

## Material for Online Appendix

### A.1 Out-of-sample evaluation of the pre-commitment and time-consistent strategies

#### A.1.1 Sharpe ratios

Tables A1-A9 contain the results for Sharpe ratios of portfolios under different strategies when  $T = 2, 4, 8$ ,  $\omega = 0.1, 0.5, 1.0, 1.5, 2.0$  and  $M = 150, 180, 240$ . It indicates that the time-consistent strategies almost have higher Sharpe ratios than pre-commitment strategies in out-of-sample test, no matter that the risk-free assets are considered into capital pool or not.

**Table A1. Sharpe ratios of portfolios under different strategies (T=2, M=150)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                           | 0.0799                          | 0.2782          | 0.1129                             | 0.1487          |
| 0.5                                                           | 0.0799                          | 0.2782          | 0.1153                             | 0.1516          |
| 1.0                                                           | 0.0799                          | 0.2782          | 0.1155                             | 0.1517          |
| 1.5                                                           | 0.0799                          | 0.2782          | 0.1156                             | 0.1518          |
| 2.0                                                           | 0.0799                          | 0.2782          | 0.1156                             | 0.1518          |

**Table A2. Sharpe ratios of portfolios under different strategies (T=2, M=180)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                           | 0.1038                          | 0.2087          | 0.1404                             | 0.1758          |
| 0.5                                                           | 0.1038                          | 0.2087          | 0.1456                             | 0.1807          |
| 1.0                                                           | 0.1038                          | 0.2087          | 0.1461                             | 0.1811          |
| 1.5                                                           | 0.1038                          | 0.2087          | 0.1463                             | 0.1812          |
| 2.0                                                           | 0.1038                          | 0.2087          | 0.1464                             | 0.1812          |

**Table A3. Sharpe ratios of portfolios under different strategies (T=2, M=240)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                           | 0.0956                          | 0.1712          | 0.1152                             | 0.1352          |
| 0.5                                                           | 0.0956                          | 0.1712          | 0.1240                             | 0.1426          |
| 1.0                                                           | 0.0956                          | 0.1712          | 0.1250                             | 0.1434          |
| 1.5                                                           | 0.0956                          | 0.1712          | 0.1253                             | 0.1436          |
| 2.0                                                           | 0.0956                          | 0.1712          | 0.1255                             | 0.1437          |

**Table A4. Sharpe ratios of portfolios under different strategies (T=4, M=150)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                           | 0.0314                          | 0.3972          | -0.0762                            | -0.0832         |
| 0.5                                                           | 0.0314                          | 0.3972          | -0.0758                            | -0.0827         |
| 1.0                                                           | 0.0314                          | 0.3972          | -0.0758                            | -0.0826         |
| 1.5                                                           | 0.0314                          | 0.3972          | -0.0758                            | -0.0826         |
| 2.0                                                           | 0.0314                          | 0.3972          | -0.0757                            | -0.0826         |

**Table A5. Sharpe ratios of portfolios under different strategies (T=4, M=180)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                           | 0.0984                          | 0.3002          | 0.1789                             | 0.2075          |
| 0.5                                                           | 0.0984                          | 0.3002          | 0.1789                             | 0.2074          |
| 1.0                                                           | 0.0984                          | 0.3002          | 0.1789                             | 0.2074          |
| 1.5                                                           | 0.0984                          | 0.3002          | 0.1789                             | 0.2074          |
| 2.0                                                           | 0.0984                          | 0.3002          | 0.1789                             | 0.2074          |

**Table A6. Sharpe ratios of portfolios under different strategies (T=4, M=240)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                           | 0.0000                          | 0.2310          | 0.1787                             | 0.2210          |
| 0.5                                                           | 0.0000                          | 0.2310          | 0.1785                             | 0.2207          |
| 1.0                                                           | 0.0000                          | 0.2310          | 0.1785                             | 0.2207          |
| 1.5                                                           | 0.0000                          | 0.2310          | 0.1785                             | 0.2206          |
| 2.0                                                           | 0.0000                          | 0.2310          | 0.1785                             | 0.2206          |

**Table A7. Sharpe ratios of portfolios under different strategies (T=8, M=150)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                           | 0.1639                          | 0.5699          | 0.1484                             | 0.1388          |
| 0.5                                                           | 0.1639                          | 0.5699          | 0.1484                             | 0.1388          |
| 1.0                                                           | 0.1639                          | 0.5699          | 0.1484                             | 0.1388          |
| 1.5                                                           | 0.1639                          | 0.5699          | 0.1484                             | 0.1388          |
| 2.0                                                           | 0.1639                          | 0.5699          | 0.1484                             | 0.1388          |

**Table A8. Sharpe ratios of portfolios under different strategies (T=8, M=180)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                           | -0.0714                         | 0.4469          | -0.1153                            | -0.0899         |
| 0.5                                                           | -0.0714                         | 0.4469          | -0.1153                            | -0.0899         |
| 1.0                                                           | -0.0714                         | 0.4469          | -0.1153                            | -0.0899         |
| 1.5                                                           | -0.0714                         | 0.4469          | -0.1153                            | -0.0899         |
| 2.0                                                           | -0.0714                         | 0.4469          | -0.1153                            | -0.0899         |

