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Research Article

Uncertain Multiattribute Decision-Making Based on Interval Number with Extension-Dependent Degree and Regret Aversion

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In view of the uncertainty multiattribute decision-making problem with attribute values and weights both being interval number, a new solution based on regret theory and extension-dependent degree is proposed. It can define pass value of each attribute, which means decision-maker's acceptance for the scheme under the pass value will decline quickly. Then according to traditional regret theory, the method defines an extension-dependent function based on pass value which can improve the flexibility of the traditional utility function and the ability to describe the risk aversion actions from decision-makers. Then the extension-dependent function for interval number is built, and the perceived utility value of each scheme is obtained based on the interval's optimal value. The method can also reflect the decision-maker's reference to high or low evaluation score by setting attitude coefficients. At last, an example is presented to examine the feasibility, effectiveness, and stability of our method.

1. Introduction

Multiattribute decision-making (MADM) is aimed at finding the most desirable solution according to the evaluation results to several given attributes, which is widely used in various business applications such as online auction, investment decision, and e-commence platform evaluation. But due to the increasing complexity of socioeconomic environment and inherent knowledge restrictions of decision-makers [1], those attributes' environment information always exhibits fuzzy and uncertain features which are difficult to describe by exact numerical values. Several technologies are proposed to deal with the above problem such as interval number, fuzzy number, connection number, and grey number [2–5]. Among them, because of outstanding simple and intuitionistic features, interval number is proved to be one of the most frequently used described frameworks for uncertain information in many practical applications [6].

Early decision-making methods belong to complete rational decisions based on expected utility theory. They often use the expected utility value of each scheme to perform sorting. But many experimental studies have shown that, in the decision-making process, decision-makers usually have some psychological and behavior characteristics such as reference dependence, loss aversion, diminishing sensitivity, and expected results comparison [7]. The traditional decision methods cannot describe these psychological characteristics. So the finite rational decision methods based on decision-maker's behavior obtain more and more attentions [8]. The representatives are prospect theory and regret theory. The regret theory is one of the attractive research hotspots [9, 10]. By rejoice-regret utility function, regret theory not only considers the utility value of the alternative to be selected but also considers the results of others alternative and can describe those psychological characteristics of decision-makers. Literature [11] calculates collective utilities of each attribute in hesitant fuzzy set according to regret theory and then pursues matching result by aggregating collective utilities with relative weights. Literature [12] builds regret value matrix of comparisons between couple alternatives by regret theory and then obtains sorting results by counting total regret values between each alternative and all the other alternatives. Literature [13] defines grey perception utility function based on regret theory and then builds multiobjective optimization model which produces the maximum grey overall perceived utility values of alternatives, so that the optimal grey perceived utility value of each scheme is ranked by the sorting rules of interval grey numbers. In literature [14], the rejoice-regret value function based on the positive and negative ideal solution is constructed and the positive and negative ideal bull's eyes are obtained. So a multiobjective grey target decision model is completed. Literature [15] uses regret theory to obtain interval comprehensive utility value of alternatives in three-parameter interval number decision-making. In literature [16], an improved feedback adjustment mechanism based on regret theory is employed as the consistency model, which complements prospect theory in multiattribute group decision problems. Literature [17] combines regret theory to TOPSIS method and applies to grey stochastic multicriteria decision-making. Literature [12, 15] point out that, among those decision strategies based on decision-makers' behavior, prospect theory has the strong capacity to describe decision-makers' behavior, but has more parameters and more computational complexity. By contrast, regret theory has fewer parameters and is easy to implement, but has weaker describing capacity. Literature [18] concludes that regret theory has more advantages in practical application than prospect theory.

Although various methods adopting regret theory have been widely applied to uncertain decision-making, there are some problems for the researchers. Firstly, most of existing methods are built on the traditional basic model of regret theory. So the ability of these methods for describing decision-makers' behavior is insufficient. That is, when the risk evasion coefficient is determined, the curve of the utility function in regret theory is fixed. It has nothing to do with the range and distribution of the corresponding attribute values, so it lacks flexibility. However, in many decision-making applications, attribute values usually have a special value called pass point, below which the acceptance of decision-makers will decline rapidly. It means that decision-makers often show distinct risk chasing psychology. The basic model of regret theory is not sufficient to describe these behavior features and psychology features of the people. Secondly, many frameworks based on the basic model of regret theory often appear too definite, which means it is difficult to perform any stability test or uncertainty analysis for final decision results. So we do not know whether a tiny change of the psychology or preference of decision makes will bring different decision results. That will influence the credibility of the results. So a good method for decision-making should not only produce a sorting result but also bring some stability test processes.

To solve the above problems, the paper attempts to propose an uncertain multiattribute decision-making model based on regret theory and extension-dependent degree. Until the present, the related research has not been seen. In this model, the pass point value of the attribute can be set by decision-maker or other methods, and the extension-dependent degree of the attribute value can be obtained by using extension-dependent function based on side distance. Actually, it represents a new utility function expression and is more flexible than the basic utility function of regret theory. It is also more in line with people's decision-making habits. Then the regret-rejoice value of each attribute value is obtained by combining its dependent degree and regret theory. According to regret-rejoice values, the comprehensive utility value of each alternative will be calculated. Lastly, in our model, through setting the different value of preference attitude coefficient, the final ranking results can be given and analyzed further, which reflects the different attitude of decision-makers towards the upper and lower bounds of evaluation and reflects the ability of uncertainty analysis.

