and applications; see, for example, maximizing the worst-case expected profit as a target [6–18], minimax regret as a target [10, 19–21], and optimizing the worst-case conditional value-at-risk as a target [22–28].

The existing research has revealed that the incompleteness of demand distributional information will have substantial impact on the newsvendor's decision. In general, these works are based on the assumption that the demand is a continuous random variable. Nevertheless, the actual demand for many products is discrete. The discrete goods can be bought and sold only in integral units, such as newspapers, cars, and machines. When the discrete goods are high-cost or low-profit products, their demand could not be approximately regarded as continuous demand. So far, there is only a little research aimed at studying the newsvendor problem with limited distributional information of discrete stochastic demand. Yu et al. [29] presented a general framework for considering discrete demand in robust newsvendor models. Carrizosa et al. [30] explored the robust newsvendor problem where demand is modeled as a time series which follows an autoregressive process. The advantage of robust autoregressive approach is that there is not much information needed, and it can be predicted by using self-variable series. But this method is also limited: (1) Autocorrelation coefficient is the key. If the autocorrelation coefficient is less than 0.5, it is inappropriate to use it; otherwise the prediction result is extremely inaccurate. (2) Autoregression can only be used to predict the economic phenomena related to their own earlier period, that is, the economic phenomena which are greatly influenced by their own historical factors, such as the amount of mining and the production of various natural resources and so on. In view of the limitations of the autoregressive method, we still only use robust approach and do not assume demand as a time series. On the basis of Yu et al. [29], we made a more systematic, more specific, and more in-depth study in robust newsvendor problem with discrete demand.

Specifically, the purpose of this paper is threefold. The first is to provide the solving steps to obtain the optimal ordering decision for the robust newsvendor model with discrete demand. The existing solution with continuous demand can merely be regarded as an approximate solution with discrete demand. The second is to compare the performance of our solution with discrete demand with the existing solution with continuous demand in different situations. The third is to study the effect of profit margin on these two kinds of solutions.

The rest of the paper is organized as follows. In Section 2, we review the existing results of classical/robust newsvendor problem. In Section 3, we formulate a robust newsvendor model with discrete demand and provide the approach to solve it. In Section 4, we conduct numerical experiments to calculate the optimal order quantity with discrete demand and compare it with that with continuous demand. In Section 5, we draw our conclusions. The proof of main result is given in Appendix.

2. Existing Related Results

The related notation are as follows:

c: the item's unit cost

p: the item's unit selling price

s: the item's unit salvage value, if an item is left unsold at the end of the sales period

q: the item's order quantity

D: the item's stochastic demand

F: the distribution function of the item's demand D

μ: the item's expected demand over the sales period

σ: standard deviation of the item's demand

$\Gamma(\mu,\sigma^2)$: the class of all distribution functions with mean μ and variance σ^2

$\Gamma_+(\mu,\sigma^2)$: the subclass of distribution functions F of nonnegative random variables, i.e., $\int_0^{+\infty} dF(x) = 1$ and $\Gamma_+(\mu,\sigma^2) \subset \Gamma(\mu,\sigma^2)$

2.1. Classical Newsvendor Model. The classical newsvendor problem first proposed by Arrow et al. [31] assumes that the stochastic demand follows a known distribution F. Its model is

$$\max_q \mathrm{E}\left[\pi_F(q)\right], \tag{1}$$

where $\pi_F(q) \coloneqq (p-s)\min(q,D) - (c-s)q$ is the newsvendor's profit function.

Lemma 1 (continuous demand). *If F is a known continuous distribution, then the optimal order quantity q_F^* maximizing $\mathrm{E}[\pi_F(q)]$ satisfies*

$$q_{Fc}^* = F^{-1}\left(\frac{p-c}{p-s}\right). \tag{2}$$

Lemma 2 (discrete demand). *If F is a known discrete distribution with points $x_1, x_2, \ldots, x_n$ ($\{x_i\}$ is a sequence sorted by size) and probabilities $p_1, p_2, \ldots, p_n$, then F is completely determined by p_i for $i = 1, \ldots, n$. The optimal order quantity q_{Fd}^* can be found as follows: Let $1 \le k \le n$ be the smallest integer s.t. $\sum_{i=1}^k p_i \ge (p-c)/(p-s)$. Maximum expected profit is achieved using*

$$q_{Fd}^* = x_k. \tag{3}$$

In the degenerate case where $\sum_{i=1}^k p_i = (p-c)/(p-s)$, the expected value is constant if $q_{Fd}^* \in [x_k, x_{k+1}]$. In all other cases, the optimal order quantity is unique.

2.2. Robust Newsvendor Model with Continuous Demand. The robust newsvendor problem assumes that the newsvendor only knows the mean μ and the variance σ^2 of the demand distribution F. The newsvendor model with free distribution of continuous demand is

$$\max_q \min_{F\in\Gamma_+(\mu,\sigma^2)} \mathrm{E}\left[\pi(q,D)\right], \tag{4}$$

where

$$\pi(q,D) := (p-s)\min(q,D) - (c-s)q \tag{5}$$

is the newsvendor's profit function.

Scarf [4] solved the robust newsvendor model with continuous demand in steps. Firstly, he solved the minimization problem

$$\min_{F\in\Gamma_+(\mu,\sigma^2)} \mathrm{E}[\pi(q,D)]. \tag{M1}$$

Then he solved the maximization problem

$$\max_q \underline{\pi}(q), \tag{M2}$$

where $\underline{\pi}(q) := \min_{F\in\Gamma_+(\mu,\sigma^2)} \mathrm{E}[\pi(q,D)]$.

Lemma 3. *If F is an uncertain continuous distribution with $F\in\Gamma_+(\mu,\sigma^2)$, then the worst-case expected profit satisfies*

$$\underline{\pi}(q) = \begin{cases} \dfrac{(p-c)\mu^2-(c-s)\sigma^2}{\mu^2+\sigma^2}q, & \text{if } 0\le q < \dfrac{(\mu^2+\sigma^2)}{2\mu}; \\ \dfrac{p-s}{2}\left[(\mu-q)-\sqrt{(\mu-q)^2+\sigma^2}\right]+(p-c)q, & \text{if } q \ge \dfrac{\mu^2+\sigma^2}{2\mu}. \end{cases} \tag{6}$$

The optimal order quantity q^*_{Rc} maximizing $\underline{\pi}(q)$ satisfies

$$q^*_{Rc} = \begin{cases} 0, & \text{if } \left(\dfrac{\mu}{\sigma}\right)^2 < \dfrac{c-s}{p-c}; \\ \mu+\dfrac{\sigma}{2}\left(\sqrt{\dfrac{p-c}{c-s}}-\sqrt{\dfrac{c-s}{p-c}}\right), & \text{if } \left(\dfrac{\mu}{\sigma}\right)^2 \ge \dfrac{c-s}{p-c}, \end{cases} \tag{7}$$

which is called "Scarf's ordering rule".

3. Robust Newsvendor Model with Discrete Demand

In this section, we are interested in solving the robust newsvendor model with discrete demand. Assume that $F\in\Gamma_+(\mu,\sigma^2)$ is an uncertain discrete distribution with known points $x_1,x_2,\ldots,x_n$ ($n\ge 4$, $\{x_i\}$ is a sequence sorted by size) and unknown probabilities $p_1,p_2,\ldots,p_n$. Motivated by the approaches adopted in Scarf [4], we also solve the model in two steps.