**Table A9. Sharpe ratios of portfolios under different strategies (T=8, M=240)**

| Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                      | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                               | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                           | -0.0937                         | 0.3080          | 0.1345                             | 0.2317          |
| 0.5                                                           | -0.0937                         | 0.3080          | 0.1345                             | 0.2317          |
| 1.0                                                           | -0.0937                         | 0.3080          | 0.1345                             | 0.2317          |
| 1.5                                                           | -0.0937                         | 0.3080          | 0.1345                             | 0.2317          |
| 2.0                                                           | -0.0937                         | 0.3080          | 0.1345                             | 0.2317          |

**A.1.2 Portfolio turnovers**

Tables A10-A18 contain the results for Portfolio turnovers of portfolios under different strategies when  $T = 2, 4, 8$ ,  $\omega = 0.1, 0.5, 1.0, 1.5, 2.0$  and  $M = 150, 180, 240$ . It indicates that the time-consistent strategies almost have smaller Portfolio turnovers than pre-commitment strategies in out-of-sample test.

**Table A10. Portfolio turnovers of portfolios under different strategies (T=2, M=150)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                                 | 2.2075                          | 1.0975          | 9.6334                             | 8.5392          |
| 0.5                                                                 | 0.4415                          | 0.2195          | 9.7126                             | 8.5880          |
| 1.0                                                                 | 0.2207                          | 0.1097          | 9.7317                             | 8.6109          |
| 1.5                                                                 | 0.1472                          | 0.0732          | 9.7384                             | 8.6196          |
| 2.0                                                                 | 0.1104                          | 0.0549          | 9.7419                             | 8.6243          |

**Table A11. Portfolio turnovers of portfolios under different strategies (T=2, M=180)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                 | 1.8388                          | 0.8519          | 6.6178                             | 5.9282          |
| 0.5                                                                 | 0.3678                          | 0.1704          | 6.4892                             | 5.7951          |
| 1.0                                                                 | 0.1839                          | 0.0852          | 6.4808                             | 5.7941          |
| 1.5                                                                 | 0.1226                          | 0.0568          | 6.4783                             | 5.7947          |

|     |        |        |        |        |
|-----|--------|--------|--------|--------|
| 2.0 | 0.0919 | 0.0426 | 6.4771 | 5.7951 |
|-----|--------|--------|--------|--------|

**Table A12. Portfolio turnovers of portfolios under different strategies (T=2, M=240)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                 | 1.2105                          | 0.5394          | 4.8221                             | 4.5414          |
| 0.5                                                                 | 0.2421                          | 0.1079          | 4.7784                             | 4.5016          |
| 1.0                                                                 | 0.1211                          | 0.0539          | 4.7784                             | 4.5024          |
| 1.5                                                                 | 0.0807                          | 0.0360          | 4.7787                             | 4.5029          |
| 2.0                                                                 | 0.0605                          | 0.0270          | 4.7789                             | 4.5032          |

**Table A13. Portfolio turnovers of portfolios under different strategies (T=4, M=150)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                                 | 5.7103                          | 1.0969          | 173.8550                           | 149.7050        |
| 0.5                                                                 | 1.1421                          | 0.2194          | 173.6078                           | 149.7453        |
| 1.0                                                                 | 0.5710                          | 0.1097          | 173.5808                           | 149.7609        |
| 1.5                                                                 | 0.3807                          | 0.0731          | 173.5720                           | 149.7667        |
| 2.0                                                                 | 0.2855                          | 0.0548          | 173.5677                           | 149.7696        |

**Table A14. Portfolio turnovers of portfolios under different strategies (T=4, M=180)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                 | 4.1873                          | 0.8509          | 114.6390                           | 100.9862        |
| 0.5                                                                 | 0.8375                          | 0.1702          | 114.5000                           | 100.9599        |
| 1.0                                                                 | 0.4187                          | 0.0851          | 114.4842                           | 100.9610        |
| 1.5                                                                 | 0.2792                          | 0.0567          | 114.4790                           | 100.9616        |
| 2.0                                                                 | 0.2094                          | 0.0425          | 114.4765                           | 100.9619        |

**Table A15. Portfolio turnovers of portfolios under different strategies (T=4, M=240)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                 | 2.6017                          | 0.5424          | 87.2186                            | 80.0922         |
| 0.5                                                                 | 0.5203                          | 0.1085          | 87.1600                            | 80.0851         |
| 1.0                                                                 | 0.2602                          | 0.0542          | 87.1539                            | 80.0868         |
| 1.5                                                                 | 0.1734                          | 0.0362          | 87.1518                            | 80.0875         |
| 2.0                                                                 | 0.1301                          | 0.0271          | 87.1508                            | 80.0879         |

