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Research Article

A Novel Fifth-Degree Strong Tracking Cubature Kalman Filter for Two-Dimensional Maneuvering Target Tracking

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A novel fifth-degree strong tracking cubature Kalman filter is put forward to improve the two-dimensional maneuvering target tracking accuracy. First, a new fifth-degree cubature rule, with only one point more than the theoretical lower bound, is used to approximate the intractable nonlinear Gaussian weighted integral in the nonlinear Kalman filtering framework, and a novel fifth-degree cubature Kalman filter is proposed. Then, the suboptimal fading factor is designed for the filter to adjust the filtering gain matrix online and force the residual sequences mutually orthogonal, thus improving the ability of the filter to track the mutation state, and the fifth-degree strong tracking cubature Kalman filter is derived. The suboptimal fading factor is calculated in a new method, which reduces the number of calculations for the cubature points from three times to twice without calculating the Jacobian matrix. The simulation results indicate that the proposed filter has the ability to track the maneuvering target and achieve higher target tracking accuracy and thus verifies the effectiveness of the proposed filter.

1. Introduction

For the last several decades, the target tracking has momentous applications in many fields, such as navigation guidance, military application, and sensor networks [1]. In target tracking problem, the process model is generally linear, while the measurement model, mainly including the measured range and bearing angle, is nonlinear [2, 3]. The essence of the target tracking is to use a series of measured ranges and bearing angle information to estimate the position and velocity of the target in real time; hence, it belongs to the nonlinear filtering problem, which has always been dealt with using the nonlinear Kalman filters [4].

In nonlinear Kalman filters, the extended Kalman filter (EKF) [5] is the most widely used one. In EKF, the nonlinear function is approximated using the first-order Taylor series expansion and, then, the standard Kalman filter is applied [6]. EKF is simple in principle; however, its filtering accuracy and stability may reduce for the strong nonlinear system, and it needs to calculate the Jacobian matrix, which is difficult to accomplish at times. Julier [7] adopts a set of sigma points to approximate the posterior probability density function (PDF) and proposes the unscented Kalman filter (UKF). UKF is a derivative-free filter and can achieve third-degree filtering accuracy [8, 9]. However, there exist some tunable parameters in the selection of sigma points, and the inappropriate selection may reduce the filtering accuracy. In addition, the weight on the center point may be negative for the high-dimensional system, which may degrade the numerical stability of the filter [10, 11]. Arasaratnam [12, 13] puts forward the cubature Kalman filter (CKF), where the intractable integral in nonlinear Kalman filter is decomposed into the spherical integral and the radial integral, which are approximated using different numerical integration rules. CKF contains a set of cubature points with equal weights; thus the numerical stability is guaranteed [14–16]. CKF can be regarded as a special case of UKF [17]; however, CKF gives the rigorous reason for the selection of parameters for the first time. In order to improve the accuracy of CKF, Jia [18] adopts the symmetric spherical interpolation and moment matching method to calculate the spherical integral and radial integral, respectively, and proposes the fifth-degree CKF. And then, Wang [19] employs the transformation group of the regular simplex instead of the symmetric spherical interpolation to derive the fifth-degree spherical simplex-radial CKF. These two filters improve the filtering accuracy effectively.

However, the conventional nonlinear Kalman filters cannot achieve the effective tracking of the maneuvering target. In order to improve the ability of the filter to fast track the mutation state, Zhou [20] proposes the strong tracking filter (STF) on the basis of EKF. The STF uses the suboptimal fading factor to realize the real time adjustment of the gain matrix to force the residual sequences mutually orthogonal [21, 22]; it has the ability to track the mutation state while inheriting the inherent defects of EKF. Correspondingly, the strong tracking UKF (STUKF) and strong tracking CKF (STCKF) are put forward in succession. There are two methods to calculate the suboptimal fading factor in these two types of filters. On the one hand, the fading factor is still calculated using the Jacobian matrix [23–25], which may bring some trouble in strong nonlinear systems. On the other hand, the fading factor is calculated using the equivalent expressions derived by the statistical regression method [26, 27], in which the predicted measurement covariance and the cross-covariance are contained. It results in the nonlinear transformation carried out for three times, which increases the amount of calculation while loses a certain precision.

In this paper, a novel fifth-degree strong tracking cubature Kalman filter (5-STCKF) is proposed to further improve the two-dimensional maneuvering target tracking accuracy. A new fifth-degree cubature rule is utilized to approximate the intractable nonlinear Gaussian weighted integral, and a novel fifth-degree cubature Kalman filter is put forward. To improve the ability of the filter to track the mutation state, the suboptimal fading factor is designed to adjust the filtering gain matrix online and force the residual sequences mutually orthogonal and the 5-STCKF is derived. The suboptimal fading factor is calculated in a new method, which reduces the number of calculations for the cubature points from three times to twice without calculating the Jacobian matrix. The proposed 5-STCKF has the ability to track the mutation state and can achieve better performance compared with the 5-CKF and 5-SSRCKF in maneuvering target tracking applications. In addition, the number of cubature points needed in the 5-STCKF is less than those in the 5-CKF and 5-SSRCKF. The simulation results verify the validity of the filter.

The rest of this paper is organized as follows: the novel fifth-degree cubature Kalman filter is proposed in Section 2, the novel fifth-degree strong tracking cubature Kalman filter is derived in Section 3, the simulation results and analysis are presented in Section 4, and the conclusion is given in Section 5.

2. A Novel Fifth-Degree Cubature Kalman Filter

The following discrete time nonlinear system is considered:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{f}\left(\mathbf{x}_{k-1}\right) + \mathbf{w}_{k-1} \\ \mathbf{z}_k &= \mathbf{h}\left(\mathbf{x}_k\right) + \mathbf{v}_k \end{aligned} \tag{1}$$

where $\mathbf{x}_k \in \mathbf{R}^n$ denotes the state vector at time k, $\mathbf{z}_k \in \mathbf{R}^p$ represents the measurement vector, and $\mathbf{f}(\cdot)$ and $\mathbf{h}(\cdot)$ denote the known nonlinear process and measurement functions, respectively. $\mathbf{w}_k$ is the zero mean Gaussian white process noise, with the covariance being $\mathbf{Q}_k$, and $\mathbf{v}_k$ is the zero mean Gaussian white measurement noise, with the covariance being $\mathbf{R}_k$.

