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Research Article

Bipartite Consensus for Multiagent Systems via Event-Based Control

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This paper studies bipartite consensus for first-order multiagent systems. To improve resource utilization, event-based protocols are considered for bipartite consensus. A new type of control gain is designed in the proposed protocols. By appropriate selection of control gains, the convergence rate of the closed-loop system can be adjusted. Firstly, for structural balance case, necessary and sufficient conditions are given on communication relations and consensus gains to achieve bipartite consensus. Secondly, for structural unbalance case, necessary and sufficient conditions are proposed to ensure the stabilizing of the system. It can be found that the system will not show Zeno behavior. Numerical simulations are used to demonstrate the theoretical results.

1. Introduction

Consensus problem is one of the hot topics in coordination of multiagent systems (MASs). In the last few years, research on consensus has received considerable attention [1–9]. Consensus means that agents can reach a common value through cooperative relations among agents. However, in real applications, not only cooperative but also competitive relations among agents exist. In these circumstances, bipartite consensus is studied [10–20]. Based on the cooperation and competition among agents, bipartite consensus can be achieved if agents agree upon a certain value with the same quantity and different signs. In [10], necessary and sufficient conditions for bipartite consensus of the single-integrator MASs are given. In [12], the communication condition is first reduced to be containing a spanning tree. In [14], the communication topology is extended to the time-varying case. In [15–20], bipartite consensus with measurement noise is considered.

It is worth pointing out that the above literature adopts a time-driven control pattern, where the state of the agents is monitored continuously and the control law updates are done at any moment. In practical implementation, the embedded processors are often resource-limited and thus an event-based control fashion is more beneficial in MASs. For conventional consensus of MASs, an event-based control fashion was thoroughly studied [21–31]. In the pioneering work [21], an event-based feedback protocol was proposed. In [23], the event-based protocols for both fixed and switching topologies have been considered. In [25], the self-triggered protocol of MASs was taken into account. Then, in [28], a new event-based protocol for average consensus of MASs was proposed and continuous monitoring of agents' states was not required. The event-based consensus for general linear MASs can be found in [29–31]. Despite these productive results, works on bipartite consensus with event-based control strategy are still rare.

In this paper, we consider event-based bipartite consensus for first-order MASs. In contrast to [32, 33], a new function is introduced into the event-based protocol, such that the Laplacian-like event-based bipartite consensus protocols in [32, 33] are special cases of this paper. Due to the new function gain, the closed-loop system is time-varying. By use of state transition matrix, the closed-loop system is analyzed. For structural balance case, necessary and sufficient conditions are given on communication relations and consensus gains to achieve bipartite consensus. For structural unbalance case, necessary and sufficient conditions are proposed to ensure the MAS stabilizing.

Organization. In Section 2, we give some basic concepts on signed graph and formulate the problem. In Section 3, we prove the main results. In Section 4, we show the validity of theoretical analysis through the simulation results. In Section 5, we conclude this paper and put forward further research directions.

Notation. $R^{n\times m}$ represents all real matrices of $n\times m$ order. $\mathbf{0}$ denotes vector or matrix whose elements are 0. $\mathbf{1}_n$ represents column vector whose elements are 1. $\mathrm{sgn}(\cdot)$ represents sign function.

2. Problem Formulation

The communication relations among N agents are expressed by the signed digraph $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$, where $\mathscr{V}=\{1,\dots,N\}$ is the node set and $\mathscr{E}$ is the edge set. $\mathscr{A}=(a_{ij})\in R^{N\times N}$ is the weighted adjacency matrix of $\mathscr{G}$, where $a_{ij}>0$ and $a_{ij}<0$ represent competition relationship and cooperation relationship between i and j, respectively. Throughout this paper, we always assume that $a_{ij}\neq 0 \Longleftrightarrow (j,i)\in\mathscr{E}$, $a_{ii}=0$, and $a_{ij}a_{ji}\geq 0$, $\forall i,j\in\mathscr{V}$. $\mathscr{L}=\mathscr{C}_r-\mathscr{A}$ is the Laplacian of $\mathscr{G}$, where $\mathscr{C}_r=\mathrm{diag}(\sum_{p=1}^{N}|a_{1p}|,\dots,\sum_{p=1}^{N}|a_{Np}|)$. $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$ is structurally balanced if $\mathscr{V}$ can be divided into two subsets $\mathscr{V}_1,\mathscr{V}_2,\mathscr{V}_1\cup\mathscr{V}_2=\mathscr{V},\mathscr{V}_1\cap\mathscr{V}_2=\varnothing$, satisfying $a_{ij}\geq 0$ for $\forall i,j\in\mathscr{V}_t(t\in\{1,2\})$, and $a_{ij}\leq 0$ for $\forall i\in\mathscr{V}_t, j\in\mathscr{V}_e$ $(t\neq e\in\{1,2\})$. $\mathscr{G}$ is structurally unbalanced otherwise.