**Table A16. Portfolio turnovers of portfolios under different strategies (T=8, M=150)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                                 | 34.2867                         | 1.0859          | 2.4914 e+04                        | 2.1013e+04      |
| 0.5                                                                 | 6.8573                          | 0.2172          | 2.4914 e+04                        | 2.1013e+04      |
| 1.0                                                                 | 3.4287                          | 0.1086          | 2.4914 e+04                        | 2.1013e+04      |
| 1.5                                                                 | 2.2858                          | 0.0724          | 2.4914 e+04                        | 2.1013e+04      |
| 2.0                                                                 | 1.7143                          | 0.0543          | 2.4914 e+04                        | 2.1013e+04      |

**Table A17. Portfolio turnovers of portfolios under different strategies (T=8, M=180)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                 | 19.6955                         | 0.8365          | 5.6155 e+03                        | 4.9898 e+03     |
| 0.5                                                                 | 3.9391                          | 0.1673          | 5.6152 e+03                        | 4.9897 e+03     |
| 1.0                                                                 | 1.9695                          | 0.0836          | 5.6152 e+03                        | 4.9897 e+03     |
| 1.5                                                                 | 1.3130                          | 0.0558          | 5.6152 e+03                        | 4.9897 e+03     |
| 2.0                                                                 | 0.9848                          | 0.0418          | 5.6152 e+03                        | 4.9897 e+03     |

**Table A18. Portfolio turnovers of portfolios under different strategies (T=8, M=240)**

| Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|---------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                            | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                     | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                 | 12.3631                         | 0.5491          | 1.3912 e+04                        | 1.3674e+04      |
| 0.5                                                                 | 2.4726                          | 0.1098          | 1.3912 e+04                        | 1.3674e+04      |
| 1.0                                                                 | 1.2363                          | 0.0549          | 1.3912 e+04                        | 1.3674e+04      |
| 1.5                                                                 | 0.8242                          | 0.0366          | 1.3912 e+04                        | 1.3674e+04      |
| 2.0                                                                 | 0.6182                          | 0.0275          | 1.3912 e+04                        | 1.3674e+04      |

**A.1.3 Maximum drawdowns**

Tables A19-A27 contain the results for Maximum drawdowns of portfolios under different strategies when  $T = 2, 4, 8$ ,  $\omega = 0.1, 0.5, 1.0, 1.5, 2.0$  and  $M = 150, 180, 240$ . It indicates that the time-consistent strategies almost have smaller Maximum drawdowns than pre-commitment strategies in out-of-sample test.

**Table A19. Maximum drawdowns of portfolios under different strategies (T=2, M=150)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                               | 45.9583                         | 23.7969         | 263.8783                           | 183.3611        |
| 0.5                                                               | 9.1917                          | 4.7594          | 249.3819                           | 168.2287        |
| 1.0                                                               | 4.5958                          | 2.3797          | 247.5699                           | 166.3372        |
| 1.5                                                               | 3.0639                          | 1.5865          | 246.9659                           | 165.7067        |
| 2.0                                                               | 2.2979                          | 1.1898          | 246.6638                           | 165.3914        |

**Table A20. Maximum drawdowns of portfolios under different strategies (T=2, M=180)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                               | 42.8837                         | 25.3822         | 176.6769                           | 141.0343        |
| 0.5                                                               | 8.5767                          | 5.0764          | 172.7793                           | 136.9842        |

|     |        |        |          |          |
|-----|--------|--------|----------|----------|
| 1.0 | 4.2884 | 2.5382 | 172.4754 | 136.9812 |
| 1.5 | 2.8589 | 1.6921 | 172.3742 | 136.9803 |
| 2.0 | 2.1442 | 1.2691 | 172.3235 | 136.9798 |

**Table A21. Maximum drawdowns of portfolios under different strategies (T=2, M=240)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                               | 44.9004                         | 21.0981         | 189.5206                           | 160.1904        |
| 0.5                                                               | 8.9801                          | 4.2196          | 189.2532                           | 159.0305        |
| 1.0                                                               | 4.4900                          | 2.1098          | 189.2198                           | 158.8855        |
| 1.5                                                               | 2.9934                          | 1.4065          | 189.2087                           | 158.8372        |
| 2.0                                                               | 2.2450                          | 1.0549          | 189.2031                           | 158.8130        |

**Table A22. Maximum drawdowns of portfolios under different strategies (T=4, M=150)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                               | 221.7973                        | 29.8781         | 8.5223 e+03                        | 5.7438 e+03     |
| 0.5                                                               | 44.3595                         | 5.9756          | 8.5167 e+03                        | 5.7395 e+03     |
| 1.0                                                               | 22.1797                         | 2.9878          | 8.5160 e+03                        | 5.7389 e+03     |
| 1.5                                                               | 14.7865                         | 1.9919          | 8.5158 e+03                        | 5.7388 e+03     |
| 2.0                                                               | 11.0899                         | 1.4939          | 8.5157 e+03                        | 5.7387 e+03     |