Firstly, we use duality theory to analyze the minimization problem (M1), which is equivalent to the following linear programming problem:

$$\begin{aligned} \min_{p_1,\ldots,p_n}\quad & \mathrm{E}[\pi(q,D)] \\ & = (p-s)\left(\sum_{k=1}^n \min(q,x_k)p_k - \frac{c-s}{p-s}q\right), \\ \text{s.t.}\quad & \sum_{k=1}^n p_k = 1, \\ & \sum_{k=1}^n x_k p_k = \mu, \\ & \sum_{k=1}^n x_k^2 p_k = \mu^2+\sigma^2, \\ & p_1,\ldots,p_n \ge 0. \end{aligned} \tag{P}$$

Its dual formulation is

$$\begin{aligned} \max_{y_1,y_2,y_3}\quad & y_1+\mu y_2+(\mu^2+\sigma^2)y_3, \\ \text{s.t.}\quad & y_1+x_k y_2+x_k^2 y_3 \\ & \le (p-s)\left(\min(q,x_k)-\frac{c-s}{p-s}q\right), \\ & \text{for each } k=1,\ldots,n, \end{aligned} \tag{D}$$

where y_0, y_1, and y_2 are the dual variables corresponding to the probability-mass, mean, and variance constraints.

The main idea is to construct a pair of primal-dual feasible solutions $(p_1^*,p_2^*,\ldots,p_n^*)$ for (P) and (y_1^*,y_2^*,y_3^*) for (D), and make sure they satisfy the complementary slackness condition:

$$\begin{aligned} p_k^*\Big[& y_1^*+x_k y_2^*+x_k^2 y_3^* \\ & -(p-s)\left(\min(q,x_k)-\frac{c-s}{p-s}q\right)\Big]=0, \\ & \text{for each } k=1,\ldots,n. \end{aligned} \tag{8}$$

In the case of linear programming, this ensures optimality.

That is to say, the point x_k has nonzero probability if and only if the dual optimal solution (y_1^*,y_2^*,y_3^*) satisfies

$$y_1^*+x_k y_2^*+x_k^2 y_3^* = (p-s)\left(\min(q,x_k)-\frac{c-s}{p-s}q\right). \tag{9}$$

In other words, the same points of the smooth curve $g(x) := y_1+y_2x+y_3x^2$ and the two-piece line $\pi(q,x)$ should be the points where the primal optimal solution places all its masses.

A fundamental result of linear programming theory asserts that there exists an optimal solution such that the number of nonzero variables is no more than the number of constraints. Therefore, for (P), the number of nonzero p_k^* is no more than three. It indicates that the worst-case distribution is a three-point distribution. Therefore, we restrict ourselves to three-point distributions.

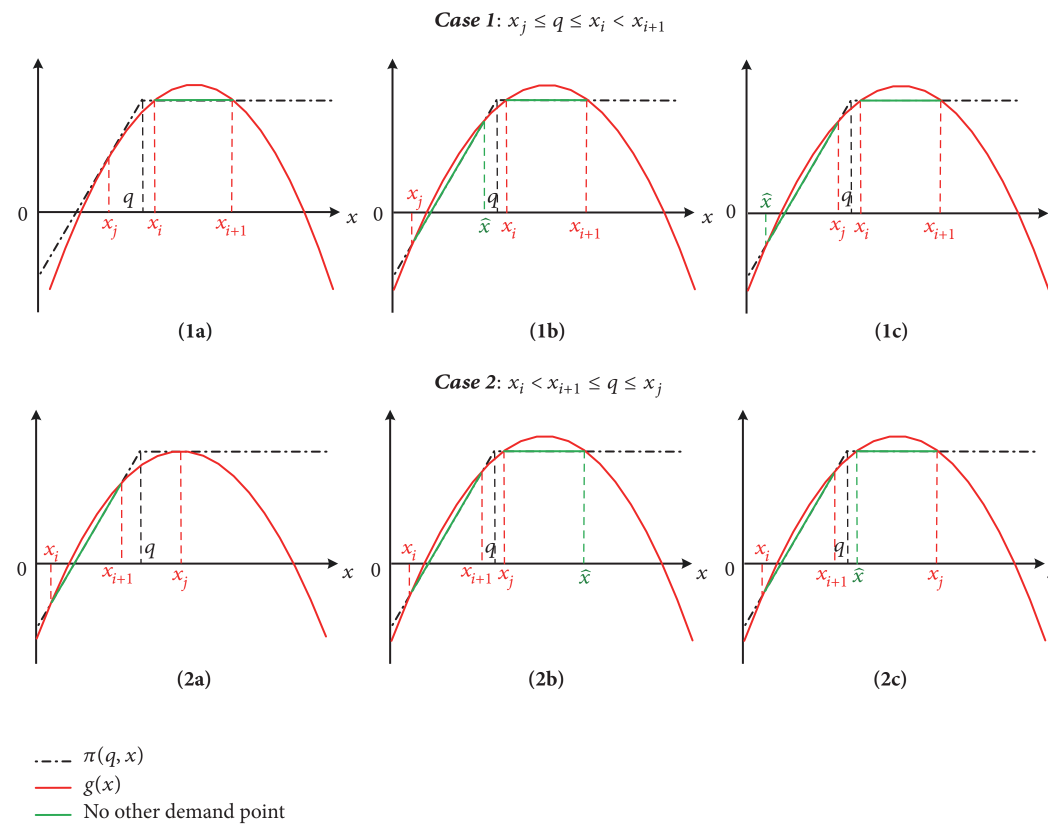

Figure 1: The cases that satisfy the dual feasible condition.

As illustrated by Figure 1, the two functions $g(x) := y_1 + xy_2 + x^2 y_3$ and $\pi(q, x)$ will have four intersection points at most. Based on the above analysis, we restrict ourselves to three corresponding demand points. Besides, in order to meet the dual feasibility, at least two of the three demand points are adjacent.

On the basis of the above analysis, we provide the close-form expression for the worst-case expected profit $\underline{\pi}(q) := \min_{p_1,\dots,p_n} \mathrm{E}[\pi(q, D)]$, in the following theorem.

Theorem 4. *For three points x_i, x_{i+1}, x_j in all known demand points, assume their probabilities as*

$$\begin{aligned} p_j^* &= \frac{\sigma^2 + (\mu - x_i)(\mu - x_{i+1})}{(x_j - x_i)(x_j - x_{i+1})}, \\ p_i^* &= \frac{\sigma^2 + (\mu - x_j)(\mu - x_{i+1})}{(x_i - x_j)(x_i - x_{i+1})}, \\ p_{i+1}^* &= \frac{\sigma^2 + (\mu - x_j)(\mu - x_i)}{(x_{i+1} - x_j)(x_{i+1} - x_i)}, \\ p_k^* &= 0, \quad k = 1, \dots, n, \ k \neq j, i, i+1. \end{aligned} \tag{10}$$

Case 1. For arbitrary $2 \leq i \leq n-1$, $1 \leq j \leq i-1$ and a fixed q, if the primal feasibility is satisfied,

$$\begin{aligned} &\max\left\{(x_i - \mu)(\mu - x_j), (x_{i+1} - \mu)(\mu - x_i)\right\} \leq \sigma^2 \\ &\quad \leq (x_{i+1} - \mu)(\mu - x_j), \end{aligned} \tag{11}$$

and the dual feasibility is satisfied,

$$\begin{aligned} &\frac{x_i x_{i+1} - x_{j-1} x_j}{(x_i + x_{i+1}) - (x_{j-1} + x_j)} \leq q \\ &\quad \leq \frac{x_i x_{i+1} - x_j x_{j+1}}{(x_i + x_{i+1}) - (x_j + x_{j+1})}, \quad \text{for } 2 \leq j \leq i-1; \\ &x_j < q \leq \frac{x_i x_{i+1} - x_j x_{j+1}}{(x_i + x_{i+1}) - (x_j + x_{j+1})}, \quad \text{for } j = 1, \end{aligned} \tag{12}$$

then the worst-case expected profit satisfies

$$\underline{\pi}(q) = (p - s) x_j p_j^* + \left[(p - s)(1 - p_j^*) - (c - s)\right] q. \tag{13}$$