Consider an MAS with N agents, whose dynamics obey the following equation:

$$\dot{x}_i(t)=u_i(t),\quad i=1,\dots,N, \tag{1}$$

where $x_i(t)\in R$ and $u_i(t)\in R$ are the state and control input of the ith agent.

We use $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$ to express the communication relations among the N agents.

Considering the limit resources, people would like to reduce the frequency of control law updates. In this case, an event-based control law is more favorable. Our aim here is to provide an event-based control in order that all agents' states converge to values with the same modulus and different signs regardless of initial states.

To achieve this goal, we assume that each agent only updates its control law at discrete times indexed by $t_0,t_1,\dots$. We define the event-based control as follows:

$$u_i(t)=c(t)\sum_{j=1}^{N}\left|a_{ij}\right|\left[\mathrm{sgn}\left(a_{ij}\right)x_j(t_l)-x_i(t_l)\right],\quad \forall t\in\left[t_l,t_{l+1}\right), \tag{2}$$

where $i=1,\dots,N, l=0,1,\dots,c(t)>0$ is a piecewise continuous function.

Remark 1. The control law will be actuated at discrete event times. A proper function $c(t)$ will be designed to improve the convergence performance of the closed-loop system. In particular, when $c(t)=1$, protocol (2) is reduced to event-based bipartite consensus protocols in [32, 33].

The state measurement error of the ith agent is denoted by $\epsilon_i(t)=x_i(t_l)-x_i(t)$, $i=1,\dots,N,t\in[t_l,t_{l+1})$, $l=0,1,\dots$. Let $\Omega(t)=(\epsilon_1(t),\dots,\epsilon_N(t))^T$ and, hence,

$$\dot{X}(t)=-c(t)\mathscr{L}(X(t)+\Omega(t)), \tag{3}$$

where $X(t)=(x_1(t),\dots,x_N(t))^T$.

We introduce the following definition to characterize the behavior of (3).

Definition 2. System (1) is said to achieve bipartite consensus via event-based protocol $\mathscr{U}=\{u_i, i=1,\dots,N\}$, if for system (1) with any given $X(0)\in R^N$, there exists $f=(f_1,\dots,f_N)^T\in R^N$ $(f_i\in\{\pm1\}, i=1,\dots,N)$ and $\mu^*\in R$ such that $\lim_{t\longrightarrow\infty}\|X(t)-\mu^* f\|=0$, where $\mu^*\in R$ depends on $X(0)$ and the communication relations among agents.

We provide the following event triggering conditions:

$$\left|\epsilon_i(t)\right|\leq Me^{-\gamma t},\quad i=1,\dots,N, \tag{4}$$

where $M>0, 0<\gamma<\min_{\lambda(L)\neq0}\{\mathrm{Re}\lambda(\mathscr{L})\}$.

When the measurement error $\epsilon_i(t)$ is over the threshold, the controller is triggered and updates itself. To analyze (3), we introduce assumptions as follows:

($\mathbf{T_1}$) $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$ is structurally balanced.

($\mathbf{T_2}$) $\mathscr{G}=(\mathscr{V},\mathscr{E},\mathscr{A})$ contains a spanning tree.

($\mathbf{T_3}$) $\int_0^\infty c(s)ds=\infty$.

The following lemma is highly related to the subsequent results.

Lemma 3. *If system (1) can achieve bipartite consensus via event-based protocol (2), then there exist $f=(f_1,\dots,f_N)^T\in R^N$, $f_i\in\{\pm1\}$, $i=1,\dots,N$, and $\phi=(\phi_1,\dots,\phi_N)^T\in R^N$, such that $\lim_{t\longrightarrow\infty}\Theta(t,0)=f\phi^T$, where $\Theta(t,0)$ is the state transition matrix of (3).*

Proof. The proof is omitted due to space limit. □

3. Main Results

The following result is the main result of this section.