**Table A23. Maximum drawdowns of portfolios under different strategies (T=4, M=180)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                               | 139.3234                        | 25.1925         | 6.4422 e+03                        | 6.0267 e+03     |
| 0.5                                                               | 27.8647                         | 5.0385          | 6.4366 e+03                        | 6.0208 e+03     |
| 1.0                                                               | 13.9323                         | 2.5192          | 6.4359 e+03                        | 6.0201 e+03     |
| 1.5                                                               | 9.2882                          | 1.6795          | 6.4357 e+03                        | 6.0199 e+03     |
| 2.0                                                               | 6.9662                          | 1.2596          | 6.4356 e+03                        | 6.0197 e+03     |



**Table A24. Maximum drawdowns of portfolios under different strategies (T=4, M=240)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                               | 148.7466                        | 29.3552         | 5.7490 e+03                        | 5.6636 e+03     |
| 0.5                                                               | 29.7493                         | 5.8710          | 5.7450 e+03                        | 5.6597 e+03     |
| 1.0                                                               | 14.8747                         | 2.9355          | 5.7445 e+03                        | 5.6592 e+03     |
| 1.5                                                               | 9.9164                          | 1.9570          | 5.7444 e+03                        | 5.6590 e+03     |
| 2.0                                                               | 7.4373                          | 1.4678          | 5.7443 e+03                        | 5.6589 e+03     |

**Table A25. Maximum drawdowns of portfolios under different strategies (T=8, M=150)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | Pre-commitment                  | Time-consistent | Pre-commitment                     | Time-consistent |
| 0.1                                                               | 413.5974                        | 33.6192         | 1.5680 e+06                        | 1.2553e+06      |
| 0.5                                                               | 82.7195                         | 6.7238          | 1.5680 e+06                        | 1.2553e+06      |
| 1.0                                                               | 41.3597                         | 3.3619          | 1.5680 e+06                        | 1.2553e+06      |
| 1.5                                                               | 27.5732                         | 2.2413          | 1.5680 e+06                        | 1.2553e+06      |
| 2.0                                                               | 20.6799                         | 1.6810          | 1.5680 e+06                        | 1.2553e+06      |

**Table A26. Maximum drawdowns of portfolios under different strategies (T=8, M=180)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                               | 1.1443e+03                      | 26.2444         | 1.2219 e+05                        | 1.3265 e+05     |
| 0.5                                                               | 0.2289e+03                      | 5.2489          | 1.2219 e+05                        | 1.3265 e+05     |
| 1.0                                                               | 0.1144e+03                      | 2.6244          | 1.2219 e+05                        | 1.3265 e+05     |
| 1.5                                                               | 0.0763e+03                      | 1.7496          | 1.2219 e+05                        | 1.3265 e+05     |
| 2.0                                                               | 0.0572e+03                      | 1.3122          | 1.2219 e+05                        | 1.3265 e+05     |

**Table A27. Maximum drawdowns of portfolios under different strategies (T=8, M=240)**

| Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                          | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                   | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                               | 297.2511                        | 30.4434         | 2.9086 e+04                        | 1.3674 e+04     |
| 0.5                                                               | 59.4502                         | 6.0887          | 2.9086 e+04                        | 1.3674 e+04     |
| 1.0                                                               | 29.7251                         | 3.0443          | 2.9086 e+04                        | 1.3674 e+04     |
| 1.5                                                               | 19.8167                         | 2.0296          | 2.9086 e+04                        | 1.3674 e+04     |
| 2.0                                                               | 14.8626                         | 1.5222          | 2.9086 e+04                        | 1.3674 e+04     |

## A.2 Out-of-sample evaluation of the recalculated pre-commitment and recalculated time-consistent strategies

### A.2.1 Sharpe ratios

Tables A28-A36 contain the results for Mean of Sharpe ratios of portfolios under different strategies when  $H = 1000$ ,  $T = 2, 4, 8$ ,  $\omega = 0.1, 0.5, 1.0, 1.5, 2.0$  and  $M = 150, 180, 240$ . It indicates that the recalculated time-consistent strategies almost have larger Mean of Sharpe ratios than the recalculated pre-commitment strategies in out-of-sample test.

**Table A28. Mean of Sharpe ratios of portfolios under different strategies (T=2, M=150)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                              |                                    |                              |
|-----------------------------------------------------------------------|---------------------------------|------------------------------|------------------------------------|------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                              | Portfolio without risk-free assets |                              |
|                                                                       | Recalculated pre-commitment     | Recalculated time-consistent | Recalculated pre-commitment        | Recalculated time-consistent |
| 0.1                                                                   | 0.2490                          | 0.2408                       | 0.1479                             | 0.1481                       |
| 0.5                                                                   | 0.2490                          | 0.2408                       | 0.1394                             | 0.1397                       |
| 1.0                                                                   | 0.2490                          | 0.2408                       | 0.1382                             | 0.1385                       |
| 1.5                                                                   | 0.2490                          | 0.2408                       | 0.1378                             | 0.1381                       |
| 2.0                                                                   | 0.2490                          | 0.2408                       | 0.1376                             | 0.1379                       |

