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Research Article

Integrating Dynamic Pricing and Inventory Control for Fresh Agriproduct under Multinomial Logit Choice

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In this article, we investigate a joint pricing and inventory problem for a retailer selling fresh agriproducts (FAPs) with two-period shelf lifetime in a dynamic stochastic setting, where new and old FAPs are on sale simultaneously. At the beginning of each period the retailer makes ordering decision for new FAP and sets regular and discount price for new and old inventories, respectively. After demand realization, the expired leftover is disposed and unexpired inventory is carried to the next period, continuing selling. Unmet demand of all FAPs is backordered. The objective is to maximize the total expected discount profit over the whole planning horizon. We present a price-dependent, stochastic dynamic programming model taking into account zero lead time, linear ordering costs, inventory holding, and backlogging costs, as well as disposal cost. Considering the influence of the perishability, we integrate a Multinomial Logit (MNL) choice model to describe the consumer behavior on purchasing fresh or nonfresh product. By way of the inverse of the price vector, the original formulation can be transferred to be jointly concave and tractable. Finally we characterize the optimal policy and develop effective methods to solve the problem and conduct a simple numerical illustration.

1. Introduction

Motivated by local fresh agriproducts (FAPs) store retailing issues in its operations management, this paper investigates a revenue problem in a stochastic setting where a retailer considers a jointly pricing and inventory control issue under consumer choice. In retailing practice, FAPs, such as fresh fruits or vegetables, are usually delivered to the store or market every day in the morning based on the retailer's order, submitted in the previous evening, and to be selling at a regular price in the following one or two more days. As the perishability nature FAPs are always deteriorating along time going, gradually being perceived nonfresh and less attractive to customers, which weakens the demand and results in the loss of revenue. To reduce the amount of mismatch between supply and demand, the retailer always sets a discount price for the nonfresh FAPs at a certain time point, which otherwise are deposed if they remain unsold at the end of selling. In the meantime the store shelf is replenished with a new batch of FAPs, ordered by the seller and charged a regular price on selling simultaneously with those unsold nonfresh products. While the regular price and discount price are determined centrally on a long-term basis, some fundamental decisions are at the store manager's discretion in such an environment, including how much new FAPs to order and what pricing policies to follow for selling. It is not a trivial problem for the seller to sell deteriorated items on discount paralleled with fresh products with regular price. As every coin has two sides, on one hand, the price cutting sales can generate some revenue for the otherwise disposed items since offering a price discount on those items can induce purchasing, while, on the other hand, in codisplay selling simultaneously mode, discount policy results in competition among consumers to purchase either fresh or nonfresh products on the basis of its utility from combination of quality and price. Some of the consumers who can afford the full price may be attracted by the price discount to buy nonfresh products, which has a negative impact on the revenue from selling at the full price.

In some form or other, jointly pricing and inventory control problem arises in a lot of instances. For example, Stater Bros, a well-known grocery store in America, always sells nonfresh vegetables or meat at a discount along with fresh ones and disposes those unsold items at the end of their shelf lifetime. That is, the Stater Brothers store's manager should make optimal order policy for fresh vegetables or meat and charge for both fresh and nonfresh products differently every day so that the inventory and demand can be extremely matched. In essence, these examples exhibit one homogeneous competition issue which is an important concern and common in retailing practice. Meanwhile, pricing is also a double-edged sword in that pricing can not only influence the demand but also have great impact on the sellers' profit, especially a poor pricing policy, which can easily damage the seller's benefit. For example, Tesco, the world's third-biggest supermarket chain, failed to revive its sales performance despite spending £500 million on price cuts in Eurozone (http://www.ft.com/intl/cms/s/0/dee4d20c-2172-11e1-a1d8-00144feabdc0.html#axzz3Iua2R9jr). On the contrary, Stater Bros. Markets reported its modest profit gains in appropriate price strategy in fiscal 2011 despite higher commodity costs and competitive pressures (http://www.pe.com/articles/million-634066-store-year.html). Therefore, instead of simple pricing of changing prices dramatically, sellers should be more prudent to adjust the price in a dynamic fashion, which enable price change in line with the availability of inventory and products residual shelf-life time. Nevertheless, as consumers may pick any product available and selects his preferred choice based on the utility of an appealing quality-price combination due to the codisplaying mode on sale, managers need to carefully take account of consumer behavior when coordinating discount sales with inventory ordering decisions.

In this study, we formulate a novel model in which the retailer makes simultaneous decisions dynamically on charging appropriate price and optimal inventory policy for perishable products, in the presence of homogeneously selling cannibalization and demand uncertainty. The retail time of any batch of FAPs is divided into two periods, a regular sale period and a discount pricing period, which implies that FAPs have two periods of shelf lifetime. At every period the retailer starts with ordering new FAPs and charging price policy for two different ages of FAPs which compete among customers in their attributes of quality and price and ends with disposing all unsold leftovers with zero shelf lifetime and carrying the leftover inventory of unexpired products to the next period for continuing selling. Generally, new FAPs are perceived to have a higher quality than the old ones. The utility for customers is mainly determined by quality-price combination. Demand in each period is random and depends on the current price, which takes a form of price function plus an additive random perturbation. The seller's objective is to maximize the total expected discount profit over the whole planning horizon. We formulate a stochastic, price-dependent demand, dynamic programming model taking into account zero lead time, linear ordering costs, inventory holding, and backlogging costs. Our model assumes a population of homogeneous (statistically identical and independent) customers making mutually exclusive choices from two different ages of FAPs. Meanwhile, we integrate a Multinomial Logit Choice model to describe dynamic consumer behavior on choosing new or old products available in inventory and prices adjustment. The retailer, who should consider both the influence of perishable nature of fresh agricultural products and the homogeneous retailing cannibalization, integrates inventory control and dynamic pricing policy to maximize his profit on the whole planning horizon. To the best of our knowledge, this is the first stochastic and dynamic inventory model on perishable fresh agricultural products, taking account of consumer behavior with joint ordering and discount decisions. The problem is extremely complicated even if the products are not perishable. Unlike standard inventory models of general products with same attributes, the perishable property plays an important role in driving the dynamic pricing adjustments which affects the demand share of new and old FAPs. Traditional dynamic inventory models cannot be suitable to our problem in that even the single period profit function is neither jointly convex nor concave in the decision variables, therefore intractable. In addition, the homogeneous competition results in interdependence across the planning horizon, which leads to complicated dynamics in how inventory is controlled over from one period to another. To deal with the complexity, we work with the transformation technique and exploiting specific properties by way of the inverse of the price vector. We also conduct numerical studies to further characterize the optimal policy.

The remainder of this paper is organized as follows. In the next section, we provide a literature review and introduce the model and formulate the problems a dynamic program in Section 3. In Sections 4 and 5, we perform the structural analysis and present an optimal pricing policy coordinated with inventory control strategies, respectively. A numerical study is conducted to show the policy efficiency in Section 6. We conclude our work in Section 7.

2. Literature Review

Our research is mostly related to jointly pricing and inventory control on perishable products, including three streams of literatures: (1) dynamic inventory control for perishable products, (2) combined dynamic pricing and inventory control, and (3) customer behavior in the context of dynamic pricing.

