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*Research Article*

# Determining Equivalent Administrative Charges for Defined Contribution Pension Plans under CEV Model

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In defined contribution pension plan, the determination of the equivalent administrative charges on balance and on flow is investigated if the risk asset follows a constant elasticity of variance (CEV) model. The maximum principle and the stochastic control theory are applied to derive the explicit solutions of the equivalent equation about the charges. Using the power utility function, our conclusion shows that the equivalent charge on balance is related to the charge on flow, risk-free interest rate, and the length of accumulation phase. Moreover, numerical analysis is presented to show our results.

## 1. Introduction

The defined contribution (DC) pension plan is a scheme that contribution is fixed in individual pension systems, which help us to ensure our life after retirement. There are many literatures to study the optimal investment strategies for DC pension plan. Vigna and Haberman [1] investigate a discrete model for the DC pension plan. Devolder et al. [2] study the optimal investment strategy by using stochastic optimal control theory under the constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) utility functions before and after retirement. On the basis of works of Devolder et al. [2], Gao [3] considers the portfolio problem with the stochastic interest rate, which is based on the Cox-Ingerroll-Ross (CIR) model and the Vasicek model. Gao [4] finds out the optimal investment strategy and extends the geometric Brownian motion (GBM) model to the constant elasticity of variance (CEV) model. Wang et al. [5] discuss the CEV model in a study of optimal investment strategy for the exponential utility function by using the Legendre transform method. Chang et al. [6] apply dynamic programming principle to obtain the Hamilton-Jacobi-Bellman (HJB) equation and consider the optimal investment and consumption decisions under the CEV model. Guan and Liang [7] investigate optimal investment strategy of DC pension plan under a stochastic interest rate and stochastic volatility model, which includes the CIR, Vasicek and Heston's stochastic volatility models. The optimal investment strategies under the loss aversion and constraints of value at risk (VaR) from a perspective of risk management are discussed in Guan and Liang [8]. Recently, Sun et al. [9] discuss the optimal strategy under inflation and stochastic income by using the Heston's SV model. Sheng and Rong [10] study the optimal time consistent investment strategy for the DC pension merged with an annuity contract, which focuses on the return of premiums clauses under the Heston's stochastic volatility model. Chen et al. [11] combine the loss aversion and inflation risk and add minimum performance constraint to investigate the asset allocation problems. Luis [12] provides a methodology to compare administrative charges, i.e., the commission of the pension management in which affiliates would pay to the Pension Fund Administrator (PFA), including the charge on balance and charge on flow.

About administrative fees, Kritzer et al. [13] point out that the PFA could charge a fee on contribution as a percentage of a person's income flow (charge on flow) or charge a fee on individual pension account asset (charge on balance). There are two types of administrative fees which are charged by PFA in Latin America. On one hand, the charge on asset is utilized by Mexico, Peru, Bolivia, Costa Rica, etc. On the other hand, Colombia, Chile, Bolivia have the charge on flow. Both Bolivia and Peru have the two types. In

Whitehouse [14] and Tapia and Yermo [15], we find some specific analysis and comparison of administrative charges across different countries. Queisser [16] finds out that the charge on flow has more advantages for the PFA in the initial stages of the system, but the charge on balance tends to be more expensive in the long run. Hernandez and Stewart [17] consider a methodology, known as the charge ratio, to make a standardized international comparison.

Motivated by the works of the Luis [12], we investigate the relationship between charge on balance and charge on flow. The research approaches come from those in Gao [5]. We consider an affiliate who is involved in a DC pension plan and aim to maximize the expected utility of terminal wealth under constraints during his accumulation period. The factors of charge on balance and charge on flow are added to constraint condition. The contribution which affiliate should contribute to his personal account monthly is fixed. This amount of money is invested by a professional PFA's manager. The affiliate pays the commission to the manager at the same time and receives his pension after retirement.

Under the continuously complete market, we extend the driving equation of the risky asset to a generalized CEV model and take the power utility function of CRRA. We assume that the contribution rate is a constant. We solve three stochastic optimal control problems and obtain the corresponding certainty equivalent (CE). We compare the CE of charge on balance and charge on flow. One of the novel features of our research is that we require the risky asset to satisfy a CEV model which is different from that in Luis [12]. The GBM model is just a special case of the CEV model. The other is that we use the power utility function of CRRA to reach our goal, which is different from that in Luis [12].

The rest of this paper is organized as follows. Section 2 provides the model setting of our paper. Section 3 solves the optimization problem and provides a methodology to compare charge on balance and charge on flow. Section 4 presents a numerical analysis to demonstrate our results. Section 5 draws a conclusion.

## 2. Model Setting

In this section, we derive our model.

*2.1. Financial Market.* Our model is set up on probability space $(\Omega, \mathbb{F}, \mathbb{P})$, and the filtration $\mathbb{F} = \{\mathbb{F}_t\}$ is generated by Brownian motion $Z(t)$. Consider that there just exists two assets, a risk-free asset and a risky asset, such as the monetary account and stock. We assume that there are no transaction costs. The risk-free asset satisfies

$$dB(t) = rB(t)\,dt, \tag{1}$$

where $r$ is the risk-free interest rate.

In several previous literatures, researchers use the GBM to describe the risk asset in the study of DC pension plan. Here we use the CEV model to describe the risk asset, which is a generalization of the GBM in Luis [12]. Namely, the risky asset satisfies

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma (S(t))^{\beta}\, dZ(t), \tag{2}$$

where $\mu$ is the expected return rate of the stock and $\sigma(S(t))^{\beta}$ is the instantaneous volatility rate, $\beta$ is the elasticity parameter. Equation (2) reduces to the GBM when $\beta = 0$.

*2.2. Wealth Process.* We suppose that the contribution rate is a fixed constant rate $\theta$ per month during the accumulation phase. Let $W(t)$ denote the affiliate's wealth in his pension account at time $t \in [0, T]$. The dynamics of wealth $W(t)$ satisfies

$$\begin{aligned} dW(t) = {} & \pi(t) W(t) \frac{dS(t)}{S(t)} + (1 - \pi(t)) W(t) \frac{dB(t)}{B(t)} \\ & + \theta dt, \end{aligned} \tag{3}$$

where $\pi(t)$ is the proportion invested in risky asset. $1 - \pi(t)$ is the proportion invested in risk-free asset, which is used by the pension manager at time $t$.