2. Regret Theory Model with Extension-Dependent Degree

2.1. Extension-Dependent Function with Side Distance. In extenics, extension distance is defined to describe the distance between a point and an interval, which is able to accurately describe the relative position between the point and the interval. For the definition of distance, classical mathematics utilizes qualitative descriptions of "belonging" and "not belonging", while extenics utilizes quantitative descriptions. In classical mathematics, there are elements that in the same domain are homogenous and but heterogeneous in different domains, while in extenics elements in the same domain they can be further classified into different layers and can be given quantitative description [19, 20].

Definition 1 (see [19, 21]). Suppose R the set of real number, $X = [a, b]$, is a finite interval in R, and its optimal point is $m \in X$, then $\forall x \in R$, according to the different location of m in X, extension distance $\rho(x, m, X)$ is

$$\text{(1) when } m = \frac{a+b}{2},\ \rho(x, m, X) = |x-m| - \frac{b-a}{2} \tag{1}$$

$$\text{(2) when } m \in \left[a, \frac{a+b}{2}\right),\ \rho(x, m, X) = \begin{cases} a-x & x \le a \\ \dfrac{b-m}{a-m}(x-a) & a < x < m \\ x-b & x \ge m \end{cases} \tag{2}$$

$$\text{(3) when } m \in \left(\frac{a+b}{2}, b\right],\ \rho(x, m, X) = \begin{cases} a-x & x \le m \\ \dfrac{a-m}{b-m}(b-x) & m < x < b \\ x-b & x \ge b \end{cases} \tag{3}$$

In addition, in this paper, we agree that $\rho(a, a, X) = \rho(b, b, X) = a - b$.

Definition 2 (see [19, 22]). Suppose an interval covering is composed of standard positive interval X_0 and positive interval X. $X_0 = [a, b]$, $X = [c, d]$, $X_0 \subset X$, and optimal point is $m_0 \in X_0$, for any point $x \in X$, extension-dependent function $k(x)$ is

$$k(x) = \begin{cases} \dfrac{\rho(x, m_0, X)}{\rho(x, m_0, X) - \rho(x, m_0, X_0)} & \rho(x, m_0, X_0) \neq \rho(x, m_0, X), \ x \notin X_0 \\ \dfrac{\rho(x, m_0, X) + a - b}{\rho(x, m_0, X) - \rho(x, m_0, X_0) + a - b} & x \in X_0 \\ \dfrac{\rho(x, m_0, X)}{a - b} & \rho(x, m_0, X_0) = \rho(x, m_0, X), \ x \notin X_0 \end{cases} \quad (4)$$

then $k(x)$ satisfies following properties:

(1) $x \in X_0$, and $x \neq a, b \Longleftrightarrow k(x) > 1$.

(2) $x = a$ or $x = b \Longleftrightarrow k(x) = 1$.

(3) $x \notin X_0, x \in X$, or $x \neq a, b, c, d \Longleftrightarrow 0 < k(x) < 1$.

(4) $x = c$ or $x = d \Longleftrightarrow k(x) = 0$.

(5) When $x = m_0$, $k(x)$ reaches its maximum value.

Extension-dependent function describes the dependent degree between point x and interval covering X_0, X. Its calculation is based on the given range of values of a certain feature, and it does not need to rely on subjective judgment or empirical value from decision-makers, so it is convenient to quantitatively describe the nature of things.

2.2. Regret Theory with Extension-Dependent Function. Regret theory [9] is one of the most important behavioral decision theories in behavioral economics. Its basic idea is that decision-makers in the decision-making process are concerned not only with the outcome of the options they consider choosing, but also with the possible impact of choosing other options. Decision-makers may have the expectation of regret or delight from their decision result and try to avoid choosing the alternative that may bring regret perception and tend to choose the alternative that brings delight perception. The perceived utility function of decision-makers consists of two parts: the utility function of the current selected alternative and the rejoice-regret function for comparing with the other schemes.

In many practical decision-making problems, decision-makers are usually risk aversion. So for the benefit attributes, the utility function $v(x)$ is shown as a monotone increasing concave function and satisfies $v'(x) > 0$ and usually employs negative exponential form [12]:

$$v(x) = \frac{1 - \exp(-\beta x)}{\beta}, \quad 0 < \beta < 1 \quad (5)$$

Here β is risk aversion coefficient. The greater the β value, the greater the risk aversion of decision-makers, as shown in Figure 1. For cost attributes, the utility function takes the following form [12]:

$$v(x) = 1 - \exp(\beta x), \quad 0 < \beta < 1 \quad (6)$$

Here the greater the β value, the greater the risk aversion of decision-makers.