Firstly, our work is closely related to dynamic inventory control for perishables. As far as we know, earliest work dates back to Ghare & Schrader [1] who first considered inventory control of exponential decay product and generalized an EOQ model by assuming the lifetime of each product is exponentially distributed. Other forms of random lifetime models can be referred to Weibull distributed deterioration [2], Gamma distributed decay [3], generalized exponential decay [4], etc. These works mainly focused on static inventory control policies, instead of dynamic strategies. In retailing, the perishable product lifetime was always known deterministically. Fixed lifetime models are also concerned and can be traced to Nahmias and Pierskalls [5] who first explored the problem in a two-period lifetime setting with zero lead time and demand uncertainty. On the basis of their research framework, Nahmias [6] and Fries [7] extended the work on the case with multiperiod lifetime in periodic-review system, where only the excess inventory, expired at the end of the current period, is disposed of. They probed the characteristic of optimal policy and pointed out that the optimal order quantity was a decreasing function of on-hand inventory of different ages. And a series of papers thereafter focused on the similar topic, see Nahmias [8], Cohen [9], Nandakumar and Morton [10], Liu and Lian [11], Lian and Liu [12], Gurler and Ozkaya [13], Berk and Gurler [14], etc. Note that a suitable model might vary even for the same problem, analysis of which was lengthy and difficult to be generalized. Due to the fact that complexity, the literature thenceforward concentrates more on developing heuristics except for model analysis. Nahmias [15, 16] and Karaesmen et al. [17] have reviewed the early and lately developments. Recently, Xue et al. [18], Li et al. 2013, and Li and Yu [19] studied a perishable inventory model in a secondary market setting where the excess inventory can be cleared with certain salvage value. They provided some structure properties and then brought about an effective heuristic. Recently, Chao et al. [20], Zhang et al. [21], and Chao et al. [22] developed approximation algorithms for perishable inventory systems. Our study generalizes this literature to allow for consumer choice on two different ages of FAPs in periodic-review system. And we do focus on exploring optimal policy instead of effective algorithms.

Secondly, in terms of jointly pricing and inventory management, to the best of our knowledge, a mountain of work contributes to model on nonperishable products in recent decades, under the environment of from deterministic periodic-review inventory and pricing models to stochastic models with different cost structures, distinguishing between single period and multiperiod. See, for example, Wagner and Whitin [23], Kunreuther and Schrage [24], Zabel [25], Thomas [26], Gilbert [27], Petruzzi and Dada [28], Federgruen and Heching [29], Feng and Chen [30], Geunes et al. [31], Chen and Simchi-Levi [32, 33], Raz and Porteus [34], Huh and Janakiraman [35], Chen et al. ([36], 2010), Song et al. [37], and Simchi-Levi [38]. Among them, there are some recent surveys on different operations research and management science perspective, which refer to Eliashberg and Steinberg [39], Elmaghraby and Keskinocak [40], Yano and Gilbert [41] and Chan et al. [42], and Chen and Simchi-Levi [43]. This substantial body of literature illustrates that the retailer can benefit great from the structures of optimal policies and effective algorithms of integrated pricing and inventory strategies. Dynamic pricing, supposed to be an effective lever to manage the profitability of perishables retailing, was firstly investigated together with inventory control by Rajan et al. [4] who studied dynamic pricing and ordering decisions for a certain product that experiences a general exponential decaying under deterministic demand, whereafter Abad [44] generalized this work by allowing for partially backlogged demand with the same objective of maximizing the long-run average profit. As Nahmias [15] pointed out, developing pricing policies for perishable items is very important in a demand uncertainty context. There are a few studies pertaining to the case on perishables selling. Among them, Ferguson and Koenigsberg [45] considered a joint pricing and inventory control problem in a two-period setting, addressing the competition impact between new and old products inventory. They presented a stylized model and concentrated on deriving managerial insights. With the assumption of inventory depleted in a first-in-first-out (FIFO) sequence, Li et al. [46] explored a dynamic joint pricing and inventory control for a two-period lifetime perishable product over an infinite horizon, taking into account linear price-response demand, backlogging, and zero lead time. And they also extended the problem to a stationary system with multiperiod lifetime and developed a base-stock/list-price (BSLP) heuristic policy. Then, Li et al. [47] continued analyzing the infinite-horizon lost-sales case where new and old inventory can not be simultaneously sold and sellers can decide whether to dispose of or carry all ending inventory until it expires at the end of each period. They proposed a stationary structural policy consisting of an inventory order-up-to level, state-dependent price, and inventory clearing decisions and developed a fractional programming algorithm to obtain the optimal policy among the class of proposed structural policies. In another relevant work, Sainathan [48] studied the two-period lifetime case where new and old inventory can also be selling simultaneously, and the seller makes decisions on price polices and how much to order at the beginning of each period. Recognizing the influence of perishability on demand, Sainathan modeled the consumer choice and characterized the structure of the optimal policy under different demand context, including deterministic demand, two-point demand, and substitution demand. Extending the analysis and results to the case with arbitrary period lifetime, Chen et al. [49] presented a significant generalization of these papers by allowing for positive lead time, both backlogging and lost-sales cases, and unrestricted ordering decisions. They developed the structural results of $L^{\#}$-concavity and generalize the regularity conditions of demand functions for lost-sales inventory-pricing models of setting a single price for inventories of different ages. Wu et al. [50] study a problem that the seller dynamically makes the joint pricing and inventory replenishment decisions over multiple periods, where each periods consist of two stages for ordering and markdown pricing, respectively. Chen & Chu [51] and Chen & Cong (2018) integrated a strategic customer's behavior into joint pricing and inventory model and derived an optimal joint pricing, delivery, and inventory policy. Recently, Peng Hu et al. (2016) formulated a model on a firm's dynamic inventory and markdown decisions for perishable goods where every period consists of regular sales phase and clearance phase under strategic consumer behavior. They showed that the seller should either put the leftover inventory on discount or dispose all of them, and the choice depends on the amount of leftover inventory from the previous period.

Finally, our work involves modeling of consumer choice behavior which is an important phenomenon within revenue management system. The earliest study in the literature related to this research is pioneered by Kincaid and Darling [52], who described the probability of the charging price, by which the expected demand can be calculated. This general model has been extended by other authors, Bitran and Mondschein [53], who concentrated on applications of the model to the sale of fashion goods. van Ryzin and Mahajan [54] firstly emploied the MNL model to describe the normal demand distribution for substitute products. Aydin and Ryan [55] used the MNL model to study pricing problem assuming the firm satisfies demand by make-to-order. Recent papers related stream of literature include Aydin and Porteus (2005), Zhang and Cooper [56], Xu and Hopp [57], Perakis and Sood [58], Lin and Sibdari [59], Song and Xue [60], and Martínez-de-Albéniz and Talluri [61]. In all those papers, customers can decide which product to buy, based on prices and inventory levels at time of purchase, with the MNL model for product categories, assuming homogeneous product groups. Our work is consistent with those characteristics above, demand uncertainty, two-period shelf lifetime, codisplaying selling, and homogeneous cannibalization. We use MNL model to describe consumer discrete choice and present a jointly dynamic pricing and inventory control policy to maximize seller's profit on the whole planning horizon.

In summary, our contribution to the literature is as follows. Firstly, we generalize the perishable joint pricing and inventory models for perishable products of two-period lifetime. After characterizing the structure of model, we present an optimal solution to the dynamic pricing of perishable products with consumers incurring dynamic substitutions throughout the selling season. Second, we provide sorts of insights on the behavior of the optimal dynamic prices and highlight the complex interplay of inventory scarcity and product quality difference. Thirdly, we generalize the regularity conditions of demand functions for integration of inventory-pricing models, using the concept of concavity, and characterize the optimal policy and develop effective methods.

3. The Problem Formulation and Model

3.1. Problem Formulation. Consider a periodic-review single type of FAP inventory system over a finite planning horizon of T periods. The product is perishable with two-period shelf lifetime, fresh in the first period and nonfresh in the next period. At the beginning of each period selling, the retailer reviews the current inventories and decides simultaneously on an order quantity for new FAP as well as charging prices for both new delivery and old inventory. During the sale, customers arrive at the store or market and make purchase choices based on FAPs attributes of quality and price, which causes a drop in the inventory level. The demand for the current inventory depends on the newly charged prices, and it is always met to the maximum extent with on-hand inventories. At the end of the period, the leftover inventory with one period lifetime is carried forward to next period, becoming old products to sell while the unsold products with zero shelf-life time have to be discarded. Then, the sale moves to the next period and the above processes repeat.