Plugging (1) and (2) into (3), we obtain

$$\begin{aligned} dW(t) = {} & \left[W(t)\left(\pi(t)\left(\mu - r\right) + r\right) + \theta\right] dt \\ & + W(t)\pi(t)\sigma(S(t))^{\beta}\, dZ(t), \\ W(0) = {} & W_0. \end{aligned} \tag{4}$$

It is clear that the optimal problem is to maximize the expected utility of terminal wealth $E[U(W_T)]$. Our goal is to find out the optimal proportion $\pi^*(t)$, which is the solution of the problem

$$\max E\left[U(W(T))\right], \tag{5}$$

where $U(.)$ is strictly concave and satisfy the Inada conditions.

Now we introduce a fee rate on the basis of (4), the "charge on balance", which is denoted by $\delta$ in continuous time. The charge on balance is a percentage of the value of assets. Similar to the works of Luis [12], we introduce the stochastic differential equation (SDE) in the form

$$\begin{aligned} dW_s(t) = {} & \left[W_s(t)\left(\pi_s(t)\left(\mu - r\right) + r - \delta\right) + \theta\right] dt \\ & + W_s(t)\pi_s(t)\sigma(S(t))^{\beta}\, dZ(t), \\ W_s(0) = {} & W_0. \end{aligned} \tag{6}$$

In a similar way, we consider other fee rate on the basis of (4), the "charge on flow". Let $\alpha$ denote the charge on flow. When someone contributes $\theta$ in his account in a particular month, he pays a proportion of that contribution to the Pension Fund Administrator, i.e., $(1 - e^{-\alpha})\theta$, and the rest of it, i.e., $e^{-\alpha}\theta$, as the adjusted contribution rate. Similar to the work of Luis [12], we introduce the SDE in the form

$$\begin{aligned} dW_f(t) = {} & \left[W_f(t)\left(\pi_s(t)\left(\mu - r\right) + r\right) + e^{-\alpha}\theta\right] dt \\ & + W_f(t)\pi_s(t)\sigma(S(t))^{\beta}\, dZ(t), \\ W_f(0) = {} & e^{-\alpha}W_0. \end{aligned} \tag{7}$$

## 3. Solution to the Model

*3.1. The Solution of Affiliate Problem.* Consider an affiliate who will retire after $T$ month. He can accumulate his pension account during this period. At the beginning, he already has $W_0$ in his individual pension account. He will contribute additional money in this account which will be managed by the special pension manager to invest these money to the monetary account or stocks of the financial market. Therefore, the affiliate's problem is given by

$$\begin{aligned} \max \quad & E\left[U\left(W_T\right)\right] \\ s.t. \quad & dW(t) \\ & = \left[W(t)\left(\pi(t)(\mu-r)+r\right)+\theta\right]dt \\ & \quad + W(t)\pi(t)\sigma(S(t))^{\beta}dZ(t), \\ & W(0)=W_0. \end{aligned} \tag{8}$$

To solve this problem, we use the classical tools of stochastic optimal controls. We define the value function

$$\begin{aligned} & V(t,s,w) \\ & \quad = \max E\left[U(W(T)) \mid S(t)=s,\ W(t)=w\right], \\ & \qquad\qquad 0<t<T. \end{aligned} \tag{9}$$

Using the CRRA utility function, it is possible for us to get a close-form solution. Using Itô lemma on $V(t,s,w)$, we have

$$\begin{aligned} dV(t,s,w) &= V_t dt + V_s dS(t) + \frac{1}{2}V_{ss}d[S,S](t) \\ &\quad + V_w dW(t) + \frac{1}{2}V_{ww}d[W,W](t) \\ &\quad + V_{sw}d[S,W](t). \end{aligned} \tag{10}$$

Integrating on both sides of (10), we get

$$\begin{aligned} & V(t+h,S(t+h),W(t+h)) = V(t,s,w) + \int_t^{t+h}\Big(V_u \\ & \quad + V_s\mu S + V_w\left(W\left(\pi(\mu-r)+r\right)+\theta\right) \\ & \quad + \frac{1}{2}V_{ss}\sigma^2S^{2\beta+2} + \frac{1}{2}V_{ww}\pi^2\sigma^2S^{2\beta}W^2 \\ & \quad + V_{sw}\sigma^2\pi S^{2\beta+1}W\Big)du + \int_t^{t+h}\Big(V_s\sigma S^{\beta+1} \\ & \quad + V_w\pi\sigma S^{\beta}W\Big)dZ(t). \end{aligned} \tag{11}$$

Using (11) yields

$$\begin{aligned} & V(t,s,w) \\ & \quad = \max E\left[V(t+h,S(t+h),W(t+h)) \mid S(t)\right. \\ & \quad \left. = s,\ W(t)=w\right]. \end{aligned} \tag{12}$$

Furthermore, we get

$$\begin{aligned} 0 = \max E\Bigg[&\int_t^{t+h}\Big(V_u + V_s\mu S + V_w\left(W\left(\pi(\mu-r)+r\right)+\theta\right) + \frac{1}{2}V_{ss}\sigma^2S^{2\beta+2} + \frac{1}{2}V_{ww}\pi^2\sigma^2S^{2\beta}W^2 + V_{sw}\sigma^2\pi S^{2\beta+1}W\Big)du \mid \\ & S(t)=s,\ W(t)=w\Bigg]. \end{aligned} \tag{13}$$

Dividing by $h>0$, letting $h\longrightarrow 0$, and using mean value theorem, we get

$$\begin{aligned} 0 = \max\Big(&V_t + V_s\mu s + V_w\left(w\left(\pi(\mu-r)+r\right)+\theta\right) \\ & + \frac{1}{2}V_{ss}\sigma^2s^{2\beta+2} + \frac{1}{2}V_{ww}\pi^2\sigma^2s^{2\beta}w^2 \\ & + V_{sw}\sigma^2\pi s^{2\beta+1}w\Big). \end{aligned} \tag{14}$$

Therefore, the Hamilton-Jacobi-Bellman (HJB) equation is derived in the form

$$\begin{aligned} & V_t + \mu sV_s + (rw+\theta)V_w + \frac{1}{2}\sigma^2s^{2\beta+2}V_{ss} \\ & \quad + \max\Big(\frac{1}{2}\pi^2\sigma^2s^{2\beta}w^2V_{ww} + \pi w(\mu-r)V_w \\ & \quad + \pi\sigma^2s^{2\beta+1}wV_{ws}\Big) = 0, \end{aligned} \tag{15}$$

where $V_t$, $V_s$, $V_w$, and $V_{ww}$ denote partial derivatives of first and second orders with respect to time, stock price, and wealth, respectively. Therefore, the problem becomes to solve the maximum problem

$$\begin{aligned} \max\Big(&\frac{1}{2}\pi^2\sigma^2s^{2\beta}w^2V_{ww} + \pi w(\mu-r)V_w \\ & + \pi\sigma^2s^{2\beta+1}wV_{ws}\Big). \end{aligned} \tag{16}$$

From the first order condition of maximum principle, the optimal investment proportion of risky asset $\pi^*(t)$ is derived to satisfy

$$\pi^*(t) = -\frac{(\mu-r)v_w + \sigma^2s^{2\beta+1}V_{ws}}{\sigma^2s^{2\beta}wV_{ww}}. \tag{17}$$

We plug (17) into (15) and then obtain the following partial differential equation (PDE)

$$V_t + \mu sV_s + (rw+\theta)V_w + \frac{1}{2}\sigma^2s^{2\beta+2}V_{ss}$$

$$-\frac{\left[(\mu-r)V_w+\sigma^2 s^{2\beta+1}V_{ws}\right]^2}{2\sigma^2 s^{2\beta V_{ww}}}=0 \tag{18}$$

with the boundary condition $V(T,s,w)=U(w)$.

