Research Article

Delay-Dependent Robust H_∞ Performance for Uncertain Neutral Systems with Mixed Time-Varying Delays and Nonlinear Perturbations

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This paper deals with the problems of delay-dependent stability and H_∞ performance for uncertain neutral systems with time-varying delays, and nonlinear perturbations. The time-varying delays are neutral, discrete, and distributed time-varying delays that the upper bounds for the delays are available. The restrictions on the derivatives of the discrete and distributed time-varying delays are removed, which mean that a fast discrete time-varying delay is allowed. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively. Firstly, by applying a novel Lyapunov-Krasovskii functional approach, Wirtinger-based integral inequality, Peng-Park's integral inequality, decomposition technique of constant matrix, descriptor model transformation, Leibniz Newton formula and utilization of zero equation, and improved delay-dependent bounded real lemmas (BRL) for systems are established in terms of linear matrix inequalities (LMIs). Then, based on the obtained BRL, some less conservative delay-dependent stability criteria of uncertain neutral systems with mixed time-varying delays and nonlinear perturbations are obtained and improved H_∞ performance criterion with the framework of LMIs is introduced. Finally, some numerical examples are given to illustrate that the presented method is effective.

1. Introduction

Time delay is frequently a source of instability and a source of generation of oscillation in many dynamic systems such as hybrid systems (and practical application) [1, 2], networked control systems, biological systems, mechanical systems, and chemical or process control systems [3]. Thus, analysis and synthesis problem for systems with time-varying delay have become an important issue and large varieties of problems have been researched since the nineties by several researchers [4–8], H_∞ performance [9–11].

In some physical system, the system models can be described by functional differential equation of neutral type, in which the models depend on the state delay but also depend on the state derivatives, are often encountered in various fields, such as population ecology [12], distributed

networks containing lossless transmission lines [13], heat exchangers, and robots in contact with rigid environments [14]. For interesting research methods, stability criteria for application neutral stochastic systems and neural networks have been discussed in [15–19]. On the one hand, some LMI criteria on robust stability for uncertain ones have been deeply derived in [20–23]. Very recently, improved H_∞ performance analysis and stability for uncertain systems with time-varying delays were proposed in [10, 11, 24]. However, there are rooms for further improvements in the feasible region of criteria for H_∞ performance and stability.

Stability criteria for time-delay systems are generally divided into two classes: delay-independent one and delay-dependent one. Delay-independent stability criteria tend to be more conservative, especially for small size delay, and such criteria do not give any information on the size of the

delay. On the other hand, delay-dependent stability criteria are concerned with the size of the delay and usually provide a maximal delay size. Generally speaking, the latter ones are less conservative than the former ones when the time-delay values are small. Much time and efforts have been put into the development of some techniques and new Lyapunov-Krasovskii functional because how to choose Lyapunov-Krasovskii functional and estimate an upper bound of time-derivative of Lyapunov-Krasovskii functional play key roles to improve the feasible region of stability criteria. Delay-dependent stability criteria for these systems are established in terms of linear matrix inequalities (LMIs).

With above motivations, based on Lyapunov stability theory, improved H_∞ performance criteria and stability analysis for uncertain neutral systems with mixed time-varying delays and nonlinear perturbations are derived by the framework of LMIs which will be introduced in Theorem 11. Some numerical examples are given to illustrate that the presented method is effective.

2. Problem Formulation

We introduce some notations and lemmas that will be used throughout the paper. R^+ denotes the set of all real nonnegative numbers; R^n denotes the n -dimensional space with the vector norm $\|\cdot\|$; $\|x\|$ denotes the Euclidean vector norm of $x \in R^n$; $R^{n \times r}$ denotes the set $n \times r$ real matrices; A^T denotes the transpose of the matrix A ; A is symmetric if $A = A^T$; I denotes the identity matrix; matrix A is called semipositive definite ($A \geq 0$) if $x^T A x \geq 0$, for all $x \in R^n$; A is positive definite ($A > 0$) if $x^T A x > 0$ for all $x \neq 0$; matrix B is called seminegative definite ($B \leq 0$) if $x^T B x \leq 0$, for all $x \in R^n$; B is negative definite ($B < 0$) if $x^T B x < 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$ ($B - A < 0$); $A \geq B$ means $A - B \geq 0$ ($B - A \leq 0$); $C([-h, 0], R^n)$ denotes the space of all continuous vector functions mapping $[-h, 0]$ into R^n when $\bar{h} = \max\{r_M, h_M\}$, $r_M, h_M \in R^+$; $x_t = x(t + s)$, $s \in [-h, 0]$; $*$ represents the elements below the main diagonal of a symmetric matrix.

Consider the following uncertain neutral system with mixed time-varying delays of the form:

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)x(t-h(t)) \\ &+ C(t)\dot{x}(t-r(t)) + D(t)\int_{t-h(t)}^t x(s)ds \\ &+ f_1(t, x(t)) + f_2(t, x(t-h(t))) \\ &+ f_3(t, \dot{x}(t-r(t))) + D_1 w(t), \quad t \geq 0; \\ z(t) &= C_1 x(t) + C_2 x(t-h(t)) + D_2 w(t), \quad t \geq 0; \\ x(t) &= \phi(t), \quad t \in [-\bar{h}, 0], \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the state variable, $w(t) \in R^p$ is the disturbance input that belongs to $L_2[0, \infty]$, $z(t) \in R^q$ is the controlled output, $r(t)$ is the neutral time-varying delay, and $h(t)$ is the discrete and distributed time-varying delays satisfying

$$0 \leq r(t) \leq r_M, \quad (2)$$

$$0 \leq \dot{r}(t) \leq r_d, \quad (3)$$

$$0 \leq h(t) \leq h_M, \quad (4)$$

where r_M, r_d and h_M are positive real constants. $\phi(t)$ is initial condition function and $A(t)$, $B(t)$, $C(t)$, and $D(t)$ are uncertain matrices. The uncertainties $f_1(\cdot)$, $f_2(\cdot)$, and $f_3(\cdot)$ represent the nonlinear parameter perturbations with respect to the current state $x(t)$, the delayed state $x(t-h(t))$, and the neutral delayed state $\dot{x}(t-r(t))$, respectively, and are bounded in magnitude

$$f_1^T(t, x(t)) f_1(t, x(t)) \leq \alpha^2 x^T(t) x(t), \quad (5)$$

$$\begin{aligned} f_2^T(t, x(t-h(t))) f_2(t, x(t-h(t))) \\ \leq \beta^2 x^T(t-h(t)) x(t-h(t)), \end{aligned} \quad (6)$$

$$\begin{aligned} f_3^T(t, \dot{x}(t-r(t))) f_3(t, \dot{x}(t-r(t))) \\ \leq \eta^2 \dot{x}^T(t-r(t)) \dot{x}(t-r(t)), \end{aligned} \quad (7)$$

where α, β and η are given positive real constants. We assume that the uncertainties are norm-bounded and can be described as

$$\begin{aligned} A(t) &= A + \Delta A(t), \\ B(t) &= B + \Delta B(t), \\ C(t) &= C + \Delta C(t), \\ D(t) &= D + \Delta D(t), \\ [\Delta A(t) \quad \Delta B(t) \quad \Delta C(t) \quad \Delta D(t)] \\ &= E \Delta(t) [G_1 \quad G_2 \quad G_3 \quad G_4], \end{aligned} \quad (8)$$

where $A, B, C, C_1, C_2, D, D_1, D_2, E, G_1, G_2, G_3$, and G_4 are real constant matrices with appropriate dimensions. The uncertain matrix $\Delta(t)$ satisfies

$$\Delta(t) = F(t) [I - HF(t)]^{-1} \quad (9)$$

and is said to be admissible, where H is known matrix satisfying

$$I - HH^T > 0. \quad (10)$$

The uncertain matrix $F(t)$ satisfies

$$F(t)^T F(t) \leq I. \quad (11)$$

Definition 1. Given a scalar $\gamma > 0$, the system (1) is said to be asymptotically stable with the H_∞ performance level γ , if it is asymptotically stable and satisfies the H_∞ -norm constraint $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$, for all nonzero $w(t) \in L_2[0, \infty)$ under zero initial condition.