For convenience and tractability, we assume that the replenishment order, placed by the retailer, arrives immediately before the demand unfolds, that is to say, replenishment is instantaneous with zero lead time. Meanwhile, there could have chance to replenish fresh products during the sale from second market since in retailing practice backorder policy is prominent, which means unsatisfied demands of all products during selling are to be backlogged. And there is neither fixed ordering cost nor constraints on the supply capacity. Demand at each period is uncertain and independent across all periods, generally decreasing on current prices. In line with Petruzzi & Dada [28] and Chen & Simchi-Levi [32] we take a form of a function of prices charged at that period multiplies a stochastic variable plus a random disturbance. Nevertheless, customers are assumed to be homogeneous and decide purchasing based on current prices and product attributes alone, instead of acting strategically by adjusting their buying behavior in response to the firm's pricing policy. In the whole horizon planning, although the retailer gets revenue from selling new and old products, he bears some costs incurred in the operations, such as order decision for wholesale each order incurs a variable cost c for per new FAP. And three more other costs are incurred: leftover inventory carrying costs over from one period to the next, backlogged costs for unmet demand from on-hand inventory, and disposal costs of leftovers with zero shelf-life time. The objective is to dynamically determine ordering and pricing decisions in all periods so as to maximize the total expected discounted profit over the whole planning horizon.

3.2. Demand Model. During sales, the new and old products compete among customers by combination of their attributes and price. And each customer selects his preferred choice based on his utility from purchasing a unit of product, new or old, which is given by $U_i = \alpha\theta_i - p_i + \xi_i$ $(i = 1, 2)$, where i=1 for the fresh product and i=2 for the nonfresh product, respectively. In the formula, θ_i and p_i are defined as the average perceptive value and the charged price on product i, respectively, while ξ_i is a certain customer's deviation from θ_i, which is influenced by unobservable characteristics. The coefficient, α, is denoted as a customer's quality sensitivity. Note that, all consumers are statistically homogeneous and have the same sensitivity to price. Generally, any customer who visits the retailer has three choices: select one unit of new or old FAP, or do nothing. Here we set a virtual product and let i=3 with price zero for denoting no purchase choice. Let ξ_3 be a certain customer's deviation from θ_3. Obviously $\xi_3 = 0$ and $p_3 = 0$. Therefore, under the assumption of rationality, a customer will always purchase one product with the highest utility among these three options, that is, a customer chooses to buy product i, where $i = \arg\max_{k=1,2,3}\{U_k = \alpha\theta_k - p_k + \xi_k\}$. Suppose random variable ξ_i be i.i.d. distributed with a double exponential distribution, that is, $\Pr(\xi_i \le \delta) = e^{-e^{-\delta}}$, $\delta \in (-\infty, +\infty)$, with mean zero and variance ε_δ. See Guadagni and Little [62] and Anderson et al. [63]. Define $\beta_i(p_1, p_2)$ to be the probability that an arriving customer in a period selects fresh or nonfresh agriproduct in response to price (p_1, p_2). It can be shown that a consumer's choice probability has a simple form:

$$\beta_i(p_1, p_2) = \frac{\exp(\alpha\theta_i - p_i)}{1 + \sum_{k=1,2}\exp(\alpha\theta_k - p_k)}, \quad i = 1, 2, \quad (1)$$

which is an MNL model, often used to study a set of products differentiated in either quality or style, a term coined in the literature of MNL models (see [63]). Let (p_{1t}, p_{2t}) be the charged prices at any period $t \in \{1, 2, \ldots, T\}$. On the basis of analysis above, we denote $\beta_{it}(p_{1t}, p_{2t})$ the market share of product i at a certain period t, which is similar to equation (1) and expressed as follows:

$$\beta_{it}(p_{1t}, p_{2t}) = \frac{\exp(\alpha\theta_i - p_{it})}{1 + \sum_{k=1,2}\exp(\alpha\theta_k - p_{kt})}, \quad i = 1, 2 \tag{2}$$

Note that the total market share $\sum_{i=1,2}\beta_{it}(p_{1t}, p_{2t}) \le 1$.

Denote ω_t to be the amount of arriving customers to buy the products at period t, which is generally uncertain and independently distributed with finite mean of μ_t and variance τ_t across all time periods. Note that the expected demand of new or old product is proportional to the market share $\beta_{it}(p_{1t}, p_{2t})$. So, provided with price (p_{1t}, p_{2t}) as well as a demand disturbance on product i at period t, ε_{it}, we can write out the demand function of product i, denoted as D_{it} below, an additive form which is commonly used in the literature (see, e.g., [28, 32]).

$$D_{it} = \beta_{it}(p_{1t}, p_{2t})\,\omega_t + \varepsilon_{it}, \quad i = 1, 2, \tag{3}$$

where ε_{it} is assumed to be identically distributed over time with zero mean and finite variance as well as c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$. Denote $d_{it}(p_{1t}, p_{2t})$ as the expected demand level, then $d_{it}(p_{1t}, p_{2t}) = \beta_{it}(p_{1t}, p_{2t})E(\omega_t) + E(\varepsilon_{it}) = \beta_{it}(p_{1t}, p_{2t})\mu_t$. The selling price is generally restricted to an interval $[\underline{p}_t, \overline{p}_t]$ and $p_{1t} \ge p_{2t}$.

Note that the monotonicity of the expected demand function implies a one-to-one correspondence between the selling price and the expected demand. And we observe that if we use the inverse of the market share function to express the price variables as a function of the market share, then the revenue function becomes jointly concave under some certain condition, allowing us to analyze the structure of model. Accordingly, we have the following.

Proposition 1. *Under the MNL market share model, the price function at period t is*

$$p_{it} = P_{it}(\beta_{1t}, \beta_{2t}) = \alpha\theta_i + \ln\left(\frac{(1 - \sum_i \beta_{it})}{\beta_{it}}\right), \quad i = 1, 2. \tag{4}$$

The corresponding market share space $\Omega_t = \{(\beta_{1t}, \beta_{2t}) : \underline{p}_t \le P_{2t}(\beta_{1t}, \beta_{2t}) \le P_{1t}(\beta_{1t}, \beta_{2t}) \le \overline{p}_t,\ i = 1, 2\}$ *can be expressed as*

$$\Omega_t = \left\{ (\beta_{1t}, \beta_{2t}) : \begin{array}{c} \dfrac{\exp(\alpha\theta_1 - \overline{p}_t)}{1 + \exp(\alpha\theta_1 - \overline{p}_t)} \le \beta_{1t} \le \dfrac{\exp(\alpha\theta_1 - \underline{p}_t)}{1 + \exp(\alpha\theta_1 - \underline{p}_t)} \\ \dfrac{\sum_{i=1,2}\exp(\alpha\theta_i - \overline{p}_t)}{1 + \sum_{i=1,2}\exp(\alpha\theta_i - \overline{p}_t)} \le \beta_{1t} + \beta_{2t} \le \dfrac{\sum_{i=1,2}\exp(\alpha\theta_i - \underline{p}_t)}{1 + \sum_{i=1,2}\exp(\alpha\theta_i - \underline{p}_t)} \\ \dfrac{\exp(\alpha\theta_2 - \overline{p}_t)}{1 + \exp(\alpha\theta_2 - \overline{p}_t)} \le \beta_{2t} \le \dfrac{\exp(\alpha\theta_2 - \underline{p}_t)}{1 + \exp(\alpha\theta_2 - \underline{p}_t)} \end{array} \right\}, \tag{5}$$

which is convex and compact.

Proof. Equation (4) comes from (2) by transforming and taking ln operation on both sides. The range is derived from the fact that the prices should be nonnegative. Consider any product, new or old FAP. If we charge the lowest price, $\underline{p}_t$, for one product and the highest price, $\overline{p}_t$, for the other, then this product should occupy the highest possible market share, $(\sum_{i=1,2}\exp(\alpha\theta_i - \underline{p}_t))/(1 + \sum_{i=1,2}\exp(\alpha\theta_i - \underline{p}_t))$. □

3.3. Dynamic Programming Formulation. Based on the description aforementioned, we define some other parameters with respect to period t below.

I_t = initial inventory level of old FAP before ordering at the beginning of period t.