The utility function reflects the risk evasion attitude of the decision-maker. It can be seen that the curve slope of utility value under small risk aversion coefficient is larger than the curve slope of utility value under large risk aversion coefficient, which expresses the high sensitivity of the decision-maker for increasing utility value near the marginal point. But the traditional utility function in regret theory is not enough to describe the psychological behavior of risk aversion. Firstly, when the value of risk aversion coefficient is determined (for example, [7] suggests 0.02), the curve of utility function is fixed in the whole decision-making process, and it cannot be adjusted according to the value of different attributes. Secondly, in several practical decision-making applications, every attribute always contains a psychological pass value, under which the decision-maker's acceptance to the alternative presents a rapid decline trend. So this value can be regarded as the pass reference value of an alternative under the attribute. For example, there are two attributes with percentage ratings and their weights are the same. Suppose their pass values both are 60, below which it is considered to be unqualified. Here alternative A is evaluated as (50,80) and alternative B is evaluated as (60,70). Due to the distinct difference of the acceptance from decision-makers to the evaluations above the pass value and below the pass value, it shows decision-makers' evasion attitude to select the alternative containing the evaluation score below the pass value. So, although the total scores of alternatives A and B both are 130, the decision-maker will choose alternative B because alternative A contains the unqualified score. Therefore, the evaluation below the pass value represents a higher risk, which decision-makers will tend to avoid. But the traditional utility function in regret theory cannot describe this psychology of risk aversion. Combined with the above ideas and extension-dependent function in extenics theory, this paper adopts extension-dependent function as a new utility function. Without increasing the number of parameters, our method can describe the behavior of decision-makers more flexibly and reasonably and enhance the ability to capture risk aversion behavior.

Definition 3. Suppose the value range of a beneficial attribute is $X = [c, d]$, and its pass value is a ($c < a < d$). Then $X = [c, d]$ can be seen as positive interval and $X_0 = [a, d]$ as standard positive interval, $X_0 \subset X$, optimal value is d, according to Definition 2, for any point $x \in X$, extension-dependent function $k(x, X_0, X)$ is

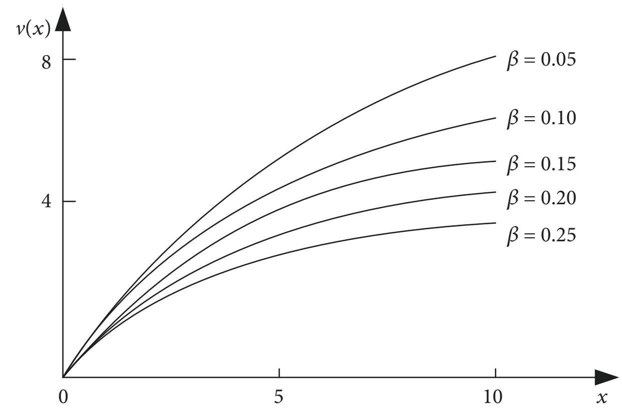

FIGURE 1: Utility function.

$$k\left(x, X_0, X\right) = \begin{cases} \dfrac{\rho\left(x, d, X\right)}{\rho\left(x, d, X\right) - \rho\left(x, d, X_0\right)} = \dfrac{c - x}{c - x - (a - x)} = \dfrac{c - x}{c - a} & x \notin X_0 \\ \dfrac{\rho\left(x, d, X\right) + a - d}{\rho\left(x, d, X\right) - \rho\left(x, d, X_0\right) + a - d} = \dfrac{c - x + a - d}{c - x - (a - x) + a - d} = \dfrac{a - x}{c - d} + 1 & x \in X_0 \end{cases} \quad (7)$$

The curve graph of $k(x, X_0, X)$ is shown in Figure 2 and has the following properties:

(1) $x \in X_0$, and $x \neq a \Longleftrightarrow k(x, X_0, X) > 1$.

(2) $x = a \Longleftrightarrow k(x, X_0, X) = 1$.

(3) $x \notin X_0, x \in X$, and $x \neq a, c, d \Longleftrightarrow 0 < k(x, X_0, X) < 1$.

(4) $x = c \Longleftrightarrow k(x, X_0, X) = 0$.

(5) When $x = d$, $k(x, X_0, X)$reaches the maximum value $(a - d)/(c - d) + 1$.

(6) The slope of the line segment $k(x, X_0, X)(x < a)$ must be larger than that of $k(x, X_0, X)(x > a)$.

As shown in Figure 2, with the pass value a as the boundary point, $k(x, X_0, X)$ consists of two segments of the line. The shape of the line segment changes adaptively with the value range of the attribute or the position of the pass value a. In Figure 2, when pass value is changed from a to a', the slope of the line segment under the pass value is automatically adjusted. When the value range of the attribute is changed from $[c, d]$ to $[c, d']$, the slope of the line segment above the pass value is also automatically adjusted. When the range of value and pass value are both changed, the whole line shape will change. So it is more flexible and adaptive than the traditional utility function in regret theory. In addition, when x is less than the pass value a, $k(x, X_0, X)$ will decrease at a faster speed. It means that the acceptance of decision-makers will decline rapidly when the evaluation of an attribute is under the pass value, which distinctly reflects a higher risk aversion attitude from decision-makers. It can be proved that, for any pass value $a \in (c, d)$, the slope of the line segment $k(x, X_0, X)(x < a)$ must be larger than that of $k(x, X_0, X)(x > a)$.