**Table A29. Mean of Sharpe ratios of portfolios under different strategies (T=2, M=180)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                              |                                    |                              |
|-----------------------------------------------------------------------|---------------------------------|------------------------------|------------------------------------|------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                              | Portfolio without risk-free assets |                              |
|                                                                       | Recalculated pre-commitment     | Recalculated time-consistent | Recalculated pre-commitment        | Recalculated time-consistent |
| 0.1                                                                   | 0.1888                          | 0.1839                       | 0.1197                             | 0.1199                       |

|     |        |        |        |        |
|-----|--------|--------|--------|--------|
| 0.5 | 0.1888 | 0.1839 | 0.1156 | 0.1159 |
| 1.0 | 0.1888 | 0.1839 | 0.1151 | 0.1154 |
| 1.5 | 0.1888 | 0.1839 | 0.1149 | 0.1152 |
| 2.0 | 0.1888 | 0.1839 | 0.1148 | 0.1151 |

**Table A30. Mean of Sharpe ratios of portfolios under different strategies (T=2, M=240)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                       | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                   | 0.3401                          | 0.3280                          | 0.1633                             | 0.1635                          |
| 0.5                                                                   | 0.3401                          | 0.3280                          | 0.1604                             | 0.1606                          |
| 1.0                                                                   | 0.3400                          | 0.3280                          | 0.1599                             | 0.1602                          |
| 1.5                                                                   | 0.3401                          | 0.3280                          | 0.1598                             | 0.1600                          |
| 2.0                                                                   | 0.3401                          | 0.3280                          | 0.1597                             | 0.1600                          |

**Table A31. Mean of Sharpe ratios of portfolios under different strategies (T=4, M=150)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                       | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                   | 0.3898                          | 0.3876                          | 0.1187                             | 0.1188                          |
| 0.5                                                                   | 0.3898                          | 0.3876                          | 0.1189                             | 0.1190                          |
| 1.0                                                                   | 0.3898                          | 0.3876                          | 0.1190                             | 0.1190                          |
| 1.5                                                                   | 0.3898                          | 0.3876                          | 0.1190                             | 0.1190                          |
| 2.0                                                                   | 0.3898                          | 0.3876                          | 0.1190                             | 0.1190                          |

**Table A32. Mean of Sharpe ratios of portfolios under different strategies (T=4, M=180)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                       | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                   | 0.3039                          | 0.3264                          | 0.1710                             | 0.1710                          |
| 0.5                                                                   | 0.3040                          | 0.3264                          | 0.1701                             | 0.1702                          |

|     |        |        |        |        |
|-----|--------|--------|--------|--------|
| 1.0 | 0.3040 | 0.3264 | 0.1700 | 0.1701 |
| 1.5 | 0.3040 | 0.3264 | 0.1700 | 0.1700 |
| 2.0 | 0.3040 | 0.3264 | 0.1700 | 0.1700 |

**Table A33. Mean of Sharpe ratios of portfolios under different strategies (T=4, M=240)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                       | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                   | 0.2047                          | 0.2451                          | 0.1326                             | 0.1327                          |
| 0.5                                                                   | 0.2046                          | 0.2451                          | 0.1322                             | 0.1323                          |
| 1.0                                                                   | 0.2048                          | 0.2451                          | 0.1322                             | 0.1323                          |
| 1.5                                                                   | 0.2046                          | 0.2451                          | 0.1322                             | 0.1323                          |
| 2.0                                                                   | 0.2046                          | 0.2451                          | 0.1322                             | 0.1323                          |

**Table A34. Mean of Sharpe ratios of portfolios under different strategies (T=8, M=150)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                       | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                   | 0.4636                          | 0.5236                          | -0.1973                            | -0.1791                         |
| 0.5                                                                   | 0.4636                          | 0.5236                          | -0.1973                            | -0.1790                         |
| 1.0                                                                   | 0.4636                          | 0.5236                          | -0.1972                            | -0.1791                         |
| 1.5                                                                   | 0.4636                          | 0.5236                          | -0.1973                            | -0.1792                         |
| 2.0                                                                   | 0.4636                          | 0.5236                          | -0.1974                            | -0.1792                         |

**Table A35. Mean of Sharpe ratios of portfolios under different strategies (T=8, M=180)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                       | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                   | 0.5078                          | 0.4658                          | -0.1524                            | -0.1516                         |

|     |        |        |         |         |
|-----|--------|--------|---------|---------|
| 0.5 | 0.5078 | 0.4658 | -0.1524 | -0.1516 |
| 1.0 | 0.5078 | 0.4658 | -0.1523 | -0.1515 |
| 1.5 | 0.5078 | 0.4658 | -0.1524 | -0.1517 |
| 2.0 | 0.5079 | 0.4658 | -0.1525 | -0.1517 |

**Table A36. Mean of Sharpe ratios of portfolios under different strategies (M=240)**

| Mean of Sharpe ratios for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                              | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                       | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                   | 0.1236                          | 0.3533                          | 0.0934                             | 0.0938                          |
| 0.5                                                                   | 0.1236                          | 0.3533                          | 0.0934                             | 0.0937                          |
| 1.0                                                                   | 0.1236                          | 0.3533                          | 0.0935                             | 0.0938                          |
| 1.5                                                                   | 0.1236                          | 0.3533                          | 0.0934                             | 0.0938                          |
| 2.0                                                                   | 0.1236                          | 0.3533                          | 0.0933                             | 0.0937                          |

### A.2.2 Portfolio turnovers

Tables A37-A45 contain the results for Mean of Portfolio turnovers of portfolios under different investment strategies when  $H = 1000$ ,  $T = 2, 4, 8$ ,  $\omega = 0.1, 0.5, 1.0, 1.5, 2.0$  and  $M = 150, 180, 240$ . It indicates that the recalculated time-consistent strategies almost have smaller Mean of Portfolio turnovers than the recalculated pre-commitment strategies in out-of-sample test.