Given the measurements $\mathbf{Z}_k = \{\mathbf{z}_1, \mathbf{z}_2, \cdots, \mathbf{z}_k\}$, the minimum mean square error (MMSE) estimate of the state $\mathbf{x}_k$ with the estimate error covariance is given below:

$$\widehat{\mathbf{x}}_k^{MMSE} = E\left(\mathbf{x}_k \mid \mathbf{Z}_k\right) = \int \mathbf{x}_k p\left(\mathbf{x}_k \mid \mathbf{Z}_k\right) d\mathbf{x}_k \tag{2}$$

$$\mathbf{P}_x = \int \left(\mathbf{x}_k - \widehat{\mathbf{x}}_k^{MMSE}\right)\left(\mathbf{x}_k - \widehat{\mathbf{x}}_k^{MMSE}\right)^{\mathrm{T}} p\left(\mathbf{x}_k \mid \mathbf{Z}_k\right) d\mathbf{x}_k \tag{3}$$

where $p(\mathbf{x}_k \mid \mathbf{Z}_k)$ denotes the posterior PDF.

For nonlinear Kalman filters, the posterior PDF is assumed to be Gaussian distribution and the following five integrals in the filtering framework are required to be calculated [12]:

$$\widehat{\mathbf{x}}_k^- = \int_{\mathbf{R}^n} \mathbf{f}\left(\mathbf{x}_{k-1}\right) N\left(\mathbf{x}_{k-1}; \widehat{\mathbf{x}}_{k-1}^+, \mathbf{P}_{k-1}^+\right) d\mathbf{x}_{k-1} \tag{4}$$

$$\begin{aligned} \mathbf{P}_k^- = &\int_{\mathbf{R}^n} \mathbf{f}\left(\mathbf{x}_{k-1}\right) \mathbf{f}^{\mathrm{T}}\left(\mathbf{x}_{k-1}\right) N\left(\mathbf{x}_{k-1}; \widehat{\mathbf{x}}_{k-1}^+, \mathbf{P}_{k-1}^+\right) d\mathbf{x}_{k-1} \\ &- \widehat{\mathbf{x}}_k^-\left(\widehat{\mathbf{x}}_k^-\right)^{\mathrm{T}} + \mathbf{Q}_k \end{aligned} \tag{5}$$

$$\widehat{\mathbf{z}}_k = \int_{\mathbf{R}^n} \mathbf{h}\left(\mathbf{x}_k\right) N\left(\mathbf{x}_k; \widehat{\mathbf{x}}_k^-, \mathbf{P}_k^-\right) d\mathbf{x}_k \tag{6}$$

$$\begin{aligned} \mathbf{P}_{z,k} = &\int_{\mathbf{R}^n} \mathbf{h}\left(\mathbf{x}_{k-1}\right) \mathbf{h}^{\mathrm{T}}\left(\mathbf{x}_{k-1}\right) N\left(\mathbf{x}_k; \widehat{\mathbf{x}}_k^-, \mathbf{P}_k^-\right) d\mathbf{x}_k \\ &- \widehat{\mathbf{z}}_k \widehat{\mathbf{z}}_k^{\mathrm{T}} + \mathbf{R}_k \end{aligned} \tag{7}$$

$$\mathbf{P}_{xz,k} = \int_{\mathbf{R}^n} \mathbf{x}_k \mathbf{h}^{\mathrm{T}}\left(\mathbf{x}_k\right) N\left(\mathbf{x}_k; \widehat{\mathbf{x}}_k^-, \mathbf{P}_k^-\right) d\mathbf{x}_k - \widehat{\mathbf{x}}_k^- \widehat{\mathbf{z}}_k \tag{8}$$

where $N(\mathbf{x}; \widehat{\mathbf{x}}, \mathbf{P}_x)$ denotes the Gaussian distribution with mean $\widehat{\mathbf{x}}$ and covariance $\mathbf{P}_x$, $\widehat{\mathbf{x}}_k^-$ and $\mathbf{P}_k^-$ represent the prior state estimate and estimate error covariance, respectively, $\widehat{\mathbf{z}}_k$ is the predicted measurement, and $\mathbf{P}_{z,k}$ and $\mathbf{P}_{xz,k}$ denote the measurement covariance and cross-covariance, respectively.

It can be seen from (4) to (8) that the key problem in nonlinear Kalman filters is to calculate the intractable integral in the form of $I_N = \int_{\mathbf{R}^n} \mathbf{g}(\mathbf{x}) N(\mathbf{x}; \widehat{\mathbf{x}}, \mathbf{P}_x) d\mathbf{x}$, where $\mathbf{g}(\mathbf{x})$ is arbitrary nonlinear function. In general, it is hard to achieve the analytical solution of I_N, and the numerical approximation should be considered. For this, the integral $I_e = \int_{\mathbf{R}^n} \mathbf{g}(\mathbf{x}) \exp(-\mathbf{x}^{\mathrm{T}}\mathbf{x}) d\mathbf{x}$ is taken into account first.

There already exist some cubature rules, including the third-degree spherical-radial cubature rule and fifth-degree spherical-radial cubature rule, to calculate I_e, and it is proved that the fifth-degree cubature rule can achieve higher accuracy than the third-degree one. For more details, please refer to [12, 18].

In this paper, through linear transformation, a novel fifth-degree cubature rule for calculating the integral I_N is given below.

It is pointed out in [28] that I_e can be calculated approximately using the following fifth-degree cubature rule:

$$\begin{aligned} I_e = {} & A\underbrace{\left[g(\eta,\eta,\cdots,\eta)+g(-\eta,-\eta,\cdots,-\eta)\right]}_{2} \\ & + \underbrace{B\sum_{perm}\left[g(\lambda,\xi,\cdots,\xi)+g(-\lambda,-\xi,\cdots,-\xi)\right]}_{2C_n^1} \\ & + \underbrace{C\sum_{perm}\left[g(\mu,\mu,\gamma,\cdots,\gamma)+g(-\mu,-\mu,-\gamma,\cdots,-\gamma)\right]}_{2C_n^2} \end{aligned} \tag{9}$$

where perm means all distinct permutations, C_n^1 and C_n^2 represent the binomial coefficients that equal to n and $n(n-1)/2$, respectively, and the parameters η, μ, and γ satisfy the following equations. For more details, please refer to [28, 29].