Equation (18) is an nonlinear PDE, which is very hard to be solved. Usually we can conjecture a solution of this complex PDE.

In this paper, we consider the power utility function, which is a special case of the CRRA utility; namely,

$$U(w)=\frac{w^p}{p},\quad \text{with } p<1,\ p\neq 0. \tag{19}$$

The power utility function is a constant relative risk aversion (CRRA) utility, and $p$ is so-called CRRA coefficient.

**Theorem 1.** *The optimal strategy is*

$$\pi^*(t)=\left(1+\frac{\theta a_1}{w}\right)M(\sigma_t)N(t), \tag{20}$$

*where* $\sigma_t=\sigma(s(t))^\beta$, $M(\sigma_t)=(\mu-r)/(1-p)\sigma_t^2$, $N(t)=1-2\beta(1-p)I(t)/(\mu-r)$, $I(t)=(\lambda_1-\lambda_1 e^{2\beta^2(\lambda_1-\lambda_2)(T-t)})/(1-(\lambda_1/\lambda_2)e^{2\beta^2(\lambda_1-\lambda_2)(T-t)})$, *and* $a_1(t)=(1-e^{-r(T-t)})/r$.

*The value function is*

$$V(t,s,w)=\left\{\exp\left\{\left[\lambda_1\beta(2\beta+1)+\frac{rp}{1-p}\right](T-t)+\frac{I(t)}{\sigma^2 s^{2\beta}}\right\}C(t)\right\}^{1-p}\frac{(w+\theta a_1)^p}{p}, \tag{21}$$

*where* $C(t)=((\lambda_2-\lambda_1)/(\lambda_2-\lambda_1 e^{2\beta^2(\lambda_2-\lambda_1)(T-t)}))^{(2\beta+1)/2\beta}$.

*If* $\overline{W}(T)$ *is the terminal wealth when investment proportion approaches to optimal strategy* $\pi^*(t)$, *then*

$$\begin{aligned}E\left[U\left(\overline{W}(T)\right)\right]&=V(0,s,w_0)\\&=\left\{\exp\left\{\left[\lambda_1\beta(2\beta+1)+\frac{rp}{1-p}\right]T+\frac{I(0)}{\sigma^2 s^{2\beta}}\right\}\cdot C(0)\right\}^{1-p}\frac{(w_0+\theta a_1(0))^p}{p}.\end{aligned} \tag{22}$$

*The certainty equivalent of* $\overline{W}(T)$ *satisfies the relations*

$$\begin{aligned}CE\left[\overline{W}(T)\right]&=\left\{\exp\left\{\left[\lambda_1\beta(2\beta+1)+\frac{rp}{1-p}\right]T+\frac{I(0)}{\sigma^2 s^{2\beta}}\right\}\cdot C(0)\right\}^{(1-p)/p}\frac{(w_0+\theta a_1(0))^p}{p},\end{aligned} \tag{23}$$

*where* $I(0)=(\lambda_1-\lambda_1 e^{2\beta^2(\lambda_1-\lambda_2)T})/(1-(\lambda_1/\lambda_2)e^{2\beta^2(\lambda_1-\lambda_2)T})$, $C(0)=((\lambda_2-\lambda_1)/(\lambda_2-\lambda_1 e^{2\beta^2(\lambda_2-\lambda_1)T}))^{(2\beta+1)/2\beta}$, *and* $a_1(0)=(1-e^{-rT})/r$.

*Proof.* Following the methods in Gao [5], we conjecture a solution of (19) in the form

$$V(t,s,w)=b(t,s)\frac{(w-a(t))^p}{p} \tag{24}$$

with the boundary condition $a(T)=0$, $b(T,s)=1$.

Plugging every derivatives of the $V(t,s,w)$ into (18), we get the equation

$$\begin{aligned}&\left\{b_t+\frac{\mu-rp}{1-p}b_s+\frac{1}{2}\sigma^2 s^{2\beta+2}b_{ss}+\frac{p\sigma^2 s^{2\beta+2}}{2(1-p)}\frac{b_s^2}{b}+\frac{p(\mu-r)^2}{2\sigma^2 s^{2\beta}(1-p)}b+rpb\right\}(w-a)^p+pb\{\theta+ra\\&-a_t\}(w-a)^{p-1}=0.\end{aligned} \tag{25}$$

We split (25) into two equations

$$a_t-ra-\theta=0, \tag{26}$$

$$b_t+\frac{\mu-rp}{1-p}b_s+\frac{1}{2}\sigma^2 s^{2\beta+2}b_{ss}+\frac{p\sigma^2 s^{2\beta+2}}{2(1-p)}\frac{b_s^2}{b}+\frac{p(\mu-r)^2}{2\sigma^2 s^{2\beta}(1-p)}b+rpb=0. \tag{27}$$

Equation (26) is a simple first ordinary differential equation (ODE) which has the solution

$$a(t)=-\theta\left(\frac{1-e^{-r(T-t)}}{r}\right), \tag{28}$$

where $a(t)\equiv-\theta a_1$, $a_1=(1-e^{-r(T-t)})/r$.

Equation (27) is a nonlinear second-order PDE and hard to find an explicit solution. We follow the method of power transformation and variable change proposed by Cox [18] and Gao [5]. Then we can transform (27) into a linear PDE.