Theorem 4. *System (1) can achieve bipartite consensus via event-based protocol (2) if and only if* ($\mathbf{T_1}$), ($\mathbf{T_2}$), *and* ($\mathbf{T_3}$) *hold.*

Proof.

Sufficiency. Assume $\Theta(t,t_0)$ is the state transition matrix of (3). Then, $\Theta(t,t_0)=e^{-\int_{t_0}^{t}c(s)ds\mathscr{L}}$. Since ($\mathbf{T_1}$) and ($\mathbf{T_2}$) hold, Lemma 1 of [17] implies that $\mathscr{L}$ has exactly one zero eigenvalue and all nonzero eigenvalues are in the open right half plane. Thus, there exists Q satisfying

$$Q^{-1}\mathscr{L}Q = B = \operatorname{diag}(0, B_2, \ldots, B_w), \tag{5}$$

where B_i $(i = 2, \ldots, w)$ is $R_i \times R_i$ dimensional Jordan block with ς_i on its diagonal, $R_2 + \cdots + R_w = N - 1$. Obviously, $\varsigma_2, \ldots, \varsigma_w$ are the eigenvalues of $\mathscr{L}$, and $\operatorname{Re}(\varsigma_i) > 0$, $i = 2, \ldots, w$. Combining this, we can get the state transition matrix:

$$\Theta(t, t_0) = Q \operatorname{diag}\left(1, \Theta_{B_2^{\varsigma_2}}(t, t_0), \ldots, \Theta_{B_w^{\varsigma_w}}(t, t_0)\right) Q^{-1}, \tag{6}$$

where $\Theta_{B_k^{\varsigma_k}}$, $k = 2, \ldots, w$, is defined as in Lemma 3 of [17]. Thus, from $(\mathbf{T_3})$, we know that $\lim_{t\longrightarrow\infty}\Theta(t, t_0) = Q\operatorname{diag}(1, 0, \ldots, 0)Q^{-1}$. By (3), $X(t) = \Theta(t, 0)X(0) - X_\Omega$, where $X_\Omega = \int_0^t c(s)\Theta(t, s)\mathscr{L}\Omega(s)ds$. By (6), one has

$$X_\Omega = Q \int_0^t c(s) \operatorname{diag}\left(1, \Theta_{B_2^{\varsigma_2}}(t, s), \ldots, \Theta_{B_w^{\varsigma_w}}(t, s)\right) Q^{-1}\mathscr{L}\Omega(s)\,\mathrm{d}s. \tag{7}$$

Furthermore, by (5), one gets

$$Q^{-1}\mathscr{L}\Omega(s) = \operatorname{diag}(0, B_2, \ldots, B_w) Q^{-1}\Omega(s). \tag{8}$$

Assume $Q^{-1} = (Q_{ij})$. Then, $Q^{-1}\Omega(s) = (\sum_{j=1}^N Q_{1j}\epsilon_j(s), \ldots, \sum_{j=1}^N Q_{Nj}\epsilon_j(s))^T$. This together with (8) gives

$$Q^{-1}\mathscr{L}\Omega(s) = \left(0, \Omega_2^*(s), \ldots, \Omega_N^*(s)\right)^T, \tag{9}$$

where $\Omega_i^*(s)$ $(i = 2, \ldots, N)$ is the linear combination of $\varsigma_2, \ldots, \varsigma_w$ and $\sum_{j=1}^N Q_{kj}\epsilon_j(s)$, $k = 1, \ldots, N$. Then, by (4), there must exist $q^* > 0$, such that

$$\left|\Omega_i^*(s)\right| \le q^* e^{-\gamma s}, \quad i = 2, \ldots, N. \tag{10}$$

Considering the specific form of $\Theta_{B_i^{\varsigma_i}}(t, s)$ $(i = 2, \ldots, w)$ in Lemma 3 of [17], we get that $X_\Omega = Q(0, \Lambda_2^\Omega(s), \ldots, \Lambda_N^\Omega(s))^T$, where $\Lambda_p^\Omega(s)(p = 2, \ldots, N)$ is the linear combination of $\int_0^t c(s)e^{-\varsigma_i\int_s^t c(\tau)\mathrm{d}\tau}(\int_s^t c(\tau)\mathrm{d}\tau)^m\Omega_q^*(s)\mathrm{d}s$, $m = 0, 1, \ldots, R_i - 1$; $i = 2, \ldots, w$; $q = 2, \ldots, N$.