Q_t = order quantity of new FAP at the beginning of period t.

c_t = unit procurement cost of new FAP.

h_{1t} = unit holding cost for the excess inventory.

h_{2t} = unit disposal cost of remaining inventory with zero shelf-life time.

l_{it} = backordered cost for unit shortage of new and old FAP, $i = 1, 2$.

γ = profit discount factor, $\gamma \in (0, 1]$.

Here, we do not factor dynamic demand substitution that the customer may buy the other product type when his preferred one is out of stock. At the beginning of period t, $t \in \{1, 2, \ldots, T\}$, the retailer reviews the current inventory I_t of old FAP which comes from the unsold inventory of new products at the previous period and places replenishment order Q_t for new FAP, as well as setting product prices, p_{1t}&p_{2t}, for new and old FAPs, respectively. Obviously, under zero lead time assumption, the inventory of new FAPs is instantaneously replenished to be Q_t. After demand realization and the retailer satisfies the demand to the maximum extent, the remaining inventory levels of the new and old products are $(Q_t - D_{1t})^+$ and $(I_t - D_{2t})^+$, respectively. The $(Q_t - D_{1t})^+$ of leftover that remains one period of shelf-life time will be carried to next period for sale as old FAP, incurring holding cost and becoming the initial inventory of nonfresh product, I_{t+1}, at period t+1, while the $(I_t - D_{2t})^+$ of old products will be disposed of on account of zero shelf-life time, incurring disposal cost. Otherwise the relevant backordered amount of new FAPs and old FAPs for period t is $(Q_t - D_{1t})^-$ and $(I_t - D_{2t})^-$, respectively, when demand is greater than on-hand inventories, where $x^+ = \max(0, x)$ and $x^- = \max(0, -x)$.

At period t, the retailer revenue, $\widehat{R}_t((p_{1t}, p_{2t}))$, can be calculated as follows:

$$\widehat{R}_t((p_{1t}, p_{2t})) = (p_{1t}, p_{2t})(D_{1t}, D_{2t})^T \tag{6}$$

Substitute the price function (5) into (6), we can obtain the expected revenue, $R_t((\beta_{1t}, \beta_{2t}))$.

$$\begin{aligned} R_t((\beta_{1t}, \beta_{2t})) &= E\left(\widehat{R}_t(p_{1t}, p_{2t})\right) \\ &= \sum_{i=1,2}\left(\alpha\theta_i + \ln\left(\frac{(1-\sum_i \beta_{it})}{\beta_{it}}\right)\right)\beta_{\mathrm{it}}(p_{1t}, p_{2t})\mu_t \end{aligned} \tag{7}$$

After demand realization at period t, some more costs incurs, backordered cost of $l_{1t}(Q_t - D_{1t})^-$ or holding cost of $h_{1t}(Q_t - D_{1t})^+$ for new FAPs, and backlogged cost of $l_{2t}(I_t - D_{2t})^-$ or disposal cost of $h_{2t}(I_t - D_{2t})^+$ for old FAPs, totally denoted as $\widehat{H}_t(Q_t, (\beta_{1t}, \beta_{2t}))$.

$$\begin{aligned} \widehat{H}_t(Q_t, (\beta_{1t}, \beta_{2t})) &= (h_{1t}, h_{2t}) \cdot \begin{pmatrix} (Q_t - \beta_{1t}\omega_t - \varepsilon_{1t})^+ \\ (I_t - \beta_{2t}\omega_t - \varepsilon_{2t})^+ \end{pmatrix} \\ &+ (l_{1t}, l_{1t}) \\ &\cdot \begin{pmatrix} (Q_t - \beta_{1t}\omega_t - \varepsilon_{1t})^- \\ (I_t - \beta_{2t}\omega_t - \varepsilon_{2t})^- \end{pmatrix} \end{aligned} \tag{8}$$

And the corresponding expected function is $H_t(Q_t, (\beta_{1t}, \beta_{2t})) = E(\widehat{H}_t(Q_t, (\beta_{1t}, \beta_{2t})))$.

Let $\widehat{G}_t(Q_t, (\beta_{1t}, \beta_{2t}))$ be the profit of period t; we have

$$\begin{aligned} \widehat{G}_t(Q_t, (\beta_{1t}, \beta_{2t})) &= \widehat{R}_t(Q_t, (\beta_{1t}, \beta_{2t})) \\ &- \widehat{H}_t(Q_t, (\beta_{1t}, \beta_{2t})) - c_t Q_t \end{aligned} \tag{9}$$

Accordingly, the expected profit function is as below.

$$\begin{aligned} G_t(Q_t, (\beta_{1t}, \beta_{2t})) &= E\widehat{G}_t(Q_t, (\beta_{1t}, \beta_{2t})) \\ &= R_t(\beta_{1t}, \beta_{2t}) - H_t(Q_t, (\beta_{1t}, \beta_{2t})) \\ &- c_t Q_t \end{aligned} \tag{10}$$

Denote $\pi_t(I_t)$ to be the maximum total expected profit-to-go function from period t to T in state, I_t, with a discount factor γ. We have the recursive equation $\pi_t(I_t) = \max_{Q_t \geq 0, (\beta_{1t}, \beta_{2t}) \in \Omega_t}\{G_t(Q_t, (\beta_{1t}, \beta_{2t})) + \gamma E\pi_{t+1}(I_{t+1})\}$. With transfer function, $I_{t+1} = (Q_t - \beta_{1t}\omega_t)^+$, by defining $\Phi_t(Q_t, (\beta_{1t}, \beta_{2t}))$,

$$\begin{aligned} \Phi_t(Q_t, (\beta_{1t}, \beta_{2t})) &= G_t(Q_t, (\beta_{1t}, \beta_{2t})) \\ &+ \gamma E\pi_{t+1}(I_{t+1}), \end{aligned} \tag{11}$$

we have the following dynamic programming model:

$$\pi_t(I_t) = \max_{Q_t \geq 0, (\beta_{1t}, \beta_{2t}) \in \Omega_t} \Phi_t(Q_t, (\beta_{1t}, \beta_{2t})) \tag{12}$$

$$s.t \quad \pi_{T+1} = -h_{2,T+1} I_{T+1}, \tag{13}$$

where (13) is the boundary condition since all leftovers will be disposed of at the end of horizon planning. That is to say, there are no second markets where some unexpired inventory can be cleared with some salvage value. On the contrary, the firm needs to pay on deposing expired inventories, even unexpired leftovers. Our analysis can be readily extended to address this situation.

4. Analysis of Model Structure

To analyze the structure of the optimal policy, we need to know the concavity property of $\Phi_t(Q_t, (\beta_{1t}, \beta_{2t}))$ which is strictly relative to revenue function $R_t((\beta_{1t}, \beta_{2t}))$ and inventory cost function $H_t(Q_t, (\beta_{1t}, \beta_{2t}))$.

Lemma 2. *$R_t((\beta_{1t}, \beta_{2t}))$ is continuously, twice differentiable, and jointly concave in (β_{1t}, β_{2t}).*