Letting $b(t,s)=f(t,y)^{1-p}$, $y=s^{-2\beta}$, from (27), we get

$$f_t+\beta\left[(2\beta+1)\sigma^2-\frac{2(\mu-rp)y}{1-p}\right]f_y+2\delta^2\beta^2 yf_{yy}+\frac{p(\mu-r)^2}{2\delta^2(1-p)^2}yf+\frac{rp}{1-p}f=0. \tag{29}$$

Adding $f(t,y)=A(t)e^{B(t)y}$ into (29) yields

$$\left\{\frac{A_t}{A}+\beta(2\beta+1)\delta^2 B+\frac{rp}{1-p}\right\}+\left\{B_t-\frac{2\beta(\mu-rp)}{1-p}B+2\delta^2\beta^2B^2+\frac{p(\mu-r)^2}{2\delta^2(1-p)^2}\right\}y=0. \tag{30}$$

Here we split (30) into two equations in order to eliminate the dependent $t$ and $y$. Then

$$\frac{A_t}{A} + \beta(2\beta+1)\delta^2 B + \frac{rp}{1-p} = 0, \quad (31)$$

$$B_t - \frac{2\beta(\mu - rp)}{1-p}B + 2\delta^2\beta^2 B^2 + \frac{p(\mu-r)^2}{2\delta^2(1-p)^2} = 0. \quad (32)$$

Thus we get

$$B(t) = \frac{1}{\delta^2} I(t), \quad (33)$$

where $I(t) = (\lambda_1 - \lambda_1 e^{2\beta^2(\lambda_1-\lambda_2)(T-t)})/(1 - (\lambda_1/\lambda_2)e^{2\beta^2(\lambda_1-\lambda_2)(T-t)})$, $\lambda_1 = ((\mu - rp) - \sqrt{(1-p)(\mu^2 - r^2 p)})/2\beta(1-p)$, and $\lambda_2 = ((\mu - rp) + \sqrt{(1-p)(\mu^2 - r^2 p)})/2\beta(1-p)$.

$$A(t) = e^{[\lambda_1\beta(2\beta+1)+rp/(1-p)](T-t)}\left(\frac{\lambda_2-\lambda_1}{\lambda_2-\lambda_1 e^{2\beta^2(\lambda_1-\lambda_2)(T-t)}}\right)^{(2\beta+1)/2\beta}. \quad (34)$$

Finally, we get the explicit solution of $b(t,s)$:

$$b(t,s) = \left\{\exp\left\{\left[\lambda_1\beta(2\beta+1) + \frac{rp}{1-p}\right](T-t) + \frac{I(t)}{\sigma^2 s^{2\beta}}\right\} C(t)\right\}^{1-p}, \quad (35)$$

where $C(t) = ((\lambda_2-\lambda_1)/(\lambda_2-\lambda_1 e^{2\beta^2(\lambda_2-\lambda_1)(T-t)}))^{(2\beta+1)/2\beta}$. □

**Corollary 2.** *If $\beta = 0$, then the CEV model reduces to the GBM model. Therefore the optimal investment strategy is*

$$\pi^*(t) = \left(1 + \frac{\theta a_1(t)}{w}\right)\left(\frac{\mu - r}{\sigma^2(1-p)}\right). \quad (36)$$

*If $\overline{W}(T)$ is the terminal wealth when investment proportion approaches to optimal strategy $\pi^*(t)$, then*

$$CE\left[\overline{W}(T)\right] = \exp\left\{\left[r - \frac{1}{2(p-1)}\left(\frac{\mu-r}{\sigma}\right)^2\right]T\right\} \cdot (W_0 + \theta a_1(0)). \quad (37)$$

*Remark 3.* Here, we want to show the results when the CEV model degenerates into the GBM model of the power utility function. Because of the difference of the assumption of the utility function, we find that the result is different from that in Luis [12]. However, it is noted that the CE is related to the parameters of risk-free interest rate $r$, risk aversion $p$, shape ratio $(\mu - r)/\sigma$, the initial wealth $W_0$, and the length of accumulation phase $T$. This relationship is similar to the CE in Luis [12].

*3.2. The Solution of Charge on Balance.* Our optimal problem becomes that we consider the charge on balance $\delta$ under the constraint condition (6)

$$\begin{aligned} \max \quad & E[U(W_s(T))] \\ s.t. \quad & dW_s(t) \\ & = [W_s(t)(\pi_s(t)(\mu - r) + r - \delta) + \theta]\,dt \\ & \quad + W_s(t)\pi_s(t)\sigma(S(t))^{\beta}\,dZ(t), \\ & W_s(0) = W_0. \end{aligned} \quad (38)$$

Similar to the method we use in the above, we define the value function

$$V(t,s,w) = \max E[U(W_s(T)) \mid S(t) = s,\ W_s(t) = w]. \quad (39)$$

Then the HJB equation is derived in the form

$$\begin{aligned} & V_t + \mu s V_s + ((r-\delta)w + \theta)V_w + \frac{1}{2}\sigma^2 s^{2\beta+2}V_{ss} \\ & + \max\left(\frac{1}{2}\pi^2\sigma^2 s^{2\beta} w^2 V_{ww} + \pi w(\mu - r)V_w \right. \\ & \left. + \pi\sigma^2 s^{2\beta+1} w V_{ws}\right) = 0. \end{aligned} \quad (40)$$

The maximum problem, which is just relative with $\pi$, is the same as that of (16). Therefore, we get the same optimal investment strategy $\pi_s^*(t)$ of risky asset

$$\pi_s^*(t) = -\frac{(\mu - r)v_w + \sigma^2 s^{2\beta+1}V_{ws}}{\sigma^2 s^{2\beta} w V_{ww}}. \quad (41)$$

Adding (41) into (40), we have

$$\begin{aligned} & V_t + \mu s V_s + ((r-\delta)w + \theta)V_w + \frac{1}{2}\sigma^2 s^{2\beta+2}V_{ss} \\ & - \frac{\left[(\mu - r)V_w + \sigma^2 s^{2\beta+1}V_{ws}\right]^2}{2\sigma^2 s^{2\beta V_{ww}}} = 0 \end{aligned} \quad (42)$$

with the boundary condition $V(T,s,w) = U(w)$.

**Theorem 4.** *The optimal strategy is*

$$\pi_s^*(t) = \left(1 + \frac{\theta a_1}{w}\right)M(\sigma_t)N(t). \quad (43)$$

*The value function is*

$$V(t,s,w) = \left\{\exp\left\{\left[\lambda_1\beta(2\beta+1) + \frac{(r-\delta)p}{1-p}\right](T-t) + \frac{I(t)}{\sigma^2 s^{2\beta}}\right\} C(t)\right\}^{1-p}\frac{(w+\theta a_2)^p}{p}, \quad (44)$$

*where $C(t) = ((\lambda_2 - \lambda_1)/(\lambda_2 - \lambda_1 e^{2\beta^2(\lambda_2-\lambda_1)(T-t)}))^{(2\beta+1)/2\beta}$.*