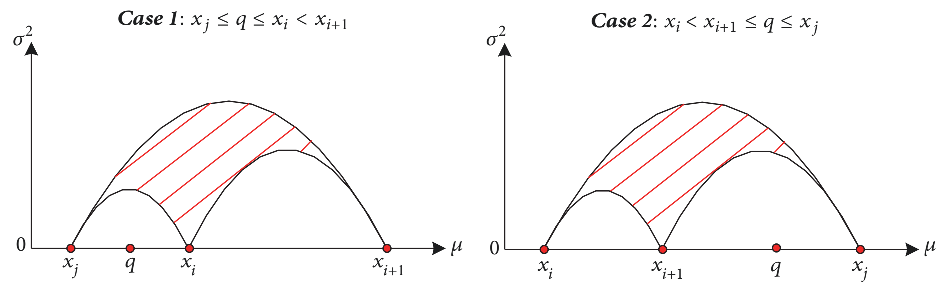

FIGURE 2: Characterization of the primal feasibility in Theorem 4.

Case 2. For arbitrary $1 \le i \le n-2$, $i+2 \le j \le n$ *and a fixed* q*, if the primal feasibility is satisfied,*

$$\max\left\{\left(x_{i+1}-\mu\right)\left(\mu-x_i\right),\left(x_j-\mu\right)\left(\mu-x_{i+1}\right)\right\} \le \sigma^2 \le \left(x_j-\mu\right)\left(\mu-x_i\right), \tag{14}$$

and the dual feasibility is satisfied,

$$\begin{aligned} &\frac{x_{j-1}x_j - x_i x_{i+1}}{\left(x_{j-1}+x_j\right)-\left(x_i+x_{i+1}\right)} \le q \le \frac{x_j x_{j+1} - x_i x_{i+1}}{\left(x_j+x_{j+1}\right)-\left(x_i+x_{i+1}\right)}, \quad \text{for } i+2 \le j \le n-1; \\ &\frac{x_{j-1}x_j - x_i x_{i+1}}{\left(x_{j-1}+x_j\right)-\left(x_i+x_{i+1}\right)} \le q < x_j, \quad \text{for } j = n, \end{aligned} \tag{15}$$

then the worst-case expected profit satisfies

$$\begin{aligned} \underline{\pi}(q) &= (p-s)\left(x_i p_i^* + x_{i+1} p_{i+1}^*\right) \\ &\quad + \left[(p-s)\,p_j^* - (c-s)\right] q. \end{aligned} \tag{16}$$

Figure 2 provides a graphical representation of the primal feasibility in Theorem 4 in the mean-variance space. If (μ, σ^2) is in the red striped area, then the primal feasibility is satisfied. Table 1 provides the dual feasible intervals of q with respect to fixed i and different j in a list.

To sum up, Theorem 4 tells us that we can find out the worst-case distributions and the worst-case expected profit $\underline{\pi}(q)$ by the following steps:

(1) Judge whether arbitrary three demand points x_i, x_{i+1}, and x_j satisfy the primal feasibility. If they satisfy it, go to step 2; or else, consider other three points.

(2) Find out the dual feasible interval of q.

(3) Obtain the close-form expression of $\underline{\pi}(q)$ on the dual feasible interval.

Theorem 4 and Table 1 reveal that $\underline{\pi}(q)$ must be a piecewise linear function. According to the expression for $\underline{\pi}(q)$ in Theorem 4, we can draw the graph of the worst-case expected profit, to help decision-makers make order decisions.

Secondly, we consider the maximization problem (M2). We can not obtain the explicit expression of the optimal order quantity q^*_{Rd} maximizing $\underline{\pi}(q)$, since it is completely determined by the values of demand points. Fortunately, we can use MATLAB to draw the graph of the worst-case expected profit and find the optimal order quantity q^*_{Rd} that maximizes the worst-case expected profit.

4. Numerical Experiments

In contrast to the existing robust newsvendor models, the model presented by us takes discrete demand into consideration. Therefore, it is expected that our model may behave differently from the existing robust newsvendor models. In the following numerical experiments, we will first figure out the concrete value of q^*_{Rd} according to the solving steps in the previous section. Second, we will compare the value and performance of q^*_{Rd} with q^*_{Rc}, where q^*_{Rc} is obtained by "Scarf's ordering rule" [4]. Third, we will study the impact of various parameters on q^*_{Rd} and q^*_{Rc}.

4.1. Optimal Ordering Decisions. Set the benchmark values of parameters as $p = 50$, $c = 35$, $s = 25$, $\mu = 1000$, $\sigma = 500$ and the known demand points are $x_1 = 100$, $x_2 = 500$, $x_3 = 1100$, $x_4 = 1500$, $x_5 = 2000$.

As indicated in Figure 3, the worst-case expected profit $\underline{\pi}(q)$ is a piecewise linear function, and the worst-case demand distribution corresponding to each segment in the piecewise linear function is a three-point distribution. For example, the worst-case distribution corresponding to the first segment is the three-point distribution of (x_1, x_3, x_4). That is to say, the worst-case distribution of the demand is a piecewise combination of several three-point cases. The results of our numerical experiments are consistent with Theorem 4. The decision maker can find the highest worst-case expected profit point through Figure 3 and make a robust order quantity decision.

4.2. Comparison with Existing Research Result. Our second numerical study aims to compare the value and performance of q^*_{Rd} with q^*_{Rc}.

Firstly, we compare the value of q^*_{Rd} with q^*_{Rc} in Table 2. Set the benchmark values of parameters as $p = 50$, $c = 35$, $s = 25$, $\mu = 1000$, and $\sigma = 500$. From Table 2, it is clear that, for the same information of mean and variance, q^*_{Rc} is always unchanging, but q^*_{Rd} varies according to different information of demand points.

TABLE 1: The interval of fixed q to make sure of the dual feasibility.

Fixed q	Dual feasible interval
$x_1 \le q \le x_i < x_{i+1}$	$q \in \left(x_1, \dfrac{x_i x_{i+1} - x_1 x_2}{(x_i + x_{i+1}) - (x_1 + x_2)}\right]$
$x_2 \le q \le x_i < x_{i+1}$	$q \in \left[\dfrac{x_i x_{i+1} - x_1 x_2}{(x_i + x_{i+1}) - (x_1 + x_2)}, \dfrac{x_i x_{i+1} - x_2 x_3}{(x_i + x_{i+1}) - (x_2 + x_3)}\right]$
$\cdots$	$\cdots$
$x_{i-1} \le q \le x_i < x_{i+1}$	$q \in \left[\dfrac{x_i x_{i+1} - x_{i-2} x_{i-1}}{(x_i + x_{i+1}) - (x_{i-2} + x_{i-1})}, x_i\right]$
$x_i < x_{i+1} \le q \le x_{i+2}$	$q \in \left[x_{i+1}, \dfrac{x_{i+2} x_{i+3} - x_i x_{i+1}}{(x_{i+2} + x_{i+3}) - (x_i + x_{i+1})}\right]$
$\cdots$	$\cdots$
$x_i < x_{i+1} \le q \le x_{n-1}$	$q \in \left[\dfrac{x_{n-2} x_{n-1} - x_i x_{i+1}}{(x_{n-2} + x_{n-1}) - (x_i + x_{i+1})}, \dfrac{x_{n-1} x_n - x_i x_{i+1}}{(x_{n-1} + x_n) - (x_i + x_{i+1})}\right]$
$x_i < x_{i+1} \le q \le x_n$	$q \in \left[\dfrac{x_{n-1} x_n - x_i x_{i+1}}{(x_{n-1} + x_n) - (x_i + x_{i+1})}, x_n\right)$