Proof. The continuity and twice differentiability of $R_t((\beta_{1t}, \beta_{2t}))$ is quite intuitive since it is expressed by some elementary operations of β_{1t} and β_{2t}. To show the concavity, we need to prove the negative semidefinite property of its Hessian Matrix. Taking the second-order derivative with respect to β_{1t} yields $\partial^2 R_t/\partial\beta_{1\mathrm{t}}^2 = -(1/(1-\beta_{1t}-\beta_{2t}) + 1/\beta_{1t} + 1/(1-\beta_{1t}-\beta_{2t})^2)\mu_t \leq 0$. Thus R_t is concave on β_{1t}. Taking the second-order derivative with respect to β_{2t} yields $\partial^2 R_t/\partial\beta_{2\mathrm{t}}^2 = -(1/(1-\beta_{1t}-\beta_{2t}) + 1/\beta_{2t} + 1/(1-\beta_{1t}-\beta_{2t})^2)\mu_t \leq 0$, which implies the concavity of R_t on β_{2t}. And taking the cross-partial derivative with respect to β_{1t} and β_{2t} yields $\partial^2 R_t/(\partial\beta_{1t}\partial\beta_{2t}) = -(1/(1-\beta_{1t}-\beta_{2t}) + 1/(1-\beta_{1t}-\beta_{2t})^2)\mu_t$. Pairing up the above terms leads to Hessian Matrix,

$$Hessian(R_t) = \begin{bmatrix} \dfrac{\partial^2 R_t}{\partial\beta_{1\mathrm{t}}^2} & \dfrac{\partial^2 R_t}{(\partial\beta_{1\mathrm{t}}\partial\beta_{2\mathrm{t}})} \\ \dfrac{\partial^2 R_t}{(\partial\beta_{2\mathrm{t}}\partial\beta_{1\mathrm{t}})} & \dfrac{\partial^2 R_t}{\partial\beta_{2\mathrm{t}}^2} \end{bmatrix}$$

$$= \begin{bmatrix} -\left(\frac{1}{1-\beta_{1t}-\beta_{2t}}+\frac{1}{\beta_{1t}}+\frac{1}{(1-\beta_{1t}-\beta_{2t})^2}\right)\mu_t & -\left(\frac{1}{1-\beta_{1t}-\beta_{2t}}+\frac{1}{(1-\beta_{1t}-\beta_{2t})^2}\right)\mu_t \\ -\left(\frac{1}{1-\beta_{1t}-\beta_{2t}}+\frac{1}{(1-\beta_{1t}-\beta_{2t})^2}\right)\mu_t & -\left(\frac{1}{1-\beta_{1t}-\beta_{2t}}+\frac{1}{\beta_{2t}}+\frac{1}{(1-\beta_{1t}-\beta_{2t})^2}\right)\mu_t \end{bmatrix}. \tag{14}$$

And the corresponding determinant is as follows:

$$\begin{aligned} |H(R_t)| &= \frac{\partial^2 R_t}{\partial\beta_{1t}^2}\cdot\frac{\partial^2 R_t}{\partial\beta_{2t}^2}-\left(\frac{\partial^2 R_t}{(\partial\beta_{1t}\partial\beta_{2t})}\right)^2 \\ &= \left(\frac{1}{1-\beta_{1t}-\beta_{2t}}+\frac{1}{(1-\beta_{1t}-\beta_{2t})^2}\right)\left(\frac{1}{\beta_{1t}}+\frac{1}{\beta_{2t}}\right) \\ &\quad + \frac{1}{\beta_{1t}\beta_{2t}} > 0, \end{aligned} \tag{15}$$

which implies that *Hessian* matrix is seminegative definite. On the basis of that, R_t is jointly concave in (β_{1t}, β_{2t}). □

Lemma 3. *$H_t(Q_t,(\beta_{1t},\beta_{2t}))$ is continuously twice differentiable and jointly convex in (β_{1t},β_{2t}) and Q_t.*

Proof. We observe that $\widehat{H}_t(Q_t,(\beta_{1t},\beta_{2t}))$ is continuously and twice differentiable for any realization of ω_t and ε_{it}. And the expectation, $H_t(Q_t,(\beta_{1t},\beta_{2t}))$, is easily justified to be continuously twice differentiable using the canonical argument (Durrett, 2010).

To show the convexity, we define some column vectors, expressed by boldface lowercase letters, such as $\mathbf{h}_t = (h_{1t}\ \ h_{2t})^T$, $\mathbf{l}_t = (l_{1t}\ \ l_{2t})^T$, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}\ \ \varepsilon_{2t})^T$. Then the demand and cost function can be expressed of vector form as follows:

$$\mathbf{D}_t = (D_{1t}\ \ D_{2t}) = \boldsymbol{\beta}_t\omega_t + \boldsymbol{\varepsilon}_t \tag{16}$$

$$\begin{aligned} \widehat{H}_t(Q_t,(\beta_{1t},\beta_{2t})) &= \widehat{H}_t(\boldsymbol{\beta}_t,Q_t) \\ &= \mathbf{h}_t\left((Q_t\ \ I_t)^T-\mathbf{D}_t\right)^+ \\ &\quad + \mathbf{l}_t\left((Q_t\ \ I_t)^T-\mathbf{D}_t\right)^- \\ &= \mathbf{h}_t\left((Q_t\ \ I_t)^T-\boldsymbol{\beta}_t\omega_t-\boldsymbol{\varepsilon}_t\right)^+ \\ &\quad + \mathbf{l}_t\left((Q_t\ \ I_t)^T-\boldsymbol{\beta}_t\omega_t-\boldsymbol{\varepsilon}_t\right)^- \end{aligned} \tag{17}$$

Define a new vector variable $\boldsymbol{u}_t = (Q_t\ \ I_t)^T - \boldsymbol{\beta}_t\omega_t - \boldsymbol{\varepsilon}_t$, and we can get $\widehat{H}_t(\boldsymbol{\beta}_t,Q_t) = \widehat{H}_t(\mathbf{u}_t) = \mathbf{h}_t(\mathbf{u}_t)^+ + \mathbf{l}_t(\mathbf{u}_t)^-$, which is convex on $\mathbf{u}_t$ (Sundaranm, R. K. 1996). Fix the random part ω_t to its realization and consider any pair $(\boldsymbol{\beta}_t, Q_t)$ and $(\boldsymbol{\beta}_t', Q_t')$, with the convexity of $\widehat{H}_t(\mathbf{u}_t)$; we have

$$\begin{aligned} &\widehat{H}_t\left(\alpha(\boldsymbol{\beta}_t,Q_t)+(1-\alpha)\left(\boldsymbol{\beta}_t',Q_t'\right)\right) = \widehat{H}_t\left(\alpha\boldsymbol{\beta}_t\right. \\ &\quad \left.+(1-\alpha)\boldsymbol{\beta}_t',\alpha Q_t+(1-\alpha)Q_t'\right) \\ &\quad = \mathbf{h}_t\left(\left(\alpha(Q_t\ \ I_t)^T+(1-\alpha)(Q_t'\ \ I_t)^T\right)\right. \\ &\quad \left.-\left(\alpha\boldsymbol{\beta}_t+(1-\alpha)\boldsymbol{\beta}_t'\right)\omega_t-(\alpha+(1-\alpha))\boldsymbol{\varepsilon}_t\right)^+ \\ &\quad +\mathbf{l}_t\left(\left(\alpha(Q_t\ \ I_t)^T+(1-\alpha)(Q_t'\ \ I_t)^T\right)\right. \\ &\quad \left.-\left(\alpha\boldsymbol{\beta}_t+(1-\alpha)\boldsymbol{\beta}_t'\right)\omega_t-(\alpha+(1-\alpha))\boldsymbol{\varepsilon}_t\right)^- \\ &\quad = \mathbf{h}_t\left(\alpha\left((Q_t\ \ I_t)^T-\boldsymbol{\beta}_t\omega_t-\boldsymbol{\varepsilon}_t\right)\right. \\ &\quad \left.+(1-\alpha)\left((Q'\ \ I_t)^T{}_t-\boldsymbol{\beta}_t'\omega_t-\boldsymbol{\varepsilon}_t\right)\right)^+ \\ &\quad +\mathbf{l}_t\left(\alpha\left((Q_t\ \ I_t)^T-\boldsymbol{\beta}_t\omega_t-\boldsymbol{\varepsilon}_t\right)\right. \\ &\quad \left.+(1-\alpha)\left((Q'\ \ I_t)^T-\boldsymbol{\beta}_t'\omega_t-\boldsymbol{\varepsilon}_t\right)\right)^- = \mathbf{h}_t\left(\alpha\mathbf{u}_t\right. \\ &\quad \left.+(1-\alpha)\mathbf{u}_t'\right)^+ + \mathbf{l}_t\left(\alpha\mathbf{u}_t+(1-\alpha)\mathbf{u}_t'\right)^- = \widehat{H}_t\left(\alpha\mathbf{u}_t\right. \\ &\quad \left.+(1-\alpha)\mathbf{u}_t'\right) \le \alpha\widehat{H}_t(\mathbf{u}_t)+(1-\alpha)\widehat{H}_t\left(\mathbf{u}_t'\right) \\ &\quad = \alpha\widehat{H}_t\left(\boldsymbol{\beta}_t,(Q_t\ \ I_t)^T\right)+(1-\alpha)\widehat{H}_t\left(\boldsymbol{\beta}_t',(Q_t'\ \ I_t)^T\right), \end{aligned} \tag{18}$$

where $0 \le \alpha \le 1$. Therefore, $\widehat{H}_t(Q_t,(\beta_{1t},\beta_{2t})) = \widehat{H}_t(\boldsymbol{\beta}_t, (Q_t\ \ I_t)^T) = \widehat{H}_t(\mathbf{u}_t)$ is jointly convex in (β_{1t},β_{2t}) and Q_t, so does the expectation function, $H_t(Q_t,(\beta_{1t},\beta_{2t}))$. □