*If $\overline{W}_s(T)$ is the terminal wealth when investment proportion approaches to optimal strategy $\pi_s^*(t)$, then*

$$\begin{aligned} E\left[U\left(\overline{W}_s(T)\right)\right] &= V(0,s,w_0) \\ &= \left\{\exp\left\{\left[\lambda_1\beta(2\beta+1)+\frac{(r-\delta)p}{1-p}\right]T+\frac{I(0)}{\sigma^2 s^{2\beta}}\right\}\right. \\ &\quad \left.\cdot C(0)\right\}^{1-p}\frac{(w_0+\theta a_2(0))^p}{p}. \end{aligned} \tag{45}$$

*Therefore, it has*

$$\begin{aligned} CE\left[\overline{W}_s(T)\right] &= \left\{\exp\left\{\left[\lambda_1\beta(2\beta+1)+\frac{(r-\delta)p}{1-p}\right]T+\frac{I(0)}{\sigma^2 s^{2\beta}}\right\}\right. \\ &\quad \left.\cdot C(0)\right\}^{(1-p)/p}\frac{(w_0+\theta a_2(0))^p}{p}, \end{aligned} \tag{46}$$

*where $I(0) = (\lambda_1 - \lambda_1 e^{2\beta^2(\lambda_1-\lambda_2)T})/(1-(\lambda_1/\lambda_2)e^{2\beta^2(\lambda_1-\lambda_2)T})$, $C(0) = ((\lambda_2-\lambda_1)/(\lambda_2-\lambda_1 e^{2\beta^2(\lambda_2-\lambda_1)T}))^{(2\beta+1)/2\beta}$, and $a_2(0) = (1-e^{-(r-\delta)T})/r$.*

*Proof.* Similarly, consider $V(t,s,w) = b(t,s)((w-a(t))^p/p)$ with the boundary condition $a(T)=0$, $b(T,s)=1$.

Plugging the derivatives of the $V(t,s,w)$ into (42), we get the equation

$$\begin{aligned} &\left\{b_t+\frac{\mu-rp}{1-p}b_s+\frac{1}{2}\sigma^2 s^{2\beta+2}b_{ss}+\frac{p\sigma^2 s^{2\beta+2}}{2(1-p)}\frac{b_s^2}{b}\right. \\ &\quad \left.+\frac{p(\mu-r)^2}{2\sigma^2 s^{2\beta}(1-p)}b+(r-\delta)pb\right\}(w-a)^p+pb\{\theta \\ &\quad +(r-\delta)a-a_t\}(w-a)^{p-1}=0. \end{aligned} \tag{47}$$

Equation (47) can be split into two equations

$$a_t-(r-\delta)a-\theta=0, \tag{48}$$

$$\begin{aligned} &b_t+\frac{\mu-rp}{1-p}b_s+\frac{1}{2}\sigma^2 s^{2\beta+2}b_{ss}+\frac{p\sigma^2 s^{2\beta+2}}{2(1-p)}\frac{b_s^2}{b} \\ &\quad +\frac{p(\mu-r)^2}{2\sigma^2 s^{2\beta}(1-p)}b+(r-\delta)pb=0. \end{aligned} \tag{49}$$

From (48), we get

$$a(t) = -\theta\left(\frac{1-e^{-(r-\delta)(T-t)}}{r}\right), \tag{50}$$

where $a(t)\equiv -\theta a_2$ and $a_2 = (1-e^{-(r-\delta)(T-t)})/r$.

Using the same method above to solve nonlinear PDE (49), we get $b(t,s)$ in the form

$$\begin{aligned} b(t,s) &= \left\{\exp\left\{\left[\lambda_1\beta(2\beta+1)+\frac{(r-\delta)p}{1-p}\right](T-t)\right.\right. \\ &\quad \left.\left.+\frac{I(t)}{\sigma^2 s^{2\beta}}\right\}C(t)\right\}^{1-p}. \end{aligned} \tag{51}$$

□

**Corollary 5.** *If $\beta=0$, then the CEV model reduces to the GBM model. Therefore the optimal investment strategy is*

$$\pi_s^*(t) = \left(1+\frac{\theta a_2(t)}{w}\right)\left(\frac{\mu-r}{\sigma^2(1-p)}\right). \tag{52}$$

*Moreover,*

$$\begin{aligned} &CE\left[\overline{W}_s(T)\right] \\ &\quad = \exp\left\{(r-\delta)-\frac{1}{2(p-1)}\left(\frac{\mu-r}{\sigma}\right)^2 T\right\} \\ &\quad \cdot(W_0+\theta a_2(0)). \end{aligned} \tag{53}$$

*Remark 6.* Corollary 5 is the result of GBM case for charge on balance, when the CEV model degenerates into the GBM model of the power utility function. Because we assume that the utility function is the CRRA, which is different from the assumption in Luis [12], we find that the result is different from that in [12]. Except the relation in Corollary 2, the CE is related to the charge on balance $\delta$. Notice that the relationship of the CE here is similar to the CE in Luis [12].

*3.3. The Solution of Charge on Flow.* Our optimal problem becomes that we consider the constraint condition equation (7), which is about charge on flow $\alpha$

$$\begin{aligned} \max\quad & E\left[U\left(W_f(T)\right)\right] \\ s.t.\quad & dW_f(t) \\ &= \left[W_f(t)\left(\pi_f(t)(\mu-r)+r\right)+e^{-\alpha}\theta\right]dt \\ &\quad +W_f(t)\pi_f(t)\sigma(S(t))^{\beta}dZ(t), \\ & W_f(0)=e^{-\alpha}W_0. \end{aligned} \tag{54}$$

Using the methods in above section, we define the value function

$$\begin{aligned} &V(t,s,w) \\ &\quad = \max E\left[U\left(W_f(T)\right)\mid S(t)=s,\ W_f(t)=w\right]. \end{aligned} \tag{55}$$

Using Itô lemma, mean value theorem, and DPP, the HJB equation is derived in the form

$$\begin{aligned} &V_t+\mu sV_s+\left(rw+e^{-\alpha}\theta\right)V_w+\frac{1}{2}\sigma^2 s^{2\beta+2}V_{ss} \\ &\quad +\max\left(\frac{1}{2}\pi^2\sigma^2 s^{2\beta}w^2V_{ww}+\pi w(\mu-r)V_w\right. \\ &\quad \left.+\pi\sigma^2 s^{2\beta+1}wV_{ws}\right)=0. \end{aligned} \tag{56}$$

From the first order condition of maximum principle, the optimal investment proportion of risky asset $\pi_f^*(t)$ is derived to satisfy

$$\pi_f^*(t) = -\frac{(\mu-r)v_w+\sigma^2 s^{2\beta+1}V_{ws}}{\sigma^2 s^{2\beta}wV_{ww}}. \tag{57}$$

Adding (57) into (56), we have

$$V_t + \mu s V_s + \left(rw + e^{-\alpha}\theta\right) V_w + \frac{1}{2}\sigma^2 s^{2\beta+2} V_{ss} - \frac{\left[\left(\mu - r\right) V_w + \sigma^2 s^{2\beta+1} V_{ws}\right]^2}{2\sigma^2 s^{2\beta} V_{ww}} = 0 \tag{58}$$

with the boundary condition $V(T, s, w) = U(w)$.