TABLE 2: Comparison of the value of q^*_{Rd} with q^*_{Rc}.

x_1	x_2	x_3	x_4	x_5	q^*_{Rd}	q^*_{Rc}
100	500	1100	1500	2000	1289	1102
50	350	700	1650	1900	700	1102
100	300	500	1000	2000	1000	1102
350	550	1200	1350	1600	1251	1102
100	200	1300	1800	2000	1300	1102

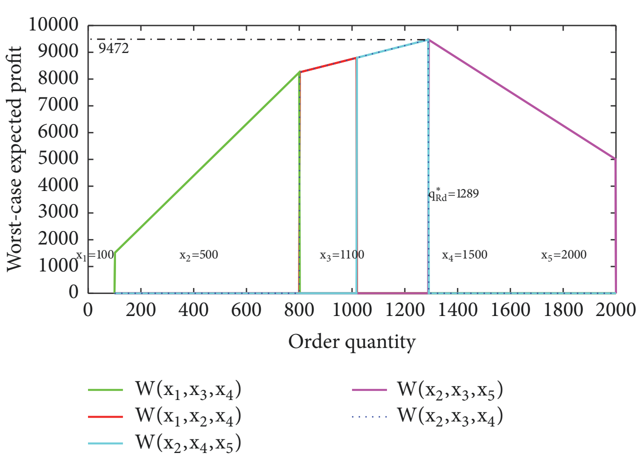

FIGURE 3: A plot of the worst-case expected profit $\underline{\pi}(q)$. PS: $W(x_i, x_{i+1}, x_j)$ represents the worst-case distribution consisting of x_i, x_{i+1}, and x_j.

Secondly, we compare the performance of q^*_{Rd} with q^*_{Rc} in view of the expected profit in real situation. Both Scarf [4] and Gallego and Moon [5] analyzed the performance of ordering decisions for high-profit and low-profit products. Motivated by their numerical experiments, we further analyze the performance of ordering decisions for products with different marginal profits.

We set parameters as $p = 10$, $c = \{1, 3, 5, 7, 9\}$, $s = 0$, and $n = \{6, 12, 24\}$. We denote profit margin by $m = (p - c)/(p - s)$; then the corresponding profit margins are $m = \{90\%, 70\%, 50\%, 30\%, 10\%\}$. For each profit margin, we randomly generate a set of 100 problems. In each instance, n demand points are randomly taken on the interval $[0, 2000]$, and their corresponding probabilities are drawn from a uniform distribution. Under each random fixed discrete distribution, we use q^*_{Rc} and q^*_{Rd} instead of q^*_{Fd} to obtain the expected profits, respectively, and then compare them in Table 3 and Figure 4, in terms of $\pi_F(q^*_{Rc})/\pi_F(q^*_{Fd})$ and $\pi_F(q^*_{Rd})/\pi_F(q^*_{Fd})$.

Table 3 shows that the number of total demand points has almost no effect on the performance of q^*_{Rd} and q^*_{Rc}. Furthermore, it is observed from both Table 3 and Figure 4 that, as the marginal profit decreases, the performance of q^*_{Rc} is getting worse and worse, but the performance of q^*_{Rd} gets worse first and then becomes better. We also found that for the super low-margin product ($m = 10\%$) q^*_{Rd} performs significantly better than q^*_{Rc}; and for the products with profit margin $m = 30\%$ and profit margin $m = 90\%$, their performances are similar; and for the products with other profit margins, the performance of q^*_{Rc} is always better than q^*_{Rd}. It is worth mentioning that both q^*_{Rd} and q^*_{Rc} perform fairly well for the super high-margin product ($m = 90\%$).

The results of the above numerical experiments conform with the reality. If we look at the discrete demand of the super low-margin product as continuous demand, the influence of the decision-making errors will be enlarged. That is to say, when the marginal profit of the product is very low, the decision maker should take the information of demand points into consideration; otherwise he should ignore this information. Therefore, we recommend the use of q^*_{Rd} as the

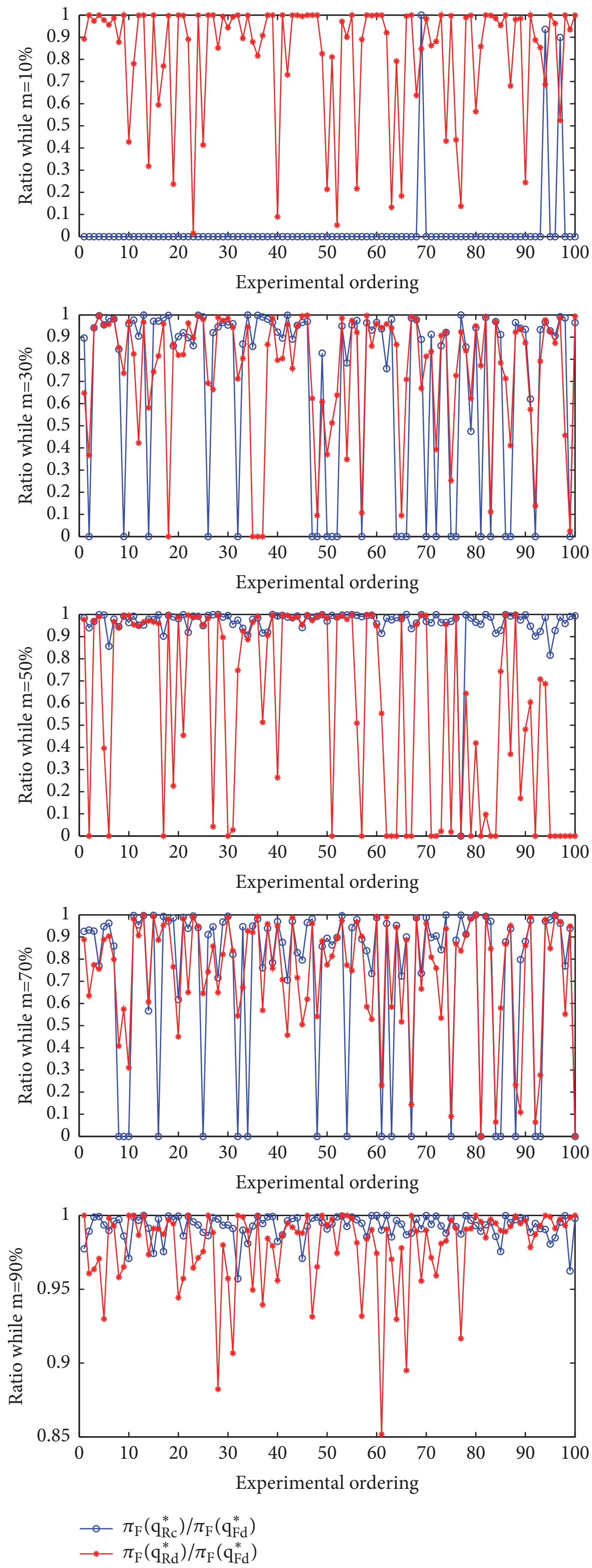

Figure 4: Comparison of the performance of q^*_{Rd} with q^*_{Rc} with respect to different profit margins. PS: $n = 12$.