Theorem 4. *At any time period t with initial inventory I_t, the expected revenue function $G_t(Q_t,(\beta_{1t},\beta_{2t}))$ is continuously twice differentiable and jointly concave in (β_{1t},β_{2t}) and Q_t.*

Proof. With Lemmas 2 and 3, we can observe that $G_t(Q_t,(\beta_{1t},\beta_{2t}))$ is continuously twice differentiable. We first observe that, for any t, the three terms of $G_t(Q_t,(\beta_{1t},\beta_{2t}))$ in (10) are jointly concave. The first term is concave in (β_{1t},β_{2t}) according to Lemma 2. The second term is jointly concave in (β_{1t},β_{2t}) and Q_t (see Lemma 3), and the third term is linear in Q_t, which implies the concavity. With the property that a nonnegative weighted sum of concave functions is concave (Stephen Boyd, Lieven Vandenberghe, 2004), $G_t(Q_t,(\beta_{1t},\beta_{2t}))$ is continuously twice differentiable and jointly concave in (β_{1t},β_{2t}) and Q_t. □

On the basis of the concavity of $G_t(Q_t,(\beta_{1t},\beta_{2t}))$, we can have Theorem 5.

Theorem 5. *Consider any time period t with initial inventory I_t; the function $\Phi_t(Q_t,(\beta_{1t},\beta_{2t}))$ is continuously twice differentiable and jointly concave in $(Q_t,(\beta_{1t},\beta_{2t}))$, and the function $\pi_t(I_t)$ is concave and nonincreasing in I_t.*

Proof. By induction, first, we prove that $\Phi_T(Q_T,(\beta_{1T},\beta_{2T}))$ is continuously twice differentiable. According to Lemma 2,

$E(Q_T - \beta_{1T}\omega_T - \varepsilon_T)^+$, as a part of $H_t(Q_t, (\beta_{1t}, \beta_{2t}))$, is thus continuously twice differentiable, which implies that $\Phi_T(Q_T, (\beta_{1T}, \beta_{2T})) = G_T(Q_T, (\beta_{1T}, \beta_{2T})) - \gamma h_{2,T+1}E(Q_T - \beta_{1T}\omega_T - \varepsilon_{2T})^+$ is continuously and twice differentiable. Assuming now that $\Phi_t(Q_t, (\beta_{1t}, \beta_{2t}))$ is continuously twice differentiable for $t = 1, 2, \ldots, T$, we prove that $\Phi_{t-1}(Q_{t-1}, (\beta_{1,t-1}, \beta_{2,t-1}))$ is also continuously twice differentiable according to Theorem 4. Let $\Lambda_T(Q_T, \beta_{1T}) = E(Q_T - \beta_{1T}\omega_T - \varepsilon_{2T})^+ = \iint_{\beta_{1T}x+y\le Q_T}(Q_T - \beta_{1T}x - y)f_\omega(x)f_\varepsilon(y)dxdy$, where $f_\omega(x)$ and $f_\varepsilon(y)$ is the p.d.f. of random variable ω_T and ε_T, respectively, which are generally assumed to be independent. For any realization of ε_{2T}, such as $\widehat{\varepsilon}_{2T}$, taking the second-order derivative of $\Lambda_T(Q_T, \beta_{1T})$ with respective to Q_T yields $\partial^2\Lambda_T(Q_T, \beta_{1T})/\partial Q_T^2 = (1/\beta_{1T})f((Q_T - \widehat{\varepsilon}_{2T})/\beta_{1T}) > 0$, which implies that $\Lambda_T(Q_T, \beta_{1T})$ is convex in Q_T. Taking the second-order derivative with respect to β_{1T} yields $\partial^2\Lambda_T(Q_T, \beta_{1T})/\partial\beta_{1T}^2 = (Q_T^2/\beta_{1T}^3)f(Q_T/\beta_{1T}) > 0$, which implies the concavity of $\Lambda_T(Q_T, \beta_{1T})$ in β_{1T}. And taking the cross-partial derivative with respect to Q_T and β_{1T} yields $\partial^2\Lambda_T(Q_T, \beta_{1T})/\partial Q_T\partial\beta_{1T} = -((Q_T - \widehat{\varepsilon}_{2T})/\beta_{1T}^2)f((Q_T - \widehat{\varepsilon}_{2T})/\beta_{1T})$. Pairing up the above three derivative terms leads to Hessian Matrix,

$$H(\Lambda_T) = \begin{bmatrix} \dfrac{\partial^2\Lambda_T}{\partial Q_T^2} & \dfrac{\partial^2\Lambda_T}{\partial Q_T\partial\beta_{1T}} \\ \dfrac{\partial^2\Lambda_T}{\partial\beta_{1T}\partial Q_T} & \dfrac{\partial^2\Lambda_T}{\partial\beta_{1T}^2} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\beta_{1T}}f\left(\dfrac{Q_T - \widehat{\varepsilon}_{2T}}{\beta_{1T}}\right) & -\dfrac{Q_T - \widehat{\varepsilon}_{2T}}{\beta_{1T}^2}f\left(\dfrac{Q_T - \widehat{\varepsilon}_{2T}}{\beta_{1T}}\right) \\ -\dfrac{Q_T - \widehat{\varepsilon}_{2T}}{\beta_{1T}^2}f\left(\dfrac{Q_T - \widehat{\varepsilon}_{2T}}{\beta_{1T}}\right) & \dfrac{(Q_T - \widehat{\varepsilon}_{2T})^2}{\beta_{1T}^3}f\left(\dfrac{Q_T - \widehat{\varepsilon}_{2T}}{\beta_{1T}}\right) \end{bmatrix}. \tag{19}$$

And the corresponding determinant, $H(\Lambda_T) = 0$, which implies matrix *Hessian* is semipositive definite. On the basis of analysis above, we can conclude that $\Lambda_T(Q_T, \beta_{1T})$ is jointly convex in (Q_T, β_{1T}). Meanwhile, the last term, $-\gamma h_{2,T+1}E(Q_T - \beta_{1T}\omega_T - \widehat{\varepsilon}_{2T})^+$, is obviously justified to be jointly concave in (Q_T, β_{1T}). According to the concavity property (Stephen Boyd, Lieven Vandenberghe, 2004), $\Phi_T(Q_T, (\beta_{1T}, \beta_{2T}))$ is jointly concave in (β_{1t}, β_{2t}) and Q_t. $\pi_T(I_T)$ is easily verified to be concave as well, and it is obviously nonincreasing.