**Table A37. Mean of Portfolio turnovers of portfolios under different strategies (T=2, M=150)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                             | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                         | 0.9353                          | 0.7634                          | 4.2749                             | 4.2730                          |
| 0.5                                                                         | 0.1871                          | 0.1527                          | 4.2412                             | 4.2382                          |
| 1.0                                                                         | 0.0935                          | 0.0763                          | 4.2438                             | 4.2407                          |
| 1.5                                                                         | 0.0624                          | 0.0509                          | 4.2453                             | 4.2422                          |
| 2.0                                                                         | 0.0468                          | 0.0382                          | 4.2462                             | 4.2430                          |

**Table A38. Mean of Portfolio turnovers of portfolios under different strategies (T=2, M=180)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                             | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                         | 0.6155                          | 0.5246                          | 3.6059                             | 3.6042                          |
| 0.5                                                                         | 0.1231                          | 0.1049                          | 3.6111                             | 3.6082                          |
| 1.0                                                                         | 0.0615                          | 0.0525                          | 3.6157                             | 3.6127                          |
| 1.5                                                                         | 0.0410                          | 0.0350                          | 3.6174                             | 3.6143                          |
| 2.0                                                                         | 0.0308                          | 0.0262                          | 3.6182                             | 3.6151                          |

**Table A39. Mean of Portfolio turnovers of portfolios under different strategies (T=2, M=240)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                             | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                         | 0.2772                          | 0.2413                          | 1.8653                             | 1.8649                          |
| 0.5                                                                         | 0.0554                          | 0.0483                          | 1.8829                             | 1.8824                          |
| 1.0                                                                         | 0.0277                          | 0.0241                          | 1.8868                             | 1.8864                          |
| 1.5                                                                         | 0.0185                          | 0.0161                          | 1.8883                             | 1.8878                          |
| 2.0                                                                         | 0.0139                          | 0.0121                          | 1.8890                             | 1.8885                          |

**Table A40. Mean of Portfolio turnovers of portfolios under different strategies (T=4, M=150)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                             | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                         | 2.9882                          | 1.4198                          | 69.8689                            | 69.7797                         |
| 0.5                                                                         | 0.5976                          | 0.2840                          | 69.7680                            | 69.6774                         |
| 1.0                                                                         | 0.2988                          | 0.1420                          | 69.7621                            | 69.6717                         |
| 1.5                                                                         | 0.1992                          | 0.0947                          | 69.7614                            | 69.6714                         |
| 2.0                                                                         | 0.1494                          | 0.0710                          | 69.7609                            | 69.6707                         |

**Table A41. Mean of Portfolio turnovers of portfolios under different strategies (T=4, M=180)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-----------------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                             | Recalculated                    | Recalculated    | Recalculated                       | Recalculated    |
|                                                                             | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                         | 2.1781                          | 1.0856          | 65.2036                            | 65.2032         |
| 0.5                                                                         | 0.4356                          | 0.2171          | 65.1184                            | 65.1171         |
| 1.0                                                                         | 0.2178                          | 0.1086          | 65.1129                            | 65.1102         |
| 1.5                                                                         | 0.1452                          | 0.0724          | 65.1110                            | 65.1083         |
| 2.0                                                                         | 0.1089                          | 0.0543          | 65.1105                            | 65.1075         |

**Table A42. Mean of Portfolio turnovers of portfolios under different strategies (T=4, M=240)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-----------------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                             | Recalculated                    | Recalculated    | Recalculated                       | Recalculated    |
|                                                                             | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                         | 1.2001                          | 0.6764          | 47.7681                            | 47.7673         |
| 0.5                                                                         | 0.2400                          | 0.1353          | 47.7222                            | 47.7214         |
| 1.0                                                                         | 0.1200                          | 0.0676          | 47.7181                            | 47.7180         |
| 1.5                                                                         | 0.0800                          | 0.0451          | 47.7172                            | 47.7172         |
| 2.0                                                                         | 0.0600                          | 0.0338          | 47.7169                            | 47.7166         |