Lemma 2 (see [25]). Suppose that $\Delta(t)$ is given by (8)-(10). Let M, L , and N be real constant matrices of appropriate dimension with $M = M^T$. Then, the inequality

$$M + L \Delta(t) N + N^T \Delta(t)^T L^T < 0 \quad (12)$$

holds if and only if for any positive real constant δ ,

$$\begin{bmatrix} M & L & \delta N^T \\ * & -\delta I & \delta H^T \\ * & * & -\delta I \end{bmatrix} < 0. \quad (12)$$

Lemma 3 (Jensen's inequality). For any symmetric positive definite matrix Q , positive real number h , and vector function $\dot{x} : [-h, 0] \rightarrow R^n$ such that the following integral is well defined, then

$$\begin{aligned} & -h \int_{-h}^0 \dot{x}^T(s+t) Q \dot{x}(s+t) ds \\ & \leq -\left(\int_{-h}^0 \dot{x}(s+t) ds \right)^T Q \left(\int_{-h}^0 \dot{x}(s+t) ds \right). \end{aligned} \quad (13)$$

Lemma 4 (Wirtinger-based integral inequality, [26]). For any matrix $Z > 0$, the following inequality holds for all continuously differentiable functions $\dot{x} : [\alpha, \beta] \rightarrow R^n$:

$$-(\beta - \alpha) \int_{\alpha}^{\beta} \dot{x}^T(s) Z \dot{x}(s) ds \leq \omega^T \Phi \omega, \quad (14)$$

where

$$\omega = \left[x^T(\beta), x^T(\alpha), \frac{1}{\beta - \alpha} \int_{\alpha}^{\beta} x^T(s) ds \right]^T \quad (15)$$

and

$$\Phi = \begin{bmatrix} -4Z & -2Z & 6Z \\ * & -4Z & 6Z \\ * & * & -12Z \end{bmatrix}. \quad (16)$$

Lemma 5 (Peng-Park's integral inequality, [27, 28]). For any matrix $\begin{bmatrix} Z & S \\ * & Z \end{bmatrix} \geq 0$, positive scalars τ and $\tau(t)$ satisfying $0 < \tau(t) < \tau$, vector function $\dot{x} : [-\tau, 0] \rightarrow R^n$, such that the concerned integrations are well defined; then

$$-\tau \int_{t-\tau}^t \dot{x}^T(s) Z \dot{x}(s) ds \leq \xi^T \Theta \xi, \quad (17)$$

where

$$\xi = [x^T(t), x^T(t - \tau(t)), x^T(t - \tau)]^T \quad (18)$$

and

$$\Theta = \begin{bmatrix} -Z & Z - S & S \\ * & -2Z + S + S^T & Z - S \\ * & * & -Z \end{bmatrix}. \quad (19)$$

Lemma 6 (see [11]). For a positive matrix M , the following inequality holds:

$$\begin{aligned} & -\frac{(\alpha - \beta)^2}{2} \int_{\beta}^{\alpha} \int_s^{\alpha} x^T(u) M x(u) dud s \\ & \leq -\left(\int_{\beta}^{\alpha} \int_s^{\alpha} x(u) dud s \right)^T M \left(\int_{\beta}^{\alpha} \int_s^{\alpha} x(u) dud s \right). \end{aligned} \quad (20)$$

Lemma 7 (see [11]). For a positive matrix M , the following inequality holds:

$$\begin{aligned} & -\frac{(\alpha - \beta)^3}{6} \int_{\beta}^{\alpha} \int_s^{\alpha} \int_u^{\alpha} x^T(\lambda) M x(\lambda) d\lambda dud s \\ & \leq -\left(\int_{\beta}^{\alpha} \int_s^{\alpha} \int_u^{\alpha} x(\lambda) d\lambda dud s \right)^T \\ & \cdot M \left(\int_{\beta}^{\alpha} \int_s^{\alpha} \int_u^{\alpha} x(\lambda) d\lambda dud s \right). \end{aligned} \quad (21)$$

Lemma 8 (see [29]). For any constant symmetric positive definite matrix $Q \in R^{n \times n}$, $h(t)$ is discrete time-varying delays with (3) and vector function $\omega : [-h_M, 0] \rightarrow R^n$ such that the integrations concerned are well defined; then

$$\begin{aligned} & -h_M \int_{-h_M}^0 \omega^T(s) Q \omega(s) ds \\ & \leq -\int_{-h(t)}^0 \omega^T(s) ds Q \int_{-h(t)}^0 \omega(s) ds \\ & - \int_{-h_M}^{-h(t)} \omega^T(s) ds Q \int_{-h_M}^{-h(t)} \omega(s) ds. \end{aligned} \quad (22)$$

Lemma 9 (see [29]). For any constant matrices $Q_1, Q_2, Q_3 \in R^{n \times n}$, $Q_1 \geq 0, Q_3 > 0, \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \geq 0$, $h(t)$ is discrete time-varying delays with (3) and vector function $\dot{x} : [-h_M, 0] \rightarrow R^n$ such that the integration is well defined, and then

$$\begin{aligned} & -h_M \int_{t-h_M}^t \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & Q_2 \\ * & Q_3 \end{bmatrix} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix} ds \\ & \leq \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \int_{t-h(t)}^t x(s) ds \\ \int_{t-h_M}^{t-h(t)} x(s) ds \end{bmatrix}^T \end{aligned} \quad (23)$$

$$\cdot \begin{bmatrix} -Q_3 & Q_3 & 0 & -Q_2^T & 0 \\ * & -Q_3 - Q_3^T & Q_3 & Q_2^T & -Q_2^T \\ * & * & -Q_3 & 0 & Q_2^T \\ * & * & * & -Q_1 & 0 \\ * & * & * & * & -Q_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \int_{t-h(t)}^t x(s) ds \\ \int_{t-h_M}^{t-h(t)} x(s) ds \end{bmatrix}.$$

Lemma 10 (see [29]). Let $x(t) \in R^n$ be a vector-valued function with first-order continuous-derivative entries. Then, the following integral inequality holds for any constant matrices $X, M_i \in R^{n \times n}$, $i = 1, 2, \dots, 5$, and $h(t)$ is discrete time-varying delays with (3):

$$\begin{aligned}
-\int_{t-h_M}^t \dot{x}^T(s) X \dot{x}(s) ds \leq & \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}^T \begin{bmatrix} M_1 + M_1^T & -M_1^T + M_2 & 0 \\ * & M_1 + M_1^T - M_2 - M_2^T & -M_1^T + M_2 \\ * & * & -M_2 - M_2^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix} \\
& + h_M \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}^T \begin{bmatrix} M_3 & M_4 & 0 \\ * & M_3 + M_5 & M_4 \\ * & * & M_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}, \quad (24)
\end{aligned}$$

where

$$\begin{bmatrix} X & M_1 & M_2 \\ * & M_3 & M_4 \\ * & * & M_5 \end{bmatrix} \geq 0. \quad (25)$$

3. Main Results

In this section, a new H_∞ performance and stability analysis for the system (1) will be introduced by using the Lyapunov functional method combining with linear matrix inequality technique. Firstly, we will establish a new version of delay-dependent BRL for the nominal system (1). We can rewrite the nominal system (1) as follows:

$$\begin{aligned}
\dot{x}(t) = & Ax(t) + Bx(t-h(t)) + C\dot{x}(t-r(t)) \\
& + D \int_{t-h(t)}^t x(s) ds + f_1(t, x(t)) \\
& + f_2(t, x(t-h(t))) + f_3(t, \dot{x}(t-r(t))) \\
& + D_1 w(t), \quad t \geq 0; \\
z(t) = & C_1 x(t) + C_2 x(t-h(t)) + D_2 w(t), \quad t \geq 0; \\
x(t) = & \phi(t), \quad t \in [-\bar{h}, 0].
\end{aligned} \quad (26)$$

We introduce the following notations for later use:

$$\sum = [\Sigma_{i,j}]_{18 \times 18}, \quad (27)$$

where $\Sigma_{i,j} = \Sigma_{j,i}^T$, $i, j = 1, 2, 3, \dots, 18$,

$$\begin{aligned}
\Sigma_{1,1} = & \epsilon_1 \alpha^2 I + W_1 + W_2 + W_1^T + W_2^T + P_2^T A_1 \\
& + P_2^T B_1 + A_1^T P_2 + B_1^T P_2 + P_3^T + P_3 + Q_1 + R_1 \\
& + h_M^2 Q_2 + M_1 + M_1^T + h_M M_3 - 4Q_4 - Q_5 \\
& + h_M^2 R_4 - R_6 + \frac{h_M^4}{4} Q_6 - 2h_M^2 Q_7 - \frac{h_M^4}{4} Q_8 \\
& + Z_3,
\end{aligned}$$

$$\begin{aligned}
\Sigma_{1,2} = & P_1 - P_2^T + A_1^T P_{14} + B_1^T P_{14} + P_{15} + R_2 \\
& + h_M^2 R_5,
\end{aligned}$$

$$\Sigma_{1,3} = A_1^T P_{11} + B_1^T P_{11} + P_{12} - P_4^T - 2Q_4 + S,$$

$$\begin{aligned}
\Sigma_{1,4} = & -W_1 + P_2^T A_2 + P_2^T B_2 + A_1^T P_5 + B_1^T P_5 - P_3^T \\
& + P_6 + P_4^T - M_1^T + M_2 + h_M M_4 + Q_5 - S \\
& + R_6,
\end{aligned}$$