By direct calculation, one obtains that

$$\begin{aligned}&\lim_{t\longrightarrow\infty}\int_0^t c(s)\, e^{-\varsigma_i\int_s^t c(\tau)\mathrm{d}\tau}\left(\int_s^t c(\tau)\,\mathrm{d}\tau\right)^m e^{-\gamma s}\mathrm{d}s\\ &\quad = \lim_{t\longrightarrow\infty}\frac{m!}{\varsigma_i^{m+1}e^{\gamma t}} = 0, \quad m = 0, 1, \ldots, R_i - 1.\end{aligned} \tag{11}$$

This together with (10) leads to

$$\begin{aligned}&\lim_{t\longrightarrow\infty}\int_0^t c(s)\, e^{-\varsigma_i\int_s^t c(\tau)\mathrm{d}\tau}\left(\int_s^t c(\tau)\,\mathrm{d}\tau\right)^m \Omega_q^*(s)\,\mathrm{d}s = 0,\\ &\qquad m = 0, 1, \ldots, R_i - 1.\end{aligned} \tag{12}$$

Therefore, $\lim_{t\longrightarrow\infty}X_\Omega = 0$, and thus $\lim_{t\longrightarrow\infty}X(t) = Q\operatorname{diag}(1, 0, \ldots, 0)Q^{-1}X(0) = Q_cQ_r^TX(0)$, where Q_c is the first column of Q, Q_r^T is the first row of Q^{-1}, and $Q_r^TQ_c = 1$. By (5), one gets that $\mathscr{L}Q_c = Q_r^T\mathscr{L} = 0$. From $(\mathbf{T_1})$, we know that there exists f such that $\mathscr{L}f = 0$. Since the eigenspace corresponding to eigenvalue 0 is 1 dimensional, $Q_c = m_0 f$, where $m_0 \ne 0$ is a constant. Denote $\mu^* = m_0Q_r^TX(0)$. Then, $\lim_{t\longrightarrow\infty}X(t) = \mu^* f$ and μ^* depends on $X(0)$ and communication relations among agents. Therefore, Sufficiency holds.

Necessity. Assume by contradiction that $(\mathbf{T_3})$ does not hold. Without loss of generality, let $\int_0^\infty c(s)ds = \bar{c} > 0$. Then, $\lim_{t\longrightarrow\infty}\Theta(t, 0) = e^{-\bar{c}\mathscr{L}}$. It is an invertible matrix and thus $\operatorname{rank}(\lim_{t\longrightarrow\infty}\Theta(t, 0)) = N$. This contradicts Lemma 3. Therefore, $(\mathbf{T_3})$ holds. Next, we prove that $\mathscr{L}$ has exactly one zero eigenvalue. If 0 is not eigenvalue $\mathscr{L}$, then $\lim_{t\longrightarrow\infty}\Theta(t, 0) = 0$. This implies that $\lim_{t\longrightarrow\infty}X(t)$ is independent of $X(0)$. It contradicts Definition 2. Thus, 0 is eigenvalue $\mathscr{L}$. If the geometric multiplicity of eigenvalue 0 is less than the algebraic multiplicity of 0, then by $(\mathbf{T_3})$ and direct calculation, one has the notion that $\lim_{t\longrightarrow\infty}\Theta(t, 0)$ does not exist. It is a contradiction with Lemma 3. So, the geometric multiplicity equals the algebraic multiplicity of eigenvalue 0. Assume 0 is p dimensional and $p > 1$. Then, with abuse of notation, $Q^{-1}\mathscr{L}Q = \operatorname{diag}(\underbrace{0, \ldots, 0}_{p}, B_{p+1}, \ldots, B_w)$. Hence, $\lim_{t\longrightarrow\infty}\Theta(t, 0) = Q\operatorname{diag}(\underbrace{1, \ldots, 1}_{p}, \Theta_{B_{p+1}^{\varsigma_{p+1}}}(t, 0), \ldots, \Theta_{B_w^{\varsigma_w}}(t, 0))Q^{-1}$ and $\operatorname{rank}(\lim_{t\longrightarrow\infty}\Theta(t, 0)) = p > 1$. This contradicts Lemma 3. So, $p = 1$; i.e., $\mathscr{L}$ has exactly one zero eigenvalue. With abuse of notation, (5) and (6) hold. Noticing Lemma 3, one gets $Q\operatorname{diag}(1, 0, 0, \ldots, 0)Q^{-1} = f\phi^T$. Therefore, $Q_c = fm_1$, where $m_1 = \phi^TQ_c \ne 0$. Since $\mathscr{L}Q_c = 0$, $\mathscr{L}f = 0$. By the definition of Laplacian $\mathscr{L}$, for any q, one gets $f_q\sum_{k\ne q}|a_{qk}| = f_ka_{qk}, q = 1, \ldots, N$. Noticing $f_q = \pm 1$, one has $f_qf_ka_{qk} = |a_{qk}| \ge 0$. Let $V_1 = \{q \mid f_q = 1\}$ and $V_2 = \{q \mid f_q = -1\}$; then $V_1 \cap V_2 = \varnothing, V_1 \cup V_2 = V$. If $q \in \mathscr{V}_j$, $j \in \{1, 2\}$, $a_{qk} \ge 0, k \in \mathscr{V}_j$ or $a_{qk} \le 0, k \in \mathscr{V}_r, r \ne j, r \in \{1, 2\}$. By definition, $\mathscr{G}$ is structurally balanced; i.e., $(\mathbf{T_1})$ holds. From Lemma 1 of [17], $\mathscr{G}$ has a spanning tree; i.e., $(\mathbf{T_2})$ holds. □