$$\frac{\mu}{\gamma} = -3 \pm \sqrt{16-2n} \tag{10}$$

$$\gamma^2 = \frac{3 \pm \sqrt{7-n}}{2\left(16-n \mp 4\sqrt{16-2n}\right)} \tag{11}$$

$$\eta^2 = \frac{n(n-7) \mp \left(n^2-3n-16\right)\sqrt{7-n}}{2\left(n^3-7n^2-16n+128\right)} \tag{12}$$

However, (9) is not suitable for calculating I_N in its current form. In order to simplify (9), we define $\mathbf{e}$ as an n-order identity matrix and the matrix subscript $[\cdot]_i$ is utilized to denote the ith column and, then, (9) can be written in the following form:

$$\begin{aligned} I_e = {} & A\left[\mathbf{g}\left(\sum_{i=1}^{n}\eta\mathbf{e}_i\right)+\mathbf{g}\left(-\sum_{i=1}^{n}\eta\mathbf{e}_i\right)\right] \\ & + B\sum_{i=1}^{n}\left[\mathbf{g}\left(\lambda\mathbf{e}_i+\sum_{j=1,j\neq i}^{n}\xi\mathbf{e}_j\right)\right. \\ & \left. + \mathbf{g}\left(-\lambda\mathbf{e}_i-\sum_{j=1,j\neq i}^{n}\xi\mathbf{e}_j\right)\right] \\ & + C\sum_{i=1}^{C_n^2}\left[\mathbf{g}\left(\mu\mathbf{e}_j+\mu\mathbf{e}_k+\sum_{l=1,l\neq j,l\neq k}^{n}\gamma\mathbf{e}_l\right)\right. \\ & \left. + \mathbf{g}\left(-\mu\mathbf{e}_j-\mu\mathbf{e}_k-\sum_{l=1,l\neq j,l\neq k}^{n}\gamma\mathbf{e}_l\right)\right], \\ & j<k;\quad j,k=1,2,\cdots,n \end{aligned} \tag{13}$$

Further, we define the following variables:

$$\mathbf{p} = \sum_{i=1}^{n}\eta\mathbf{e}_i \tag{14}$$

$$\mathbf{q}_i = \lambda\mathbf{e}_i + \sum_{j=1,j\neq i}^{n}\xi\mathbf{e}_j \tag{15}$$

$$\mathbf{s}_i = \mu\mathbf{e}_j + \mu\mathbf{e}_k + \sum_{l=1,l\neq j,l\neq k}^{n}\gamma\mathbf{e}_l \tag{16}$$

Then, (13) is transformed into the following simplified form:

$$\begin{aligned} I_e = {} & A\left[\mathbf{g}(\mathbf{p})+\mathbf{g}(-\mathbf{p})\right] + B\sum_{i=1}^{n}\left[\mathbf{g}(\mathbf{q}_i)+\mathbf{g}(-\mathbf{q}_i)\right] \\ & + C\sum_{i=1}^{C_n^2}\left[\mathbf{g}(\mathbf{s}_i)+\mathbf{g}(-\mathbf{s}_i)\right] \end{aligned} \tag{17}$$

It can be proved that I_N has the following equivalent form [12]:

$$I_N = \frac{1}{\sqrt{\pi^n}}\int_{\mathbf{R}^n}\mathbf{g}\left(\sqrt{2\mathbf{P}_x}\mathbf{x}+\hat{\mathbf{x}}\right)\exp\left(-\mathbf{x}^{\mathrm{T}}\mathbf{x}\right)d\mathbf{x} \tag{18}$$

such that I_N can be approximated using the cubature rule below:

$$\begin{aligned} I_N = {} & \frac{A}{\sqrt{\pi^n}}\left[\mathbf{g}\left(\sqrt{2\mathbf{P}_x}\mathbf{p}+\hat{\mathbf{x}}\right)+\mathbf{g}\left(-\sqrt{2\mathbf{P}_x}\mathbf{p}+\hat{\mathbf{x}}\right)\right] \\ & + \frac{B}{\sqrt{\pi^n}}\sum_{i=1}^{n}\left[\mathbf{g}\left(\sqrt{2\mathbf{P}_x}\mathbf{q}_i+\hat{\mathbf{x}}\right)+\mathbf{g}\left(-\sqrt{2\mathbf{P}_x}\mathbf{q}_i+\hat{\mathbf{x}}\right)\right] \\ & + \frac{C}{\sqrt{\pi^n}}\sum_{i=1}^{C_n^2}\left[\mathbf{g}\left(\sqrt{2\mathbf{P}_x}\mathbf{s}_i+\hat{\mathbf{x}}\right)+\mathbf{g}\left(-\sqrt{2\mathbf{P}_x}\mathbf{s}_i+\hat{\mathbf{x}}\right)\right] \\ = {} & \sum_{i=1}^{n^2+n+2}\omega_i\mathbf{g}\left(\hat{\mathbf{x}}^{(i)}\right) \end{aligned} \tag{19}$$

where the cubature points and corresponding weights are given below and the specific values of A, B, and C are given in [29].

$$\hat{\mathbf{x}}^{(i)} = \begin{cases} \sqrt{2\mathbf{P}_x}\,[\mathbf{p},-\mathbf{p}]_i+\hat{\mathbf{x}}, & i=1,2 \\ \sqrt{2\mathbf{P}_x}\,[\mathbf{q},-\mathbf{q}]_{i-2}+\hat{\mathbf{x}}, & i=3,\cdots,2n+2 \\ \sqrt{2\mathbf{P}_x}\,[\mathbf{s},-\mathbf{s}]_{i-2n-2}+\hat{\mathbf{x}}, & i=2n+3,\cdots,n^2+n+2 \end{cases} \tag{20}$$

$$\omega_i = \begin{cases} \dfrac{A}{\sqrt{\pi^n}}, & i=1,2 \\ \dfrac{B}{\sqrt{\pi^n}}, & i=3,\cdots,2n+2 \\ \dfrac{C}{\sqrt{\pi^n}}, & i=2n+3,\cdots,n^2+n+2 \end{cases} \tag{21}$$

where $\mathbf{q} = [\mathbf{q}_1,\mathbf{q}_2,\cdots,\mathbf{q}_n]$ and $\mathbf{s} = [\mathbf{s}_1,\mathbf{s}_2,\cdots,\mathbf{s}_{C_n^2}]$ represent the matrices constituted by $\mathbf{q}_i$ and $\mathbf{s}_i$, respectively.

Take $n = 6$ as an example, which will be used in the simulation section; the parameters in (19) are listed in Table 1.

