Research Article

Determining the Optimal Order Quantity with Compound Erlang Demand under (T,Q) Policy

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Received 10 March 2018; Revised 25 June 2018; Accepted 12 July 2018; Published 19 August 2018

Academic Editor: Neale R. Smith

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Management of electric equipment has a direct impact on companies' performance and profitability. Considering the critical role that electric power materials play in supporting maintenance operations and preventing equipment failure, it is essential to maintain an inventory to a reasonable level. However, a majority of these electric power materials exhibit an intermittent demand pattern characterized by random arrivals interspersed with time periods with no demand at all. These characteristics cause additional difficulty for companies in managing these electric power material inventories. In response to the above problem, this paper, based on the electric power material demand data of Shanghai Electric Power Company, develops a new method to determine the optimal order quantity Q^* in a cost-oriented periodic review (T,Q) system, whereby unsatisfied demands are backordered and demand follows a compound Erlang distribution. Q^* corresponds to the value of Q that gives the minimum expected total inventory holding and backordering cost. Subsequently, an empirical investigation is conducted to compare this method with the Newsvendor model. Results verify its superiority in cost savings. Ultimately, considering the complicated calculation and low efficiency of that algorithm, this paper proposes an approximation and a heuristic algorithm which have a higher level of utility in a real industrial context. The approximation algorithm simplifies the calculation process by reducing iterative times while the heuristic algorithm achieves it by generalizing the relationship between the optimal order quantity Q^* and mean demand interarrival rate λ .

1. Introduction

When managing electric power material inventories, companies need to decide when to order and how much to order so as to maintain the appropriate inventory level. Considering that a majority of electric power materials exhibit an intermittent demand pattern, effective management of these electric power materials is essential to electric power companies. Intermittent demand appears at random, with time intervals during which no demand occurs. These characteristics make inventory control a rather challenging task. These electric power materials are usually characterized by low usage frequency, long intervals, and irregular demand. Furthermore, demand arrivals and sizes vary with usage and maintenance patterns—further adding to the degree of intermittence. Moreover, the historical data of electric power materials are limited, coupled with constraints on the forecasting method, demand forecast for these electric power materials is difficult.

However, such electric power materials are usually essential components of expensive electric equipment. In the case of equipment failure, when electric power materials are unavailable and need to be ordered immediately, the cost of these urgent orders is higher than regular orders, significantly increasing the maintenance cost. In addition to this, the failure of a component in the electrical equipment is likely to lead to equipment failure, generating a high backordering cost and economic loss. On the other hand, excessive stock of electric power materials leads to backlogs and erosion, resulting in a significant waste of capital as well as the corresponding corporate cost. Some electric power materials stored in warehouses for a long time erode before they are put into use, while some are permanently idle due to the continuous changes and updates of electrical equipment. Since traditional inventory control is generally designed for regular demand, its application to intermittent demand is likely to lead to frequent stock-out events or excessive stock,

making the inventory control of electric power materials a more challenging task. Thus, it is essential for electric power companies to control their intermittent electric power material inventories.

2. Research Background

In this section, a brief review of the literature related to intermittent material inventory control is undertaken.

Traditional inventory control systems generally focus on regular demand and are not suitable for intermittent demand (Adelson [1]; Friend [2]). As a representative of traditional inventory control models, the Newsvendor model had been extensively used since it was introduced in 1962 (Hanssman [3]). This model states that if inventory is larger than demand, the remaining inventory will be sold at a discounted price or disposed at the end of period; however, if inventory is smaller than demand, some profit will be lost. Thus this model focuses on optimizing cost or profit and develops relevant inventory policy. Subsequently a great number of scholars make extensions to this traditional model which can be divided into three categories. Firstly, traditional models usually assume a fixed purchase price which departs from the reality. Thus price-sensitive demand is introduced. For instance, Xiao et al. [4] study the price-dependent demand in a multi-product Newsvendor model and propose corresponding methods of obtaining the optimal order quantity and optimal sales price. Kébé et al. [5] consider a two-echelon inventory lot-sizing problem with price-dependent demand and develop a mixed-integer programming formulation as well as a Lagrangian relaxation solution procedure for solving this problem. Secondly, considering the profound influence of demand variation on expected profits or costs, plenty of scholars focus on the modelling of demand when studying Newsvendor model. Uduman [6] employs demand distributions satisfying the SCBZ (clock back to zero) property to model the single demand for a single product, namely, newspaper, and subsequently obtain the optimal order quantity. Fathima et al. [7] study the generalization of Uduman's model with several individual demands for a single product, followed by numerical illustrations. Furthermore, Faithma & Uduman [8] propose an approximation closed-form optimal solution for three cases of single period inventory model, in which the demand varies with the SCBZ property. In addition to SCBZ property, Kamburowski [9] developed a theoretical framework for analyzing the distribution-free Newsboy problems due to incomplete information. Similarly, Chen [10] considers unknown demand and demonstrates the appropriateness of applying the bootstrap method to Newsvendor model. Hnaïen et al. [11] focus on discrete distributions of probabilities of demand. Besides, Priyan & Uthayakumar [12] consider a two-echelon multi-product multi-constraint inventory model with product returns and recovery. Lastly, two-level Newsvendor model including inventories on raw materials and finished products has recently received increasingly attention. In such model, Kera-matpour et al. [13] develop a meta-heuristic algorithm to obtain the optimal solution which optimize profit and service level simultaneously.

Axsäter [14] reveals that a majority of inventory control models including Newsvendor model are built on some assumptions, for instance, that lead-time demand follows the normal distribution. However, they point out that although this assumption is reasonable in most cases, it is inconsistent with the reality in certain circumstances and leads to a waste of capital or low service levels. Thus, they propose the (R, Q) policy with the compound Poisson demand. Considering that the compound distribution model can determine arrival and demand sizes, respectively, and its structure is similar to the intermittent demand process, a vast number of scholars propose that compound distributions, compound Poisson distribution in particular, can offer a good fit for intermittent material inventory (Adelson [1]; Friend [2]). Similarly, Matheus and Gelder [15] also employ compound Poisson distribution to model demand and determine the order quantity and reorder points. Babai et al. [16] have developed a method to determine the optimal order-up-to level, subject to a compound Poisson distribution.

Dunsmuir and Snyder [17] present a method for determining an optimal reorder point for the continuous review (S, Q) inventory policy, subject to a compound Bernoulli demand. However, they fail to take the impact of lead-time demand on the service level into account. To fill the gap, Janssen et al. [18] make some modifications and adopt the (R, s, Q) inventory model to calculate the reorder point with a service-level restriction. Teunter et al. [19] restrict their study to the calculation of the mean and variance of lead time demand and derive closed-form expressions for the service level and the expected total cost under the (T, S) policy with a compound binomial demand, thereby avoiding complicated calculation of the mean and variance.

Of all the literature with compound demand distribution assumptions, the compound Poisson is the most widely utilized. Its popularity can be mainly attributed to the simplicity arising from the memoryless propensity of the exponential distribution of the demand interarrivals. Gupta et al. [20] introduce the Palm's theorem to justify such demand modelling for whole complicated systems. However, in the case of one single material, this modelling is reported to lead to excess stock and imposes an additional financial burden on companies (Smith and Dekker [21]). The memoryless propensity implies a constant failure rate, which is inconsistent with the reality, where the likelihood of the failure of an item and the demand for it grows with time. Smith and Dekker [21] initially propose a more realistic renewal demand arrival process which has an Increasing Failure Rate (IFR), but they do not propose specific demand distribution to model the renewal demand arrival process. Subsequently Larsen and Kiesmuller [22] provide the first insight into the employment of compound Erlang distribution- k in the inventory control system and they derive a closed-form cost expression of the optimal reorder level for an (R, s, nQ) policy. Based on the same demand distribution, Larsen and Thorstenson [23] consider a continuous review base stock policy and demonstrate the use of a computer program to obtain the base stock level needed to achieve the specified order fill rate (OFR) and volume fill rate (VFR), respectively. Moreover, the respective performances of the OFR and VFR

for different inventory control systems are compared (Larsen and Thorstenson [24]). With demand arrivals following the Erlang distribution, in conjunction with a Gamma distribution for demand size, Saidane et al. [25, 26] propose an algorithm to determine the optimal base stock level under a cost-oriented continuous review inventory system and conduct numerical investigations to compare the performance of this model in cost reduction with that of the compound Poisson distribution and the classic Newsvendor model, respectively. Syntetos et al. [27] not only demonstrate the superiority of deviating from the memoryless demand model, but also take the degree of intermittence into consideration and explore its connection with forecast accuracy through a numerical investigation. In sum, the above literature under a compound Erlang process all assume that the lead time is constant.