Theorem 4. *In Definition 3, the slope of the line segment $k(x, X_0, X)(x < a)$ must be larger than that of $k(x, X_0, X)(x > a)$.*

Proof. As shown in Figure 2, $\forall a \in (c, d)$, the slope of the line segment of $k(x, X_0, X)(x < a)$ is $1/(a - c)$, and that of $k(x, X_0, X)(x > a)$ is $((a - d)/(c - d))/(d - a) = 1/(d - c)$. Due to $c < a < d$, there must be $1/(a - c) > 1/(d - c)$. □

After getting the dependent degree of the attribute value, the regret value of each attribute value and its optimal value can be calculated for each alternative. Suppose the regret-rejoice function is $R(\Delta k)$, due to decision-makers' risk aversion attribute to regret, function $R(\Delta k)$ is a monotonously incrementally concave function and satisfies $R(\Delta k)' > 0$, $R(\Delta k)$ can be expressed as [12]:

$$R\left(\Delta k\right) = 1 - \exp\left(-\delta \Delta k\right) \quad (8)$$

Here, δ is regret aversion coefficient and satisfies $\delta > 0$. The setting of δ is decided by the experience value and the psychology of decision-makers and usually taken in 0~1. As shown in Figure 3, the larger the value δ, the greater the extent of regret evasion from the decision-maker. When people has higher risk awareness and regret evasion trend, he should take a greater value of δ. If people has medium regret evasion psychology, δ is often taken as 0.3 [12, 15]. Δk denotes the difference of dependent degree of the same attribute under two alternatives. When $R(\Delta k) > 0$, $R(\Delta k)$ denotes rejoice value, and when $R(\Delta k) < 0$, $R(\Delta k)$ denotes regret value.

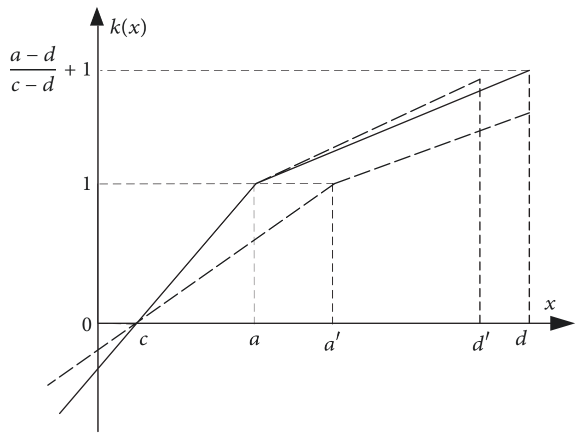

Figure 2: Extension-dependent function.

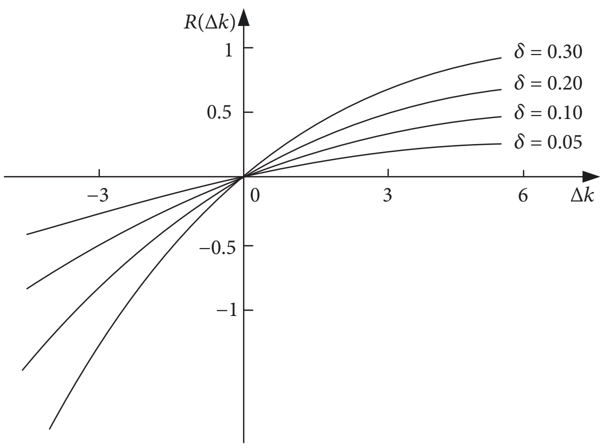

Figure 3: Regret-rejoice function.

According to formula (8), when $\Delta k > 0$, there is $|R(-\Delta k)| > R(\Delta k)$. It indicates that decision-makers are more sensitive to the psychological perception of $-\Delta k$ than to Δk; that is, decision-makers are always regret aversion.

For simplifying the calculating process and without losing its validity, we obtain regret value through comparing the attribute value of each alternative to the optimal value of the attribute. Here, according formula (8), for a beneficial attribute with its positive interval $X = [c, d]$ and standard positive interval $X_0 = [a, d]$, clearly its ideal value is d, then the regret value of attribute value x is

$$R(x, X_0, X) = 1 - \exp(-\delta(k(x, X_0, X) - k(d, X_0, X))) \tag{9}$$

The perception utility value of x is

$$u(x, X_0, X) = k(x, X_0, X) + R(x, X_0, X) \tag{10}$$

2.3. Interval Extension-Dependent Degree. In many decision-making applications, due to the complexity and uncertainty of objective things and the vague and finite features of human thinking, people often cannot give the definite value of attributes of things, but can only give an interval range. It is called uncertain decision-making problem. When the definite values of attributes are generalized to intervals, the computational model based on definite values will be transformed into interval-based computing models. So according to Definition 3, here we give a computing method of interval extension-dependent degree under beneficial attribute.