Remark 1. The number of points needed in the fifth-degree cubature rule proposed in [18] is $2n^2+1$, and that in the spherical simplex cubature rule proposed in [19] is n^2+3n+3. However, the number of points needed in the proposed

Table 1: Parameters of the cubature rule with n=6.

Parameters	Values
η	1
λ	$\sqrt{2}$
ξ	0
μ	−1
γ	1
A	$0.0078125\,\pi^{n/2}$
B	$0.0625\,\pi^{n/2}$
C	$0.078125\,\pi^{n/2}$

cubature rule (19) is $2 + 2C_n^1 + 2C_n^2 = n^2 + n + 2$, which is less than those of the aforementioned two filters and only one more than the theoretical lower bound of the fifth-degree rules, that is, $n^2 + n + 1$ [29].

Remark 2. The cubature rule (19) can be applied in the condition that $2 \leq n \leq 7$, and for the conventional two-dimensional maneuvering target tracking problem, the maximum state dimension is seven (including two position variables, two velocity variables, two acceleration variables, and an optional turn rate); hence, the proposed cubature rule is suitable in the application of maneuvering target tracking.

Based on the cubature rule and the points and weights, the novel fifth-degree cubature Kalman filter is proposed in the nonlinear Kalman framework as follows.

Step 1 (filter initialization). Take the initial values $\hat{\mathbf{x}}_0^+$ and $\mathbf{P}_0^+$ into consideration.

Cycle $k = 1, 2, \cdots$, and complete the following steps.

Step 2 (time update). The posterior state estimate $\hat{\mathbf{x}}_{k-1}^+$ and estimate error covariance $\mathbf{P}_{k-1}^+$ are used instead of $\hat{\mathbf{x}}$ and $\mathbf{P}_x$ in (20) to calculate the cubature points $\hat{\mathbf{x}}_{k-1}^{(i)}$, which are propagated using the nonlinear process function $\mathbf{f}(\cdot)$ to obtain the following points $\mathbf{X}_k^{(i)}$:

$$\mathbf{X}_k^{(i)} = \mathbf{f}\left(\hat{\mathbf{x}}_{k-1}^{(i)}\right) \tag{22}$$

The prior state estimate and the estimate error covariance are calculated below:

$$\hat{\mathbf{x}}_k^- = \sum_{i=1}^{n^2+n+2} \omega_i \mathbf{X}_k^{(i)} \tag{23}$$

$$\mathbf{P}_k^- = \sum_{i=1}^{n^2+n+2} \omega_i \left(\mathbf{X}_k^{(i)} - \hat{\mathbf{x}}_k^-\right)\left(\mathbf{X}_k^{(i)} - \hat{\mathbf{x}}_k^-\right)^{\mathrm{T}} + \mathbf{Q}_{k-1} \tag{24}$$

where ω_i is given in (21).

Step 3 (measurement update). The prior state estimate $\hat{\mathbf{x}}_k^-$ and estimate error covariance $\mathbf{P}_k^-$ are used instead of $\hat{\mathbf{x}}$ and $\mathbf{P}_x$ in (20) to calculate the cubature points $\hat{\mathbf{x}}_k^{(i)}$, which are propagated using the nonlinear measurement function $\mathbf{h}(\cdot)$ to obtain the following points $\mathbf{Z}_k^{(i)}$:

$$\mathbf{Z}_k^{(i)} = \mathbf{h}\left(\hat{\mathbf{x}}_k^{(i)}\right) \tag{25}$$

The predicted measurement $\hat{\mathbf{z}}_k$, the predicted measurement covariance $\mathbf{P}_{z,k}$ and the cross-covariance $\mathbf{P}_{xz,k}$ are calculated as follows, respectively:

$$\hat{\mathbf{z}}_k = \sum_{i=1}^{n^2+n+2} \omega_i \mathbf{Z}_k^{(i)} \tag{26}$$

$$\mathbf{P}_{z,k} = \sum_{i=1}^{n^2+n+2} \omega_i \left(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k\right)\left(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k\right)^{\mathrm{T}} + \mathbf{R}_k \tag{27}$$

$$\mathbf{P}_{xz,k} = \sum_{i=1}^{n^2+n+2} \omega_i \left(\hat{\mathbf{x}}_k^{(i)} - \hat{\mathbf{x}}_k^-\right)\left(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k\right)^{\mathrm{T}} \tag{28}$$

The Kalman filtering gain $\mathbf{K}_k$, the posterior state estimate $\hat{\mathbf{x}}_k^+$, and the posterior estimate error covariance $\mathbf{P}_k^+$ are calculated, respectively.

$$\mathbf{K}_k = \mathbf{P}_{xz,k}\mathbf{P}_{z,k}^{-1} \tag{29}$$

$$\hat{\mathbf{x}}_k^+ = \hat{\mathbf{x}}_k^- + \mathbf{K}_k\left(\mathbf{z}_k - \hat{\mathbf{z}}_k\right) \tag{30}$$

$$\mathbf{P}_k^+ = \mathbf{P}_k^- - \mathbf{K}_k\mathbf{P}_{z,k}\mathbf{K}_k^{\mathrm{T}} \tag{31}$$

3. A Novel Fifth-Degree Strong Tracking Cubature Kalman Filter

The maneuvering target tracking problem can be regarded as a mutation state tracking problem. It can be seen from (30) that once the state has an abrupt change, the residual $\boldsymbol{\varepsilon}_k = \mathbf{z}_k - \hat{\mathbf{z}}_k$ may increase thereupon, and if $\mathbf{K}_k$ remains minimum as it tends to be as the filter is stable, the proposed fifth-degree cubature Kalman filter may lose the ability to track the mutation state. Therefore, we should adjust $\mathbf{K}_k$ online to satisfy the following two criterions:

$$(a)\quad E\left[\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^+\right)\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^+\right)^{\mathrm{T}}\right] = \min \tag{32}$$

$$(b)\quad E\left(\boldsymbol{\varepsilon}_{k+j}\boldsymbol{\varepsilon}_k^{\mathrm{T}}\right) = 0, \quad j = 1, 2, \cdots \tag{33}$$

The first criterion ensures the optimal filter, and the second criterion, which is called the orthogonality principle [20], plays a key role in tracking the mutation state. It has been proved that the residual series in Kalman filter are mutually orthogonal, which can be regarded as metrics to evaluate the performance of the filter. If we adjust $\mathbf{K}_k$ online to force the criterion (b) established once the state takes an abrupt change, the filter has the ability to track the mutation state then and is named strong tracking filter.