**Theorem 7.** *The optimal strategy is*

$$\pi_f^*(t) = \left(1 + \frac{e^{-\alpha}\theta a_1}{w}\right) M(\sigma_t) N(t). \tag{59}$$

*The value function is*

$$V(t, s, w) = \left\{\exp\left\{\left[\lambda_1 \beta (2\beta + 1) + \frac{rp}{1-p}\right](T - t) + \frac{I(t)}{\sigma^2 s^{2\beta}}\right\} C(t)\right\}^{1-p} \frac{\left(w + e^{-\alpha}\theta a_2\right)^p}{p}, \tag{60}$$

*where* $C(t) = ((\lambda_2 - \lambda_1)/(\lambda_2 - \lambda_1 e^{2\beta^2(\lambda_2-\lambda_1)(T-t)}))^{(2\beta+1)/2\beta}$.

*If* $\overline{W}_f(T)$ *is the terminal wealth when investment proportion approaches to optimal strategy* $\pi_f^*(t)$, *then*

$$\begin{aligned} E\left[U\left(\overline{W}_f(T)\right)\right] &= V(0, s, w_0) \\ &= \left\{\exp\left\{\left[\lambda_1 \beta (2\beta + 1) + \frac{rp}{1-p}\right] T + \frac{I(0)}{\sigma^2 s^{2\beta}}\right\} \cdot C(0)\right\}^{1-p} \frac{\left(w_0 + e^{-\alpha}\theta a_2(0)\right)^p}{p}. \end{aligned} \tag{61}$$

*Therefore, it has*

$$\begin{aligned} CE\left[\overline{W}_f(T)\right] &= \left\{\exp\left\{\left[\lambda_1 \beta (2\beta + 1) + \frac{rp}{1-p}\right] T + \frac{I(0)}{\sigma^2 s^{2\beta}}\right\} \cdot C(0)\right\}^{(1-p)/p} \frac{\left(e^{-\alpha} w_0 + e^{-\alpha}\theta a_2(0)\right)^p}{p}, \end{aligned} \tag{62}$$

*where* $I(0) = (\lambda_1 - \lambda_1 e^{2\beta^2(\lambda_1-\lambda_2)T})/(1 - (\lambda_1/\lambda_2)e^{2\beta^2(\lambda_1-\lambda_2)T})$, $C(0) = ((\lambda_2 - \lambda_1)/(\lambda_2 - \lambda_1 e^{2\beta^2(\lambda_2-\lambda_1)T}))^{(2\beta+1)/2\beta}$, *and* $a_1(0) = (1 - e^{-rT})/r$.

*Proof.* Consider $V(t, s, w) = b(t, s)((W - a(t))^p/p)$ with the boundary condition $a(T) = 0$, $b(T, s) = 1$.

Plugging the derivatives of the $V(t, s, w)$ into (58), we get the equation

$$\left\{b_t + \frac{\mu - rp}{1-p} b_s + \frac{1}{2}\sigma^2 s^{2\beta+2} b_{ss} + \frac{p\sigma^2 s^{2\beta+2}}{2(1-p)}\frac{b_s^2}{b} + \frac{p(\mu - r)^2}{2\sigma^2 s^{2\beta}(1-p)} b + rpb\right\}(w - a)^p + pb\left\{e^{-\alpha}\theta + ra - a_t\right\}(w - a)^{p-1} = 0. \tag{63}$$

Splitting (63) into two equations

$$a_t - ra - e^{-\alpha}\theta = 0, \tag{64}$$

$$b_t + \frac{\mu - rp}{1-p} b_s + \frac{1}{2}\sigma^2 s^{2\beta+2} b_{ss} + \frac{p\sigma^2 s^{2\beta+2}}{2(1-p)}\frac{b_s^2}{b} + \frac{p(\mu - r)^2}{2\sigma^2 s^{2\beta}(1-p)} b + rpb = 0. \tag{65}$$

From (64), we get

$$a(t) = -e^{-\alpha}\theta\left(\frac{1 - e^{-r(T-t)}}{r}\right), \tag{66}$$

where $a(t) \equiv -e^{-\alpha}\theta a_1$, $a_1 = (1 - e^{-r(T-t)})/r$.

Use the same method above to solve that nonlinear PDE (65), we get $b(t, s)$ in the form

$$b(t, s) = \left\{\exp\left\{\left[\lambda_1 \beta (2\beta + 1) + \frac{rp}{1-p}\right](T - t) + \frac{I(t)}{\sigma^2 s^{2\beta}}\right\} C(t)\right\}^{1-p}. \tag{67}$$

□

**Corollary 8.** *If* $\beta = 0$, *then the CEV model reduces to the GBM model. Therefore the optimal investment strategy is*

$$\pi_f^*(t) = \left(1 + \frac{e^{-\alpha}\theta a_1(t)}{w}\right)\left(\frac{\mu - r}{\sigma^2(1-p)}\right). \tag{68}$$

*Moreover,*

$$CE\left[\overline{W}_f(T)\right] = \exp\left\{r - \frac{1}{2(p-1)}\left(\frac{\mu - r}{\sigma}\right)^2 T\right\} \cdot \left(e^{-\alpha} W_0 + e^{-\alpha}\theta a_1(0)\right). \tag{69}$$

*Remark 9.* Similar to Corollary 5, Corollary 8 is the result of GBM case for charge on flow, when the CEV model degenerates into the GBM model of the power utility function. Because of the difference of the assumption of the utility function, we find that the result is different from that in Luis [12]. Except the relation in Corollary 2, the CE is related to the charge on flow $\alpha$. Notice that the relationship of the CE here is similar to the CE in Luis [12].

*3.4. Compare "Charge on Balance" and "Charge on Flow".* In this section, we compare two charges and figure out the relation between the "charge on balance" and "charge on flow". We know that an affiliate is to seek an optimal investment strategy to maximize his terminal expected utility.