Definition 5. Suppose the value range of a beneficial attribute is $X = [c, d]$, and its pass value is a ($c < a < d$). Then $X = [c, d]$ can be seen as positive interval and $X_0 = [a, d]$ as standard positive interval, $X_0 \subset X$, optimal value is d, for any interval $Q = [q^-, q^+]$ and $Q \subseteq X$, extension-dependent function $k(Q, X_0, X)$ of interval Q and interval covering X_0, X is

$$k(Q, X_0, X) = \alpha k(q^-, X_0, X) + (1 - \alpha) k(q^+, X_0, X), \quad \alpha \in [0, 1] \tag{11}$$

Here, α is the preference attitude coefficient, which reflects the preference extent of decision-makers to the dependent degree of upper and lower bounds of the attribute index. That is to say, whether decision-makers attach more attention to the high evaluation of the attribute or to the low evaluation. When $\alpha = 0$, according to formula (11), it means that decision-makers only consider the upper bounds of evaluations, and vice versa. When $\alpha = 0.5$, it means that decision-makers pay equal attentions to the upper and lower bounds of evaluations. So the value of α is decided by the preference attitude of decision-makers to the two bounds. $k(Q, X_0, X)$ satisfies the following properties:

(1) When $q^- = q^+$, $k(Q, X_0, X)$ will degenerate into Definition 3.

(2) When $q^- = q^+ = d$, $k(Q, X_0, X)$ has the maximum value $(a-d)/(c-d)+1$. When $q^- = q^+ = c$, $k(Q, X_0, X)$ reaches the minimum value 0.

(3) When $Q \subset X_0$, $1 < k(Q, X_0, X) < (a - d)/(c - d) + 1$. When $Q \subset X$ and $Q \not\subset X_0$, $0 < k(Q, X_0, X) < 1$.

2.4. Pass Value Selecting. Pass value selecting of attributes can adopt subjective choosing or objective choosing. Subjective choosing is generally based on the experience of experts or some industry standards. Objective choosing can be carried out by choosing mean value, median value, expectation value, and two-eight principle.

2.5. Weight Calculating Based on Maximum Deviation of Deviation Degree. In view of sorting alternatives, if the difference of attribute value of all alternatives under a certain attribute is smaller, then the distinguishing effect of the attribute to all alternatives is smaller. So the smaller the weight should be given to the attribute. On the contrary, the greater the difference of the attribute value of each alternative under a certain attribute, the greater the distinguishing effect of the attribute, the greater the weight that should be given to the attribute. Here weight calculating model based on maximum deviation of interval deviation degree is built. Suppose the evaluation matrix is $(Q_{ij})_{m\times n}$ $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$, its evaluation value is interval value $Q_{ij} = [q_{ij}^{-}, q_{ij}^{+}]$, W is weight vector of attributes, and Φ is the range constraint of attribute weight values. The single object optimization model will be built:

$$\max \quad D(W) = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{m} \left(\left| q_{ij}^{-} - q_{kj}^{-} \right| + \left| q_{ij}^{+} - q_{kj}^{+} \right| \right) w_j$$

$$\text{s.t.} \quad W \in \Phi \tag{12}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} w_j = 1$$

Solving this model, the optimal weight vector $W = (w_1, w_2, \ldots\ldots, w_n)$ is obtained.

3. Process of Decision Algorithm

Suppose alternative set of multiattribute decision-making is $S = \{s_1, s_2, \ldots\ldots, s_m\}$, attribute set is $U = \{u_1, u_2, \ldots\ldots, u_n\}$, the evaluation value of alternative s_i to attribute u_j is interval value $P_{ij} = [p_{ij}^{-}, p_{ij}^{+}]$, then evaluation matrix is $(P_{ij})_{m\times n}$ $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$.

Step 1. By normalization of evaluation matrix $(P_{ij})_{m\times n}$, the new normalized evaluation matrix $(Q_{ij})_{m\times n}$ $(Q_{ij} = [q_{ij}^{-}, q_{ij}^{+}])$ is obtained. The standardized formula for the benefit attributes is $Q_{ij} = [q_{ij}^{-} = p_{ij}^{-}/\max_i(p_{ij}^{+}),\ q_{ij}^{+} = p_{ij}^{+}/\max_i(p_{ij}^{+})]$, $i = 1, 2, \ldots, m$. The standardized formula for the cost attributes is $Q_{ij} = [q_{ij}^{-} = (\min_i(p_{ij}^{-}))/p_{ij}^{+},\ q_{ij}^{+} = (\min_i(p_{ij}^{-}))/p_{ij}^{-}]$, $i = 1, 2, \ldots, m$. After the normalization, each attribute well be transformed into benefit attribute with its value range is $[0, 1]$.

Step 2. Get the value range of each attribute u_j and its pass value q_j^{*}, and then get the positive interval X_j and the standard positive interval X_{j0} of each attribute. Due to normalization of evaluation matrix, the value range of each attribute is mapped to $[0, 1]$ which is noted as the positive interval X_j. Clearly, the ideal point of X_j is 1. The pass value q_j^{*} of each attribute u_j is decided as required by decision-makers. Then standard positive interval X_{j0} of each attribute is $[q_j^{*}, 1]$. For formula (7), the extension-dependent degree $k(q_{ij}^{-}, X_{j0}, X_j)$ and $k(q_{ij}^{+}, X_{j0}, X_j)$ of each attribute value Q_{ij} are obtained. Then, for formula (11), the interval-dependent degree $k(Q_{ij}, X_{j0}, X_j)$ of each evaluation value Q_{ij} is calculated. Here, the preference attitude coefficient α should be decided as required by decision-makers.