It has been proved that (33) has the following expression:

$$\begin{aligned} E&\left(\boldsymbol{\varepsilon}_{k+j}\boldsymbol{\varepsilon}_k^{\mathrm{T}}\right) \\ &= \mathbf{H}_{k+j}\mathbf{F}_{k+j}\left[\prod_{i=k+1}^{k+j-1}\left(\mathbf{I} - \mathbf{K}_i\mathbf{H}_i\right)\mathbf{F}_i\right]\left(\mathbf{P}_k^-\mathbf{H}_k^{\mathrm{T}} - \mathbf{K}_k\mathbf{V}_k\right) \end{aligned} \tag{34}$$

where $\mathbf{F}_k = \partial \mathbf{f}/\partial \mathbf{x}|_{\mathbf{x}=\hat{\mathbf{x}}_{k-1}^{+}}$ and $\mathbf{H}_k = \partial \mathbf{h}/\partial \mathbf{x}|_{\mathbf{x}=\hat{\mathbf{x}}_k^{-}}$ denote the Jacobian matrix and $\mathbf{V}_k = E(\boldsymbol{\varepsilon}_k \boldsymbol{\varepsilon}_k^{\mathrm{T}})$ represents the residual covariance.

It can be seen from (34) that, in order to force $E(\boldsymbol{\varepsilon}_{k+j}\boldsymbol{\varepsilon}_k^{\mathrm{T}}) = 0$, the following must hold:

$$\mathbf{P}_k^{-}\mathbf{H}_k^{\mathrm{T}} - \mathbf{K}_k \mathbf{V}_k = 0 \tag{35}$$

By using the statistical linear regression method, the following can be achieved:

$$\begin{aligned}\mathbf{P}_{z,k} &= E\left[\left(\mathbf{z}_k - \hat{\mathbf{z}}_k\right)\left(\mathbf{z}_k - \hat{\mathbf{z}}_k\right)^{\mathrm{T}}\right] \\ &= E\left[\left(\mathbf{H}_k\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right) + \mathbf{v}_k\right)\left(\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right)^{\mathrm{T}}\mathbf{H}_k^{\mathrm{T}} + \mathbf{v}_k^{\mathrm{T}}\right)\right] \\ &= \mathbf{H}_k E\left[\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right)\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right)^{\mathrm{T}}\right]\mathbf{H}_k^{\mathrm{T}} + E\left(\mathbf{v}_k\mathbf{v}_k^{\mathrm{T}}\right) \\ &= \mathbf{H}_k\mathbf{P}_k^{-}\mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k\end{aligned} \tag{36}$$

$$\begin{aligned}\mathbf{P}_{xz,k} &= E\left[\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right)\left(\mathbf{z}_k - \hat{\mathbf{z}}_k\right)^{\mathrm{T}}\right] \\ &= E\left[\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right)\left(\mathbf{H}_k\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right) + \mathbf{v}_k\right)^{\mathrm{T}}\right] \\ &= E\left[\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right)\left(\mathbf{x}_k - \hat{\mathbf{x}}_k^{-}\right)^{\mathrm{T}}\right]\mathbf{H}_k^{\mathrm{T}} = \mathbf{P}_k^{-}\mathbf{H}_k^{\mathrm{T}}\end{aligned} \tag{37}$$

Combined with (37), (35) is transformed into the following form:

$$\mathbf{P}_{xz,k} - \mathbf{K}_k\mathbf{V}_k = 0 \tag{38}$$

Then, by substituting (29) into (38), we have

$$\mathbf{P}_{xz,k} - \mathbf{K}_k\mathbf{V}_k = \mathbf{K}_k\mathbf{P}_{z,k} - \mathbf{K}_k\mathbf{V}_k = \mathbf{K}_k\left(\mathbf{P}_{z,k} - \mathbf{V}_k\right) = 0 \tag{39}$$

On account of $\mathbf{K}_k$ which is a nonzero matrix, (39) turns into

$$\mathbf{P}_{z,k} - \mathbf{V}_k = 0 \tag{40}$$

In order to adjust $\mathbf{K}_k$ online, the time-varying suboptimal fading factor is introduced into the prior estimate error covariance and (24) is modified as follows:

$$\mathbf{P}_k^{-} = \lambda_k\left[\sum_{i=1}^{n^2+n+2}\omega_i\left(\mathbf{X}_k^{(i)} - \hat{\mathbf{x}}_k^{-}\right)\left(\mathbf{X}_k^{(i)} - \hat{\mathbf{x}}_k^{-}\right)^{\mathrm{T}} + \mathbf{Q}_{k-1}\right] \tag{41}$$

where $\lambda_k \geq 1$ is the time-varying suboptimal fading factor.

Similarly, $\mathbf{P}_{z,k}$ and $\mathbf{P}_{xz,k}$ are modified according to (36) and (37) as below:

$$\mathbf{P}_{z,k} = \lambda_k\sum_{i=1}^{n^2+n+2}\omega_i\left(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k\right)\left(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k\right)^{\mathrm{T}} + \mathbf{R}_k \tag{42}$$

$$\mathbf{P}_{xz,k} = \lambda_k\sum_{i=1}^{n^2+n+2}\omega_i\left(\hat{\mathbf{x}}_k^{(i)} - \hat{\mathbf{x}}_k^{-}\right)\left(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k\right)^{\mathrm{T}} \tag{43}$$

By substituting (42) into (40), we obtain that

$$\lambda_k\sum_{i=1}^{n^2+n+2}\omega_i\left(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k\right)\left(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k\right)^{\mathrm{T}} + \mathbf{R}_k - \mathbf{V}_k = 0 \tag{44}$$

Define $\mathbf{M}_k = \sum_{i=1}^{n^2+n+2}\omega_i(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k)(\mathbf{Z}_k^{(i)} - \hat{\mathbf{z}}_k)^{\mathrm{T}}$ and $\mathbf{N}_k = \mathbf{V}_k - \mathbf{R}_k$, and (44) turns into

$$\lambda_k\mathbf{M}_k = \mathbf{N}_k \tag{45}$$

By calculating the trace of both sides of (45), we achieve the following suboptimal fading factor:

$$\begin{aligned}\lambda_k &= \begin{cases}\lambda_0, & \lambda_0 \geq 1 \\ 1, & \lambda_0 < 1,\end{cases} \\ \lambda_0 &= \frac{\operatorname{tr}\left(\mathbf{N}_k\right)}{\operatorname{tr}\left(\mathbf{M}_k\right)}\end{aligned} \tag{46}$$

The residual covariance $\mathbf{V}_k = E(\boldsymbol{\varepsilon}_k\boldsymbol{\varepsilon}_k^{\mathrm{T}})$ can be estimated as follows:

$$\mathbf{V}_k = \begin{cases}\boldsymbol{\varepsilon}_1\boldsymbol{\varepsilon}_1^{\mathrm{T}}, & k = 1 \\ \dfrac{\rho\mathbf{V}_{k-1} + \boldsymbol{\varepsilon}_k\boldsymbol{\varepsilon}_k^{\mathrm{T}}}{1+\rho}, & k \geq 2\end{cases} \tag{47}$$

where ρ denotes the forgetting factor and is generally set to be $0.95 \leq \rho \leq 0.99$.