Assume now that $\Phi_t(Q_t, (\beta_{1t}, \beta_{2t}))$ is jointly concave for some $t = 1, 2, \ldots, T-1$ and that $\pi_t(I_t)$ is also concave and nonincreasing. Then, $\Phi_{t-1}(Q_{t-1}, (\beta_{1,t-1}, \beta_{2,t-1}))$ is jointly concave: joint concavity of the first term of (10) is verified for the case $t = T$ above. For any pair of points (Q_t, β_{1t}) and (Q_t', β_{1t}') with given value of ω_t, we have

$$\begin{aligned} &\pi_{t+1}\left(\left(\alpha Q_t + (1-\alpha)Q_t'\right)_t - \left(\alpha\beta_{1t} + (1-\alpha)\beta_{1t}'\right)\omega_t \right. \\ &\quad \left. - \left(\alpha\varepsilon_{1t} + (1-\alpha)\varepsilon_{1t}\right)\right) = \pi_{t+1}\left(\alpha\left(Q_t - \beta_{1t}\omega_t - \varepsilon_{1t}\right)\right. \\ &\quad \left. + (1-\alpha)\left(Q_t' - \beta_{1t}'\omega_t - \varepsilon_{1t}\right)\right) \ge \alpha\pi_{t+1}\left(Q_t - \beta_{1t}\omega_t\right. \\ &\quad \left. - \varepsilon_{1t}\right) + (1-\alpha)\pi_{t+1}\left(Q_t' - \beta_{1t}'\omega_t - \varepsilon_{1t}\right), \end{aligned} \tag{20}$$

by the concavity of $\pi_{t+1}(\cdot)$, which implies that $E\pi_{t+1}(Q_t - \beta_{1t}\omega_t - \varepsilon_{1t})$ is jointly concave in Q_t and β_{1t}. With above argument, the desired result holds. □

5. Optimal Policy

Following the structure property of model, we can obtain the following result.

Theorem 6. *Consider any period t, with the initial inventory I_t, there is a unique maximizer, $(Q_t^*, (\beta_{1t}^*, \beta_{2t}^*))$, to maximize the constrained expected periodical profit function,*

$$(Q_t^*, (\beta_{1t}^*, \beta_{2t}^*)) \in \arg\max_{Q_t\ge 0, (\beta_{1t}, \beta_{2t})\in\Omega_t} \Phi_t(Q_t, (\beta_{1t}, \beta_{2t})). \tag{21}$$

Proof. It can be easily proved on the basis of Theorem 5. □

On the basis of the dynamic programming analysis, including the concavity property, we can construct an optimization problem for any period t with constrained conditions as follows:

$$\begin{aligned} &\textit{Minimize} \quad -\Phi_t\left(Q_t, (\beta_{1t}, \beta_{2t})\right) \\ &\textit{s.t.} \quad \begin{cases} \left(\beta_{1t} - \dfrac{\exp\left(\alpha\theta_1 - \underline{p}_t\right)}{1 + \exp\left(\alpha\theta_1 - \underline{p}_t\right)}\right)\left(\beta_{1t} - \dfrac{\exp\left(\alpha\theta_1 - \overline{p}_t\right)}{1 + \exp\left(\alpha\theta_1 - \overline{p}_t\right)_t}\right) \le 0 \\ \left(\beta_{2t} - \dfrac{\exp\left(\alpha\theta_2 - \underline{p}_t\right)}{1 + \exp\left(\alpha\theta_2 - \underline{p}_t\right)}\right)\left(\beta_{2t} - \dfrac{\exp\left(\alpha\theta_2 - \overline{p}_t\right)}{1 + \exp\left(\alpha\theta_2 - \overline{p}_t\right)}\right) \le 0 \\ \left(\beta_{1t} + \beta_{2t} - \dfrac{\sum_{i=1,2}\exp\left(\alpha\theta_i - \overline{p}_t\right)}{1 + \sum_{i=1,2}\exp\left(\alpha\theta_i - \overline{p}_t\right)}\right)\left(\beta_{1t} + \beta_{2t} - \dfrac{\sum_{i=1,2}\exp\left(\alpha\theta_i - \underline{p}_t\right)}{1 + \sum_{i=1,2}\exp\left(\alpha\theta_i - \underline{p}_t\right)}\right) \le 0 \\ -Q_t \le 0 \end{cases} \end{aligned} \tag{22}$$

The *Lagrangian* and the *Karush-Kuhn-Tucker*(*KTT*) conditions provide an efficient solution approach to this optimization problem. Denote $\underline{A}_t = (\exp(\alpha\theta_1 - \underline{p}_t))/(1+\exp(\alpha\theta_1 - \underline{p}_t))$, $\overline{A}_t = (\exp(\alpha\theta_1 - \overline{p}_t))/(1+\exp(\alpha\theta_1 - \overline{p}_t))$, $\underline{B}_t = (\exp(\alpha\theta_2 - \underline{p}_t))/(1+\exp(\alpha\theta_2 - \underline{p}_t))$, $\overline{B}_t = (\exp(\alpha\theta_2 - \overline{p}_t))/(1+\exp(\alpha\theta_2 - \overline{p}_t))$, $\underline{C}_t = (\sum_{i=1,2}\exp(\alpha\theta_i - \overline{p}_t))/(1+\sum_{i=1,2}\exp(\alpha\theta_i - \overline{p}_t))$, and $\overline{C}_t = (\sum_{i=1,2}\exp(\alpha\theta_i - \underline{p}_t))/(1+\sum_{i=1,2}\exp(\alpha\theta_i - \underline{p}_t))$. We construct for each period t the *Lagrangian* of the optimization problem as

$$\begin{aligned} &L_t\left(Q_t, (\beta_{1t}, \beta_{2t}), (\lambda_{t1}, \lambda_{t2}, \lambda_{t3}, \lambda_{t4})\right) \\ &\quad = -\Phi_t\left(Q_t, (\beta_{1t}, \beta_{2t})\right) + \lambda_{t1}\left(\beta_{1\mathrm{t}} - \underline{A}_t\right)\left(\beta_{1\mathrm{t}} - \overline{A}_t\right) \\ &\qquad + \lambda_{t2}\left(\beta_{2\mathrm{t}} - \underline{B}_t\right)\left(\beta_{2\mathrm{t}} - \overline{B}_t\right) \\ &\qquad + \lambda_{t3}\left(\beta_{1\mathrm{t}} + \beta_{2t} - \underline{C}_t\right)\left(\beta_{1\mathrm{t}} + \beta_{2t} - \overline{C}_t\right) - \lambda_{t4}Q_t, \end{aligned} \tag{23}$$

where the *Lagrange* multipliers are $\lambda_{ti} \geq 0$, $i = 1, 2, 3, 4$. Let $(Q_t^*, (\beta_{1t}^*, \beta_{2t}^*))$ be the optimizing solution; we can construct the *KKT* conditions as follows:

$$\begin{aligned} &-\nabla\Phi_t\left(Q_t, (\beta_{1t}, \beta_{2t})\right) + \lambda_{t1}\left(0, 2\beta_{1\mathrm{t}} - \underline{A}_t - \overline{A}_t, 0\right) \\ &\quad + \lambda_{t2}\left(0, 0, 2\beta_{2\mathrm{t}} - \underline{B}_t - \overline{B}_t\right) + \lambda_{t3}\left(0, 2\beta_{1\mathrm{t}} + 2\beta_{2t}\right. \\ &\quad \left. - \underline{C}_t - \overline{C}_t, 2\beta_{1\mathrm{t}} + 2\beta_{2t} - \underline{C}_t - \overline{C}\right) - \lambda_{t4}(1, 0, 0) = 0 \\ &\lambda_{t1}\left(\beta_{1\mathrm{t}} - \underline{A}_t\right)\left(\beta_{1\mathrm{t}} - \overline{A}_t\right) = 0 \\ &\lambda_{t2}\left(\beta_{2\mathrm{t}} - \underline{B}_t\right)\left(\beta_{2\mathrm{t}} - \overline{B}_t\right) = 0 \\ &\lambda_{t3}\left(\beta_{1\mathrm{t}} + \beta_{2t} - \underline{C}_t\right)\left(\beta_{1\mathrm{t}} + \beta_{2t} - \overline{C}_t\right) = 0 \\ &\lambda_{t4}Q_t = 0, \end{aligned} \tag{24}$$

where the first equation represents *F.O.C.* of *Lagrangian* and the latter four equations represent the conditions of complementary slackness.