Step 3. According to formula (9) and $k(Q_{ij}, X_{j0}, X_j)$ from Step 2, for each alternative, the regret value $R(Q_{ij}, X_{j0}, X_j)$ of each attribute value Q_{ij} is given. Here $R(Q_{ij}, X_0, X) = 1 - \exp(-\delta(k(Q_{ij}, X_0, X) - k(1, X_0, X)))$ because the ideal value is 1, and the setting of regret aversion coefficient δ is decided by the regret psychology of decision-makers. Combining $k(Q_{ij}, X_{j0}, X_j)$, $R(Q_{ij}, X_{j0}, X_j)$ and formula (10), the perception utility value u_{ij} $(u_{ij} = u(Q_{ij}, X_{j0}, X_j))$ of each attribute value Q_{ij} is obtained. Finally, the perception utility value matrix $(u_{ij})_{m\times n}$ is obtained.

Step 4. Counting deviation matrix $(D_{ij})_{m\times n}$ of interval deviation degree from the normalized evaluation matrix $(Q_{ij})_{m\times n}$, in which $D_{ij} = \sum_{k=1}^{m}(|q_{ij}^{-} - q_{kj}^{-}| + |q_{ij}^{+} - q_{kj}^{+}|)$. Then solving formula (12) to get optimal weight vector $W = (w_1, w_2, \ldots\ldots, w_n)$.

Step 5. From Steps 3 and 4, the optimal weight vector W of attributes and the perception utility value matrix $(u_{ij})_{m\times n}$ are obtained. Calculating the comprehensive perception utility value $U(s_i)$ of each alternative s_i by formula $U(s_i) = \sum_{j=1}^{n} w_j u_{ij}$, and then getting sorting results from $U(s_i)$.

Step 6. Performing uncertain analysis by different setting of preference attitude coefficient α and regret aversion coefficient δ. Here we recommend the setting ranges of α and δ are 0.2~0.8. α reflects the preference extent of decision-makers to the upper and lower evaluation bounds of the attribute. δ represents the psychology of regret evasion from decision-makers.

4. Example Analysis

For easy of comparison and illustration, the example is based on the data of [23]. An electronic commerce website intends to evaluate the different brands of air-conditionings sold online and to determine the ranking of brands recommended to users. The website examines five brands (s_1, s_2, s_3, s_4, s_5). The evaluation contains eight attributes, that is, fixed cost u_1, operation cost u_2, performance u_3, noise u_4, maintainability u_5, reliability u_6, flexibility u_7, and security u_8. Among them, the evaluations of attributes u_3, u_5, u_7, u_8 come from website users, and the scoring range is from 1 to 10. In addition, u_1, u_2, u_4 are cost attributes, and the others are benefit attributes. Based on attribute weights and evaluation matrix shown in Table 1, brand sorting will need to be determined. The example is performed by MATLAB program.

(1) According to Step 1, normalize the original evaluation matrix and get the standardized evaluation matrix, as shown in Table 2. The process not only eliminates the dimension differences of each attribute, but also transforms each attribute into benefit attribute.

(2) Getting the value interval of each attribute and its pass value: after standardization, evaluation value interval is changed to $[0, 1]$, so that the positive interval of each attribute value also is $[0, 1]$. Here, we take the median value as the pass value of each attribute, which represents the decision-maker's aversion attitude towards the evaluation below the median value, as shown in Table 3. According to Step 2, the interval extension-dependent degree of each evaluation value is calculated, and the interval-dependent degree matrix is obtained, as shown in Table 4.

(3) According to Step 3, the regret value and perception utility value of each attribute value and its optimal value are calculated, and the perception utility matrix is obtained, as shown in Table 5. The regret aversion coefficient δ here takes a recommended value of 0.3 [12].

Table 1: Evaluation matrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
s_1	[3.7, 4.7]	[5.9, 6.9]	[8, 10]	[30, 40]	[3, 5]	[90, 100]	[3, 5]	[6, 8]
s_2	[1.5, 2.5]	[4.7, 5.7]	[4, 6]	[65, 75]	[3, 5]	[70, 80]	[7, 9]	[4, 6]
s_3	[3.0, 4.0]	[4.2, 5.2]	[4, 6]	[60, 70]	[7, 9]	[80, 90]	[7, 9]	[5, 7]
s_4	[3.5, 4.5]	[4.5, 5.5]	[7, 9]	[35, 45]	[8, 10]	[85, 95]	[6, 8]	[7, 9]
s_5	[2.5, 3.5]	[5.0, 6.0]	[6, 8]	[50, 60]	[5, 7]	[85, 95]	[4, 6]	[8, 10]
w	[0.042, 0.049]	[0.084, 0.098]	[0.121, 0.137]	[0.121, 0.137]	[0.168, 0.182]	[0.214, 0.229]	[0.040, 0.046]	[0.159, 0.171]