In practical applications, $\mathbf{N}_k$ is often modified as $\mathbf{N}_k = \mathbf{V}_k - \beta\mathbf{R}_k$, where $\beta \geq 1$ denotes the softening factor to smooth the state estimate.

Thus, the novel fifth-degree strong tracking cubature Kalman filter is derived by substituting (42), (43) instead of (27), and (28) and (41) into (31), and the calculation process is listed in Figure 1.

Remark 3. In the previous nonlinear strong tracking filters, $\mathbf{P}_{z,k}$ and $\mathbf{P}_{xz,k}$ are contained in the calculation of λ_k [26, 27] and results in the cubature points being calculated three times in a filtering cycle, which may reduce the filtering accuracy due to the loss of the higher order moment information. However, in the proposed 5-STCKF, $\mathbf{M}_k$ and $\mathbf{N}_k$ are calculated using a different method and the cubature points are calculated only twice in a filtering cycle as the conventional CKF, which may achieve more accuracy estimate and better computational efficiency.

Remark 4. There is no need to calculate the Jacobian matrix; thus, the proposed 5-STCKF is a derivative-free filter.

4. Simulation Results and Analysis

In this section, a maneuvering target tracking simulation is taken to test the performance of the proposed filter

4.1. Maneuvering Target Tracking Models. The dynamic model of the two-dimensional maneuvering target is given below:

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \boldsymbol{\Gamma}\mathbf{u}_{k-1} + \mathbf{G}\mathbf{w}_{k-1} \tag{48}$$

where $\mathbf{x}_k = (x_k, \dot{x}_k, y_k, \dot{y}_k)^{\mathrm{T}}$ denotes the target state, $\mathbf{u}_k = (\ddot{x}_k, \ddot{y}_k)^{\mathrm{T}}$ represents the control input, and $\mathbf{w}_{k-1}$ is the zero mean Gaussian process noise. $\mathbf{F}$, $\boldsymbol{\Gamma}$, and $\mathbf{G}$ denote the state

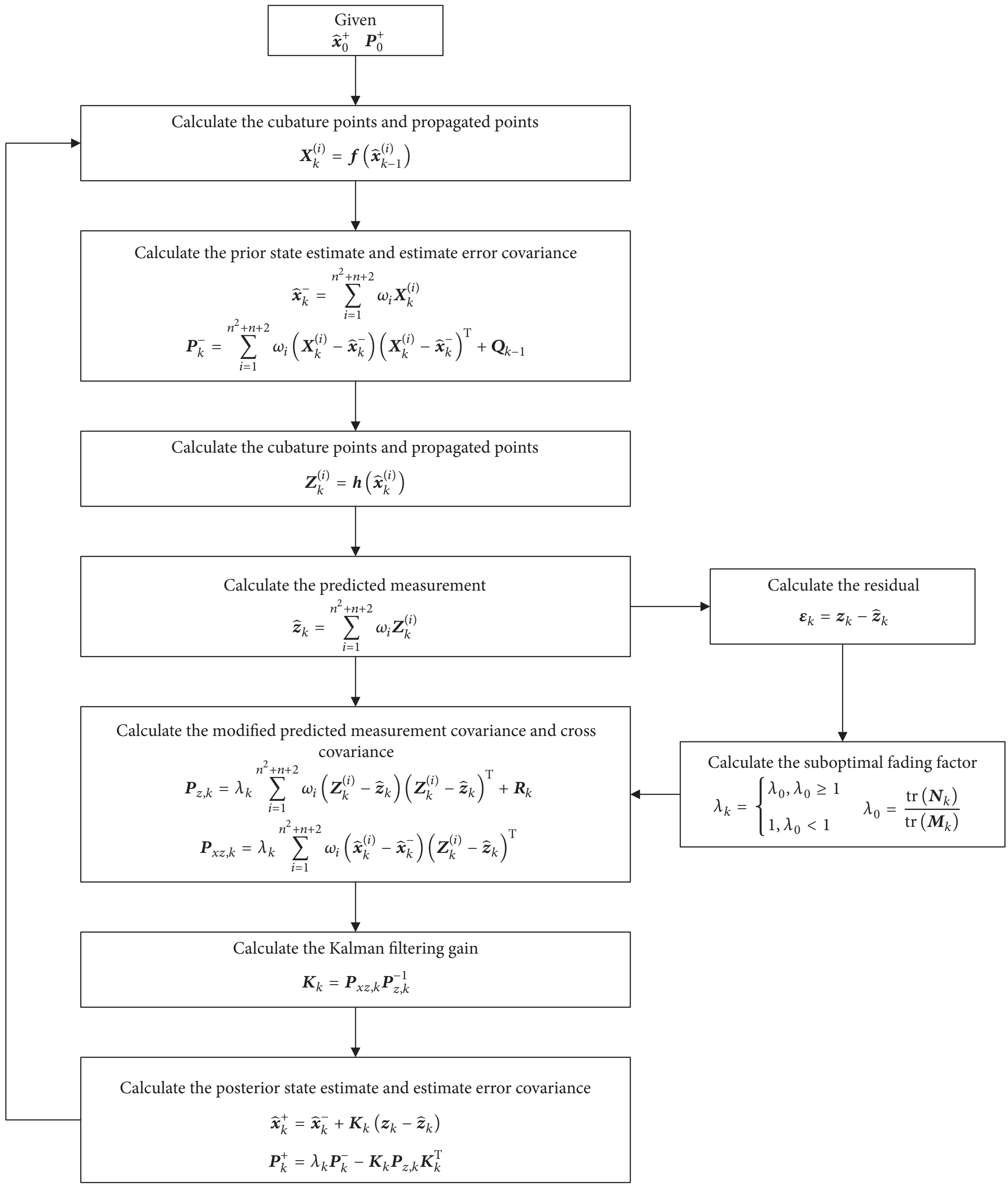

FIGURE 1: The calculation process of the proposed 5-STCKF.

transformation matrix, the control input matrix, and the noise input matrix, respectively, which are given below:

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{\Gamma} = \mathbf{G} = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix} \tag{49}$$

where T is the time interval.