Remark 5. From (11), one obtains that the convergence rate of the closed-loop system is closely related to eigenvalues of $\mathscr{L}$ and the rate of $\int_0^t c(s)ds$ converging to infinity. It follows that, by appropriate selection of $c(t)$, the convergence rate of (3) can be adjusted.

From Theorem 4, we can see that structural balance is a necessary and sufficient condition to ensure bipartite consensus. When $\mathscr{G}$ is structurally unbalanced, we investigate the evolution of the MAS:

$(\mathbf{T_1'})$ $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ is structurally unbalanced.

$(\mathbf{T_2'})$ $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ does not contain structurally balanced input solitary subgraphs.

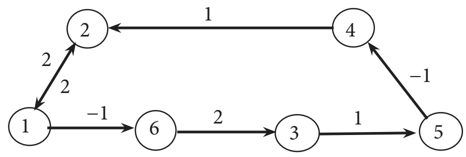

Figure 1: The communication relations $\mathscr{G}_1$.

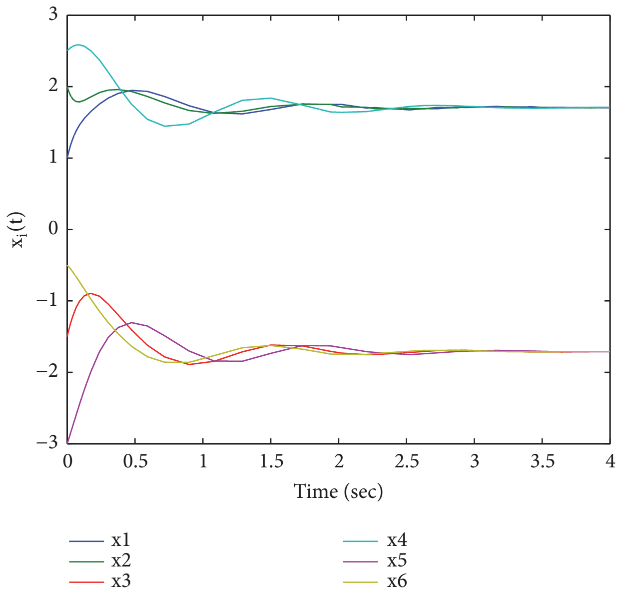

Figure 2: Evolution of the states for six agents.