In spite of the many contributions to the material inventory management field, there are some shortcomings that need to be addressed. Firstly, the majority of research focuses on the service-oriented rather than cost-oriented inventory systems. For instance, Matheus and Gelder [15] demonstrate a simpler method to calculate the optimal reorder point subject to a compound Poisson demand due to the assumption of a target service level. Fung et al. [28] build a model of period-review with an order-up-to-level (T, S) policy for single-item inventory systems under compound Poisson demands and service-level constraints. Zhao [29] evaluates an Assemble-to-Order (ATO) system under compound Poisson demand with a batch ordering policy by means of service level, including the delivery lead-times and the order-based fill rates. Topan and Bayindir [30] explore the multi-item two-echelon material inventory system with compound Poisson demand and constraints on the aggregate mean response time, which is an indicator of a target service level. In addition, the proposed algorithm is not able to be implemented since it is complicated and does not fit bulk operation.

In response to these shortcomings, this paper is based on the electric power material demand data of Shanghai Electric Power Company and develops a method to determine the optimal order quantity with the (T, Q) policy, subject to the compound Erlang demand. Empirical analysis is conducted to verify the superiority of the proposed theoretical model in cost reductions, as compared to the Newsvendor model. This paper also proposes an approximation and a heuristic algorithm to overcome the inconvenience and low efficiency of existing algorithms in the real world.

3. The Optimal Order Quantity Model with Compound Erlang Demand and (T, Q) Policy

3.1. Problem Description and Model Assumptions

3.1.1. Problem Description. This paper explores the inventory control of electric power materials with intermittent demand, which is a rather challenging task with tremendous cost implications for companies holding these inventories. The relevant cost mainly consists of holding cost incurred for each unit kept in stock and backordering cost arising from excess demand. Intermittent demand is modelled as a more realistic compound Erlang- k distribution which has an IFR. We

consider a cost-oriented single echelon single-item inventory system where unfilled demand is backordered and lead time is deterministic. The stock level is controlled according to a periodic review (T, Q) policy, which means that an order with fixed-amount quantity Q is triggered every inspection interval T . The stock level is recorded as holding cost if Q exceeds the total demand sizes within the lead time and as backordering cost if not.

3.1.2. Model Assumptions. This section shows relevant notations and assumptions of the inventory control model proposed in this paper.

We introduce the following notations:

L : lead time (constant)

Q : order quantity

X : demand size (random variable)

θ_X : probability distribution function of demand sizes

β : mean of nonzero demand sizes

λ : mean demand arrival rate

k : the number of Erlang stages

T : inspection interval

h : inventory holding cost per SKU (Stock Keeping Unit) per unit of time

b : inventory backordering cost per SKU per unit of time

i : the complex number

For the proposed model, the following assumptions are made.

(1) A single echelon single-item inventory system, which means that only a particular item in a particular echelon of the supply chain is considered in the inventory system.

(2) A periodic review (T, Q) policy, where periodic inspections are performed every cycle time of T and a fixed-amount Q order quantity is subsequently generated. This policy means that organizations do nothing to control inventory within a cycle.

(3) The compound Erlang demand process, where demand arrivals are governed by Erlang distribution and demand sizes are governed by an arbitrary nonnegative probability distribution.

(3) Constant lead time.

(4) The total inventory cost consisting of inventory holding cost and backordering cost. Inventory holding cost is incurred for each unit kept in stock while backordering cost is due to the excess demand.

3.2. Model Analysis. The model proposed in this paper presents demand as a compound Erlang process where demand arrivals are governed by an Erlang- k distribution and demand sizes are governed by an arbitrary nonnegative probability distribution. According to the above assumptions, the total expected inventory cost consists of the holding cost h and backordering cost b . Assuming that the probability of no-demand arrivals during the lead time is P_0 , the expected

cost equals hQP_0 . When there are m demand arrivals with a total size of X_m during the lead time, the inventory holding cost constitutes the total cost when $Q - X_m > 0$, while the backordering cost constitutes the total cost when $Q - X_m < 0$. Thus, the expected total cost is given as follows:

$$\begin{aligned} E[C(Q)] &= hQP_0 \\ &+ \sum_{m=1}^{+\infty} \left[h \int_0^Q (Q - X_m) P_m d\theta_{X_m}^{(m)}(X_m) \right. \\ &\left. + b \int_Q^{+\infty} (X_m - Q) P_m d\theta_{X_m}^{(m)}(X_m) \right] \end{aligned} \quad (1)$$

Note that the probability distribution of X_m demand size $\theta_{X_m}^{(m)}$ is the m -fold iterated convolution of θ_X . Besides, the probability of m demand arrivals within the lead time P_m in formula (1) can be derived based on the queuing and renewal theories (Saidane et al. [26]):

$$P_m = \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(km+i)}}{(km+i)!} e^{-k\lambda L} \quad (m \geq 0) \quad (2)$$

The objective of this paper is to derive the optimal order quantity Q^* , minimizing the expected total cost. Since $E[C(Q)]$ is a convex function, it can be obtained by setting the derivative of the expected total cost with respect to Q equal to zero:

$$\frac{dE[C(Q)]}{dQ} = hP_0 + \sum_{m=1}^{+\infty} [(h+b)\theta_{X_m}^{(m)}(Q) - b] P_m = 0 \quad (3)$$

Proposition 1. The optimal order quantity Q^* is the solution of

$$\begin{aligned} &\sum_{m=1}^{+\infty} \theta_{X_m}^{(m)}(Q^*) \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(km+i)}}{(km+i)!} e^{-k\lambda L} \\ &= \frac{b}{h+b} - \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(i)}}{i!} e^{-k\lambda L} \end{aligned} \quad (4)$$

Proof. The optimal order quantity Q^* is the solution of the derivative of the expected total cost with respect to Q which equals zero:

$$\frac{dE[C(Q)]}{dQ} = 0 \quad (5)$$

which is equivalent to

$$\frac{dE[C(Q)]}{dQ} = hP_0 + \sum_{m=1}^{+\infty} [(h+b)\theta_{X_m}^{(m)}(Q) - b] P_m = 0 \quad (6)$$

Thus

$$\sum_{m=1}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m = \frac{b}{h+b} \sum_{m=1}^{+\infty} P_m - \frac{h}{h+b} P_0 \quad (7)$$

Considering that $\sum_{m=1}^{+\infty} P_m = 1 - P_0$, the optimal order quantity Q^* can be derived from

$$\begin{aligned} &\sum_{m=1}^{+\infty} \theta_{X_m}^{(m)}(Q^*) \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(km+i)}}{(km+i)!} e^{-k\lambda L} \\ &= \frac{b}{h+b} - \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(i)}}{i!} e^{-k\lambda L} \end{aligned} \quad (8)$$

Since formula (8) is the infinite sum, this paper uses an upper bound Q_U and a lower bound Q_L to approximate the optimal order quantity Q^* . In order to derive Q_U and Q_L , we denote that

$$\begin{aligned} G(X_m, n) &= \sum_{m=1}^n \theta_{X_m}^{(m)}(Q) P_m - \frac{b}{h+b} + P_0 \\ &+ \left(1 - \sum_{m=0}^n P_m \right) \end{aligned} \quad (9)$$

$$H(X_m, n) = \sum_{m=1}^n \theta_{X_m}^{(m)}(Q) P_m - \frac{b}{h+b} + P_0$$

$$O(X_m) = \sum_{m=1}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m - \frac{b}{h+b} + P_0$$

□

Property 2. $H(X_m, n) \leq O(X_m) \leq G(X_m, n)$, for all $X_m, n \geq 0$.