For the tracking system, the target state and the control input are unknown; in this case, the target state and the control input can be used to form the augmented state vector $\mathbf{x}_k^a = (\mathbf{x}_k^{\mathrm{T}}, \mathbf{u}_k^{\mathrm{T}})^{\mathrm{T}}$, and (48) is modified as follows:

$$\mathbf{x}_k^a = \mathbf{F}^a \mathbf{x}_{k-1}^a + \mathbf{G}^a \mathbf{w}_{k-1} \tag{50}$$

where $\mathbf{F}^a$ and $\mathbf{G}^a$ denote the augmented state transformation matrix and noise input matrix, respectively, which are given below:

$$\mathbf{F}^a = \begin{bmatrix} \mathbf{F} & \mathbf{\Gamma} \\ \mathbf{0}_{2\times 4} & \mathbf{I}_2 \end{bmatrix}, \qquad \mathbf{G}^a = \begin{bmatrix} \mathbf{G} \\ \mathbf{0}_{2\times 2} \end{bmatrix} \tag{51}$$

The measurement model is given as follows:

$$\mathbf{z}_k = \begin{bmatrix} \sqrt{x_k^2 + y_k^2} \\ \operatorname{atan} 2\left(y_k, x_k\right) \end{bmatrix} + \mathbf{v}_k \tag{52}$$

where atan 2 denotes the four-quadrant inverse tangent function and $\mathbf{v}_k$ is the zero mean white Gaussian measurement noise.

4.2. Simulation Results and Analysis. In this simulation, the initial location of the target is $(x_0, y_0) = (100, 400)$ and the initial velocity of the target is $(\dot{x}_0, \dot{y}_0) = (15, 20)$. The location of the radar is $(100, 0)$. The total simulation time is 400s, and the target takes a high maneuvering with the acceleration given below. The trajectory of the target and the location of the radar is shown in Figure 2.

$$\mathbf{u}_k = \begin{cases} (0,0)^{\mathrm{T}}, & 0 < k \le 40 \\ (6,-8)^{\mathrm{T}}, & 40 < k \le 120 \\ (-3,7)^{\mathrm{T}}, & 120 < k \le 200 \\ (5,2)^{\mathrm{T}}, & 200 < k \le 400 \end{cases} \tag{53}$$

The initial filtering state is $\hat{\mathbf{x}}_0^+ = (100, 15, 400, 20, 0, 0)^{\mathrm{T}}$, and the initial covariance is $\mathbf{P}_0^+ = \operatorname{diag}(2500, 400, 2500, 100, 10, 10)$. The process noise covariance is $\mathbf{Q}_k = 0.1 \times \operatorname{diag}(1, 1)$, and the standard deviations of range measurement noise and bearing measurement noise are 25m and 0.02°, respectively. The forgetting factor is $\rho = 0.98$, and the softening factor is set to be $\beta = 6$.

The CKF, the proposed 5-CKF, the STCKF in [26] (denoted as STCKF-1), the STCKF (CKF combined with the new strong tracking filter structure in this paper), and the proposed 5-STCKF are taken into account in this simulation. The metrics used to compare the performance of various filters are the root mean square error (RMSE) and average RMSE (ARMSE), which are defined as follows:

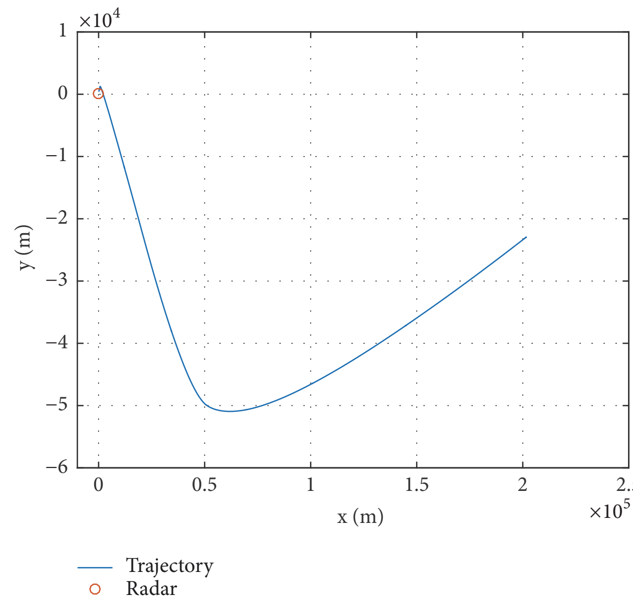

FIGURE 2: The trajectory of the target and the location of the radar.

$$RMSE_{pos}(k) = \sqrt{\frac{1}{M} \sum_{m=1}^{M} \left(\left(x_k - \hat{x}_{m,k}^+\right)^2 + \left(y_k - \hat{y}_{m,k}^+\right)^2 \right)} \tag{54}$$

$$ARMSE_{pos} = \frac{1}{N} \sum_{k=1}^{N} RMSE_{pos}(k) \tag{55}$$

where M denotes the number of Monte-Carlo runs, N represents the total simulation times, x_k and y_k denote the true position at time k, and $\hat{x}_{m,k}^+$ and $\hat{y}_{m,k}^+$ denote the estimated position at time k in the mth Monte-Carlo simulation. The velocity RMSE and velocity ARMSE are defined similarly.

The Monte-Carlo simulations are implemented 200 times, and the results are shown in Figures 3–6. It can be seen from Figures 3 and 4 that the position RMSE and velocity RMSE of all the filters have a sudden jump at 40s, 120s, and 200s on account of the maneuverings taken. However, the RMSEs of the three strong tracking filters achieve convergence after a short time, while those of the CKFs achieve divergence, indicating that the three strong tracking filters have the ability to track the maneuvering target, while the CKF and 5-CKF cannot. The reason is that the suboptimal fading factor in the strong tracking filters can adjust the filtering gain matrix in real time to force the residual sequences mutually orthogonal, thus to enhance the ability of the filter to track the mutation state, while no fading factor exists in the CKF and 5-CKF. From Figures 5 and 6, we see that the three strong tracking filters can estimate the accelerations effectively, while CKF and 5-CKF cannot achieve the estimations of the accelerations.