With solution of the *KKT* conditions, $(Q_t^*, (\beta_{1t}^*, \beta_{2t}^*))$, and (4), we can get the optimization pricing and ordering policy of $(Q_t^*, (p_{1t}^*, p_{2t}^*))$ at period t, where (p_{1t}^*, p_{2t}^*) is as follows:

$$p_{it}^* = P_{it}\left(\beta_{1t}^*, \beta_{2t}^*\right) = \alpha\theta_i + \ln\left(\frac{\left(1 - \sum_i \beta_{it}^*\right)}{\beta_{it}^*}\right), \qquad i = 1, 2. \tag{25}$$

Note that $(Q_t^*, (\beta_{1t}^*, \beta_{2t}^*))$ depends on state variable, I_t, and parameters of (h_{1t}, h_{2t}) and (l_{1t}, l_{2t}) as well as μ_t, which implies that the retailer decides the optimal policy on the basis of these parameters at every period t. With backward induction, we get the optimal strategy, a sequence of $[(Q_1^*, (p_{11}^*, p_{21}^*)), (Q_2^*, (p_{12}^*, p_{22}^*)), \ldots, (Q_T^*, (p_{1T}^*, p_{2T}^*))]$.

6. Numerical Experiment

We now conduct a numerical example to illustrate our results and select the parameters as follows. The demand follows the multiplicative form with i.i.d. terms $\omega_t \sim U(0, 100)$ for all periods. We assume $(\theta_1\ \ \theta_2)^T = (10\ \ 5)^T$, since fresh product has a higher perceptive value than nonfresh ones. The cost structure is symmetric: $\mathbf{h}_t = (0.5\ \ 0.5)^T$, $\mathbf{l}_t = (4.5\ \ 4.5)$, and $c_t = 10$. For simplicity to compute the optimal prices, we consider a finite horizon with $T = 5$, the discount rate is $\gamma = 0.9$, and the initial inventory level $I_t = 0$. Using Matlab to program the dynamic model and get the following results with *KKT* condition method, we can obtain a sequence of order quantity and optimal prices, such as $[(78, (14.8, 9.7))$, $(68, (12.6, 8.6))$, $(92, (17.5, 11.4))$, $(43, (10.9, 6.2))$, $(52, (11.5, 10.2))]$, at which the maximum profit $\pi^* = 4,235.6$ occurs.

In Table 1, we compare the performance between dynamic pricing policy and a general policy under one period lifetime setting where all products are sold at a regular price at any period of time. Similarly, we assume $(\theta_1\ \ \theta_2)^T = (10\ \ 5)^T$ in dynamic pricing policy while $(\theta_1\ \ \theta_2)^T = (10\ \ 10)^T$ in general policy. Note that in general setting the optimal pricing and ordering policy can be obtained by way of general dynamic programming method. Three examples of different finite horizon time ($T = 5$,10,20) are conducted with the same cost structure and discount rate mentioned above. Accordingly, the optimal solutions, such as pricing and order quantity, as well as dynamic inventory, are shown in Table 1, where superscript g and d represent regular policy and dynamic policy, respectively.

A summary of interesting observations is as follows. The dynamic policy is obviously better than general pricing and inventory policy, with which the retailer's profit increases by 15-20%. Further, the demand is also matched to a more certain extent by the supply since the order quantity is almost more than that under general policy while the. Based on the results, we can conclude that dynamic pricing and inventory policy is beneficial for retailer's revenue management in selling fresh agriculture products. Furthermore, from the perspective of agriculture industrial development, it also can maximally reduce the operating costs, such as holding costs or lost costs, since the optimal match between products supply and demand.

7. Concluding Remark

In this paper, we address a joint pricing and inventory control problem for stochastic fresh agriproduct inventory systems under consumer choice over a finite horizon in the backlogging case. In the model the products are perishable along time and the customers are sensitive to the ages of inventories, which result in consumer choice behavior based

Table 1: The optimal solution under dynamic pricing and regular pricing.

Periods T	Order quantity Q^d	Order quantity Q^g	Pricing (p_1^d, p_2^d)	Pricing p^g	Inventory (I_1^d, I_2^d)	Inventory I^g	π^d	π^g	$\frac{(\pi^d - \pi^g)}{\pi^g}$
5	78	72	(14.8,9.7)	12.5	(49,0)	9	4235.6	3578.2	18.4%
	68	65	(12.6,8.6)	10.4	(31,2)	8			
	92	80	(17.5,11.4)	12.8	(56,11)	13			
	43	45	(10.9,6.2)	8.1	(22,0)	3			
	52	45	(11.5,8.2)	9.3	(12,1)	5			
10	81	70	(15.6,10.7)	12.2	(52,0)	10	8343.8	6984.4	19.5%
	72	61	(13.6,8.4)	10.3	(38,6)	7			
	83	58	(16.5,10.4)	13.2	(56,9)	8			
	76	59	(11.9,7.8)	8.5	(32,2)	4			
	70	73	(12.5,9.2)	8.1	(34,6)	4			
	56	41	(16.5,9.8)	14.6	(30,1)	9			
	68	59	(12.2,8.8)	9.5	(31,2)	8			
	65	47	(15.4,10.4)	13.8	(33,9)	7			
	58	56	(12.4,7.3)	9.2	(23,0)	3			
	49	39	(12.8,6.4)	9.7	(19,1)	2			
20	90	86	(15.5,9.8)	13.1	(54,0)	16	17532.8	14963.2	17.2%
	72	65	(14.3,8.5)	10.4	(36,6)	8			
	83	57	(16.5,10.1)	11.1	(40,9)	9			
	66	75	(13.2,7.6)	8.6	(31,4)	5			
	72	54	(12.9,9.5)	9.2	(34,6)	4			
	56	51	(16.3,9.2)	13.2	(35,3)	7			
	68	62	(13.2,8.7)	9.8	(31,3)	4			
	65	45	(15.1,12.2)	13.6	(33,5)	4			
	57	69	(13.8,7.9)	9.7	(28,0)	3			
	48	46	(15.5,9.2)	10.2	(21,1)	2			
	81	72	(13.8,7.1)	9.8	(43,0)	2			
	70	58	(13.9,9.2)	10.4	(38,6)	7			
	78	50	(15.4,6.8)	11.9	(49,2)	5			
	71	48	(13.2,8.8)	9.5	(33,3)	0			
	61	63	(14.5,8.2)	8.4	(35,6)	8			
	56	52	(16.5,7.8)	14.6	(37,3)	10			
	76	48	(11.3,8.8)	9.9	(32,3)	6			
	66	64	(14.5,9.4)	12.8	(39,5)	8			
	52	49	(13.7,7.8)	9.8	(23,0)	3			
	50	46	(14.5,6.9)	9.1	(22,0)	1			

on product freshness. By using MNL model to describe consumer discrete choice we construct a stochastic and dynamic inventory model facing strategic customer behavior with joint ordering and discounting decisions. We are able to characterize the firm's optimal policy with the use of a transformation technique and employing the concept of the concavity, and finally develop a *Lagrangian* and the *Karush-Kuhn-Tucker* (*KTT*) conditions to provide an efficient solution approach to this optimization problem.

A limitation in our model is that we assume that the lifetime of the product is only two periods. Although this is true for some fresh agriproducts in the market, there are other products that are perishable but yet have longer lifetimes. To extend our model to the products with more than two periods lifetimes, the state space has to be increased more to specify the amounts of inventory that are going to expire at different points of time in the future. This definitely makes the problem quite hard. Another issue is to incorporate an operating cost for discounting pricing practice, which, however, will introduce additional technical complexity to the dynamic program. There are several future research directions. One is to extend the model to the environment where all the demand can be lost or only non-FAP demand can be lost while the fresh FAP can be backlogged instead. Another is to consider the case where the product has a fixed finite lifetime of exactly $n (n \geq 3)$ periods or random lifetime. Finally, it would be interesting to account for stock-out based substitutions.

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

This paper was previously presented as an abstract in 2016 International Symposium for Facing New Conditions and Challenges: the Agricultural Development in USA and China.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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