Proof.

$$\begin{aligned} O(X_m) &= \sum_{m=1}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m - \frac{b}{h+b} + P_0 \\ &= \sum_{m=1}^n \theta_{X_m}^{(m)}(Q) P_m - \frac{b}{h+b} + P_0 \\ &+ \sum_{m=n}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m \\ &= H(X_m, n) + \sum_{m=n}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m \end{aligned} \quad (10)$$

Since $0 \leq \theta_{X_m}^{(m)}(Q) \leq 1$ for any $X_m, m \geq 0$,

$$0 \leq \sum_{m=n}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m \leq \sum_{m=n}^{+\infty} P_m, \quad \text{for all } X_m, n \geq 0 \quad (11)$$

Thus $H(X_m, n) \leq O(X_m, n)$:

$$\begin{aligned} O(X_m) &= \sum_{m=1}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m - \frac{b}{h+b} + P_0 \\ &= \sum_{m=1}^n \theta_{X_m}^{(m)}(Q) P_m - \frac{b}{h+b} + P_0 \end{aligned}$$

$$\begin{aligned}
& + \left(1 - \sum_{m=0}^n P_m\right) + \sum_{m=n+1}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m \\
& - \left(1 - \sum_{m=0}^n P_m\right) \\
& = G(X_m, n) - \sum_{m=n+1}^{+\infty} [1 - \theta_{X_m}^{(m)}(Q)] P_m
\end{aligned} \tag{12}$$

Since $0 \leq 1 - \theta_{X_m}^{(m)}(Q) \leq 1$, $\sum_{m=n+1}^{+\infty} [1 - \theta_{X_m}^{(m)}(Q)] P_m \geq 0$.

Thus $O(X_m) \leq G(X_m, n)$.

To sum up, $H(X_m, n) \leq O(X_m) \leq G(X_m, n)$. $\square$

Property 3. $H(X_m, n)$, $O(X_m)$, and $G(X_m, n)$ are strictly increasing functions of X_m .

Proof. Since $H(X_m, n)$, $O(X_m)$, and $G(X_m, n)$ are sums of varied cumulative probability distribution functions which are strictly increasing, these three functions are strictly increasing. $\square$

Property 4.

$$\begin{aligned}
\lim_{n \rightarrow +\infty} G(X_m, n) & \rightarrow O(X_m), \\
\lim_{n \rightarrow +\infty} H(X_m, n) & \rightarrow O(X_m)
\end{aligned} \tag{13}$$

Proof. When n tends to infinity, $\sum_{m=n}^{+\infty} \theta_{X_m}^{(m)}(Q) P_m$ and $\sum_{m=n+1}^{+\infty} [1 - \theta_{X_m}^{(m)}(Q)] P_m$ tend to zero.

Assume that Q_L , Q_U , and Q^* are the solutions of $G(X_m, n) = 0$, $H(X_m, n) = 0$, and $O(X_m) = 0$, respectively. Based on the above three properties, the following Proposition 5 is correct when n tends to infinity. $\square$

Proposition 5. Q^* can be approximated by an upper bound Q_U and a lower bound Q_L based on $Q_L \leq Q^* \leq Q_U$, where Q_U is the solution of

$$\begin{aligned}
& \sum_{m=1}^n \theta_{X_m}^{(m)}(Q) \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(km+i)}}{(km+i)!} e^{-k\lambda L} \\
& = \frac{b}{h+b} - \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(i)}}{i!} e^{-k\lambda L}
\end{aligned} \tag{14}$$

and Q_L is the solution of

$$\begin{aligned}
& \sum_{m=1}^n \theta_{X_m}^{(m)}(Q) \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(km+i)}}{(km+i)!} e^{-k\lambda L} \\
& = \frac{b}{h+b} - \sum_{i=0}^{k-1} \frac{(k\lambda L)^{(i)}}{i!} e^{-k\lambda L} \\
& - \left(1 - \sum_{m=0}^n \left(\sum_{i=0}^{k-1} \frac{(k\lambda L)^{(km+i)}}{(km+i)!} e^{-k\lambda L}\right)\right)
\end{aligned} \tag{15}$$

3.3. Basic Algorithm. Based the above analysis, the algorithm to compute the optimal order quantity Q^* is comprised the following steps.

Step 0. For a given arbitrarily small value δ ($\delta > 0$), initialize $n = 1$.

Step 1. Compute $1 - \sum_{m=0}^n P_m = 1 - \sum_{m=0}^n (\sum_{i=0}^{k-1} ((k\lambda L)^{(km+i)}) / ((km+i)!) e^{-k\lambda L})$.

Step 2

(i) If $1 - \sum_{m=0}^n P_m \leq \delta$, go to Step 3.

(ii) Otherwise do $n = n + 1$ and go to Step 1.

Step 3. Utilize the derived n to solve (15) so as to obtain Q^* .

Step 4. Stop.

In order to help understand the basic algorithm, all above procedures are illustrated in the flow chart, i.e., Figure 1.

In this flow chart, the optimal order quantity Q^* is approximated by an upper bound Q_U and a lower bound Q_L , both of which are calculated with finite varying n . As n gets larger, the lower and upper bounds get tighter and the gap between them gets smaller. The algorithm cannot be stopped until the gap is smaller than the given stopping criterion δ . In this circumstance, Q_L is obtained by substituting the corresponding n into formula (15), which is considered as an alternative to the optimal order quantity Q^* . In addition, it should be noted that Q^* can be approximately as closely as desired by setting varied stopping criterion δ .

FIGURE 2: The scatter diagram of demand interarrival rate of electric power materials.

4. Numerical Investigation

4.1. Data Processing. There are 3,839 types of electric power materials in the State Grid Shanghai Electric Power Company. Because most of the 3,839 electrical power materials are relatively less important (e.g., circuits), we do not need to pay much attention to the inventory management for these electrical power materials with the model proposed in our paper. Moreover, for those electrical power materials which are also suitable for our model, some of them are not standardized and therefore eliminated (or are being eliminated). For standardized electrical power materials, due to similarity, only representative electrical power materials are selected. Thereby, we gather the historical data of demands of 53 types of electric power materials from 2015 to 2017, the demand arrivals of which follow an Erlang distribution in conjunction with a Gamma distribution of demand sizes. Since the Gamma distribution covers a large spectrum of probability distributions, such modelling is more appropriate. Basic information of the 53 types of electric power materials is presented in Table 1.

As shown in Table 1, the mean nonzero demand sizes β of the 53 types of electric power materials vary sharply, for example, the minimum mean nonzero demand size β is 3.19 while the maximum mean nonzero demand size β is 10437.16. Moreover, the variance of the mean nonzero demand size β is 3041691. In a word, there exists a high volatility of mean nonzero demand size. And demand arrivals of the 53 types of electric power materials mostly follow a two-stage Erlang distribution. There are only four electric power materials consisting of 500014661, 500017325, 500020638 and 500064132 whose demand interarrivals follow three stage Erlang distribution.

Figure 2 shows the scatter diagram of demand arrival rate of the 53 types of electric power materials

As shown in Figure 2, the mean demand interarrival rate λ of these 53 types of electric power materials within 3 years is between 0.01 and 0.52, denoted by $\lambda \in [0.01, 0.52]$. The variance of the mean demand interarrival rate λ is 0.019, reflecting its low volatility. According to the classification method proposed by Eaves [31], these electric power materials exhibit a high level of intermittence.

Subsequently, we adopt the above model to model the inventory cost of these materials, compute the optimal order

FIGURE 3: Average inventory costs calculated with the two models.

quantity of every electric power material and the corresponding average inventory cost and then compare them with results of the classic Newsvendor model (Trabka and Marchand [32]) to examine the validity of this method.

4.2. Comparison with the Newsvendor Model. The aim of this subsection is to examine the performance of the proposed theoretical model in cost reduction. We initially employed this new model to obtain the optimal order quantity of each electric power material and the corresponding average inventory cost and followed this with a comparison to the classic Newsvendor model under a Gamma demand process. Considering that the backordering cost is higher than the inventory holding cost, this paper assumes that $h = 1, b = 10$. Lead-time L is assumed to be 1. Results are as in Table 2.