Table 2: Standardized evaluation matrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
s_1	[0.319, 0.405]	[0.609, 0.712]	[0.800, 1.000]	[0.750, 1.000]	[0.300, 0.500]	[0.900, 1.000]	[0.333, 0.556]	[0.600, 0.800]
s_2	[0.600, 1.000]	[0.737, 0.894]	[0.400, 0.600]	[0.400, 0.462]	[0.300, 0.500]	[0.700, 0.800]	[0.778, 1.000]	[0.400, 0.600]
s_3	[0.375, 0.500]	[0.808, 1.000]	[0.400, 0.600]	[0.429, 0.500]	[0.700, 0.900]	[0.800, 0.900]	[0.778, 1.000]	[0.500, 0.700]
s_4	[0.333, 0.429]	[0.764, 0.933]	[0.700, 0.900]	[0.667, 0.857]	[0.800, 1.000]	[0.850, 0.950]	[0.667, 0.889]	[0.700, 0.900]
s_5	[0.429, 0.600]	[0.700, 0.840]	[0.600, 0.800]	[0.500, 0.600]	[0.500, 0.700]	[0.850, 0.950]	[0.444, 0.667]	[0.800, 1.000]
w	[0.042, 0.049]	[0.084, 0.098]	[0.121, 0.137]	[0.121, 0.137]	[0.168, 0.182]	[0.214, 0.229]	[0.040, 0.046]	[0.159, 0.171]

Table 3: Positive interval and pass value of each attribute.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
Positive interval	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[0, 1]
Pass value	0.429	0.764	0.600	0.500	0.500	0.850	0.667	0.700
Standard positive interval	[0.429, 1]	[0.764, 1]	[0.600, 1]	[0.500, 1]	[0.500, 1]	[0.850, 1]	[0.667, 1]	[0.700, 1]

Table 4: Interval-dependent degree matrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
s_1	0.8438	0.8645	1.3000	1.3750	0.8000	1.1000	0.6664	0.9786
s_2	1.3710	1.0473	0.8333	0.8620	0.8000	0.8824	1.2220	0.7143
s_3	0.9726	1.1400	0.8333	0.9290	1.3000	0.9956	1.2220	0.8571
s_4	0.8881	1.0845	1.2000	1.2620	1.4000	1.0500	1.1110	1.1000
s_5	1.0855	0.9961	1.1000	1.0500	1.1000	1.0500	0.8328	1.2000

Table 5: Perception utility matrix.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
s_1	0.6000	0.7466	1.2695	1.3368	0.5663	1.0849	0.4450	0.8773
s_2	1.3092	0.9891	0.6480	0.6511	0.5663	0.7987	1.1881	0.5222
s_3	0.7759	1.1108	0.6480	0.7422	1.2382	0.9482	1.1881	0.7151
s_4	0.6608	1.0380	1.1382	1.1880	1.3695	1.0195	1.0421	1.0382
s_5	0.9287	0.9215	1.0058	0.9055	0.9725	1.0195	0.6709	1.1695

(4) According to Step 4, deviation matrix of deviation degree can be counted. For each attribute, the deviation value of the attribute value of each alternative and that of the other alternatives is obtained, as shown in Table 6.

According to formula (12) and deviation matrix, a single object optimization model is established.

$$\begin{aligned}
\max \quad & D(W) \\
& = 7.843w_1 + 4.524w_2 + 8.800w_3 + 9.484w_4 \\
& \quad + 11.200w_5 + 3.600w_6 + 9.780w_7 \\
& \quad + 8.000w_8 \\
\text{s.t.} \quad & 0.042 \le w_1 \le 0.049, 0.084 \le w_2 \le 0.098, 0.121 \\
& \le w_3 \le 0.137, 0.121 \le w_4 \le 0.137 \\
& 0.168 \le w_5 \le 0.182, 0.214 \le w_6 \le 0.229, 0.040 \\
& \le w_7 \le 0.046, 0.159 \le w_8 \le 0.171 \\
\text{s.t.} \quad & \sum_{j=1}^{8} w_j = 1
\end{aligned} \tag{13}$$

Table 6: Deviation matrix of deviation degree.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8
s_1	1.370	1.392	2.200	2.585	2.200	0.800	2.667	1.200
s_2	3.010	0.644	1.800	1.855	2.200	1.200	1.778	2.000
s_3	1.143	1.043	1.800	1.654	2.200	0.600	1.778	1.400
s_4	1.256	0.710	1.600	1.907	2.800	0.500	1.556	1.400
s_5	1.297	0.735	1.400	1.483	1.800	0.500	2.001	2.000

Table 7: Comprehensive perceived utility value sorting.

	s_1	s_2	s_3	s_4	s_5	Sorting result
$U(s_i)$	0.9389	0.7271	0.9123	1.1129	0.9893	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$

Table 8: Sorting results under different attitude coefficient α.