$$\Sigma_{1,5} = -W_2 + L_1,$$

$$\Sigma_{1,6} = -W_1 - P_2^T B_1 + P_2^T A_2 - P_3^T + L_2,$$

$$\Sigma_{1,7} = -W_2 + L_3,$$

$$\Sigma_{1,8} = -P_4^T,$$

$$\Sigma_{1,11} = -R_5^T + P_2^T D + A_1^T P_8 + B_1^T P_8 + P_9,$$

$$\Sigma_{1,12} = \frac{6}{h_M} Q_4,$$

$$\Sigma_{1,13} = \sqrt{2} h_M Q_7,$$

$$\Sigma_{1,14} = \frac{h_M}{2} Q_8,$$

$$\Sigma_{1,15} = P_2^T C,$$

$$\Sigma_{1,16} = \Sigma_{1,17} = \Sigma_{1,18} = P_2^T,$$

$$\begin{aligned}
\Sigma_{2,2} = & r_M Q_9 - P_{14}^T - P_{14} + R_3 + h_M Q_3 + h_M^2 Q_4 \\
& + h_M^2 Q_5 + h_M^2 R_6 + \frac{h_M^4}{4} Q_7 + \frac{h_M^6}{36} Q_8 + h_M^2 Z_1 \\
& + r_M^2 Z_2,
\end{aligned}$$

$$\Sigma_{2,3} = -P_{11} - P_{16}^T,$$

$$\Sigma_{2,4} = -P_5 + P_{14}^T A_2 + P_{14}^T B_2 - P_{15}^T + P_{16},$$

$$\Sigma_{2,6} = -P_{14}^T B_1 + P_{14}^T A_2 - P_{15}^T,$$

$$\Sigma_{2,8} = -P_{16}^T,$$

$$\begin{aligned}
\Sigma_{2,11} &= P_{14}^T D - P_8, \\
\Sigma_{2,15} &= P_{14}^T C, \\
\Sigma_{2,16} &= \Sigma_{2,17} = \Sigma_{2,18} = P_{14}^T, \\
\Sigma_{3,3} &= -P_{13}^T - P_{13} - Q_1 - R_1 - M_2 - M_2^T + h_M M_5 \\
&\quad - 4Q_4 - Q_5 - R_6, \\
\Sigma_{3,4} &= P_{11}^T A_2 + P_{11}^T B_2 - P_{12}^T - P_7 + P_{13}^T - M_1 + M_2^T \\
&\quad + h_M M_4^T + Q_5^T - S^T + R_6, \\
\Sigma_{3,6} &= -P_{11}^T B_1 + P_{11}^T A_2 - P_{12}^T, \\
\Sigma_{3,8} &= P_{13}^T, \\
\Sigma_{3,9} &= -R_2, \\
\Sigma_{3,10} &= R_5^T, \\
\Sigma_{3,11} &= P_{11}^T D - P_{10}, \\
\Sigma_{3,12} &= \frac{6}{h_M} Q_4, \\
\Sigma_{3,15} &= P_{11}^T C, \\
\Sigma_{3,16} &= \Sigma_{3,17} = \Sigma_{3,18} = P_{11}^T, \\
\Sigma_{4,4} &= \epsilon_2 \beta^2 I + P_5^T A_2 + P_5^T B_2 + A_2^T P_5 + B_2^T P_5 - P_6^T \\
&\quad - P_6 + P_7^T + P_7 + M_1 + M_1^T - M_2 - M_2^T \\
&\quad + S^T + h_M M_3 + h_M M_5 - 2Q_5 + S - R_6 \\
&\quad - R_6^T, \\
\Sigma_{4,6} &= -P_5^T B_1 + P_5^T A_2 - P_6^T - L_2, \\
\Sigma_{4,8} &= -P_7^T, \\
\Sigma_{4,10} &= -R_5^T, \\
\Sigma_{4,11} &= R_5^T + P_5^T D + A_2^T P_8 + B_2^T P_8 - P_9 + P_{10}, \\
\Sigma_{4,15} &= P_5^T C, \\
\Sigma_{4,16} &= \Sigma_{4,17} = \Sigma_{4,18} = P_5^T, \\
\Sigma_{5,5} &= -L_1^T - L_1 - Z_3 + r_d Z_3, \\
\Sigma_{5,7} &= -L_1^T - L_3, \\
\Sigma_{6,6} &= -L_2^T - L_2 - Z_1, \\
\Sigma_{6,8} &= -Z_1, \\
\Sigma_{6,11} &= -B_1^T P_8 + A_2^T P_8 - P_9, \\
\Sigma_{7,7} &= -L_3^T - L_3 - Z_2,
\end{aligned}$$

$$\begin{aligned}
\Sigma_{8,8} &= -Z_1, \\
\Sigma_{8,11} &= -P_{10}, \\
\Sigma_{9,9} &= -R_3, \\
\Sigma_{10,10} &= -Q_2 - R_4, \\
\Sigma_{11,11} &= -Q_2 - R_4 + P_8^T D + D^T P_8, \\
\Sigma_{11,15} &= P_8^T C, \\
\Sigma_{11,16} &= \Sigma_{11,17} = \Sigma_{11,18} = P_8^T, \\
\Sigma_{12,12} &= \frac{-12}{h^2} Q_4, \\
\Sigma_{13,13} &= -Q_7, \\
\Sigma_{14,14} &= -Q_6 - Q_8, \\
\Sigma_{15,15} &= \epsilon_3 \eta^2 I - r_M (1 - r_d) Q_9, \\
\Sigma_{16,16} &= \epsilon_1 I, \\
\Sigma_{17,17} &= \epsilon_2 I, \\
\Sigma_{18,18} &= \epsilon_3 I, \\
W_1 &= P_1 J, \\
W_2 &= P_2 K,
\end{aligned} \tag{28}$$

and other terms are 0.

$$\widehat{\Sigma} = [\widehat{\Sigma}_{(i,j)}]_{19 \times 19}, \tag{29}$$

where $\widehat{\Sigma}_{i,j} = \widehat{\Sigma}_{j,i}^T = \Sigma_{i,j}$, $i, j = 1, 2, 3, \dots, 19$, except

$$\begin{aligned}
\widehat{\Sigma}_{1,1} &= \epsilon_1 \alpha^2 I + W_1 + W_2 + W_1^T + W_2^T + P_2^T A_1 \\
&\quad + P_2^T B_1 + A_1^T P_2 + B_1^T P_2 + P_3^T + P_3 + Q_1 \\
&\quad + R_1 + h_M^2 Q_2 + M_1 + M_1^T + h_M M_3 \\
&\quad - 4Q_4 - Q_5 + h_M^2 R_4 - R_6 + \frac{h_M^4}{4} Q_6 \\
&\quad - 2h_M^2 Q_7 - \frac{h_M^4}{4} Q_8 + Z_3 + C_1^T C_1, \\
\widehat{\Sigma}_{1,4} &= -W_1 + P_2^T A_2 + P_2^T B_2 + A_1^T P_5 + B_1^T P_5 - P_3^T \\
&\quad + P_6 + P_4^T - M_1^T + M_2 + h_M M_4 + Q_5 - S \\
&\quad + R_6 + C_1^T C_2, \\
\widehat{\Sigma}_{1,19} &= C_1^T D_2 + P_2^T D_1, \\
\widehat{\Sigma}_{2,19} &= P_{14}^T D_1, \\
\widehat{\Sigma}_{3,19} &= P_{11}^T D_1,
\end{aligned}$$

$$\begin{aligned}
\hat{\Sigma}_{4,4} &= \epsilon_2 \beta^2 I + P_5^T A_2 + P_5^T B_2 + A_2^T P_5 + B_2^T P_5 - P_6^T \\
&\quad - P_6 + P_7^T + P_7 + M_1 + M_1^T - M_2 - M_2^T \\
&\quad + S^T + h_M M_3 + h_M M_5 - 2Q_5 + S - R_6 \\
&\quad - R_6^T + C_2^T C_2, \\
\hat{\Sigma}_{4,19} &= C_2^T D_2 + P_5^T D_1, \\
\hat{\Sigma}_{11,19} &= P_8^T D_1, \\
\Sigma_{19,19} &= D_2^T D_2 - \gamma^2 I,
\end{aligned} \tag{30}$$

and other terms are 0.