Table 3: Comparison of the performance of q^*_{Rd} with q^*_{Rc} over 100 instances.

	m	90%	70%	50%	30%	10%
n=6	$E\left[\frac{\pi_F(q^*_{Rc})}{\pi_F(q^*_{Fd})}\right]$	0.9853	0.9627	0.9415	0.6137	0.1875
n=6	$E\left[\frac{\pi_F(q^*_{Rd})}{\pi_F(q^*_{Fd})}\right]$	0.9573	0.7321	0.4898	0.6236	0.8902
n=12	$E\left[\frac{\pi_F(q^*_{Rc})}{\pi_F(q^*_{Fd})}\right]$	0.9922	0.9757	0.9617	0.7068	0.0283
n=12	$E\left[\frac{\pi_F(q^*_{Rd})}{\pi_F(q^*_{Fd})}\right]$	0.9786	0.7833	0.6040	0.7446	0.8238
n=24	$E\left[\frac{\pi_F(q^*_{Rc})}{\pi_F(q^*_{Fd})}\right]$	0.9967	0.9817	0.9797	0.7810	0.0085
n=24	$E\left[\frac{\pi_F(q^*_{Rd})}{\pi_F(q^*_{Fd})}\right]$	0.9832	0.7775	0.5309	0.7429	0.5327

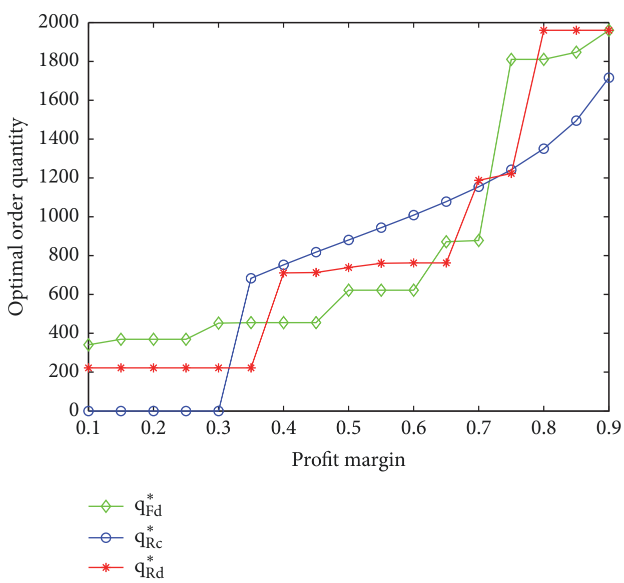

Figure 5: Sensitivity of different optimal order quantities against the profit margin.

optimal ordering decision for the decision maker in those circumstances when the product's marginal profit is super low and only partial information of discrete demand is known.

4.3. Effect of the Profit Margin. To gain more insights, in our third numerical experiment we study the effect of the profit margin $m = (p-c)/(p-s)$ on three different order decisions q^*_{Rd}, q^*_{Rc}, and q^*_{Fd}. We set parameters as $p = 10$ and $s = 0$. In this example, 12 demand points are randomly taken on the interval $[0, 2000]$, and their corresponding probabilities are drawn from a uniform distribution.

Figure 5 demonstrates that q^*_{Rd}, q^*_{Rc}, and q^*_{Fd} are all increasing in profit margin m. That is to say, the higher the marginal profit of the product is, the higher the optimal order quantity is, no matter what kind of optimal order quantity is. Specifically, in this random case, under different profit margins, q^*_{Rd} is always closer to q^*_{Fd} than q^*_{Rc}; namely, q^*_{Rd} performs better than q^*_{Rc}.

5. Conclusion

Most previous researches about the robust newsvendor problem are based on continuous demand. However, in practice, the demand of many products is discrete. This paper integrates a distribution-free design with discrete demand, into a robust optimization approach. The close-form of expression for the worst-case expected profit is provided, and solving steps to obtain the optimal decision q^*_{Rd} are given. The important findings are summarized as follows:

(1) For the same information of mean and variance, q^*_{Rc} is constant, but q^*_{Rd} adjusts according to the different information of demand points.

(2) The number of total demand points has almost no effect on the performance of q^*_{Rd} and q^*_{Rc}.

(3) The performance of q^*_{Rd} is significantly better than q^*_{Rc} only if the product's marginal profit is very low.

The managerial implications of these findings are as follows: When the product's marginal profit is super low and only partial distributional information of the discrete demand is known, q^*_{Rd} is the most appropriate optimal ordering decision for the decision maker. Under this situation, besides the information of mean and variance, the information of demand points is also very important to the decision maker. In addition, the number of total demand points has no effect on the decision.

There are many questions that need to be further explored. For example, other extensions of our research include studying newsvendor problem with other partial information about the discrete demand distribution, e.g., range, median, symmetry, unimodality, and partial information of demand points.

Appendix

Proof of Theorem 4. Set $\pi_1(D) := (p-s)D - (c-s)q$ and $\pi_2(D) := (p-c)q$; then

$$\pi(q,D) = \begin{cases} \pi_1(D), & \text{for } 0 \le D < \dfrac{c-s}{p-s}q \\ \pi_2(D), & \text{for } D \ge q. \end{cases} \tag{A.1}$$

Let x_j be the demand point different from the adjacent two points x_i and x_{i+1}. We divide $x_j < x_i$ and $x_j > x_{i+1}$ into Case 1 and Case 2.

Case 1. We consider a fixed q satisfying $x_j < q \le x_i$, where $2 \le i \le n-1, 1 \le j \le i-1$. If $\max\{(x_i-\mu)(\mu-x_j), (x_{i+1}-\mu)(\mu-x_i)\} \le \sigma^2 \le (x_{i+1}-\mu)(\mu-x_j)$, then the three-point distribution is constructed as

$$\begin{aligned} p_j^* &= \frac{\sigma^2 + (\mu - x_i)(\mu - x_{i+1})}{(x_j - x_i)(x_j - x_{i+1})}, \\ p_i^* &= \frac{\sigma^2 + (\mu - x_j)(\mu - x_{i+1})}{(x_i - x_j)(x_i - x_{i+1})}, \\ p_{i+1}^* &= \frac{\sigma^2 + (\mu - x_j)(\mu - x_i)}{(x_{i+1} - x_j)(x_{i+1} - x_i)}, \\ p_k^* &= 0, \quad k = 1, \dots, n, \ k \ne j, i, i+1. \end{aligned} \tag{A.2}$$

in order to satisfy the primal feasibility.

From the complementary slackness condition

$$\begin{aligned} y_1^* + x_j y_2^* + x_j^2 y_3^* &= (p-s)\left(x_j - \frac{c-s}{p-s}q\right), \\ y_1^* + x_i y_2^* + x_i^2 y_3^* &= (p-s)\left(q - \frac{c-s}{p-s}q\right), \\ y_1^* + x_{i+1} y_2^* + x_{i+1}^2 y_3^* &= (p-s)\left(q - \frac{c-s}{p-s}q\right), \end{aligned} \tag{A.3}$$

we can get that

$$\begin{aligned} y_1^* &= (p-s)\frac{x_j\left[x_i x_{i+1} - q\left(x_i + x_{i+1} - x_j\right)\right]}{(x_i - x_j)(x_{i+1} - x_j)} \\ &\quad - (c-s)q, \\ y_2^* &= \frac{(p-s)(q - x_j)(x_{i+1} + x_i)}{(x_i - x_j)(x_{i+1} - x_j)}, \\ y_3^* &= -\frac{(p-s)(q - x_j)}{(x_i - x_j)(x_{i+1} - x_j)} < 0, \end{aligned} \tag{A.4}$$

In this case, we still need to guarantee that the solution (y_1^*, y_2^*, y_3^*) also satisfies the dual feasibility condition. It is easy to check that

$$\begin{aligned} g'(x_i) &> \pi_1'(x_i), \\ g'(x_{i+1}) &< \pi_1'(x_{i+1}). \end{aligned} \tag{A.5}$$

and

$$\begin{aligned} &g'(x_j) = \pi_1'(x_j), \\ &\qquad \text{if } x_j < q = \frac{x_i x_{i+1} - x_j^2}{(x_i + x_{i+1}) - 2x_j} < x_i; \\ &g'(x_j) > \pi_1'(x_j), \\ &\qquad \text{if } x_j < \frac{x_i x_{i+1} - x_j^2}{(x_i + x_{i+1}) - 2x_j} < q < x_i; \\ &g'(x_j) < \pi_1'(x_j), \\ &\qquad \text{if } x_j < q < \frac{x_i x_{i+1} - x_j^2}{(x_i + x_{i+1}) - 2x_j} < x_i. \end{aligned} \tag{A.6}$$

Case 1a. In this case, we see $g'(x_j) = \pi_1'(x_j)$, i.e., $q = (x_i x_{i+1} - x_j^2)/((x_i + x_{i+1}) - 2x_j)$. The dual feasibility is satisfied.