α	s_1	s_2	s_3	s_4	s_5	Sorting result
0.3	1.0002	0.7936	0.9668	1.1594	1.0334	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
0.4	0.9696	0.7604	0.9396	1.1362	1.0113	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
0.5	0.9389	0.7271	0.9123	1.1129	0.9893	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
0.6	0.9082	0.6937	0.8850	1.0896	0.9671	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
0.7	0.8773	0.6603	0.8576	1.0663	0.9450	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$

Table 9: Sorting results under different regret aversion coefficient δ.

δ	s_1	s_2	s_3	s_4	s_5	Sorting result
0.2	0.9746	0.7814	0.9496	1.1325	1.0195	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
0.3	0.9389	0.7271	0.9123	1.1129	0.9893	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
0.5	0.8615	0.6088	0.8324	1.0723	0.9254	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
0.7	0.7751	0.4762	0.7449	1.0292	0.8567	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
0.8	0.7281	0.4040	0.6980	1.0068	0.8204	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$

The optimal weight vector W is obtained by solving this model.

$$W = (0.042, 0.084, 0.136, 0.137, 0.182, 0.214, 0.046, 0.159) \quad (14)$$

(5) According to Step 5, calculating the comprehensive perceived utility value of each alternative and sorting by the values, the result is shown in Table 7.

(6) Performing uncertain analysis: the attitude coefficient α represents the decision-maker's preference towards the upper and lower bounds of attribute evaluation. Table 8 shows the decision sorting results under five different α settings. It shows that the sorting results under different attitude coefficients are still the same, which indicates that the current sorting does not change with the decision-makers' preference to the evaluation bounds. Table 9 shows the decision results under different regret aversion coefficient. It can be seen that alternatives sorting remains unchanged, which illustrates the current sorting has higher stability.

(7) Comparison of decision results under different algorithms: Table 10 shows the decision results under five algorithms. In our algorithm, the pass value is median value and mean value, respectively, attitude coefficient is 0.5, and regret aversion coefficient is 0.3. In regret theory, risk aversion coefficient is 0.02 and regret aversion coefficient is 0.3. In set pair analysis, the uncertain number i is 0 and 0.5, respectively. The sorting results of these algorithms are consistent. The results of TOPSIS and relative membership degree algorithms are slightly different, mainly on the sorting of alternatives s_1, s_3, s_5. It can be seen that, for sorting of s_3 and s_5, TOPSIS is consistent with our method and the conclusion of relative membership degree method are different. But for sorting of s_1 and s_3, relative membership degree method and our method are consistent and are disagreement with TOPSIS. So in the whole, the sorting result of our method is more reasonable.

(8) Comparison with the traditional regret theory: the line shape of utility function in regret theory is uniquely determined by the risk aversion coefficient β and is not flexible enough. Moreover, its setting is difficult due to often lack sufficient basis. By comparison, the extension-dependent function of our algorithm can be adjusted automatically according to the pass value and the value range of attribute, and so it is more flexible. In many cases, setting pass value is more in line with people's habit in decision-making. It is also easy to find a certain basis for its value taking, whether it follows subjective method or objective method. Pass value setting will have an impact on the result of the decision. In

Table 10: Sorting results under different algorithms.

	s_1	s_2	s_3	s_4	s_5	Sorting result
TOPSIS [24]	0.4176	0.2671	0.4188	0.5777	0.4498	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
elative membership degree [25]	0.4502	0.0327	0.3262	0.9167	0.3381	$s_4 \succ s_1 \succ s_5 \succ s_3 \succ s_2$
set pair analysis ($i = 0$) [23]	0.6036	0.4781	0.5827	0.6974	0.6203	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
set pair analysis ($i = 0.5$) [23]	0.7270	0.5916	0.6958	0.8227	0.7354	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
regret theory	0.6268	0.4465	0.5873	0.7579	0.6369	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
regret-extension (median value)	0.9389	0.7271	0.9123	1.1129	0.9893	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$
regret-extension (mean value)	0.8663	0.6450	0.8353	1.0474	0.9052	$s_4 \succ s_5 \succ s_1 \succ s_3 \succ s_2$

this case, assuming that the decision-maker has a subjective setting for pass value of 0.4, 0.4, and 0.6 for the attributes u_3, u_4, u_5 in Table 3, then alternatives s_1 and s_3 in the decision result will be reversed. But in traditional regret theory, the reversion will not happen.

5. Conclusion

In summary, the main work of the paper is as follows. (1) For uncertain multiattribute decision-making, a new extension-dependent degree decision-making method based on regret theory is proposed. (2) Aiming at the problem that the utility function in traditional regret theory is not strong enough to describe risk aversion behavior, an extension-dependent degree function combining pass value is given. (3) For quantitative describing uncertain attribute value information, the expression of interval extension-dependent degree function is given. The function can reflect the degree of decision-maker's preference towards the upper and lower bounds of evaluation interval through preference attitude coefficient setting. (4) For attribute weight being taken as interval number, an objective weight assignment method based on the maximum deviation of interval deviation degree is given. (5) An example is given to verify the feasibility and applicability of our method, and the uncertain analysis for the decision result is carried out. The future research work is very rich. The regret extension-dependent decision method in the paper is considered to be applied to various decision scenarios such as mixed value type, incomplete information, dynamic stochastic, fuzzy set, and in further study its extension transformation [26] in the dynamic decision model.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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