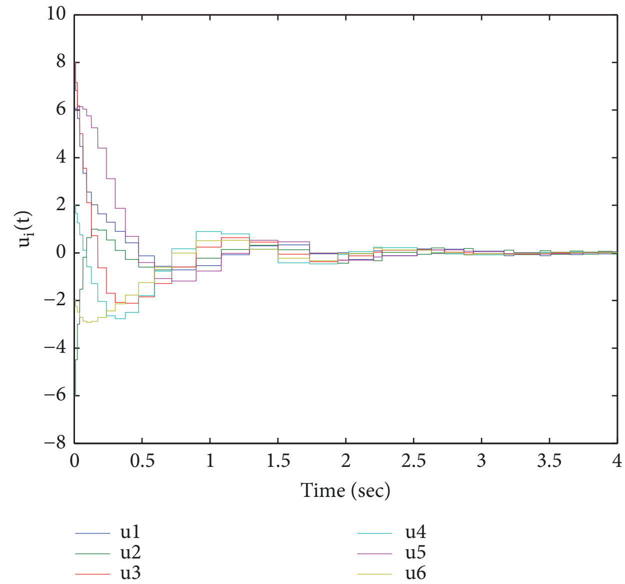

Figure 3: Evolution of control inputs for six agents.

An input solitary subgraph of $\mathscr{G}$ indicates that agents of the subgraph cannot obtain information from other agents of $\mathscr{G}$.

Theorem 6. *System (2) can be stabilizing via event-based protocol (2), i.e.,* $\lim_{t\longrightarrow\infty} X(t) = 0$ *if and only if* $(\mathbf{T}_1')$, $(\mathbf{T}_2')$, *and* $(\mathbf{T}_3)$ *hold.*

Proof.

Necessity. The necessity is similar to the proof of Theorem 4.

Sufficiency. From $(\mathbf{T}_1')$, $(\mathbf{T}_2')$, and Lemma 2 of [17], we can see that all eigenvalues of $\mathscr{L}$ have positive real parts. Since $(\mathbf{T}_3)$ holds, one has $\lim_{t\longrightarrow\infty} X(t) = 0$ by repeating the sufficiency proof of Theorem 4. □

4. Numerical Simulation

Example 1. Six agents' communication relations are expressed by Figure 1, where $\mathscr{G}_1 = (\mathscr{V}, \mathscr{E}_1, \mathscr{A}_1)$, $\mathscr{V} = \{1, \ldots, 6\}$, $a_{12} = a_{21} = a_{36} = 2$, $a_{45} = a_{61} = -1$, and $a_{24} = a_{53} = 1$. Obviously, $\mathscr{G}_1$ satisfies assumptions $(\mathbf{T}_1)$ and $(\mathbf{T}_2)$. Laplacian $\mathscr{L}_1$ has eigenvalues $\lambda_1(\mathscr{L}_1) = 0$, $\lambda_2(\mathscr{L}_1) = 4.5698$, $\lambda_3(\mathscr{L}_1) = 0.6852 + 0.8449\imath$, $\lambda_4(\mathscr{L}_1) = 0.6852 - 0.8449\imath$, $\lambda_5(\mathscr{L}_1) = 2.0299 + 0.5637\imath$, and $\lambda_6(\mathscr{L}_1) = 2.0299 - 0.5637\imath$ ($\imath^2 = -1$). It can be seen that 0 is a simple eigenvalue of $\mathscr{L}_1$, and $\min_{\lambda(\mathscr{L}_1)\neq 0}\{\mathrm{Re}\lambda(\mathscr{L}_1)\} = 0.6852$. Agents' dynamics satisfy (1). Assume $X(0) = (1, 2, -1.5, 2.5, -3, -0.5)$. If we choose $M = 1.5$, $\gamma = 0.65$, and $c(t) = 1$ in (2), then $c(t)$ satisfies $(\mathbf{T}_3)$ and state trajectories are given in Figure 2. One can find that agents 1, 2, and 4 converge to 1.7 while agents 3, 5, and 6 converge to −1.7; i.e., system (1) achieves bipartite consensus via event-based protocol (2). From Figure 3, we know that the inputs are constants between the event triggering time intervals. Furthermore, as shown in Figure 4, error norm of each agent converges to zero. This means that the MAS avoids the Zeno behavior.