If we want to compare which charge is better, it allows us to compare the expected utility of the two types of charges, respectively. We have known that the expected utility can be replaced by the certainty equivalent. Therefore, this problem

becomes a comparison about which CE is bigger. We denote a ratio which is used in Luis [12] in the form

$$R_{sf} \equiv \frac{CE\left(\overline{W}_s(T)\right)}{CE\left(\overline{W}_f(T)\right)}. \tag{70}$$

If $R_{sf} > 1$, it means that the charge on balance is better. If $R_{sf} < 1$, it means that the charge on flow is better. If $R_{sf} = 1$, it means that both of them are indifferent, i.e., equivalent. Hence, plugging (46) and (62) into (70), we obtain

$$R_{sf} = e^{\alpha-\delta T}\frac{W_0 + (\theta/r)\left(1 - e^{-(r-\delta)T}\right)}{W_0 + (\theta/r)\left(1 - e^{-rT}\right)}. \tag{71}$$

For simplicity, we consider $W_0 = 0$, which means that, initially, the pension account has no wealth. Then we get

$$R_{sf} = e^{\alpha-\delta T}\frac{1 - e^{-(r-\delta)T}}{1 - e^{-rT}}. \tag{72}$$

From (72), we find that $R_{sf}$ is determined by $\alpha$, $\delta$, and accumulation time $T$. Interestingly, this has nothing to do with the parameter $\beta$ no matter how $\beta$ changes. Namely, when we compare these two commissions, i.e., charge on balance and charge on flow, even if the risky asset satisfies GBM, it does not affect our final comparison, which is not affected by the volatility risk $\beta$.

We study the case when $R_{sf} = 1$, because we want to know what relationship between two types of commission when they are equivalent. Letting $R_{sf} = 1$, we have

$$e^{\alpha^*-\delta^* T} = \frac{1 - e^{-rT}}{1 - e^{-(r-\delta^*)T}}, \tag{73}$$

where $\alpha^*$ and $\delta^*$ are called the equivalent charge on flow and on balance, respectively. Thus, if one equivalent charge is given, then another can be figured out. It is our goal which determines the equivalent charges in our DC pension plan under the CEV model of risky asset and the CRRA utility function. Our conclusion shows that the equivalent charge on balance is related to charge on flow, risk-free interest rate, and the length of accumulation phase.

## 4. Numerical Analysis

In this section, we show the numerical analysis of our solution if $W_0 = 0$. In Peruvian Private Pension System, Peru had finished its reform of administrative charges which substituted from charge on flow to charge on balance of DC pension plan. It is a significant reform that allows affiliates to choose the commissions they want.

Similar to the works in Luis [12], we consider a person who will retire at age of 65, and we cite the data of 2014 that is used by Luis [12]. The data we used are as follows: $f_{min} = 1.47\%$, $f_{avg} = 1.58\%$, and $f_{max} = 1.69\%$ yearly. We can calculate the parameter $\alpha_i$ from $\alpha_i = -\ln(1 - 10f_i)$, and the monthly risk-free interest rate is $r = 0.037\%$. The accumulation phase $T$ is $(65 - Age) * 12 months$. A mandatory contribution rate is 10% of the affiliate's salary.

Table 1: For a charge on balance $\delta^*$ such that $R_{sf} = 1$ for different charge on flow and charge on balance, we give three charge on flow, and $r = 0.037\%$, the constant contribution rate is 10% of the affiliate's salary.

| Age (years) | Equivalent charge on balance (in % and yearly) | | |
|---|---|---|---|
| | f=1.47% | f=1.58% | f=1.69% |
| 20 | 0.005304 | 0.005734 | 0.006170 |
| 21 | 0.005315 | 0.005746 | 0.006183 |
| 22 | 0.005326 | 0.005758 | 0.006196 |
| 23 | 0.005337 | 0.005771 | 0.006209 |
| 24 | 0.005349 | 0.005783 | 0.006222 |
| 25 | 0.005360 | 0.005795 | 0.006235 |
| 26 | 0.005372 | 0.005808 | 0.006249 |
| 27 | 0.005383 | 0.005820 | 0.006262 |
| 28 | 0.005395 | 0.005832 | 0.006275 |
| 29 | 0.005406 | 0.005845 | 0.006289 |
| 30 | 0.005418 | 0.005857 | 0.006302 |
| 31 | 0.005429 | 0.005870 | 0.006316 |
| 32 | 0.005441 | 0.005882 | 0.006329 |
| 33 | 0.005453 | 0.005895 | 0.006343 |
| 34 | 0.005464 | 0.005908 | 0.006356 |
| 35 | 0.005476 | 0.005920 | 0.006370 |
| 36 | 0.005488 | 0.005933 | 0.006384 |
| 37 | 0.005500 | 0.005946 | 0.006397 |
| 38 | 0.005511 | 0.005959 | 0.006411 |
| 39 | 0.005523 | 0.005971 | 0.006425 |
| 40 | 0.005535 | 0.005984 | 0.006439 |
| 41 | 0.005547 | 0.005997 | 0.006452 |
| 42 | 0.005559 | 0.006010 | 0.006466 |
| 43 | 0.005571 | 0.006023 | 0.006480 |
| 44 | 0.005583 | 0.006036 | 0.006494 |
| 45 | 0.005595 | 0.006049 | 0.006508 |
| 46 | 0.005607 | 0.006062 | 0.006522 |
| 47 | 0.005619 | 0.006075 | 0.006536 |
| 48 | 0.005632 | 0.006088 | 0.006551 |
| 49 | 0.005644 | 0.006101 | 0.006565 |
| 50 | 0.005656 | 0.006115 | 0.006579 |
| 51 | 0.005668 | 0.006128 | 0.006593 |
| 52 | 0.005681 | 0.006141 | 0.006607 |
| 53 | 0.005693 | 0.006155 | 0.006622 |
| 54 | 0.005705 | 0.006168 | 0.006636 |
| 55 | 0.005718 | 0.006181 | 0.006651 |
| 56 | 0.005730 | 0.006195 | 0.006665 |
| 57 | 0.005743 | 0.006208 | 0.006680 |
| 58 | 0.005755 | 0.006222 | 0.006694 |
| 59 | 0.005768 | 0.006235 | 0.006709 |
| 60 | 0.005780 | 0.006249 | 0.006723 |
| 61 | 0.005793 | 0.006263 | 0.006738 |
| 62 | 0.005806 | 0.006276 | 0.006753 |
| 63 | 0.005819 | 0.006290 | 0.006767 |
| 64 | 0.005831 | 0.006304 | 0.006782 |
| 65 | NAN | NAN | NAN |

By calculation, we get Table 1 and Figure 1. Figure 1 describes an affiliate who has power utility and will retire at 65, seeking the maximum expected terminal wealth, for different accumulation phase and three given charge on flows. As can be seen from the figure, on one hand, giving a certain charge on flow, the equivalent charge on balance