Table 2 illustrates the performances of the two models with respect to the optimal order quantity and inventory cost. Results show that the proposed theoretical model, i.e., new model suggests a higher optimal order quantity and lower average inventory cost compared with the Newsvendor model. Take the electric power material 500065644; for example, the optimal order quantity Q obtained with the new model is 32, which is approximately 4.5 times of that obtained with the Newsvendor model which equals 7. However, in terms of average inventory cost C , the new model suggests a cost of 30.63 which is significantly less than that suggested by the Newsvendor model which equals 2162199. According to Table 2, other electric power materials show similar pattern as the above material 500065644.

In order to more vividly demonstrate the significant difference of average inventory cost C between these two models, we make the line chart, i.e., Figure 3.

Since that there is huge gap between the average inventory cost C calculated with the above two models, we use the natural logarithm of average inventory cost C as the ordinate so as to narrow the gap. As shown in the Figure 3, the average inventory cost of new model is significantly less than that of Newsvendor model, which is consistent with the finding revealed by the Table 2. In a word, larger optimal order quantity and lower average inventory cost are advised by the new model. The conflicting change in these two variables can be explained by the fact that although higher order quantity increases the amount of electric power materials kept in stock and thus holding cost, the backordering cost decreases due

TABLE 1: Basic information of 53 types of electric power materials.

State grid code of spare parts	Class	Division	Section	k	β	λ
500012723	Hardware	Mechanical hardware	Bolt	2	2137.77	0.47
500012771	Hardware	Mechanical hardware	Bolt	2	840.70	0.39
500012797	Hardware	Mechanical hardware	Bolt	2	352.31	0.06
500012801	Hardware	Mechanical hardware	Bolt	2	118.76	0.13
500014661	Installation	Wire	Aerial insulation wire	3	9.47	0.51
500014662	Installation	Wire	Aerial insulation wire	2	14.37	0.23
500014670	Installation	Wire	Aerial insulation wire	2	37.59	0.46
500014711	Installation	Wire	Aerial insulation wire	2	5.18	0.41
500014713	Installation	Wire	Aerial insulation wire	2	4.53	0.35
500014805	Installation	Cable	Cloth wire	2	10437.16	0.05
500016589	Installation	Cable	Control cable	2	1161.67	0.20
500016592	Installation	Cable	Control cable	2	1080.33	0.16
500017293	Installation	Insulator	Post porcelain insulator	2	254.76	0.33
500017320	Installation	Insulator	Strain insulator	2	279.17	0.19
500017324	Installation	Fitting	Shackle insulator	2	6474.62	0.42
500017325	Installation	Fitting	Shackle insulator	3	1849.46	0.52
500020609	Installation	Cable accessories	Splicing fitting—GT-G	2	27.05	0.26
500020612	Installation	Cable accessories	Splicing fitting—GT-G	2	3091.22	0.07
500020638	Installation	Cable accessories	Splicing fitting—GTL	3	22.04	0.01
500021412	Installation	Cable accessories	35kV cable connector	2	6.64	0.19
500021803	Installation	Cable accessories	Cable terminals	2	79.97	0.24
500021830	Installation	Low voltage apparatus	Cable terminals	2	280.81	0.33
500021862	Installation	Low voltage apparatus	Cable terminals	2	1363.48	0.30
500021869	Installation	Surge arrester	Cable terminals	2	305.76	0.33
500022269	Low voltage apparatus	Traverse, earth wire	Low-tension fuse	2	218.86	0.19
500022271	Low voltage apparatus	Fitting	Low-tension fuse	2	2293.08	0.18
500027151	Primary equipment	High-pressure fuse fittings	AC surge arrester	2	210.94	0.43
500027464	Installation	High-tension fuse accessories	Aerial insulation traverse	2	5.05	0.26
500029690	Installation	High-tension fuse accessories	Guy wire fitting—wedge clamps	2	280.30	0.47
500032681	Spare part	High-tension fuse accessories	Fuse	2	85.64	0.31
500032682	Spare part	Iron fittings	Fuse	2	27.35	0.03
500032684	Spare part	Mechanical hardware	Fuse	2	151.88	0.47
500032687	Spare part	Cable accessories	Fuse	2	60.44	0.36
500035137	Installation	Mechanical hardware	Anchor rod	2	250.67	0.44
500051829	Hardware	Low voltage apparatus	Bolt	2	349.51	0.32
500053694	Installation	Low voltage apparatus	35kV cable termination	2	8.26	0.16
500058470	Hardware	Security burglary fire	Bolt	2	1547.98	0.30
500061526	Low voltage apparatus	Security burglary fire	Low-tension fuse accessories	2	131.57	0.22
500061564	Low voltage apparatus	Fitting	Low-tension fuse accessories	2	113.72	0.31
500061758	Supplementary equipment	Fitting	Anti-lighting device	2	211.26	0.25
500062282	Supplementary equipment	Traverse, earth wire	Anti-lighting device	2	163.65	0.18
500063419	Installation	Traverse, earth wire	Strain clamps—pre-twisted type	2	166.65	0.33
500064132	Installation	Iron fittings	Guy wire fittings—guy clips	3	1465.33	0.51
500065644	Installation	Mechanical hardware	Aerial insulation traverse	2	3.19	0.48
500065658	Installation	Mechanical hardware	Aerial insulation traverse	2	31.22	0.44
500066495	Installation	Fittings	Iron connection	2	2827.08	0.31
500067251	Hardware	Iron fittings	Bolt	2	352.50	0.19
500067292	Hardware	Iron fittings	Bolt	2	469.55	0.43
500067551	Installation	Iron fittings	Guy wire fittings—guy clips	2	581.01	0.27

TABLE 1: Continued.

State grid code of spare parts	Class	Division	Section	k	β	λ
500067567	Installation	Description of the class	Cable hold hoop	2	56.25	0.13
500067652	Installation	Mechanical hardware	Semicircular hold hoop	2	92.10	0.28
500067654	Installation	Mechanical hardware	Semicircular hold hoop	2	202.46	0.29
500016521	Installation	Mechanical hardware	Control cable	2	576.27	0.15

FIGURE 4: The average inventory cost C with varying lead time L for the electric power materials with compound Erlang-distributed demand.

to the less unfilled demand, leading to a reduction in total inventory cost. Thus the new model outperforms the classic Newsboy model in cost reductions, which is more suitable for cost-oriented inventory system

4.3. Sensitivity Analysis of Lead Time. In this subsection, we vary the value of lead time L in order to analyze its impacts on the average inventory cost C . Thus we consider the various values of lead time L in the interval $[1, 10]$ and compute the corresponding average inventory cost C respectively. The outcomes are shown in Table 3 and Figure 4.

As shown in the Table 3, an overwhelming percent of electric power materials with compound Erlang-distributed demand demonstrates a nonlinear relationship between lead time L and average inventory cost C . Specifically speaking, as lead time L increases, the average inventory cost C of most electric power material rises first and then falls. For example, for electric power material 500012723, the corresponding average inventory cost C experiences an increase from 0.78 to 1.88 as the lead time L increases from 1 to 3. After reaching its peak of 1.88, the average inventory cost begins to decrease and reaches 0.29 when lead time is 10.

In order to more vividly depict the effect of lead time on the average inventory cost, we make the line chart in Figure 4 based on Table 3.

Figure 4 illustrates that, for most electric power material with compound Erlang-distributed demand, average inventory cost initially rises in the lead time and subsequently this trend reverses. However, there are three types of electric power material such as 500012797, 500020612, and 500032682 which do not show the above relationship. Instead, the average inventory cost rises with the lead time. Maybe it is because that the lead time is not large enough. If lead time is larger, the corresponding inventory cost maybe will provide

the same pattern as the other most electric power material with compound Erlang-distributed demand.

5. The Approximation and Heuristic Algorithm

Given the complexity of the basic algorithm, this paper proposes easily implementable approximation and heuristic algorithms for determining the optimal order quantity under the (T, Q) policy. These two methods simplify complicated mathematical calculation and solve the lot-sizing problem, which are of great utility in the real industrial context.

5.1. The Approximation Algorithm. This paper proposes an approximation algorithm to determine the optimal order quantity by seeking the minimum iterative numbers within the desired accuracy under the varied average demand interarrival rate λ and average nonzero demand size β . Thus, the calculation process is simplified due to reduced iterative numbers. Electric power materials vary with the iterative number corresponding to the optimal order quantity in this proposed model. If practitioners control material inventory and determine the optimal order quantity based on the above iterative numbers, the efficiency of inventory management will be rather low due to the wide range of electric power materials. Therefore, if the iterative number can be reduced while guaranteeing accuracy, the simplification can significantly increase companies' efficiency.