Example 2. When communication relationship among the six agents is given by $\mathscr{G}_2 = (\mathscr{V}, \mathscr{E}_2, \mathscr{A}_2)$ as shown in Figure 5, where $a_{12} = a_{21} = a_{32} = a_{43} = a_{54} = 1$, $a_{13} = a_{31} = -2$ and $a_{65} = 2$. Eigenvalues of Laplacian $\mathscr{L}_2$ are $\lambda_1(\mathscr{L}_2) = 0.7639$, $\lambda_2(\mathscr{L}_2) = \lambda_3(\mathscr{L}_2) = \lambda_4(\mathscr{L}_2) = 1$, $\lambda_5(\mathscr{L}_2) = 2$, and $\lambda_6(\mathscr{L}_2) = 5.2316$, respectively. $\mathscr{G}_2$ satisfies $(\mathbf{T}_1')$ and $(\mathbf{T}_2')$. It is easy to know that $\min_{\lambda(\mathscr{L}_2)\neq 0}\{\mathrm{Re}\lambda(\mathscr{L}_2)\} = 0.7639$. Let $M = 1.2$, $\gamma = 0.6$, and $c(t) = 1$; the state trajectories of the MAS can be obtained as shown in Figure 6. All states eventually converge to 0. This is consistent with Theorem 6. The evolution of control input and error norm are shown in Figures 7 and 8, respectively.

5. Conclusion

In this paper, event-driven protocols are considered for bipartite consensus of MASs. Based on them, the number of controller updates is reduced. Under necessary and sufficient conditions on protocol gain and communication