Theorem 11. For a prescribed scalar $\gamma > 0$, given scalars $r_M > 0$, $r_d > 0$ and $h_M > 0$, the system (26) is asymptotically stable with the H_∞ performance γ ; if $\|C\| + \eta < 1$, there exist positive definite symmetric matrices P_1, R_4, R_6, Q_i , and Z_j , $i = 1, 2, \dots, 9$, $j = 1, 2, 3$, any appropriate dimensional matrices, J, K, S, R_k, M_l, P_m , and $k = 1, 2, \dots, 6$, $l = 1, 2, \dots, 5$, $m = 2, 3, \dots, 16$, and positive real constants ϵ_1, ϵ_2 , and ϵ_3 satisfying the following LMIs:

$$\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} > 0, \tag{31}$$

$$\begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} > 0, \tag{32}$$

$$\begin{bmatrix} Q_5 & S \\ S^T & Q_5 \end{bmatrix} \geq 0, \tag{33}$$

$$\begin{bmatrix} Q_3 & M_1 & M_2 \\ M_1^T & M_3 & M_4 \\ M_2^T & M_4^T & M_5 \end{bmatrix} \geq 0, \tag{34}$$

$$\hat{\Sigma} < 0. \tag{35}$$

Proof. Under the conditions of the theorem, we first show the asymptotic stability of the system (26). To this end, we consider the nominal system (26) with $w(t) = 0$, that is,

$$\begin{aligned}
\dot{x}(t) &= Ax(t) + Bx(t-h(t)) + Cx(t-r(t)) \\
&\quad + D \int_{t-h(t)}^t x(s) ds + f_1(t, x(t)) \\
&\quad + f_2(t, x(t-h(t))) + f_3(t, \dot{x}(t-r(t))).
\end{aligned} \tag{36}$$

From model transformation method, we rewrite the system (36) in the following system:

$$\dot{x}(t) = y(t), \tag{37}$$

$$\begin{aligned}
0 &= -y(t) + Ax(t) + Bx(t-h(t)) + Cy(t-r(t)) \\
&\quad + D \int_{t-h(t)}^t x(s) ds + f_1(t, x(t)) \\
&\quad + f_2(t, x(t-h(t))) + f_3(t, y(t-r(t))).
\end{aligned} \tag{38}$$

In order to improve of the discrete delay $h(t)$ in (26), let us decompose constant matrices A and B as

$$A = A_1 + A_2, \tag{39}$$

$$B = B_1 + B_2, \tag{40}$$

where A_1, A_2, B_1 , and $B_2 \in R^{n \times n}$ are real constant matrices. By utilizing the zero equations, we obtain

$$0 = Jx(t) - Jx(t-h(t)) - J \int_{t-h(t)}^t \dot{x}(s) ds, \tag{41}$$

$$0 = Kx(t) - Kx(t-r(t)) - K \int_{t-r(t)}^t \dot{x}(s) ds, \tag{42}$$

where J and $K \in R^{n \times n}$ will be chosen to guarantee the asymptotic stability of the system (26). By (39)-(42), the systems (37)-(38) can be represented by the form

$$\begin{aligned}
\dot{x}(t) &= y(t) + Jx(t) - Jx(t-h(t)) - J \int_{t-h(t)}^t y(s) ds \\
&\quad + Kx(t) - Kx(t-r(t))
\end{aligned} \tag{43}$$

$$\begin{aligned}
&- K \int_{t-r(t)}^t y(s) ds, \\
0 &= -y(t) + (A_1 + B_1)x(t) \\
&\quad + (A_2 + B_2)(t)x(t-h(t)) \\
&\quad - (B_1 - A_2) \int_{t-h(t)}^t y(s) ds \\
&\quad + D \int_{t-h(t)}^t x(s) ds + f_1(t, x(t)) \\
&\quad + f_2(t, x(t-h(t))) + f_3(t, y(t-r(t))) \\
&\quad + Cy(t-r(t)).
\end{aligned} \tag{44}$$

Construct a Lyapunov-Krasovskii functional candidate for the systems (43)-(44) of the form

$$V(t) = \sum_{i=1}^9 V_i(t), \tag{45}$$

where

$$\begin{aligned}
V_1(t) &= \zeta^T(t) EP \zeta(t), \\
V_2(t) &= \int_{t-h_M}^t x^T(s) Q_1 x(s) ds \\
&\quad + \int_{t-h_M}^t \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ * & R_3 \end{bmatrix} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} ds,
\end{aligned}$$

$$\begin{aligned}
V_3(t) &= h_M \int_{-h_M}^0 \int_{t+s}^t x^T(\theta) Q_2 x(\theta) d\theta ds \\
&\quad + \int_{-h_M}^0 \int_{t+s}^t y^T(\theta) Q_3 y(\theta) d\theta ds, \\
V_4(t) &= h_M \int_{-h_M}^0 \int_{t+s}^t y^T(\theta) Q_4 y(\theta) d\theta ds \\
&\quad + h_M \int_{-h_M}^0 \int_{t+s}^t y^T(\theta) Q_5 y(\theta) d\theta ds, \\
V_5(t) &= h_M \int_{-h_M}^0 \int_{t+s}^t \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ * & R_6 \end{bmatrix} \begin{bmatrix} x(\theta) \\ y(\theta) \end{bmatrix} d\theta ds, \\
V_6(t) &= \frac{(h_M)^2}{2} \int_{t-h_M}^t \int_s^t \int_u^t x^T(\lambda) Q_6 x(\lambda) d\lambda du ds \\
&\quad + h_M^2 \int_{t-h_M}^t \int_s^t \int_u^t y^T(\lambda) Q_7 y(\lambda) d\lambda du ds, \\
V_7(t) &= \frac{(h_M)^3}{6} \int_{t-h_M}^t \int_s^t \int_u^t \int_\lambda^t y^T(\theta) Q_8 y(\theta) d\theta d\lambda du ds, \\
V_8(t) &= h_M \int_{-h_M}^0 \int_{t+s}^t y^T(\theta) Z_1 y(\theta) d\theta ds \\
&\quad + r_M \int_{-r_M}^0 \int_{t+s}^t y^T(\theta) Z_2 y(\theta) d\theta ds, \\
V_9(t) &= \int_{t-r(t)}^t x^T(s) Z_3 x(s) ds \\
&\quad + r_M \int_{t-r(t)}^t y^T(s) Q_9 y(s) ds,
\end{aligned} \tag{46}$$

with

$$E = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
P &= \begin{bmatrix} P_1 & 0 & 0 & 0 & 0 \\ P_2 & P_5 & P_8 & P_{11} & P_{14} \\ P_3 & P_6 & P_9 & P_{12} & P_{15} \\ P_4 & P_7 & P_{10} & P_{13} & P_{16} \end{bmatrix}, \\
\zeta(t) &= \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \int_{t-h(t)}^t x(s) ds \\ x(t-h_M) \\ y(t) \end{bmatrix}.
\end{aligned} \tag{47}$$

The time derivative of $V(t)$ along the trajectory of systems (43)-(44) is given by

$$\dot{V}(t) = \sum_{i=1}^9 \dot{V}_i(t). \tag{48}$$

It is noted that $\zeta^T(t)EP\zeta(t)$ is really $x^T(t)P_1x(t)$. Then the time derivative of $V_1(t)$ is calculated as

$$\begin{aligned}
\dot{V}_1(t) &= 2x^T(t)P_1\dot{x}(t) = 2\zeta^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
&= 2 \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \int_{t-h(t)}^t x(s) ds \\ x(t-h_M) \\ y(t) \end{bmatrix}^T \\
&\quad \cdot \begin{bmatrix} P_1 & P_2^T & P_3^T & P_4^T \\ 0 & P_5^T & P_6^T & P_7^T \\ 0 & P_8^T & P_9^T & P_{10}^T \\ 0 & P_{11}^T & P_{12}^T & P_{13}^T \\ 0 & P_{14}^T & P_{15}^T & P_{16}^T \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ 0 \\ 0 \\ 0 \end{bmatrix} = 2x^T(t)P_1 \\
&\quad \times \left[y(t) + Jx(t) - Jx(t-h(t)) - J \int_{t-h(t)}^t y(s) ds \right. \\
&\quad \left. + Kx(t) - Kx(t-r(t)) - K \int_{t-r(t)}^t y(s) ds \right] \\
&\quad + 2 \left[x^T(t)P_2^T + x^T(t-h(t))P_5^T \right. \\
&\quad \left. + \left(\int_{t-h(t)}^t x(s) ds \right)^T P_8^T + x^T(t-h_M)P_{11}^T \right]
\end{aligned}$$

$$\begin{aligned}
& + y^T(t) P_{14}^T \Big] \times \Big[-y(t) + (A_1 + B_1)x(t) \\
& + (A_2 + B_2)(t)x(t-h(t)) \\
& - (B_1 - A_2) \int_{t-h(t)}^t y(s) ds + D \int_{t-h(t)}^t x(s) ds \\
& + f_1(t, x(t)) + f_2(t, x(t-h(t))) \\
& + f_3(t, y(t-r(t))) + Cy(t-r(t)) \Big] \\
& + 2 \Big[x^T(t) P_3^T + x^T(t-h(t)) P_6^T \\
& + \left(\int_{t-h(t)}^t x(s) ds \right)^T P_9^T + x^T(t-h_M) P_{12}^T \\
& + y^T(t) P_{15}^T \Big] \times \Big[x(t) - x(t-h(t)) \\
& - \int_{t-h(t)}^t y(s) ds \Big] + 2 \Big[x^T(t) P_4^T \\
& + x^T(t-h(t)) P_7^T + \left(\int_{t-h(t)}^t x(s) ds \right)^T P_{10}^T \\
& + x^T(t-h_M) P_{13}^T + y^T(t) P_{16}^T \Big] \times \Big[x(t-h(t)) \\
& - x(t-h_M) - \int_{t-h_M}^{t-h(t)} y(s) ds \Big] + 2L_1^T x^T(t-r(t)) \\
& \times \Big[x(t) - x(t-r(t)) - \int_{t-r(t)}^t y(s) ds \Big] \\
& + 2L_2^T \int_{t-h(t)}^t y(s) ds \times \Big[x(t) - x(t-h(t)) \\
& - \int_{t-h(t)}^t y(s) ds \Big] + 2L_3^T \int_{t-r(t)}^t y(s) ds \times \Big[x(t) \\
& - x(t-r(t)) - \int_{t-r(t)}^t y(s) ds \Big].
\end{aligned} \tag{49}$$