Calculation is as follows.

Step 1. Adopt the basic algorithm to calculate the optimal iterative numbers of each electric power material with different λ, β, k . When the value of $1 - \sum_{m=0}^n P_m$ in formula (15) converges to zero, the right side of formula (14) and (15) is almost equal. The corresponding iterative number is the optimal, denoted by n^* . We assumed that the optimal iterative number n^* is derived when δ equals 10^{-4} or equivalently $1 - \sum_{m=0}^n P_m \leq 10^{-4}$.

Step 2. Calculate the corresponding optimal order quantity Q^* .

Step 3. Calculate the effective iterative number within the acceptable error range, denoted by $\tilde{n}$. We assume that the acceptable error range is 10%.

The effective iterative number $\tilde{n}$ is derived as in Figure 5.

In this flow chart, Q^* and $\tilde{n}$ are derived by comparing the corresponding order quantity Q_n with optimal order quantity Q^* . This algorithm can be stopped when the difference between Q_n and Q^* is small enough and conforms to the

TABLE 2: The optimal order quantity and corresponding average inventory cost of the two methods.

State grid code of materials	Optimal order quantity Q		Average inventory cost C	
	New model	Newsvendor model	New model	Newsvendor model
500012723	7475	4697	0.78	2
500012771	3669	1847	0.91	22
500012797	935	774	1.32	154
500012801	613	261	1.59	1490
500014661	77	21	24.03	244424
500014662	116	32	5.23	106020
500014670	283	83	3.91	15360
500014711	50	11	14.51	819737
500014713	44	10	14.65	1069736
500016589	3987	2552	0.99	10
500016592	3506	2374	1.02	12
500017293	1392	560	1.22	307
500017320	1332	613	1.22	253
500017325	5282	4064	0.89	3
500020609	203	59	3.63	29781
500020612	2557	6792	1.08	1
500021412	57	15	9.11	498517
500021803	505	176	1.89	3338
500021830	1507	617	1.19	250
500021862	5069	2996	0.91	7
500021869	1615	672	1.16	209
500022269	1111	481	1.30	422
500022271	6100	5038	0.96	1
500027151	1219	463	1.35	455
500027464	47	11	11.33	863026
500029690	1541	616	1.22	251
500032681	559	188	1.91	2904
500032682	110	60	4.02	29123
500032684	930	334	1.64	899
500032687	421	133	2.45	5889
500035137	1408	551	1.26	317
500051829	1794	768	1.12	157
500053694	67	18	7.43	321886
500058470	5567	3401	0.91	5
500061526	753	289	1.52	1208
500061564	707	250	1.67	1629
500061758	1145	464	1.30	454
500062282	862	360	1.41	771
500063419	980	366	1.43	743
500064132	4528	3220	0.94	5
500065644	32	7	30.63	2162199
500065658	240	69	4.39	22318
500066495	8558	6212	0.88	1
500067251	1622	775	1.16	154
500067292	2340	1032	1.01	83

TABLE 2: Continued.

State grid code of materials	Optimal order quantity Q		Average inventory cost C	
	New model	News vendor model	New model	News vendor model
500067551	2601	1277	1.02	51
500067567	329	124	2.18	6809
500067652	585	202	1.81	2505
500067654	1135	445	1.32	496

FIGURE 5: The flow chart of calculation of $\tilde{n}$ under the approximation approach.

predefined criterion. The effective iterative number $\tilde{n}$ is thus obtained.

Based on the above approximation algorithm, Q^* and $\tilde{n}$ under varied λ , β , k are calculated and the results are shown in Appendices A and B (Figures 7 and 8 correspond to the result of $\tilde{n}$ and Figures 9 and 10 correspond to the result of Q^*). It reveals that there is an overwhelming percentage of electric power materials whose effective iterative numbers $\tilde{n}$ equal 2, while the effective iterative number for the remaining electric power materials is 1. This means, for most electric power material, the approximately optimal order quantity Q^* can be derived after only two iterations under the acceptable error range of 10 percent. Given the positive relationships between the accuracy and the number of iterations, the uniform iterative number for all electric power materials of electric power companies with a compound Erlang- k demand distribution is set to be 2. Thus practitioners can calculate the order quantity through two iterations for all electric power material with compound Erlang-distributed demand.

5.2. The Heuristic Algorithm. Given the complexity of the basic algorithm and the shortage of relevant experts in enterprises, this paper proposes a more implementable heuristic

algorithm. This algorithm attempts to generalize the relationship between average demand interarrival rate λ and the optimal order quantity Q^* for varied β and k . In this paper, we utilize the scatter diagrams to illustrate their relationships in the cases of k is equal to 2 and 3.

When $k = 2$, the relationships between the independent variable λ and the dependent variable Q^* under varied β are as in Figure 6.

Because of space constraints, the relationships between λ and Q^* with respect to varied β when $k = 3$ are shown in the Figure 11 which is put in Appendix C.

As shown in Figures 6 and 11, the optimal quantity Q^* also increases when the average demand arrival rate λ increases. As λ increases, i.e., the intermittence get weaker, the optimal order quantity Q^* increases. Such relationship implies that for electric power materials whose demand exhibits a higher intermittence, it is more economic for organizations to experience stock-outs occasionally than to keep high quantities in stock. Moreover, as λ rises, the optimal order quantity tends to be deterministic. In addition, keeping λ constant, the optimal order quantity Q^* grows with the mean nonnegative demand sizes β . Moreover, the optimal order quantity Q^* also increases with k . As k increases, i.e., the degree of certainty on demand arrival increases, the order quantity increases. For instance, when β is 500 and λ is 0.5, the optimal order quantity Q^* is from 750 to 3000, respectively, when k shifts from 2 to 3.

Based on the above analysis, the relationships between λ and Q^* can be generalized as

$$Q^* = c - e^{-a\lambda} \quad (a \geq 0) \quad (16)$$

Subsequently, we employ the calculated λ and Q^* as inputs to determine parameters c and a , followed by an analysis of accuracy. Results are as in Table 4.

As the average demand sizes β increases, the parameter c increases obviously while the parameter a remains relatively stable. When λ is very small, i.e., demand interval T is large, the optimal order quantity Q^* is approximately equal to parameter c . In this case, when average demand size is large, coupled with long demand interval, organizations will order more quantity to satisfy the large demand, leading to the rise in parameter c . This is the reason why c grows with β . Moreover, parameters c and a decrease when k increases; i.e., the certainty degree on demand arrivals increases. In addition, MAPE is computed to examine the goodness of fit of these relationships to real data. The value of MAPE is mostly around 15%, indicating that this algorithm is effective in the practical application. Moreover, MAPE gets larger with

TABLE 3: Average inventory cost C of varying lead time L for the electric power materials with compound Erlang distributed demand.