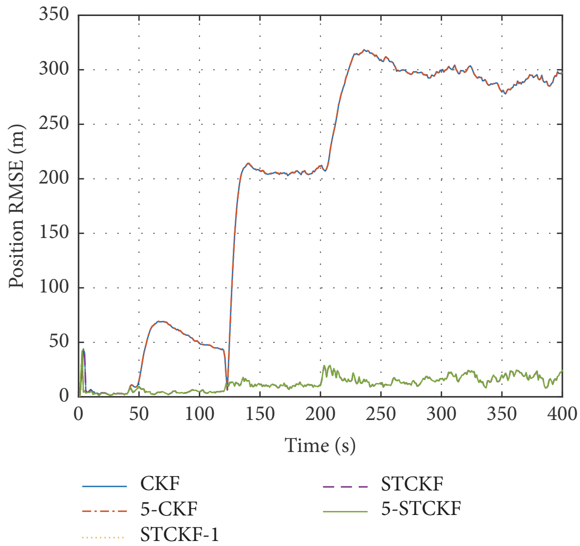

Figure 3: Position RMSEs of the five filters.

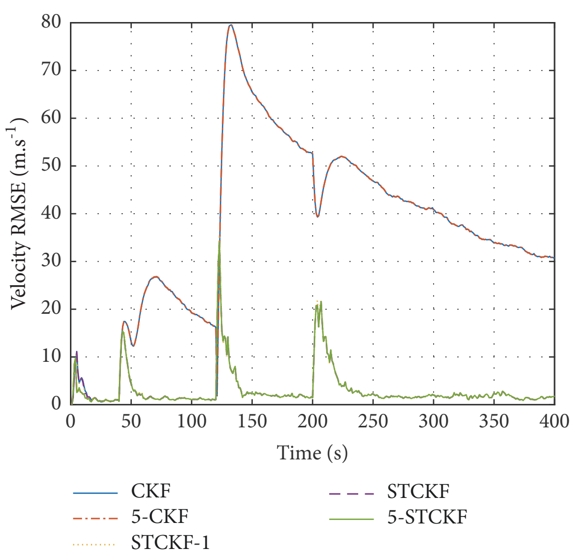

Figure 4: Velocity RMSEs of the five filters.

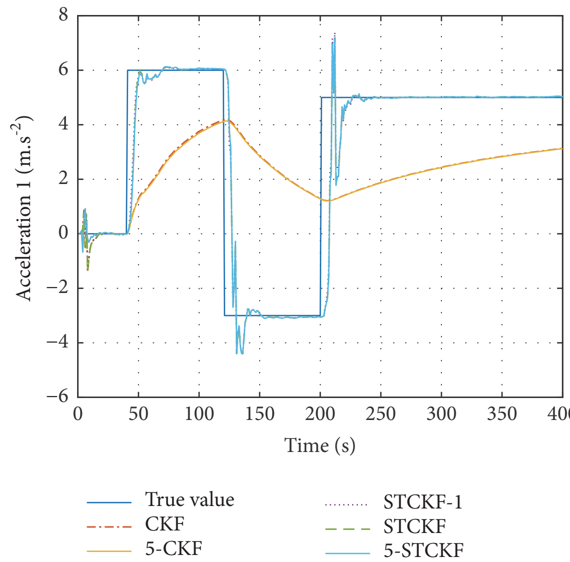

Figure 5: Estimated acceleration 1 of the five filters.

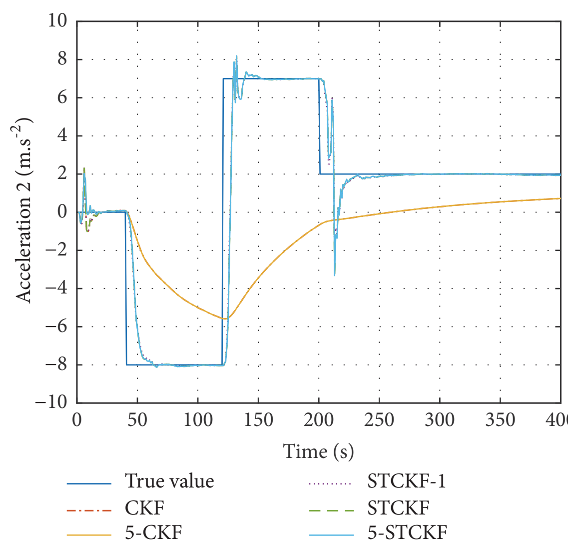

Figure 6: Estimated acceleration 2 of the five filters.

Table 2: Position ARMSEs and velocity ARMSEs of the five filters.

Filters	Position ARMSE/m	Velocity ARMSE/(m.s^{-1})
CKF	194.2629	36.4900
5-CKF	194.2501	36.4539
STCKF-1	12.1252	3.1578
STCKF	12.0985	3.1417
5-STCKF	11.9466	3.0843

The position ARMSE and velocity ARMSE are listed in Table 2. As shown in the figures, the RMSEs of the CKF and 5-CKF are significantly larger than those of the other three strong tracking filters. The RMSEs of STCKF are smaller than that of STCKF-1, indicating that the new strong tracking filter structure proposed in this paper is better than that in [26]; the reason is that the proposed structure reduces the calculation for cubature points from three times to twice, which reduces the loss of higher order moment information. Compared with STCKF, the 5-STCKF improves the position ARMSE and velocity ARMSE by 1.26% and 1.83%, respectively, indicating that the novel fifth-degree cubature rule is more accurate in approximating the Gaussian weighted integral than conventional third-degree cubature rule.

5. Conclusion

In this paper, a novel 5-STCKF is put forward to improve the two-dimensional maneuvering target tracking accuracy. For this, the intractable nonlinear Gaussian weighted integral is approximated using a novel fifth-degree cubature rule, and the suboptimal fading factor is designed in a new method to adjust the filtering gain matrix online and force the residual sequences mutually orthogonal, thus to improve the ability of the filter to track the mutation state. Simulation results show that the conventional CKF cannot track the maneuvering target, while the 5-STCKF has the ability to track the maneuvering target, and compared with STCKF, 5-STCKF can achieve higher target tracking accuracy.

Data Availability

The simulation results data used to support the method of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

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