Differentiating $V_2(t)$, we have

$$\begin{aligned}
\dot{V}_2(t) &= x^T(t) Q_1 x(t) - x^T(t-h_M) Q_1 x(t-h_M) \\
&+ \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \\
&- \begin{bmatrix} x(t-h_M) \\ y(t-h_M) \end{bmatrix}^T \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} \begin{bmatrix} x(t-h_M) \\ y(t-h_M) \end{bmatrix}.
\end{aligned} \tag{50}$$

Using Lemmas 8 and 10, $V_3(t)$ is calculated as

$$\begin{aligned}
\dot{V}_3(t) &= h_M \int_{-h_M}^0 \left(x^T(t) Q_2 x(t) - x^T(t+s) Q_2 x(t+s) \right) ds + \int_{-h_M}^0 \left(y^T(t) Q_3 y(t) - y^T(t+s) Q_3 y(t+s) \right) ds \\
&= h_M \int_{-h_M}^0 x^T(t) Q_2 x(t) ds - h_M \int_{-h_M}^0 x^T(t+s) Q_2 x(t+s) ds + \int_{-h_M}^0 y^T(t) Q_3 y(t) ds \\
&\quad - \int_{-h_M}^0 y^T(t+s) Q_3 y(t+s) ds \\
&= h_M^2 x^T(t) Q_2 x(t) - h_M \int_{t-h_M}^t x^T(s) Q_2 x(s) ds + h_M y^T(t) Q_3 y(t) - \int_{t-h_M}^t \dot{x}^T(s) Q_3 \dot{x}(s) ds \\
&\leq h_M^2 x^T(t) Q_2 x(t) + h_M y^T(t) Q_3 y(t) - \int_{t-h(t)}^t x^T(s) ds Q_2 \int_{t-h(t)}^t x(s) ds - \int_{t-h_M}^{t-h(t)} x^T(s) ds Q_2 \int_{t-h_M}^{t-h(t)} x(s) ds \\
&\quad + \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}^T \begin{bmatrix} M_1 + M_1^T & -M_1^T + M_2 & 0 \\ -M_1 + M_2^T & M_1 + M_1^T - M_2 - M_2^T & -M_1^T + M_2 \\ 0 & -M_1 + M_2^T & -M_2 - M_2^T \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix} \\
&\quad + h_M \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}^T \begin{bmatrix} M_3 & M_4 & 0 \\ M_4^T & M_3 + M_5 & M_4 \\ 0 & M_4^T & M_5 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}.
\end{aligned} \tag{51}$$

Using Lemma 4 (Wirtinger-base integral inequality) and Lemma 5 (Peng-Park's integral inequality), an upper bound of $V_4(t)$ can be obtained as

$$\begin{aligned}
 \dot{V}_4(t) &= h_M^2 y^T(t) Q_4 y(t) - h_M \int_{t-h_M}^t \dot{x}^T(s) Q_4 \dot{x}(s) ds + h_M^2 y^T(t) Q_5 y(t) - h_M \int_{t-h_M}^t \dot{x}^T(s) Q_5 \dot{x}(s) ds \\
 &\leq h_M^2 y^T(t) Q_4 y(t) + h_M^2 y^T(t) Q_5 y(t) + \begin{bmatrix} x^T(t) & x^T(t-h_M) & \frac{1}{h_M} \int_{t-h_M}^t x^T(s) ds \end{bmatrix} \begin{bmatrix} -4Q_4 & -2Q_4 & 6Q_4 \\ -2Q_4^T & -4Q_4 & 6Q_4 \\ 6Q_4^T & 6Q_4^T & -12Q_4 \end{bmatrix} \\
 &\quad \times \begin{bmatrix} x(t) \\ x(t-h_M) \\ \frac{1}{h_M} \int_{t-h_M}^t x(s) ds \end{bmatrix} + \begin{bmatrix} x^T(t) & x^T(t-h(t)) & x^T(t-h_M) \end{bmatrix} \begin{bmatrix} -Q_5 & Q_5 - S & S \\ Q_5^T - S^T & -2Q_5 + S + S^T & Q_5 - S \\ S^T & Q_5^T - S^T & -Q_5 \end{bmatrix} \\
 &\quad \times \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \end{bmatrix}.
 \end{aligned} \tag{52}$$

It is from Lemma 9 that we have

$$\begin{aligned}
 \dot{V}_5(t) &= h_M \int_{-h_M}^0 \left(\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \right. \\
 &\quad \left. - \begin{bmatrix} x(t+s) \\ y(t+s) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} x(t+s) \\ y(t+s) \end{bmatrix} \right) ds \\
 &= h_M^2 \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \\
 &\quad - h_M \int_{t-h_M}^t \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} ds \\
 &\leq h_M^2 \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \\
 &\quad + \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \int_{t-h(t)}^t x(s) ds \\ \int_{t-h_M}^{t-h(t)} x(s) ds \end{bmatrix}^T \\
 &\quad \cdot \begin{bmatrix} -R_6 & R_6 & 0 & -R_5^T & 0 \\ R_6^T & -R_6 - R_6^T & R_6 & R_5^T & -R_5^T \\ 0 & R_6^T & -R_6 & 0 & R_5^T \\ -R_5 & R_5 & 0 & -R_4 & 0 \\ 0 & -R_5 & R_5 & 0 & -R_4 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ x(t-h_M) \\ \int_{t-h(t)}^t x(s) ds \\ \int_{t-h_M}^{t-h(t)} x(s) ds \end{bmatrix}.
 \end{aligned} \tag{53}$$

By Lemma 6, we can obtain $\dot{V}_6(t)$ as follows:

$$\begin{aligned}
 \dot{V}_6(t) &= \frac{h_M^2}{2} \left(\frac{h_M^2}{2} x^T(t) Q_6 x(t) \right. \\
 &\quad \left. - \int_{t-h_M}^t \int_u^t x^T(\lambda) Q_6 x(\lambda) d\lambda du \right) \\
 &\quad + h_M^2 \left(\frac{h_M^2}{2} y^T(t) Q_7 y(t) \right. \\
 &\quad \left. - \int_{t-h_M}^t \int_u^t \dot{x}^T(\lambda) Q_7 \dot{x}(\lambda) d\lambda du \right) = \frac{h_M^4}{2} x^T(t) \\
 &\quad \cdot Q_6 x(t) - \frac{h_M^2}{2} \int_{t-h_M}^t \int_u^t x^T(\lambda) Q_6 x(\lambda) d\lambda du \\
 &\quad + \frac{h_M^4}{2} y^T(t) Q_7 y(t) \\
 &\quad - h_M^2 \int_{t-h_M}^t \int_u^t \dot{x}^T(\lambda) Q_7 \dot{x}(\lambda) d\lambda du \leq \frac{h_M^4}{4} x^T(t) \\
 &\quad \cdot Q_6 x(t) - \int_{t-h_M}^t \int_u^t x^T(\lambda) d\lambda du \\
 &\quad \cdot Q_6 \int_{t-h_M}^t \int_u^t x(\lambda) d\lambda du + \frac{h_M^4}{2} y^T(t) Q_7 y(t) \\
 &\quad - 2 \int_{t-h_M}^t \int_u^t \dot{x}^T(\lambda) d\lambda du Q_7 \int_{t-h_M}^t \int_u^t \dot{x}(\lambda) d\lambda du \\
 &= \frac{h_M^4}{4} x^T(t) Q_6 x(t) + \frac{h_M^4}{2} y^T(t) Q_7 y(t)
 \end{aligned}$$

$$\begin{aligned}
& - \int_{t-h_M}^t \int_u^t x^T(\lambda) d\lambda du Q_6 \int_{t-h_M}^t \int_u^t x(\lambda) d\lambda du \\
& - \left[\sqrt{2} h_M x^T(t) - \sqrt{2} \int_{t-h_M}^t x^T(u) du \right] \\
& \cdot Q_7 \left[\sqrt{2} h_M x(t) - \sqrt{2} \int_{t-h_M}^t x(u) du \right].
\end{aligned} \tag{54}$$