State grid code of materials	Average inventory cost C									
	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$	$L=7$	$L=8$	$L=9$	$L=10$
500012723	0.78	1.61	1.88	1.75	1.46	1.13	0.71	0.60	0.42	0.29
500012771	0.91	2.08	2.65	2.64	2.30	1.85	1.21	1.03	0.73	0.51
500012797	1.32	4.89	10.19	16.79	24.31	32.46	45.31	49.66	58.34	66.88
500012801	1.59	5.28	9.81	14.35	18.38	21.65	24.90	25.56	26.29	26.33
500014661	24.03	62.03	65.13	46.20	26.11	12.70	3.57	2.25	0.86	0.31
500014662	5.23	14.90	23.58	29.15	31.40	30.93	27.04	25.29	21.55	17.84
500014670	3.91	7.74	8.31	6.88	4.92	3.20	1.49	1.13	0.63	0.34
500014711	14.51	31.23	36.63	33.20	26.00	18.52	9.88	7.82	4.77	2.82
500014713	14.65	34.44	44.39	44.35	38.38	30.25	18.82	15.69	10.62	6.97
500016589	0.99	3.02	5.17	7.00	8.33	9.14	9.54	9.49	9.21	8.73
500016592	1.02	3.26	5.87	8.36	10.48	12.11	13.63	13.92	14.20	14.16
500017293	1.22	3.02	4.11	4.36	4.02	3.40	2.37	2.06	1.51	1.08
500017320	1.22	3.76	6.47	8.75	10.35	11.25	11.48	11.32	10.74	9.93
500017325	0.89	2.46	3.08	2.94	2.48	1.98	1.31	1.13	0.82	0.58
500020609	3.63	9.92	15.00	17.68	18.15	17.02	13.81	12.59	10.20	8.03
500020612	1.08	3.93	8.10	13.23	19.03	25.28	35.15	38.51	45.25	51.98
500021412	9.11	27.71	46.94	62.28	72.13	76.53	74.93	72.80	66.98	59.92
500021803	1.89	5.37	8.48	10.47	11.28	11.12	9.74	9.12	7.79	6.48
500021830	1.19	2.93	3.99	4.25	3.93	3.33	2.33	2.03	1.49	1.07
500021862	0.91	2.41	3.56	4.14	4.24	4.01	3.36	3.11	2.61	2.15
500021869	1.16	2.86	3.90	4.15	3.85	3.27	2.30	2.00	1.48	1.06
500022269	1.30	3.93	6.66	8.85	10.30	10.99	10.89	10.63	9.90	8.97
500022271	0.96	3.02	5.36	7.53	9.35	10.74	12.07	12.33	12.62	12.66
500027151	1.35	2.84	3.26	2.92	2.26	1.60	0.85	0.68	0.42	0.25
500027464	11.33	31.10	47.26	56.03	57.78	54.46	44.48	40.65	33.07	26.12
500029690	1.22	2.42	2.61	2.19	1.59	1.06	0.52	0.40	0.23	0.13
500032681	1.91	4.80	6.65	7.17	6.72	5.75	4.06	3.54	2.61	1.88
500032682	4.02	15.43	33.35	56.92	85.37	117.99	173.30	193.13	234.49	277.68
500032684	1.64	3.18	3.36	2.74	1.94	1.25	0.58	0.44	0.24	0.13
500032687	2.45	5.69	7.25	7.17	6.14	4.80	2.96	2.46	1.65	1.08
500035137	1.26	2.59	2.90	2.53	1.91	1.32	0.68	0.53	0.32	0.19
500051829	1.12	2.79	3.86	4.17	3.92	3.38	2.43	2.13	1.61	1.18
500053694	7.43	23.73	42.31	59.23	72.48	81.35	86.83	86.86	84.77	80.47
500058470	0.91	2.39	3.53	4.12	4.24	4.03	3.41	3.16	2.68	2.22
500061526	1.52	4.45	7.26	9.27	10.33	10.56	9.77	9.32	8.28	7.15
500061564	1.67	4.24	5.94	6.49	6.16	5.35	3.87	3.40	2.55	1.86
500061758	1.30	3.60	5.55	6.70	7.05	6.80	5.78	5.35	4.49	3.67
500062282	1.41	4.38	7.58	10.30	12.25	13.36	13.68	13.51	12.84	11.89
500063419	1.43	3.52	4.80	5.10	4.71	3.97	2.76	2.39	1.75	1.24
500064132	0.94	2.63	3.29	3.07	2.52	1.93	1.21	1.02	0.72	0.49
500065644	30.63	58.31	60.09	47.63	32.54	20.18	8.72	6.43	3.40	1.75
500065658	4.39	9.00	10.03	8.63	6.41	4.34	2.14	1.65	0.96	0.54
500066495	0.88	2.32	3.47	4.12	4.33	4.24	3.77	3.57	3.15	2.73
500067251	1.16	3.52	5.98	7.98	9.33	10.01	10.03	9.83	9.23	8.44
500067292	1.01	2.14	2.50	2.27	1.80	1.30	0.73	0.59	0.37	0.23
500067551	1.02	2.79	4.22	5.01	5.20	4.97	4.18	3.87	3.24	2.64
500067567	2.18	7.27	13.62	20.06	25.89	30.69	35.64	36.70	37.97	38.24
500067652	1.81	4.80	7.03	8.03	7.99	7.27	5.64	5.06	3.99	3.05
500067654	1.32	3.43	4.93	5.54	5.43	4.86	3.69	3.30	2.57	1.94

FIGURE 6: Continued.

FIGURE 6: The relationships between λ and Q^* with respect to varied β when $k = 2$.

λ	β	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	1000	3000	6000	
λ	β	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
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0	2	2	2	2	2	2																										

FIGURE 7: $\tilde{n}$ under varied λ, β when $k = 2$.

FIGURE 8: $\tilde{n}$ under varied λ, β when $k = 3$.

6. Conclusion and Future Research

Currently, the compound Poisson distribution is the most often utilized model for intermittent demand to control