Case 1b. In this case, we see $g'(x_j) > \pi_1'(x_j)$, i.e., $(x_i x_{i+1} - x_j^2)/((x_i + x_{i+1}) - 2x_j) < q \le x_i$. The other intersection point of $g(x)$ and $h(x)$ is $\widehat{x}$, which satisfies

$$\begin{aligned} \widehat{x} &= -\frac{y_2^* - (p-s)}{y_3^*} - x_j \\ &= \left(x_{i+1} + x_i - x_j\right) - \frac{(x_i - x_j)(x_{i+1} - x_j)}{q - x_j}. \end{aligned} \tag{A.7}$$

Then the dual feasibility is satisfied, if $\widehat{x} \le x_{j+1}$; i.e.,

$$\frac{x_i x_{i+1} - x_j^2}{(x_i + x_{i+1}) - 2x_j} < q \le \frac{x_i x_{i+1} - x_j x_{j+1}}{(x_i + x_{i+1}) - (x_j + x_{j+1})}. \tag{A.8}$$

Case 1c. In this case, we see $g'(x_j) < \pi_1'(x_j)$; i.e., $x_j < q < (x_i x_{i+1} - x_j^2)/((x_i + x_{i+1}) - 2x_j)$. For $j = 1$, the dual feasibility is already satisfied. For $2 \le j \le i-1$, the dual feasibility is satisfied, if $g(0) \le \pi_1(0)$; i.e.,

$$\frac{x_i x_{i+1}}{x_i + x_{i+1} - x_j} \le q < \frac{x_i x_{i+1} - x_j^2}{(x_i + x_{i+1}) - 2x_j}, \tag{A.9}$$

and furthermore $\widehat{x} \ge x_{j-1}$, and

$$\frac{x_i x_{i+1} - x_{j-1} x_j}{(x_i + x_{i+1}) - (x_{j-1} + x_j)} \le q < \frac{x_i x_{i+1} - x_j^2}{(x_i + x_{i+1}) - 2x_j}, \tag{A.10}$$

where

$$\begin{aligned}\widehat{x} &= -\frac{y_2^* - (p-s)}{y_3^*} - x_j \\ &= \left(x_{i+1} + x_i - x_j\right) - \frac{\left(x_i - x_j\right)\left(x_{i+1} - x_j\right)}{q - x_j}\end{aligned} \tag{A.11}$$

is the other intersection point of $g(x)$ and $\pi(q, x)$.

Therefore if

$$\begin{aligned}&\frac{x_i x_{i+1} - x_{j-1} x_j}{\left(x_i + x_{i+1}\right) - \left(x_{j-1} + x_j\right)} \le q \\ &\quad \le \frac{x_i x_{i+1} - x_j x_{j+1}}{\left(x_i + x_{i+1}\right) - \left(x_j + x_{j+1}\right)}, \quad \text{for } 2 \le j \le i-1; \\ &x_j < q \le \frac{x_i x_{i+1} - x_j x_{j+1}}{\left(x_i + x_{i+1}\right) - \left(x_j + x_{j+1}\right)}, \quad \text{for } j = 1,\end{aligned} \tag{A.12}$$

then the strong duality of (P) and (D) holds by this complementarity duality pair. The primal and dual optimal objectives are equal to

$$\begin{aligned}\underline{\pi}(q) &= \sum_{k=1}^{n} \pi\left(q, x_k\right) p_k^* = y_1^* + \mu y_2^* + \left(\mu^2 + \sigma^2\right) y_3^* \\ &= (p-s)\, x_j p_j^* \\ &\quad + \left[(p-s)\left(1 - p_j^*\right) - (c-s)\right] q.\end{aligned} \tag{A.13}$$

Case 2. We consider a fixed q satisfying $x_{i+1} \le q < x_j$, where $1 \le i \le n-2,\ i+2 \le j \le n$. If $\max\{(x_{i+1} - \mu)(\mu - x_i), (x_j - \mu)(\mu - x_{i+1})\} \le \sigma^2 \le (x_j - \mu)(\mu - x_i)$, then the three-point distribution is constructed as

$$\begin{aligned}p_i^* &= \frac{\sigma^2 + \left(\mu - x_{i+1}\right)\left(\mu - x_j\right)}{\left(x_i - x_{i+1}\right)\left(x_i - x_j\right)} \ge 0, \\ p_{i+1}^* &= \frac{\sigma^2 + \left(\mu - x_i\right)\left(\mu - x_j\right)}{\left(x_{i+1} - x_i\right)\left(x_{i+1} - x_j\right)} \ge 0, \\ p_j^* &= \frac{\sigma^2 + \left(\mu - x_i\right)\left(\mu - x_{i+1}\right)}{\left(x_j - x_i\right)\left(x_j - x_{i+1}\right)} \ge 0, \\ p_k^* &= 0, \quad k = 1, \ldots, n,\ k \ne i, i+1, j,\end{aligned} \tag{A.14}$$

in order to satisfy the primal feasibility.

From the complementary slackness condition

$$\begin{aligned}y_1^* + x_i y_2^* + x_i^2 y_3^* &= (p-s)\left(x_i - \frac{c-s}{p-s} q\right), \\ y_1^* + x_{i+1} y_2^* + x_{i+1}^2 y_3^* &= (p-s)\left(x_{i+1} - \frac{c-s}{p-s} q\right), \\ y_1^* + x_j y_2^* + x_j^2 y_3^* &= (p-s)\left(q - \frac{c-s}{p-s} q\right),\end{aligned} \tag{A.15}$$

we can get that

$$\begin{aligned}y_1^* &= (p-s)\left[\frac{x_i x_{i+1} x_j}{\left(x_i - x_{i+1}\right)\left(x_i - x_j\right)} \right. \\ &\quad + \frac{x_{i+1} x_i x_j}{\left(x_{i+1} - x_i\right)\left(x_{i+1} - x_j\right)} \\ &\quad \left. + \frac{q x_i x_{i+1}}{\left(x_j - x_i\right)\left(x_j - x_{i+1}\right)}\right] - (c-s)\, q, \\ y_2^* &= \frac{(p-s)\left[x_j^2 + x_i x_{i+1} - \left(x_i + x_{i+1}\right) q\right]}{\left(x_j - x_i\right)\left(x_j - x_{i+1}\right)}, \\ y_3^* &= -\frac{(p-s)\left(x_j - q\right)}{\left(x_j - x_i\right)\left(x_j - x_{i+1}\right)} < 0.\end{aligned} \tag{A.16}$$