By Lemma 7 and calculating $\dot{V}_7(t)$, we have

$$\begin{aligned}
\dot{V}_7(t) &= \frac{h_M^3}{6} \left(\frac{h_M^3}{6} y^T(t) Q_8 y(t) \right. \\
& - \int_{t-h_M}^t \int_u^t \int_\lambda^t \dot{x}^T(\theta) Q_8 \dot{x}(\theta) d\theta d\lambda du \Big) = \frac{h_M^6}{36} \\
& \cdot y^T(t) Q_8 y(t) - \frac{h_M^3}{6} \\
& \cdot \int_{t-h_M}^t \int_u^t \int_\lambda^t \dot{x}^T(\theta) Q_8 \dot{x}(\theta) d\theta d\lambda du \leq \frac{h_M^6}{36} y^T(t) \\
& \cdot Q_8 y(t) - \int_{t-h_M}^t \int_u^t \int_\lambda^t \dot{x}^T(\theta) d\theta d\lambda du \\
& \cdot Q_8 \int_{t-h_M}^t \int_u^t \int_\lambda^t \dot{x}(\theta) d\theta d\lambda du = \frac{h_M^6}{36} y^T(t) Q_8 y(t) \\
& - \left[\frac{h_M^2}{2} x^T(t) - \int_{t-h_M}^t \int_u^t x^T(\lambda) d\lambda du \right] \\
& \cdot Q_8 \left[\frac{h_M^2}{2} x(t) - \int_{t-h_M}^t \int_u^t x(\lambda) d\lambda du \right].
\end{aligned} \tag{55}$$

Using Lemma 3 (Jensen's Inequality) that we have

$$\begin{aligned}
\dot{V}_8(t) &\leq h_M^2 y^T(t) Z_1 y(t) - \int_{t-h_M}^t y^T(s) ds \\
& \cdot Z_1 \int_{t-h_M}^t y(s) ds + r_M^2 y^T(t) Z_2 y(t) \\
& - \int_{t-r_M}^t y^T(s) ds Z_2 \int_{t-r_M}^t y(s) ds \leq h_M^2 y^T(t) \\
& \cdot Z_1 y(t) - \left[\int_{t-h(t)}^t y^T(s) ds + \int_{t-h_M}^{t-h(t)} y^T(s) ds \right] \\
& \cdot Z_1 \left[\int_{t-h(t)}^t y(s) ds + \int_{t-h_M}^{t-h(t)} y(s) ds \right] + r_M^2 y^T(t) \\
& \cdot Z_2 y(t) - \int_{t-r(t)}^t y^T(s) ds Z_2 \int_{t-r(t)}^t y(s) ds.
\end{aligned} \tag{56}$$

Calculating $\dot{V}_9(t)$ leads to

$$\begin{aligned}
\dot{V}_9(t) &= x^T(t) Z_3 x(t) \\
& - (1 - \dot{r}(t)) x^T(t - r(t)) Z_3 x(t - r(t)) \\
& + r_M y^T(t) Q_9 y(t) \\
& - r_M (1 - \dot{r}(t)) y^T(t - r(t)) Q_9 y(t - r(t)) \\
& \leq x^T(t) Z_3 x(t) \\
& - (1 - r_d) x^T(t - r(t)) Z_3 x(t - r(t)) \\
& + r_M y^T(t) Q_9 y(t) \\
& - r_M (1 - r_d) y^T(t - r(t)) Q_9 y(t - r(t)).
\end{aligned} \tag{57}$$

From (4), (5), and (6), we obtain for any positive real constants ϵ_1, ϵ_2 , and ϵ_3 ,

$$\epsilon_1 \left(\alpha^2 x^T(t) x(t) - f_1^T(t, x(t)) f_1(t, x(t)) \right) \geq 0, \tag{58}$$

$$\begin{aligned}
& \epsilon_2 \left(\beta^2 x^T(t - h(t)) x(t - h(t)) \right. \\
& \left. - f_2^T(t, x(t - h(t))) f_2(t, x(t - h(t))) \right) \geq 0,
\end{aligned} \tag{59}$$

$$\begin{aligned}
& \epsilon_3 \left(\eta^2 y^T(t - r(t)) y(t - r(t)) \right. \\
& \left. - f_3^T(t, y(t - r(t))) f_3(t, y(t - r(t))) \right) \geq 0.
\end{aligned} \tag{60}$$

According to (48)-(60), it is straightforward to see that

$$\dot{V}(t) \leq \zeta^T(t) \sum \zeta(t), \tag{61}$$

where $\zeta^T(t) = [x^T(t), y^T(t), x^T(t - h_M), x^T(t - h(t)), x^T(t - r(t)), \int_{t-h(t)}^t y^T(s) ds, \int_{t-r(t)}^t y^T(s) ds, \int_{t-h_M}^{t-h(t)} y^T(s) ds, y^T(t - h_M), \int_{t-h_M}^{t-h(t)} x^T(s) ds, \int_{t-h(t)}^t x^T(s) ds, \int_{t-h_M}^t x^T(s) ds, \sqrt{2} \int_{t-h_M}^t x^T(u) du, \int_{t-h_M}^t \int_u^t x^T(\lambda) d\lambda du, y(t - r(t)), f_1^T(t, x(t)), \text{ and } f_2^T(t, x(t - h(t))), f_3^T(t, y(t - r(t)))]$.

If the conditions (31)-(34) and $\sum < 0$ hold, then (61) implies that there exists $\delta > 0$ such that $\dot{V}(t) \leq -\delta \|x(t)\|^2$. Therefore system (26) is asymptotically stable.

Next, we shall establish the H_∞ performance of system (26) under zero initial condition. To this end, we introduce

$$J_{zw}(t) = \int_0^t [z^T(s) z(s) - \gamma^2 w^T(s) w(s)] ds, \quad t > 0. \tag{62}$$

Noting zero initial condition, it can be shown that for any nonzero $w(t) \in L_2[0, \infty)$ and $t > 0$. This can be written as

$$\begin{aligned}
J_{zw}(t) &= \int_0^t [z^T(s) z(s) - \gamma^2 w^T(s) w(s)] ds \\
&= \int_0^t [z^T(s) z(s) - \gamma^2 w^T(s) w(s)] ds \\
&+ \int_0^t \dot{V}(s) ds - V(t) + V(0)
\end{aligned}$$

$$\begin{aligned}
&= \int_0^t [z^T(s)z(s) - \gamma^2 w^T(s)w(s) + \dot{V}(s)] ds \\
&\quad - V(t) \\
&\leq \int_0^t [z^T(s)z(s) - \gamma^2 w^T(s)w(s) + \dot{V}(s)] ds,
\end{aligned} \tag{63}$$

where $V(t)$ is define in (45). After some algebraic manipulations, we obtain

$$z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t) \leq \tilde{\zeta}^T(t) \widehat{\Sigma} \tilde{\zeta}(t), \tag{64}$$

where $\tilde{\zeta}^T(t) = [x^T(t), y^T(t), x^T(t - h_M), x^T(t - h(t)), x^T(t - r(t)), \int_{t-h(t)}^t y^T(s)ds, \int_{t-r(t)}^t y^T(s)ds, \int_{t-h_M}^{t-h(t)} y^T(s)ds, y^T(t - h_M), \int_{t-h_M}^{t-h(t)} x^T(s)ds, \int_{t-h(t)}^t x^T(s)ds, \int_{t-h_M}^t x^T(s)ds, \sqrt{2} \int_{t-h_M}^t x^T(u)du, \int_{t-h_M}^t \int_u x^T(\lambda)d\lambda du, y(t - r(t)), f_1(t, x(t)), \text{ and } f_2(t, x(t - h(t))), f_3(t, y(t - r(t))), w(t)]$.

$$\begin{aligned}
L^T &= [E^T P_2 \quad E^T P_{14} \quad E^T P_{11} \quad E^T P_5 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad E^T P_8 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\
N &= [G_1 \quad 0 \quad 0 \quad G_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad G_4 \quad 0 \quad 0 \quad 0 \quad G_3 \quad 0 \quad 0 \quad 0].
\end{aligned} \tag{68}$$