λ	β	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	1000	3000	6000
λ	β	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*
0.01	1	6	9	13	16	18	21	23	25	27	29	44	54	60	63	65	65	63	61	57	0	0	0	0	0	0	0	0	0	0	0
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λ	β	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	1000	3000	6000
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λ	β	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	1000	3000	6000
λ	β	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*	Q*
0.03	1	5	8	11	13	15	17	19	20	22	23	32	35	35	32	28	21	14	5	0	0	0	0	0	0	0	0	0	0	0	0
0.04	1	6	10	13	16	19	22	24	27	29	31	48	59	67	74	76	78	79	78	76	12	0	0	0	0	0	0	0	0	0	0
0.05	1	6	11	15	19	22	26	29	32	34	37	60	78	92	104	113	121	128	133	137	136	82	0	0	0	0	0	0	0	0	0
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0.11	1	9	15	21	27	33	38	43	48	53	58	101	139	174	206	236	265	292	318	342	546	697	814	906	977	1032	1072	1100	1117	46	0
0.12	1	9	16	22	28	34	39	45	50	55	60	105	145	182	212	249	279	309	337	363	588	759	897	1010	1102	1178	1239	1288	1335	671	0
0.13	1	9	16	22	29	35	40	46	51	57	62	109	151	190	226	260	293	324	353	382	625	816	972	1103	1214	1309	1389	1456	1512	1233	0
0.14	1	9	16	23	29	35	41	47	53	58	63	112	156	197	234	270	304	337	369	399	659	866	1040	1188	1316	1427	1524	1609	1682	1741	0
0.15	1	9	17	23	30	36	42	48	54	59	65	115	161	203	242	279	315	349	382	414	690	913	1101	1265	1408	1535	1647	1747	1836	2203	225
0.16	1	9	17	24	30	37	43	49	55	61	66	118	165	208	249	288	325	361	395	428	718	955	1158	1335	1493	1633	1760	1874	1976	2624	1067
0.17	1	10	17	24	31	38	44	50	56	62	68	121	169	213	256	296	334	371	407	441	743	993	1209	1399	1570	1723	1863	1989	2105	3010	1838
0.18	1	10	17	25	31	38	45	51	57	63	69	123	172	218	261	303	342	380	417	453	767	1029	1256	1458	1641	1806	1957	2095	2223	3364	2546
0.19	1	10	18	25	32	39	45	52	58	64	70	125	176	223	267	309	350	389	427	464	789	1061	1300	1513	1706	1882	2044	2193	2331	3689	3797
0.2	1	10	18	25	32	39	46	53	59	65	71	127	179	227	272	315	357	397	436	474	809	1091	1340	1563	1766	1952	2124	2283	2431	3899	4287
0.21	1	10	18	26	33	40	46	53	59	66	72	129	181	230	276	321	363	404	443	483	827	1119	1377	1609	1821	2017	2198	2366	2431	4266	4879
0.22	1	10	18	26	33	40	47	54	60	66	73	133	184	234	281	326	369	411	452	492	844	1144	1411	1652	1872	2076	2266	2443	2609	4522	4868
0.23	1	10	18	26	33	40	47	54	61	67	73	133	186	237	285	331	375	418	459	500	860	1168	1442	1691	1920	2132	2329	2514	2688	4759	5336
0.24	2	11	20	28	37	45	52	60	67	74	82	149	211	269	332	379	432	483	532	581	1023	1412	1768	2098	2408	2701	2980	3246	3502	7201	10242
0.25	2	11	20	29	37	45	53	61	69	76	84	153	219	277	335	391	446	498	550	601	1062	1442	1883	2242	2580	2902	3210	3505	3789	7789	11422
0.26	2	11	20	29	37	45	53	61	69	77	85	156	222	284	344	402	458	513	566	618	1098	1525	1918	2285	2633	2963	3280	3583	3876	8324	12496
0.27	2	11	21	29	38	45	54	62	69	77	86	158	224	288	348	407	464	519	574	627	1114	1530	1951	2327	2683	3022	3346	3658	3959	8574	12977
0.28	2	11	21	30	38	46	54	62	70	78	85	156	222	284	344	402	458	513	566	618	1098	1525	1918	2285	2633	2963	3280	3583	3876	8324	12496
0.29	2	11	21	30	38	47	55	63	71	79	86	158	224	288	348	407	464	519	574	627	1114	1530	1951	2327	2683	3022	3346	3658	3959	8574	12977
0.3	2	11	21	31	40	48	55	63	71	79	87	160	227	291	352	412	469	526	581	635	1130	1533	1983	2367	2730	3077	3410	3730	4039	8813	13478
0.31	2	12	21	30	39	48	56	64	72	80	88	161	229	294	356	416	475	532	588	642	1146	1596	2013	2405	2776	3131	3471	3799	4115	9042	13938
0.32	2	12	21	30	39	48	56	65	73	81	88	163	231	297	360	421	480	538	594	650	1160	1618	2043	2441	2820	3182	3530	3865	4189	9262	14379
0.33	2	12	21	31	40	48	57	65	73	81	89	164	233	300	363	425	485	543	601	657	1174	1639	2071	2477	2862	3231	3586	3928	4259	9472	14803
0.34	2	12	22	31	40	49	57	66	74	82	90	165	236	302	367	429	490	549	607	663	1188	1660	2098	2510	2903	3279	3640	3989	4326	9675	15211
0.35	2	12	22	31	40	49	58	66	74	82	90	167	237	305	370	433	494	554	612	670	1201	1679	2124	2543	2942	3324	3692	4047	4391	9870	15558
0.36	2	12	22	31	40	49	58	66	75	83	91	168	239	307	373	437	498	559	618	676	1213	1698	2149	2574	2979	3368	3742	4103	4454	10057	15933
0.37	2	12	22	31	41	50	58	67	75	84	92	169	241	310	376	440	503	564	624	682	1225	1716	2173	2604	3015	3410	3790	4158	4514	10238	16294
0.38	2	12	22	32	41	50	59	67	76	84	92	170	243	312	379	444	507	568	629	688	1237	1733	2196	2633	3050	3450	3836	4210	4575	10411	16641
0.39	2	12	22	32	41	50	59	68	76	85	93	171	245	314	382	447	511	573	634	694	1248	1750	2218	2661	3084	3489	3881	4260	4628	10579	16976
0.4	2	12	22	32	41	50	59	68	77	85	93	172	246	317	384	450	514	577	639	699	1259	1766	2240	2688	3116	3527	3924	4308	4682	10740	17299
0.41	2	12	22	32	42	51	60	68	77	86	94	173	248	319	387	453	518	581	648	709	1279	1797	2281	2739	3177	3599	4006	4400	4784	11047	17977
0.42	2	12	22	32	42	51	60	69	77	86	94	174	249	321	390	456	522	585	648	709	1279	1797	2281	2739	3177	3599	4006	4400	4784	11047	17977
0.43	2	12	23	32	42	51	60	69	78	86	95	175	251	323	392	459	525	589	652	714	1289	1811	2300	2763	3206	3633	4044	4444	4832	11192	18202
0.44	2	12	23	33	42	51	61	69	78	87	95	176	252	324	394	462	528	593	656	719	1289	1825	2319	2787	3234	3665	4082	4486	4879	11332	18555
0.45	2	12	23	33	42	51	61	70	79	87	96	177	253	326	396	465	531	597	660	723	1307	1839	2337	2809	3257	3697	4118	4527	4924	11521	18830
0.46	2	12	23	33	42	52	61	70	79	88	96	178	255	328	399	467	534	600	664	728	1316	1852	2354	2831	3264	3727	4153	4566	4968	11599	19096
0.47	2	12	23	33	43	52	61	70	79	88	97	179	256	330	401	470	537	603	668	732	1324	1865	2371	2852	3313	3757	4187	4604	5010	11726	19353
0.48	2	12	23	33	43	52	62	71	80	88	97	180	257	331	403	472	540	607	673	736	1333	1877	2387	2873	3337	3786	4219	4641	5051	11848	19602
0.49	2	12	23	33	43	52	62	71	80	88	97	181	258	333	405	475	543	610</													