In this case, we still need to guarantee that the solution (y_1^*, y_2^*, y_3^*) also satisfies the dual feasibility condition. It is easy to check that

$$\begin{aligned}g'\left(x_i\right) &> \pi_2'\left(x_i\right), \\ g'\left(x_{i+1}\right) &< \pi_2'\left(x_{i+1}\right),\end{aligned} \tag{A.17}$$

and

$$\begin{aligned}&g'\left(x_j\right) = \pi_2'\left(x_j\right), \\ &\qquad \text{if } x_{i+1} < q = \frac{x_j^2 - x_i x_{i+1}}{2x_j - \left(x_i + x_{i+1}\right)} < x_j;\end{aligned} \tag{A.18}$$

$$\begin{aligned}&g'\left(x_j\right) > \pi_2'\left(x_j\right), \\ &\qquad \text{if } x_{i+1} < \frac{x_j^2 - x_i x_{i+1}}{2x_j - \left(x_i + x_{i+1}\right)} < q < x_j;\end{aligned} \tag{A.19}$$

$$\begin{aligned}&g'\left(x_j\right) < \pi_2'\left(x_j\right), \\ &\qquad \text{if } x_{i+1} < q < \frac{x_j^2 - x_i x_{i+1}}{2x_j - \left(x_i + x_{i+1}\right)} < x_j.\end{aligned} \tag{A.20}$$

Case 2a. In this case, we see $g'(x_j) = \pi_2'(x_j)$, i.e., $q = (x_j^2 - x_i x_{i+1})/(2x_j - (x_i + x_{i+1}))$. The dual feasibility is satisfied.

Case 2b. In this case, we see $g'(x_j) > \pi_2'(x_j)$; i.e., $(x_j^2 - x_i x_{i+1})/(2x_j - (x_i + x_{i+1})) < q < x_j$. For $j = n$, the dual feasibility is already satisfied. For $i + 2 \le j \le n-1$, the dual feasibility is satisfied, if $\widehat{x} \le x_{j+1}$; i.e.,

$$\frac{x_j^2 - x_i x_{i+1}}{2x_j - \left(x_i + x_{i+1}\right)} \le q \le \frac{x_j x_{j+1} - x_i x_{i+1}}{\left(x_j + x_{j+1}\right) - \left(x_i + x_{i+1}\right)}, \tag{A.21}$$

where

$$\widehat{x} = -\frac{y_2^*}{y_3^*} - x_j = \frac{x_j^2 + x_i x_{i+1} - \left(x_i + x_{i+1}\right) q}{x_j - q} - x_j \tag{A.22}$$

is the other intersection point of $g(x)$ and $\pi(q, x)$.

Case 2c. In this case, we see $g'(x_j) < \pi_2'(x_j)$; i.e., $x_{i+1} < q < (x_j^2 - x_i x_{i+1})/(2x_j - (x_i + x_{i+1}))$. Then the dual feasibility is satisfied, if $\widehat{x} \geq x_{j-1}$; i.e.,

$$\frac{x_{j-1}x_j - x_i x_{i+1}}{\left(x_{j-1} + x_j\right) - \left(x_i + x_{i+1}\right)} \leq q < \frac{x_j^2 - x_i x_{i+1}}{2x_j - \left(x_i + x_{i+1}\right)}, \quad (A.23)$$

where

$$\widehat{x} = -\frac{y_2^*}{y_3^*} - x_j = \frac{x_j^2 + x_i x_{i+1} - \left(x_i + x_{i+1}\right) q}{x_j - q} - x_j \quad (A.24)$$

is the other intersection point of $g(x)$ and $\pi_2(x)$.

Therefore if

$$\begin{aligned} \frac{x_{j-1}x_j - x_i x_{i+1}}{\left(x_{j-1} + x_j\right) - \left(x_i + x_{i+1}\right)} &\leq q \\ &\leq \frac{x_j x_{j+1} - x_i x_{i+1}}{\left(x_j + x_{j+1}\right) - \left(x_i + x_{i+1}\right)}, \\ &\qquad \text{for } i + 2 \leq j \leq n - 1; \\ \frac{x_{j-1}x_j - x_i x_{i+1}}{\left(x_{j-1} + x_j\right) - \left(x_i + x_{i+1}\right)} &\leq q < x_j, \quad \text{for } j = n, \end{aligned} \quad (A.25)$$

then the strong duality of (P) and (D) holds by this complementarity duality pair. The primal and dual optimal objectives are equal to

$$\begin{aligned} \underline{\pi}(q) &= \sum_{k=1}^{n} \pi\left(q, x_k\right) p_k^* = y_1^* + \mu y_2^* + \left(\mu^2 + \sigma^2\right) y_3^* \\ &= \left(p - s\right)\left(x_i p_i^* + x_{i+1} p_{i+1}^*\right) \\ &\quad + \left[\left(p - s\right) p_j^* - \left(c - s\right)\right] q. \end{aligned} \quad (A.26)$$

□

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] M. Khouja, "The single-period (news-vendor) problem: literature review and suggestions for future research," *Omega* , vol. 27, no. 5, pp. 537–542, 1999.

[2] Y. Qin, R. Wang, A. J. Vakharia, Y. Chen, and M. M. H. Seref, "The newsvendor problem: review and directions for future research," *European Journal of Operational Research*, vol. 213, no. 2, pp. 361–374, 2011.

[3] T. M. Choi, Ed., *Handbook of newsvendor problems: Models, Extensions and Applications*, Springer, 2012.

[4] H. Scarf, "A min-max solution of an inventory problem.K," in *Studies in The Mathematical Theory of Inventory and Production*, Arrow., S. Karlin, and H. Scarf, Eds., pp. 201–209, Stanford University Press, Palo Alto, Calif, USA, 1958.

[5] G. Gallego and I. Moon, "Distribution free newsboy problem: review and extensions," *Journal of the Operational Research Society*, vol. 44, no. 8, pp. 825–834, 1993.

[6] I. Moon and S. Choi, "The distribution free newsboy problem with balking," *Journal of the Operational Research Society*, vol. 46, no. 4, pp. 537–542, 1995.

[7] I. Moon and S. Choi, "Distribution free procedures for make-to-order (MTO), make-in-advance (MIA), and composite policies," *International Journal of Production Economics*, vol. 48, no. 1, pp. 21–28, 1997.

[8] H. K. Alfares and H. H. Elmorra, "The distribution-free newsboy problem: extensions to the shortage penalty case," *International Journal of Production Economics*, vol. 93-94, pp. 465–477, 2005.

[9] J. Mostard, R. de Koster, and R. Teunter, "The distribution-free newsboy problem with resalable returns," *International Journal of Production Economics*, vol. 97, no. 3, pp. 329–342, 2005.

[10] J. Yue, B. Chen, and M. Wang, "Expected value of distribution information for the newsvendor problem," *Operations Research*, vol. 54, no. 6, pp. 1128–1136, 2006.

[11] Y. Liao, A. Banerjee, and C. Yan, "A distribution-free newsvendor model with balking and lost sales penalty," *International Journal of Production Economics*, vol. 133, no. 1, pp. 224–227, 2011.

[12] C.-M. Lee and S.-L. Hsu, "The effect of advertising on the distribution-free newsboy problem," *International Journal of Production Economics*, vol. 129, no. 1, pp. 217–224, 2011.

[13] J. Andersson, K. Jörnsten, S. Nonås, L. Sandal, and J. Ubøe, "A maximum entropy approach to the newsvendor problem with partial information," *European Journal of Operational Research*, vol. 228, no. 1, pp. 190–200, 2013.