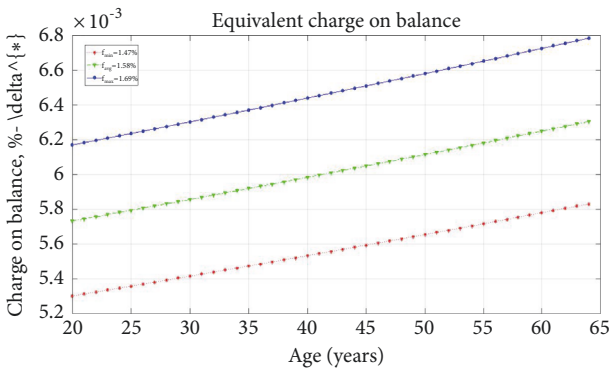


Figure 1: Equivalent charge on balance.

gradually increases with age. On the other hand, at a certain age, charge on balance increases with charge on flow. This trend is consistent with the conclusion in Luis [12]. However, we need to note that this increment is very small, and the value of charge on balance is very small in our model, which is different from the result in Luis [12]. Because his model is based on the assumption that the utility function is exponential, we assume here that the utility function is power utility. Hence we can see that the power utility function yields results similar to that of the exponential utility function in [12]. That is, the charge is smaller, the cost is lower. For an affiliate of 20 years old, $\delta^*$ is 0.005304% per year when $f_{min}$ is the corresponding charge on flow. This shows that a charge on balance $\delta^* \geq 0.005304\%$ would make this pension plan to be preferred for all affiliates of Peru.

## 5. Conclusion

In this paper, we consider an affiliate who is involved in a defined contribution (DC) pension plan and aim to maximize the expected utility of terminal wealth under the constraint of his accumulation period. Fixed contribution is contributed to his personal pension account monthly, which is invested by a professional Pension Fund Administrator (PFA)'s manager and a certain commission is paid to the manager at the same time. The pension is invested in risk-free asset and risky asset such as monetary account and stock in order to preserve and increase the value of the pension account. The affiliate will receive his pension after retirement. For the commission, charge on balance, charge on flow, or both of them are prevalent in Latin America recently. The relationship between these two kinds of administrative charges, when they are equivalent, is based on the study in Luis [12]. Under the continuously complete market, we expand the driving equation of the risky asset to a general one which captures the implied volatility skew, namely, the constant elasticity of variance (CEV) model, and take the power utility function of CRRA in our considerations. It is noted that the contribution rate is a constant in our paper. The general optimization problems are listed. Two kinds of charge factors are added to our SDE, respectively, to solve the three stochastic optimal control problems, so as to obtain the certainty equivalent. We utilize the methodology in Luis [12] to determine equivalent administrative charges. Our conclusion shows that, even if the risky asset is assumed to follow the CEV model, there is no elasticity parameter in our comparison. Namely, it has nothing to do with the elasticity parameter, i.e., the implied volatility skew, and there is no concern with the risk aversion coefficient.

## Data Availability

Table 1 has the data to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

The article is a joint work of three authors who contributed equally to the final version of the paper. They read and approved the final manuscript.

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## References

[1] E. Vigna and S. Haberman, "Optimal investment strategy for defined contribution pension schemes," *Insurance: Mathematics and Economics*, vol. 28, no. 2, pp. 233–262, 2001.

[2] P. Devolder, M. B. Princep, and I. D. Fabian, "Stochastic optimal control of annuity contracts," *Insurance: Mathematics and Economics*, vol. 33, no. 2, pp. 227–238, 2003.

[3] J. W. Gao, "Stochastic optimal control of DC pension funds," *Insurance: Mathematics and Economics*, vol. 42, no. 3, pp. 1159–1164, 2008.

[4] J. Gao, "Optimal portfolios for DC pension plans under a CEV model," *Insurance: Mathematics & Economics*, vol. 44, no. 3, pp. 479–490, 2009.

[5] A. Wang, L. S. Yong, Y. Wang, and X. Luo, "The CEV model and its application in a study of optimal investment strategy," *Mathematical Problems in Engineering*, vol. 2014, Article ID 317071, 7 pages, 2014.

[6] H. Chang, X.-m. Rong, H. Zhao, and C.-b. Zhang, "Optimal investment and consumption decisions under the constant elasticity of variance model," *Mathematical Problems in Engineering*, vol. 2013, Article ID 974098, 11 pages, 2013.

[7] G. Guan and Z. Liang, "Optimal management of DC pension plan in a stochastic interest rate and stochastic volatility framework," *Insurance: Mathematics and Economics*, vol. 57, pp. 58–66, 2014.

[8] G. Guan and Z. Liang, "Optimal management of DC pension plan under loss aversion and Value-at-Risk constraints," *Insurance: Mathematics and Economics*, vol. 69, pp. 224–237, 2016.

[9] J. Sun, Z. Li, and Y. Li, "Equilibrium investment strategy for DC pension plan with inflation and stochastic income under Heston's SV model," *Mathematical Problems in Engineering*, vol. 2016, Article ID 2391849, 18 pages, 2016.

[10] D.-L. Sheng and X. Rong, "Optimal time-consistent investment strategy for a DC pension plan with the return of premiums clauses and annuity contracts," *Discrete Dynamics in Nature and Society*, Art. ID 862694, 13 pages, 2014.

[11] Z. Chen, Z. Li, Y. Zeng, and J. Sun, "Asset allocation under loss aversion and minimum performance constraint in a DC pension plan with inflation risk," *Insurance: Mathematics and Economics*, vol. 75, pp. 137–150, 2017.

[12] L. Chávez-Bedoya, "Determining equivalent charges on flow and balance in individual account pension systems," *Journal of Economics, Finance and Administrative Science*, vol. 21, no. 40, pp. 2–7, 2016.

[13] B. E. Kritzer, S. J. Kay, and T. Sinha, "Next generation of individual account pension reforms in Latin America," *Social Security Bulletin*, vol. 71, no. 11, pp. 35–76, 2011.

[14] E. Whitehouse, "Administrative charges for funded pensions: comparison and assessment of 13 countries," *Social Protection Discussion Papers*, pp. 85–154, 2001.

[15] W. Tapia and J. Yermo, "Fees in individual account pension systems: A cross country comparison," *OECD Working Papers on Insurance and Private Pensions*, vol. 27, 2008.

[16] M. Queisser, "Regulation and supervision of pension funds: Principles and practices," *International Social Security Review*, vol. 51, no. 2, pp. 39–55, 1998.

[17] D. G. Hernanez and F. Stewart, *Comparison of Costs And Fees in Countries with Private Defined Contribution Pension Systems*, vol. 6, Internantional Organisation of Pension Supervisors, Paris, France, 2008.

[18] J. C. Cox, "The constant elasticity of variance option pricing model," *The Journal of Portfolio Management*, vol. 22, pp. 16-17, 1996.