Theorem 12. For a prescribed scalar $\gamma > 0$, given scalars $r_M > 0, r_d > 0$ and $h_M > 0$, the system (1) is robustly asymptotically stable with the H_∞ performance γ ; if $\|C(t)\| + \eta < 1$, there exist positive definite symmetric matrices P_1, R_4, R_6, Q_i , and $Z_j, i = 1, 2, \dots, 9, j = 1, 2, 3$, any appropriate dimensional matrices, J, K, S, R_k, M_l , and $P_m, k = 1, 2, \dots, 6, l = 1, 2, \dots, 5, m = 2, 3, \dots, 16$ and positive real constants $\delta, \epsilon_1, \epsilon_2$, and ϵ_3 satisfying the following LMIs:

$$\begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix} > 0, \tag{69}$$

$$\begin{bmatrix} R_4 & R_5 \\ R_5^T & R_6 \end{bmatrix} > 0, \tag{70}$$

$$\begin{bmatrix} Q_5 & S \\ S^T & Q_5 \end{bmatrix} \geq 0, \tag{71}$$

$$\begin{bmatrix} Q_3 & M_1 & M_2 \\ M_1^T & M_3 & M_4 \\ M_2^T & M_4^T & M_5 \end{bmatrix} \geq 0, \tag{72}$$

$$\begin{bmatrix} \widehat{\Sigma} & L & \delta N^T \\ * & -\delta I & \delta H^T \\ * & * & -\delta I \end{bmatrix} < 0. \tag{73}$$

If the condition (35) holds, we have $z^T(t)z(t) - \gamma^2 w^T(t)w(t) + \dot{V}(t) \leq 0$. Thus,

$$\dot{V}(t) \leq -z^T(t)z(t) + \gamma^2 w^T(t)w(t). \tag{65}$$

Integrating both sides of (65) from t_0 to t , we obtain

$$V(t) - V(t_0) \leq - \int_{t_0}^t z^T(s)z(s) ds + \int_{t_0}^t \gamma^2 w^T(s)w(s) ds. \tag{66}$$

Then, letting $t \rightarrow \infty$ and under zero initial condition, we have $V(t_0) = V(0) = 0$ and $V(\infty) = 0$, which leads to

$$\int_{t_0}^{\infty} z^T(t)z(t) dt \leq \int_{t_0}^{\infty} \gamma^2 w^T(t)w(t) dt. \tag{67}$$

Therefore $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ is satisfied for any nonzero $w(t) \in L_2[0, \infty)$.

The proof of theorem is complete. $\square$

According to Theorem 11, we can obtain delay-dependent robust asymptotic stability criteria with the H_∞ performance level γ of system (1). We introduce the following notations for later use

Proof. Replacing A, B, C , and D in (35) with $A = A + E\Delta(t)G_1, B = B + E\Delta(t)G_2, C = C + E\Delta(t)G_3$, and $D = D + E\Delta(t)G_4$, respectively, we find that condition (35) is equivalent to the following condition:

$$\widehat{\Sigma} + L\Delta(t)N + N^T\Delta(t)^T L^T < 0. \tag{74}$$

By using Lemma (1), we can find that (74) is equivalent to the LMIs as follows:

$$\begin{bmatrix} \widehat{\Sigma} & L & \delta N^T \\ * & -\delta I & \delta H^T \\ * & * & -\delta I \end{bmatrix} < 0, \tag{75}$$

where δ is a positive real constant. From Theorem (26) and conditions (69)-(73), system (1) is robustly asymptotically stable and satisfies $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$. The proof of theorem is complete. $\square$

4. Numerical Examples

Example 1. Consider the following uncertain neutral system with mixed time-varying delays (26). We consider robust asymptotic stability with H_∞ performance γ of system (26) by using Theorem 11. The system (26) is specified as follows:

$$\begin{aligned}
A &= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \\
B &= \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}, \\
C &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
C_1 &= [1 \ 0], \\
C_2 &= [1 \ 0], \\
D &= \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \\
D_1 &= \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \\
D_2 &= 0.1, \\
I &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\end{aligned} \tag{76}$$

$$\begin{aligned}
f_1(t, x(t)) &= \begin{bmatrix} 0.1 \cos(t) x_1(t) \\ 0.1 \cos(t) x_2(t) \end{bmatrix}, \\
f_2(t, x(t-h(t))) &= \begin{bmatrix} 0.1 \sin(t) x_1(t-h(t)) \\ 0.1 \sin(t) x_2(t-h(t)) \end{bmatrix}, \\
f_3(t, \dot{x}(t-r(t))) &= \begin{bmatrix} 0.1 \cos(t) \dot{x}_1(t-r(t)) \\ 0.1 \sin(t) \dot{x}_2(t-r(t)) \end{bmatrix}.
\end{aligned}$$

Decompose the matrices $A = A_1 + A_2$ and $B = B_1 + B_2$, where

$$\begin{aligned}
A_1 &= \begin{bmatrix} -1.5 & -0.5 \\ -0.1 & -1.5 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} -0.5 & 0.5 \\ 0.1 & -0.5 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} -0.6 & 0 \\ -0.5 & -1.1 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0.6 & 0.4 \\ 0.9 & 1.1 \end{bmatrix}, \\
h(t) &= |\cos(t)|, \\
r(t) &= \sin^2(0.6t), \\
\phi(t) &= \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \\
t &\in [-1, 0].
\end{aligned} \tag{77}$$

we use (31)-(35) in Theorem 11. This example shows that the solutions of LMIs are given as follows:

$$\begin{aligned}
P_1 &= \begin{bmatrix} 168.3110 & -44.5433 \\ -44.5433 & 85.4171 \end{bmatrix}, \\
P_2 &= \begin{bmatrix} 127.4372 & -29.2853 \\ -29.2853 & 96.9456 \end{bmatrix}, \\
P_3 &= \begin{bmatrix} 5.8222 & 34.8937 \\ 34.8937 & 24.4550 \end{bmatrix}, \\
P_4 &= \begin{bmatrix} -4.3890 & 2.1594 \\ 2.1594 & -0.9552 \end{bmatrix}, \\
P_5 &= \begin{bmatrix} -9.5746 & -7.5072 \\ -7.5072 & -1.7680 \end{bmatrix}, \\
P_6 &= \begin{bmatrix} -0.1185 & 0.1299 \\ 0.1299 & -0.0008 \end{bmatrix}, \\
P_7 &= \begin{bmatrix} -0.5126 & 0.2522 \\ 0.2522 & -0.2031 \end{bmatrix}, \\
P_8 &= \begin{bmatrix} 5.1725 & 11.9120 \\ 11.9120 & -1.9884 \end{bmatrix}, \\
P_9 &= \begin{bmatrix} 11.1571 & 12.7192 \\ 12.7192 & 31.2439 \end{bmatrix}, \\
P_{10} &= \begin{bmatrix} -0.1533 & 0.1197 \\ 0.1197 & 0.1812 \end{bmatrix}, \\
P_{11} &= \begin{bmatrix} 27.3684 & -4.8605 \\ -4.8605 & 8.2454 \end{bmatrix}, \\
P_{12} &= \begin{bmatrix} 15.0797 & 4.8737 \\ 4.8737 & 16.5943 \end{bmatrix}, \\
P_{13} &= \begin{bmatrix} 4.1994 & -2.0019 \\ -2.0019 & 1.0377 \end{bmatrix}, \\
P_{14} &= \begin{bmatrix} 75.3600 & -12.6160 \\ -12.6160 & 49.4866 \end{bmatrix}, \\
P_{15} &= \begin{bmatrix} -6.5616 & 39.9896 \\ 39.9896 & 29.4163 \end{bmatrix}, \\
P_{16} &= \begin{bmatrix} -23.5965 & -7.6251 \\ -7.6251 & 3.1193 \end{bmatrix}, \\
Q_1 &= \begin{bmatrix} 62.0192 & -23.0061 \\ -23.0061 & 11.6538 \end{bmatrix},
\end{aligned}$$

It is easy to see that $\alpha = 0.1$, $\beta = \eta = 0.05$, $h_M = 1$, $r_M = 1$, $r_d = 0.6$, and $\gamma = 11$. By using LMI Toolbox in MATLAB,

$$Q_2 = \begin{bmatrix} 15.1796 & -10.2201 \\ -10.2201 & 21.4165 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 6.9057 & -3.1644 \\ -3.1644 & 1.7515 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 15.5854 & 6.9868 \\ 6.9868 & 19.8625 \end{bmatrix},$$

$$Q_5 = \begin{bmatrix} 14.1662 & -5.1961 \\ -5.1961 & 6.1072 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 64.9034 & -31.5581 \\ -31.5581 & 21.8466 \end{bmatrix},$$

$$Q_7 = \begin{bmatrix} 10.6144 & -5.0964 \\ -5.0964 & 3.0301 \end{bmatrix},$$

$$Q_8 = \begin{bmatrix} 24.1919 & -11.7046 \\ -11.7046 & 7.6700 \end{bmatrix},$$

$$Q_9 = \begin{bmatrix} 14.9436 & -2.3454 \\ -2.3454 & 12.6511 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 44.5756 & 0.7540 \\ 0.7540 & 45.5384 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 50.6259 & 2.5076 \\ 2.5076 & 24.1936 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 46.8341 & -0.3771 \\ -0.3771 & 46.2886 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 62.0196 & -23.0062 \\ -23.0062 & 11.6539 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} -0.0320 & 0.0152 \\ 0.0152 & -0.0079 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 2.8943 & -1.3916 \\ -1.3916 & 0.8241 \end{bmatrix},$$