**Table A43. Mean of Portfolio turnovers of portfolios under different strategies (T=8, M=150)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                 |                                    |                 |
|-----------------------------------------------------------------------------|---------------------------------|-----------------|------------------------------------|-----------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                 | Portfolio without risk-free assets |                 |
|                                                                             | Recalculated                    | Recalculated    | Recalculated                       | Recalculated    |
|                                                                             | pre-commitment                  | time-consistent | pre-commitment                     | time-consistent |
| 0.1                                                                         | 8.2969                          | 1.8308          | 6.3223 e+03                        | 6.3227 e+03     |
| 0.5                                                                         | 1.6591                          | 0.3662          | 6.3223 e+03                        | 6.3226 e+03     |
| 1.0                                                                         | 0.8297                          | 0.1831          | 6.3223 e+03                        | 6.3227 e+03     |
| 1.5                                                                         | 0.5531                          | 0.1221          | 6.3223 e+03                        | 6.3227 e+03     |
| 2.0                                                                         | 0.4148                          | 0.0915          | 6.3223 e+03                        | 6.3227 e+03     |

**Table A44. Mean of Portfolio turnovers of portfolios under different strategies (T=8, M=180)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                             | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                         | 5.7590                          | 1.3782                          | 8.5621 e+03                        | 8.5653 e+03                     |
| 0.5                                                                         | 1.1517                          | 0.2756                          | 8.5622 e+03                        | 8.5651 e+03                     |
| 1.0                                                                         | 0.5758                          | 0.1378                          | 8.5624 e+03                        | 8.5654 e+03                     |
| 1.5                                                                         | 0.3839                          | 0.0919                          | 8.5622 e+03                        | 8.5651 e+03                     |
| 2.0                                                                         | 0.2879                          | 0.0689                          | 8.5624 e+03                        | 8.5652 e+03                     |

**Table A45. Mean of Portfolio turnovers of portfolios under different strategies (T=8, M=240)**

| Mean of Portfolio turnovers for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|-----------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                    | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                             | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                         | 2.8316                          | 0.8567                          | 1.1767 e+04                        | 1.1771 e+04                     |
| 0.5                                                                         | 0.5663                          | 0.1713                          | 1.1766 e+04                        | 1.1772 e+04                     |
| 1.0                                                                         | 0.2832                          | 0.0857                          | 1.1766 e+04                        | 1.1772 e+04                     |
| 1.5                                                                         | 0.1888                          | 0.0571                          | 1.1767 e+04                        | 1.1771 e+04                     |
| 2.0                                                                         | 0.1416                          | 0.0428                          | 1.1767 e+04                        | 1.1772 e+04                     |

### A.2.3 Maximum drawdowns

Tables A46-A54 contain the results for Mean of Maximum drawdowns of portfolios under different investment strategies when  $H = 1000$ ,  $T = 2, 4, 8$ ,  $\omega = 0.1, 0.5, 1.0, 1.5, 2.0$  and  $M = 150, 180, 240$ . It indicates that the recalculated time-consistent strategies almost have smaller Mean of Maximum drawdowns than the recalculated the pre-commitment strategies in out-of-sample test.

**Table A46. Mean of Maximum drawdowns of portfolios under different strategies (T=2, M=150)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                    |
|---------------------------------------------------------------------------|---------------------------------|------------------------------------|
| $\omega$                                                                  | Portfolio with risk-free assets | Portfolio without risk-free assets |



|     | Recalculated<br>pre-commitment | Recalculated<br>time-consistent | Recalculated<br>pre-commitment | Recalculated<br>time-consistent |
|-----|--------------------------------|---------------------------------|--------------------------------|---------------------------------|
| 0.1 | 13.8548                        | 12.8811                         | 77.2186                        | 77.2014                         |
| 0.5 | 2.7708                         | 2.5762                          | 79.8838                        | 79.8565                         |
| 1.0 | 1.3855                         | 1.2881                          | 80.2178                        | 80.1884                         |
| 1.5 | 0.9236                         | 0.8587                          | 80.3274                        | 80.2987                         |
| 2.0 | 0.6927                         | 0.6441                          | 80.3834                        | 80.3540                         |

**Table A47. Mean of Maximum drawdowns of portfolios under different strategies (T=2, M=180)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|---------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
|                                                                           | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
| $\omega$                                                                  | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                       | 14.4687                         | 13.5266                         | 82.9933                            | 82.9719                         |
| 0.5                                                                       | 2.8934                          | 2.7053                          | 85.2455                            | 85.2055                         |
| 1.0                                                                       | 1.4468                          | 1.3527                          | 85.5274                            | 85.4846                         |
| 1.5                                                                       | 0.9645                          | 0.9018                          | 85.6218                            | 85.5777                         |
| 2.0                                                                       | 0.7234                          | 0.6763                          | 85.6678                            | 85.6241                         |

**Table A48. Mean of Maximum drawdowns of portfolios under different strategies (T=2, M=240)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|---------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
|                                                                           | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
| $\omega$                                                                  | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                       | 11.1367                         | 10.6189                         | 45.0787                            | 45.0777                         |
| 0.5                                                                       | 2.2274                          | 2.1238                          | 45.6988                            | 45.6970                         |
| 1.0                                                                       | 1.1137                          | 1.0619                          | 45.7763                            | 45.7744                         |
| 1.5                                                                       | 0.7425                          | 0.7079                          | 45.8022                            | 45.8002                         |
| 2.0                                                                       | 0.5568                          | 0.5309                          | 45.8151                            | 45.8131                         |