[14] B. Pal, S. S. Sana, and K. Chaudhuri, "A distribution-free newsvendor problem with nonlinear holding cost," *International Journal of Systems Science*, vol. 46, no. 7, pp. 1269–1277, 2015.

[15] K. Kwon and T. Cheong, "A minimax distribution-free procedure for a newsvendor problem with free shipping," *European Journal of Operational Research*, vol. 232, no. 1, pp. 234–240, 2014.

[16] K. Kysang and C. Taesu, "A minimax distribution-free procedure for a newsvendor problem with free shipping," *European Journal of Operational Research*, vol. 232, pp. 234–240, 2014.

[17] H. Yu and J. Zhai, "The distribution-free newsvendor problem with balking and penalties for balking and stockout," *Journal of Systems Science and Systems Engineering*, vol. 23, no. 2, pp. 153–175, 2014.

[18] H. Yu, J. Zhai, and G.-Y. Chen, "Robust Optimization for the Loss-Averse Newsvendor Problem," *Journal of Optimization Theory and Applications*, vol. 171, no. 3, pp. 1008–1032, 2016.

[19] G. Perakis and G. Roels, "Regret in the newsvendor model with partial information," *Operations Research*, vol. 56, no. 1, pp. 188–203, 2008.

[20] H. Jiang, S. Netessine, and S. Savin, "Robust newsvendor competition under asymmetric information," *Operations Research*, vol. 59, no. 1, pp. 254–261, 2011.

[21] Z. Zhu, J. Zhang, and Y. Ye, "Newsvendor optimization with limited distribution information," *Optimization Methods and Software*, vol. 28, no. 3, pp. 640–667, 2013.

[22] D. Huang, S. Zhu, F. J. Fabozzi, and M. Fukushima, "Portfolio selection with uncertain exit time: a robust CVaR approach," *Journal of Economic Dynamics and Control (JEDC)*, vol. 32, no. 2, pp. 594–623, 2008.

[23] D. Huang, S. Zhu, F. J. Fabozzi, and M. Fukushima, "Portfolio selection under distributional uncertainty: a relative robust CVaR approach," *European Journal of Operational Research*, vol. 203, no. 1, pp. 185–194, 2010.

[24] S. Zhu and M. Fukushima, "Worst-case conditional value-at-risk with application to robust portfolio management," *Operations Research*, vol. 57, no. 5, pp. 1155–1168, 2009.

[25] K. Natarajan, D. Pachamanova, and M. Sim, "Constructing risk measures from uncertainty sets," *Operations Research*, vol. 57, no. 5, pp. 1129–1141, 2009.

[26] K. Natarajan, M. Sim, and J. Uichanco, "Asymmetry and Ambiguity in Newsvendor Models," *Remote Method Invocation (RMI)*, no. 09/07, 2009.

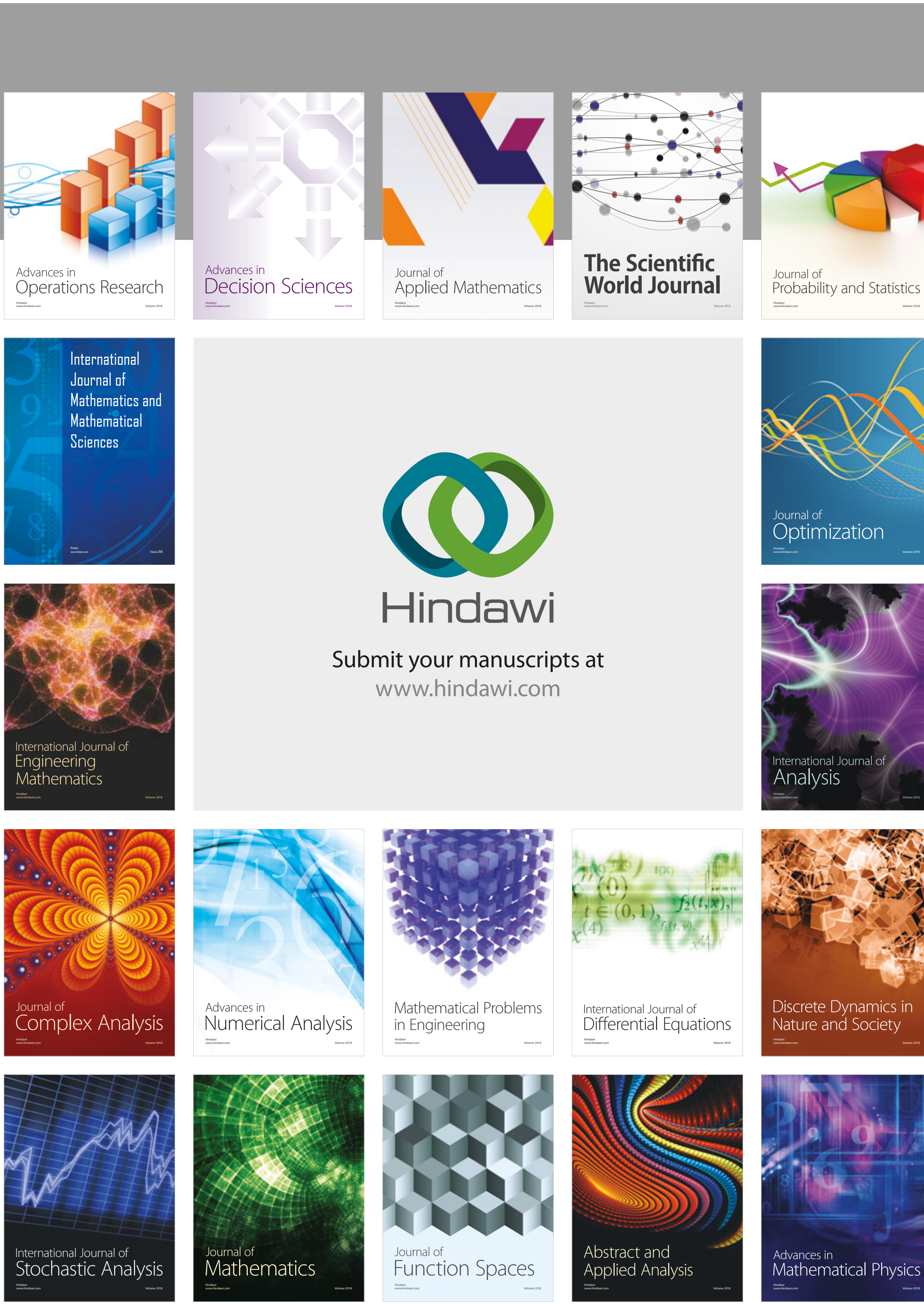
[27] G. A. Hanasusanto, D. Kuhn, S. W. Wallace, and S. Zymler, "Distributionally robust multi-item newsvendor problems with multimodal demand distributions," *Mathematical Programming*, vol. 152, no. 1-2, pp. 1–32, 2015.

[28] L. Xiao and Y. Chen, "Robust optimal decision for the newsvendor problem with uncertain market demand," *Journal of Uncertain Systems*, vol. 11, no. 1, pp. 68–80, 2017.

[29] H. Yu, J. Zhai, and C. H. Sun, "Robust newsvendor problem with discrete demand," in *Proceedings of the 11th International Conference on Service Systems and Service Management (ICSSSM '14)*, Beijing, China, 2014.

[30] E. Carrizosa, A. V. Olivares-Nadal, and P. Ramírez-Cobo, "Robust newsvendor problem with autoregressive demand," *Computers & Operations Research*, vol. 68, pp. 123–133, 2016.

[31] K. A. Arrow, T. E. Harris, and J. Marshak, "Optimal inventory policy," *Econometrica*, vol. 19, no. 3, pp. 250–272, 1951.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