$$R_4 = \begin{bmatrix} 28.2168 & -12.4294 \\ -12.4294 & 138.1825 \end{bmatrix},$$

$$R_5 = \begin{bmatrix} 0.3352 & 0.4558 \\ 0.4558 & 12.2682 \end{bmatrix},$$

$$R_6 = \begin{bmatrix} 7.1169 & -3.1950 \\ -3.1950 & 2.8227 \end{bmatrix},$$

$$J = \begin{bmatrix} 0.1922 & 0.3186 \\ 0.5884 & 0.2678 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.3073 & -0.1700 \\ -0.1716 & -0.6207 \end{bmatrix},$$

$$S = \begin{bmatrix} -8.1980 & 2.4201 \\ 2.4201 & -4.6544 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} -6.7588 & 3.0952 \\ 3.0952 & -1.7091 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 6.7510 & -3.0908 \\ -3.0908 & 1.7066 \end{bmatrix},$$

$$M_3 = \begin{bmatrix} 10.7350 & -5.0861 \\ -5.0861 & 2.8906 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} -7.1468 & 3.2947 \\ 3.2947 & -1.9222 \end{bmatrix},$$

$$M_5 = \begin{bmatrix} 10.9985 & -5.2411 \\ -5.2411 & 2.9857 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 4.8285 & -2.2335 \\ -2.2335 & 1.1915 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} -0.4052 & -0.1913 \\ -0.1913 & 0.1191 \end{bmatrix},$$

$$Z_3 = \begin{bmatrix} 17.9504 & -8.9631 \\ -8.9631 & 5.9259 \end{bmatrix},$$

$$\epsilon_1 = 1.3735 \times 10^3,$$

$$\epsilon_2 = 1.2228 \times 10^3,$$

$$\epsilon_3 = 609.7809.$$

(78)

Example 2. Consider system (26) with $D = 0$, $\omega(t) \equiv 0$, $f_1(\cdot) = f_2(\cdot) = f_3(\cdot) \equiv 0$, and $C_1 = C_2 = 0$, which means neutral system with time-varying delay

$$\dot{x}(t) - C\dot{x}(t - r(t)) = Ax(t) + Bx(t - h(t)),$$

 $t \geq 0;$ (79)

$$x(t) = \phi(t), \quad t \in [-\bar{h}, 0],$$

with the parameters

$$A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.5 & 0 \\ 0.5 & -0.5 \end{bmatrix}, \quad (80)$$

$$C = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}.$$

TABLE 1: Maximum allowable upper bounds h_M with $r_M = 1$ and given r_d for different μ_m in Example 2.

Method				
μ_m	0.7	0.9	1.1	Unknown
$r_d=0.5$				
Kwon et al. (2008) [30]	-	-	-	1.0864
Guo et al. (2010) [6]	-	-	-	1.2223
Liu et al. (2015) [32]	1.655	1.615	-	-
Liu (2016) [31]	1.864	1.742	-	-
Liu (2016) [31]	-	-	-	1.8611
Wang et al. (2017) [33]	1.895	1.862	1.820	-
This paper	-	-	-	2.0447

Decompose matrix A and B as follows: $A = A_1 + A_2$ and $B = B_1 + B_2$, where

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -1.5 & -0.5 \\ -0.1 & -0.5 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} 0.5 & 1.5 \\ 0.1 & -0.5 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} -0.6 & 0 \\ -0.5 & -1.1 \end{bmatrix}, \\
 B_2 &= \begin{bmatrix} 1.1 & 0 \\ 1 & 0.6 \end{bmatrix}.
 \end{aligned} \tag{81}$$

Table 1 lists the comparison of the upper bounds delays for asymptotic stability of system (79) by different methods. And the derivative of the discrete time-varying delay is unknown, because we removed the derivative of the discrete time-varying delay. We can see from Table 1 that our results are superior to those in [6, 30–33].

Example 3. Consider system (26) with $D = 0, \omega(t) \equiv 0$ and $C_1 = C_2 = 0$,

$$\begin{aligned}
 \dot{x}(t) - C\dot{x}(t - h(t)) &= Ax(t) + Bx(t - h(t)) \\
 &\quad + f_1(t, x(t)) \\
 &\quad + f_2(t, x(t - h(t))) \\
 &\quad + f_3(t, \dot{x}(t - h(t))),
 \end{aligned} \tag{82}$$

$$t \geq 0,$$

$$x(t) = \phi(t), \quad t \in [-\bar{h}, 0],$$

with the parameters

$$\begin{aligned}
 A &= \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \\
 B &= \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix},
 \end{aligned}$$

TABLE 2: Maximum allowable upper bounds h_M with $r_M = 1$ and given r_d for different μ_m in Example 3.

Method				
μ_m	0.5	0.9	1.1	Unknown
$r_d=0.6$				
Lakshmanan et al. (2011) [34]	-	-	-	3.9563
Cheng et al. (2013) [22]	-	-	-	4.6235
Qiu et al. (2015) [35]	-	-	-	4.9423
Ours	-	-	-	8.7375
$r_d=0$				
Liu et al. (2015) [32]	8.975	8.820	-	-
Qiu et al. (2015) [35]	9.646	9.225	-	-
Cheng et al. (2013) [22]	9.975	9.756	9.685	-
This paper	-	-	-	9.7967

$$C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$\alpha = 0.1,$$

$$\beta = \eta = 0.05.$$

(83)

Decompose matrix A and B as follows: $A = A_1 + A_2$ and $B = B_1 + B_2$, where

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -1.5 & -0.5 \\ -0.1 & -1.5 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} -0.5 & 0.5 \\ 0.1 & -0.5 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} -0.6 & 0 \\ -0.5 & -1.1 \end{bmatrix}, \\
 B_2 &= \begin{bmatrix} 0.6 & 0.4 \\ 0.9 & 1.1 \end{bmatrix}.
 \end{aligned} \tag{84}$$

Table 2 lists the comparison of the upper bounds delays for asymptotic stability of system (82) by different methods. And the derivative of the discrete time-varying delay is unknown, because we removed the derivative of the discrete time-varying delay. It is clear that our results are superior to those in [22, 32, 34, 35].

Example 4. We consider system (82) with the parameters

$$\begin{aligned}
 A &= \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, \\
 B &= \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, \\
 C &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
 \beta &= 0.1.
 \end{aligned} \tag{85}$$

TABLE 3: Maximum allowable upper bounds h_M with $\mu_m = 0.5$, $r_M = 1$ and different values of η in Example 4.

Method	$\alpha = 0$			$\alpha = 0.1$		
η	0.1	0.2	0.3	0.1	0.2	0.3
$r_d=0$						
Lakshmanan et al. (2011) [34]	1.3386	1.0816	0.8563	1.1950	0.9734	0.7760
Cheng et al. (2013) [22]	1.3721	1.0923	0.8613	1.2466	0.9996	0.7804
This paper (μ_m is unknown)	1.3496	1.1232	0.9027	1.2479	1.0450	0.8522

Decompose matrix A and B as follows: $A = A_1 + A_2$ and $B = B_1 + B_2$, where

$$\begin{aligned}
 A_1 &= \begin{bmatrix} -1 & -0.5 \\ -0.1 & -0.5 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} -0.2 & 0.6 \\ 0 & -0.5 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} -0.6 & 0.3 \\ -1.1 & -1.1 \end{bmatrix}, \\
 B_2 &= \begin{bmatrix} 0 & 0.4 \\ 0.1 & 0.3 \end{bmatrix}.
 \end{aligned} \tag{86}$$

Table 3 shows the comparison of the upper bounds delay allowed obtained for asymptotic stability of system (82) by other methods. And the derivative of the discrete time-varying delay is unknown, because we removed the derivative of the discrete time-varying delay. It can be found from Table 3 that our results are significantly better than those in [22, 34].

5. Conclusions

The problem of robust stability and H_∞ performance for uncertain neutral systems with time-varying delays was studied. The restriction on the derivative of the discrete time-varying delay is removed. The uncertainties under consideration are nonlinear time-varying parameter perturbations and norm-bounded uncertainties, respectively. By applying a novel Lyapunov-Krasovskii functional approach, Wirtinger-based integral inequality and Peng-Park's integral inequality, decomposition technique of constant matrix, descriptor model transformation, Leibniz-Newton formula and utilization of zero equation, and improved delay-dependent bounded real lemmas (BRL) for systems are established in terms of linear matrix inequalities (LMIs). Then, based on the obtained BRL, some less conservative delay-dependent stability criteria of uncertain neutral systems with mixed time-varying delays and nonlinear perturbations are obtained and improved H_∞ performance criterion with the framework of LMIs is introduced. Numerical examples have shown significant improvements over some existing results.

Conflicts of Interest

The authors declare no conflicts of interest.

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