λ	β	1	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	100	200	300	400	500	600	700	800	900	1000	3000	6000
λ	β	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*	Q^*
0.55	2	13	24	34	44	54	63	72	82	91	100	185	265	341	415	487	558	627	694	761	1383	1952	2488	2998	3488	3961	4420	4885	5302	12602	21146
0.56	2	13	24	34	44	54	63	73	82	91	100	185	266	343	416	489	560	629	697	764	1389	1962	2500	3014	3507	3983	4446	4895	5333	12696	21340
0.57	2	13	24	34	44	54	63	73	82	91	100	186	267	344	419	491	561	632	700	767	1399	1967	2513	3029	3525	4005	4470	4922	5364	12787	21529
0.58	2	13	24	34	44	54	64	73	82	91	100	187	268	345	420	493	564	636	703	770	1401	1980	2524	3044	3543	4025	4493	4939	5393	12876	21713
0.59	2	13	24	34	44	54	64	73	83	92	101	187	268	346	421	495	566	638	705	773	1407	1988	2536	3058	3560	4045	4516	4975	5442	12961	21891
0.6	2	13	24	34	44	54	64	73	83	92	101	191	270	348	423	497	569	640	709	777	1415	2000	2552	3079	3585	4075	4550	5013	5464	13105	22063
0.61	3	13	24	34	45	54	64	74	83	93	102	192	271	350	426	500	569	640	711	780	1420	2009	2563	3092	3618	4113	4594	5062	5519	13369	22394
0.62	3	13	24	34	45	55	64	74	83	93	102	192	271	350	426	500	569	642	714	782	1426	2017	2574	3106	3618	4113	4594	5062	5519	13369	22394
0.63	3	13	24	35	45	55	64	74	83	93	102	193	272	351	427	502	570	642	716	785	1431	2025	2584	3119	3633	4131	4612	5085	5545	13448	22553
0.64	3	13	24	35	45	55	65	74	84	93	102	193	273	352	429	503	573	648	718	788	1436	2032	2594	3131	3648	4149	4635	5108	5570	13424	22706
0.65	3	13	24	35	45	55	65	74	84	93	102	193	273	353	430	505	575	650	720	790	1441	2040	2604	3144	3663	4149	4654	5130	5595	13498	22856
0.66	3	13	24	35	45	55	65	75	84	93	103	193	274	354	431	506	576	652	723	792	1446	2047	2614	3156	3677	4166	4673	5152	5619	13570	23001
0.67	3	13	24	35	45	55	65	75	84	94	103	194	276	356	433	509	580	655	727	797	1455	2064	2632	3167	3691	4182	4692	5172	5642	13640	23278
0.68	3	13	24	35	45	55	65	75	84	94	103	194	276	356	433	509	580	655	727	797	1455	2064	2632	3167	3691	4182	4692	5172	5642	13640	23278
0.69	3	13	24	35	45	55	65	75	85	94	103	194	276	357	434	510	581	657	729	799	1459	2067	2641	3189	3718	4230	4727	5212	5686	13774	23411
0.7	3	13	24	35	45	55	65	75	85	94	104	194	277	358	435	512	582	659	731	801	1463	2073	2649	3200	3730	4244	4744	5231	5707	13838	23540
0.71	3	13	24	35	45	56	65	75	85	94	104	195	277	358	437	513	583	661	732	803	1467	2079	2657	3210	3743	4259	4761	5250	5728	13900	23666
0.72	3	13	24	35	46	56	66	75	85	95	104	195	278	359	437	514	583	662	734	805	1471	2085	2665	3220	3755	4273	4777	5268	5748	13960	23788
0.73	3	13	24	35	46	56	66	76	85	95	104	195	278	359	437	514	583	662	734	805	1471	2085	2665	3220	3755	4273	4777	5268	5748	13960	23788
0.74	3	13	24	35	46	56	66	76	85	95	104	195	279	361	439	516	586	665	737	809	1479	2097	2680	3239	3781	4286	4792	5285	5767	14019	23906
0.75	3	13	24	35	46	56	66	76	86	95	105	195	280	361	440	517	587	667	739	811	1483	2102	2688	3248	3788	4292	4807	5295	5786	14076	24021
0.76	3	13	25	35	46	56	66	76	86	95	105	195	280	362	441	518	588	668	741	813	1486	2107	2695	3257	3789	4305	4819	5304	5804	14131	24133
0.77	3	13	25	35	46	56	66	76	86	95	105	196	282	364	444	521	591	671	744	816	1493	2113	2702	3266	3799	4337	4849	5330	5839	14237	24246
0.78	3	13	25	35	46	56	66	76	86	96	105	196	281	363	442	519	589	669	742	814	1490	2117	2708	3274	3809	4348	4863	5365	5866	14287	24349
0.79	3	13	25	36	46	56	66	76	86	96	105	196	282	364	444	521	591	672	745	818	1496	2122	2715	3282	3819	4359	4876	5379	5872	14336	24458
0.8	3	13	25	36	46	56	66	76	86	96	105	196	283	365	444	522	592	673	747	819	1499	2127	2721	3290	3829	4370	4888	5393	5888	14383	24644
0.81	3	13	25	36	46	56	66	77	86	96	106	197	283	365	446	524	594	676	749	822	1505	2136	2733	3305	3847	4391	4912	5420	5918	14474	24828
0.82	3	13	25	36	46	57	67	77	86	96	106	197	283	365	446	524	594	676	749	822	1505	2136	2733	3305	3847	4391	4912	5420	5918	14474	24828
0.83	3	13	25	36	46	57	67	77	87	96	106	198	284	367	447	525	595	677	751	823	1508	2140	2739	3312	3856	4401	4923	5433	5934	14518	24916
0.84	3	13	25	36	46	57	67	77	87	96	106	198	284	367	447	525	595	677	751	823	1508	2140	2739	3312	3856	4401	4923	5433	5934	14518	24916
0.85	3	13	25	36	46	57	67	77	87	96	106	198	284	367	448	527	597	679	753	826	1513	2148	2744	3325	3881	4420	4945	5458	5959	14600	25085
0.86	3	13	25	36	46	57	67	77	87	97	106	198	285	368	449	527	598	680	754	827	1516	2152	2749	3332	3889	4429	4955	5469	5972	14640	25165
0.87	3	13	25	36	46	57	67	77	87	97	106	198	285	368	449	528	598	681	755	829	1518	2156	2759	3338	3896	4438	4965	5480	5984	14678	25243
0.88	3	13	25	36	46	57	67	77	87	97	106	199	285	369	450	529	599	682	756	830	1521	2159	2764	3344	3903	4446	4975	5491	5997	14715	25318
0.89	3	13	25	36	47	57	67	77	87	97	107	199	286	369	450	529	599	683	757	831	1523	2163	2769	3350	3910	4455	4984	5502	6008	14751	25391
0.9	3	13	25	36	47	57	67	77	87	97	107	199	286	370	451	530	599	684	758	832	1525	2166	2773	3355	3917	4462	4993	5512	6020	14785	25462
0.91	3	13	25	36	47	57	67	77	87	97	107	199	286	370	452	531	599	685	759	833	1527	2170	2778	3361	3924	4470	5002	5522	6030	14819	25531
0.92	3	13	25	36	47	57	67	77	87	97	107	199	287	371	452	531	599	685	760	834	1530	2173	2782	3366	3930	4477	5010	5531	6041	14851	25597
0.93	3	13	25	36	47	57	67	77	87	97	107	200	287	371	453	532	599	686	761	835	1532	2176	2786	3371	3936	4487	5019	5541	6051	14882	25661
0.94	3	13	25	36	47	57	67	77	88	97	107	200	287	371	453	533	599	687	762	836	1534	2179	2790	3376	3942	4491	5027	5549	6061	14913	25724
0.95	3	13	25	36	47	57	67	77	88	97	107	200	288	372	454	534	599	688	763	837	1535	2181	2794	3381	3948	4498	5034	5558	6071	14942	25784
0.96	3	13	25	36	47	57	67	77	88	97	107	200	288	372	454	534	599	688	763	837	1535	2181	2794	3381	3948	4498	5034	5558	6071	14942	25784
0.97	3	13	25	36	47	57	67	77	88	97	107	200	288	373	454	534	599	689	765	839	1539	2187	2801	3390	3958	4511	5049	5574	6089	14977	25898
0.98	3	13	25	36	47	57	67	77	88	97	107	201	289	373	455	535	599	690	766	841	1541	2189	2804	3398	3968	4522	5052	5582	6097	15023	25952
0.99	3	13	25	36	47	57	67	77	88	97</

FIGURE 11: Continued.

FIGURE 11: The relationships between λ and Q^* with respect to varied β when $k = 3$.

material inventory. However, its memoryless property is unrealistic. The Erlang is an IFR distribution, which better fits the nature of intermittent demand. Therefore, this paper proposes an algorithm to determine the optimal order quantity in a single echelon single electric power material inventory system for (T, Q) policy. An empirical investigation was conducted to collaborate the validity of this new method by means of comparison with the classic Newsvendor model. The findings indicate that this new method outperforms the classic one in achieving cost reduction, which makes it more economic and practical. In addition, considering the complexity of this new method, an approximation and

a heuristic algorithm is proposed. To sum up, the paper develops three algorithms for the optimal order quantity, each of which has their own advantages and disadvantages.

(1) The basic algorithm: this method has great advantages with regard to accuracy. However, it is time-consuming, computation-intensive, and not suitable for bulk operation.

(2) The approximation algorithm: it simplifies computational complexity by reducing iterative numbers within an acceptable error range. Such a method enables companies to enhance their efficiency of stock management with guaranteed accuracy, which is of great validity and utility in a real-world context.

TABLE 4: Estimations of parameters c and a with respect to varied β .

$k = 2$				$k = 3$			
β	c	a	MAPE	β	c	a	MAPE
1	12.48	3.68	14.29%	1	12.08	3.08	14.29%
2	22.96	3.45	8.00%	2	22.13	3.01	12.00%
3	32.85	3.42	10.81%	3	31.58	3.01	11.11%
4	42.32	3.45	10.64%	4	40.71	3.00	12.77%
5	51.68	3.44	10.34%	5	49.58	3.00	13.79%
6	60.71	3.44	10.29%	6	58.25	2.98	14.71%
7	69.63	3.43	11.39%	7	66.81	3.00	15.19%
8	78.46	3.44	12.36%	8	75.15	2.99	15.73%
9	87.03	3.41	12.12%	9	83.35	2.98	16.16%
10	95.84	3.40	11.93%	10	91.46	3.00	16.51%
20	176.43	3.43	13.30%	20	174.26	3.00	16.91%
30	252.31	3.43	13.70%	30	245.37	2.97	17.40%
50	394.41	3.43	14.53%	50	382.59	3.02	17.88%
100	643.37	3.40	14.84%	100	533.04	2.99	18.37%
500	3021.17	3.43	15.12%	500	2982.11	3.04	19.86%
1000	4873.52	3.44	17.21%	1000	4638.47	3.01	21.35%

(3) The heuristic algorithm: it derives the optimal order quantity by establishing a functional relation where the independent variable is λ and the dependent variable is Q^* . This method is the simplest but has the least degree of accuracy among all algorithms.

In addition, these algorithms can also be applied to other electric power materials with intermittent demand within other industries.

Appendix

A. Results of $\tilde{n}$ under Varied λ, β, k

See Figures 7 and 8.

B. Results of Q^* under Varied λ, β, k

See Figures 9 and 10.

C. The Relationships between λ and Q^* with respect to Varied β When $k = 3$

See Figure 11.

Data Availability

The data used to support the findings of this study were supplied by Shanghai Electric Power Company under license and so cannot be made freely available. Requests for access to these data should be made to Aiping Jiang, Chengzhong Rd., JiaDing Dist., Shanghai, China.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The research described in this paper has been funded by the National Science Foundation of China (Grant no. 71302053).

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