**Table A49. Mean of Maximum drawdowns of portfolios under different strategies (T=4, M=150)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|---------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                  | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                           | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                       | 38.6826                         | 27.6503                         | 2.6109 e+03                        | 2.6110 e+03                     |
| 0.5                                                                       | 7.7374                          | 5.5301                          | 2.6151 e+03                        | 2.6151 e+03                     |
| 1.0                                                                       | 3.8679                          | 2.7650                          | 2.6156 e+03                        | 2.6157 e+03                     |
| 1.5                                                                       | 2.5786                          | 1.8434                          | 2.6157 e+03                        | 2.6159 e+03                     |
| 2.0                                                                       | 1.9341                          | 1.3825                          | 2.6158 e+03                        | 2.6159 e+03                     |

**Table A50. Mean of Maximum drawdowns of portfolios under different strategies (T=4, M=180)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|---------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                  | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                           | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                       | 36.2965                         | 21.7705                         | 3.0988 e+03                        | 3.0977 e+03                     |
| 0.5                                                                       | 7.2615                          | 4.3541                          | 3.1007 e+03                        | 3.0995 e+03                     |
| 1.0                                                                       | 3.6323                          | 2.1770                          | 3.1009 e+03                        | 3.0998 e+03                     |
| 1.5                                                                       | 2.4207                          | 1.4514                          | 3.1011 e+03                        | 3.0999 e+03                     |
| 2.0                                                                       | 1.8155                          | 1.0885                          | 3.1012 e+03                        | 3.0998 e+03                     |

**Table A51. Mean of Maximum drawdowns of portfolios under different strategies (T=4, M=240)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|---------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                  | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                           | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                       | 33.5615                         | 26.1484                         | 4.3600 e+03                        | 4.3555 e+03                     |
| 0.5                                                                       | 6.7128                          | 5.2297                          | 4.3615 e+03                        | 4.3571 e+03                     |
| 1.0                                                                       | 3.3563                          | 2.6148                          | 4.3617 e+03                        | 4.3574 e+03                     |
| 1.5                                                                       | 2.2373                          | 1.7432                          | 4.3617 e+03                        | 4.3574 e+03                     |
| 2.0                                                                       | 1.6782                          | 1.3074                          | 4.3617 e+03                        | 4.3574 e+03                     |

**Table A52. Mean of Maximum drawdowns of portfolios under different strategies (T=8, M=150)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|---------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                  | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                           | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                       | 96.8919                         | 31.8224                         | 1.7835 e+05                        | 1.7943 e+05                     |
| 0.5                                                                       | 19.3750                         | 6.3645                          | 1.7835 e+05                        | 1.7941 e+05                     |
| 1.0                                                                       | 9.6895                          | 3.1822                          | 1.7835 e+05                        | 1.7942 e+05                     |
| 1.5                                                                       | 6.4611                          | 2.1215                          | 1.7834 e+05                        | 1.7941 e+05                     |
| 2.0                                                                       | 4.8463                          | 1.5911                          | 1.7835 e+05                        | 1.7942 e+05                     |

**Table A53. Mean of Maximum drawdowns of portfolios under different strategies (T=8, M=180)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|---------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                  | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                           | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                       | 61.4483                         | 26.2698                         | 4.9044 e+04                        | 4.9212 e+04                     |
| 0.5                                                                       | 12.2868                         | 5.2540                          | 4.9050 e+04                        | 4.9217 e+04                     |
| 1.0                                                                       | 6.1448                          | 2.6270                          | 4.9051 e+04                        | 4.9219 e+04                     |
| 1.5                                                                       | 4.0958                          | 1.7513                          | 4.9050 e+04                        | 4.9218 e+04                     |
| 2.0                                                                       | 3.0721                          | 1.3135                          | 4.9051 e+04                        | 4.9217 e+04                     |

**Table A54. Mean of Maximum drawdowns of portfolios under different strategies (T=8, M=240)**

| Mean of Maximum drawdowns for different risk aversion parameters $\omega$ |                                 |                                 |                                    |                                 |
|---------------------------------------------------------------------------|---------------------------------|---------------------------------|------------------------------------|---------------------------------|
| $\omega$                                                                  | Portfolio with risk-free assets |                                 | Portfolio without risk-free assets |                                 |
|                                                                           | Recalculated<br>pre-commitment  | Recalculated<br>time-consistent | Recalculated<br>pre-commitment     | Recalculated<br>time-consistent |
| 0.1                                                                       | 82.6546                         | 28.6053                         | 7.2667 e+05                        | 7.2801 e+05                     |
| 0.5                                                                       | 16.5318                         | 5.7211                          | 7.2663 e+05                        | 7.2804 e+05                     |
| 1.0                                                                       | 8.2658                          | 2.8605                          | 7.2664 e+05                        | 7.2806 e+05                     |
| 1.5                                                                       | 5.5099                          | 1.9070                          | 7.2668 e+05                        | 7.2799 e+05                     |
| 2.0                                                                       | 4.1331                          | 1.4303                          | 7.2669 e+05                        | 7.2805 e+05                     |