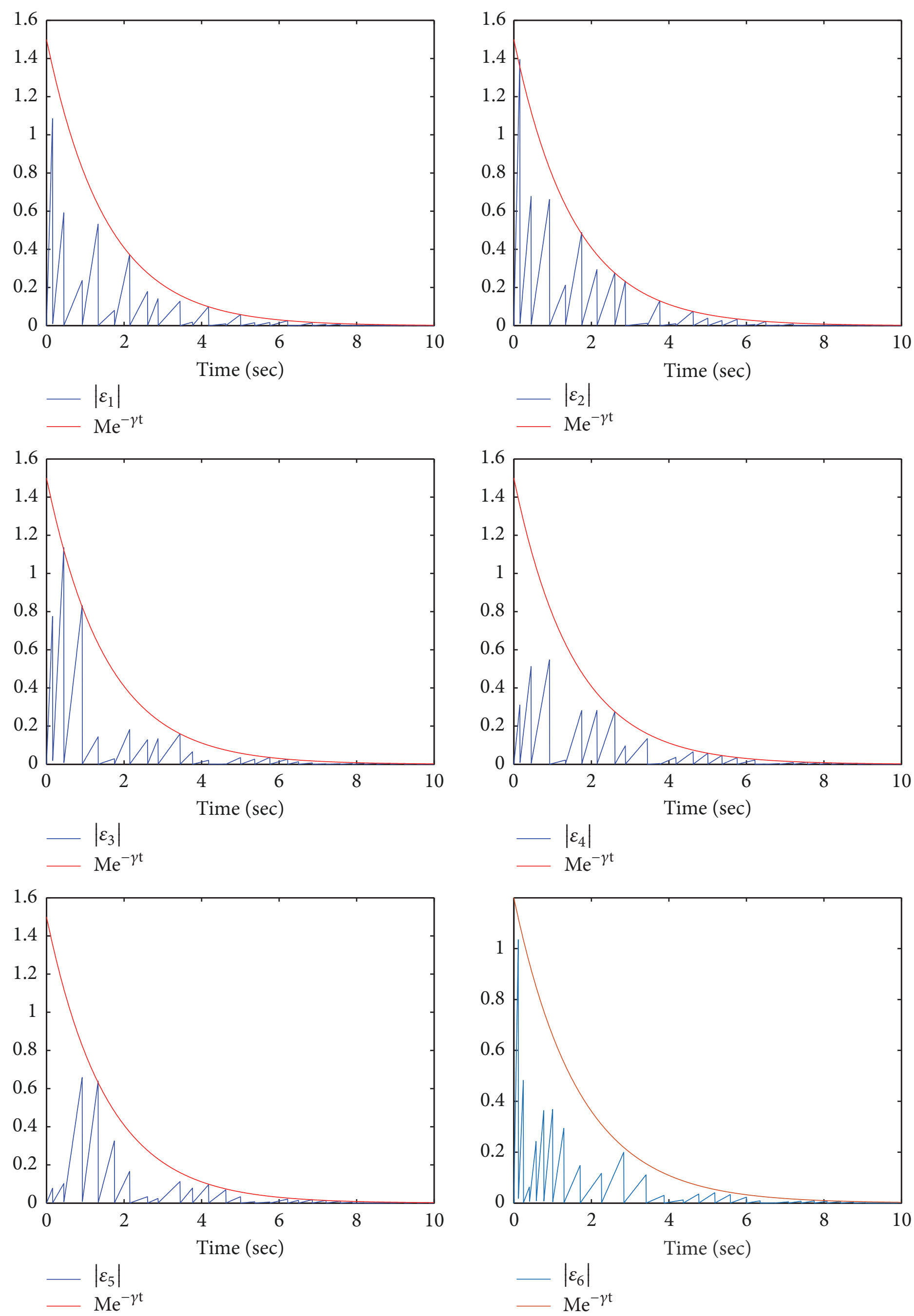

FIGURE 4: Six agents' error norms.

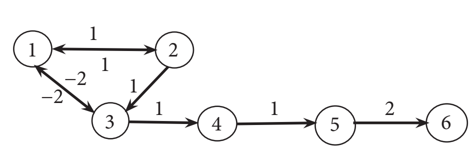

FIGURE 5: The communication relations $\mathcal{G}_2$.

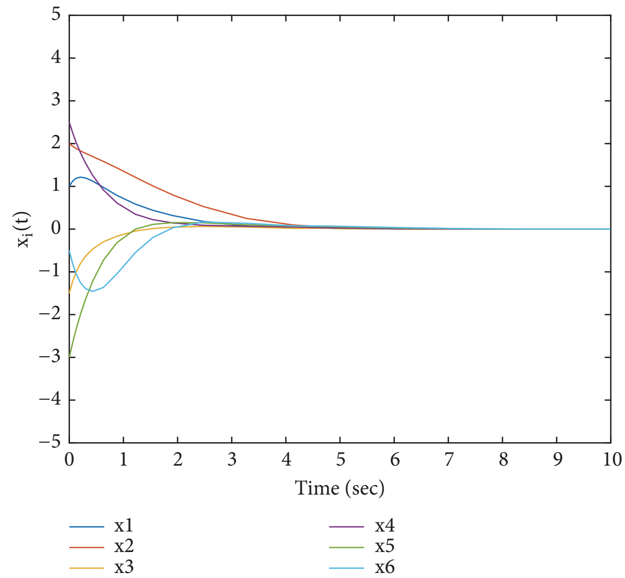

FIGURE 6: Six agents' states under $\mathscr{G}_2$.

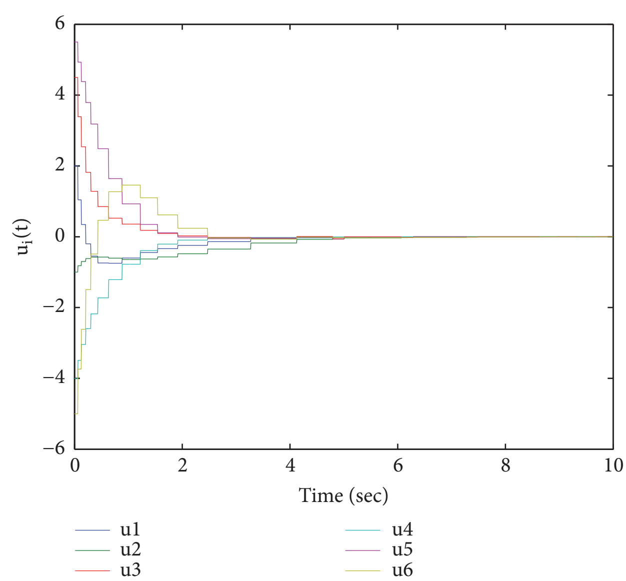

FIGURE 7: Six agents' control inputs under $\mathscr{G}_2$.

topology, the MAS is shown to reach event-based bipartite consensus. When the graph is structurally unbalanced, the MAS is proved to be stabilizing. The further research is related to MASs with time-varying topology and time delays.

Data Availability

The MatLab based models used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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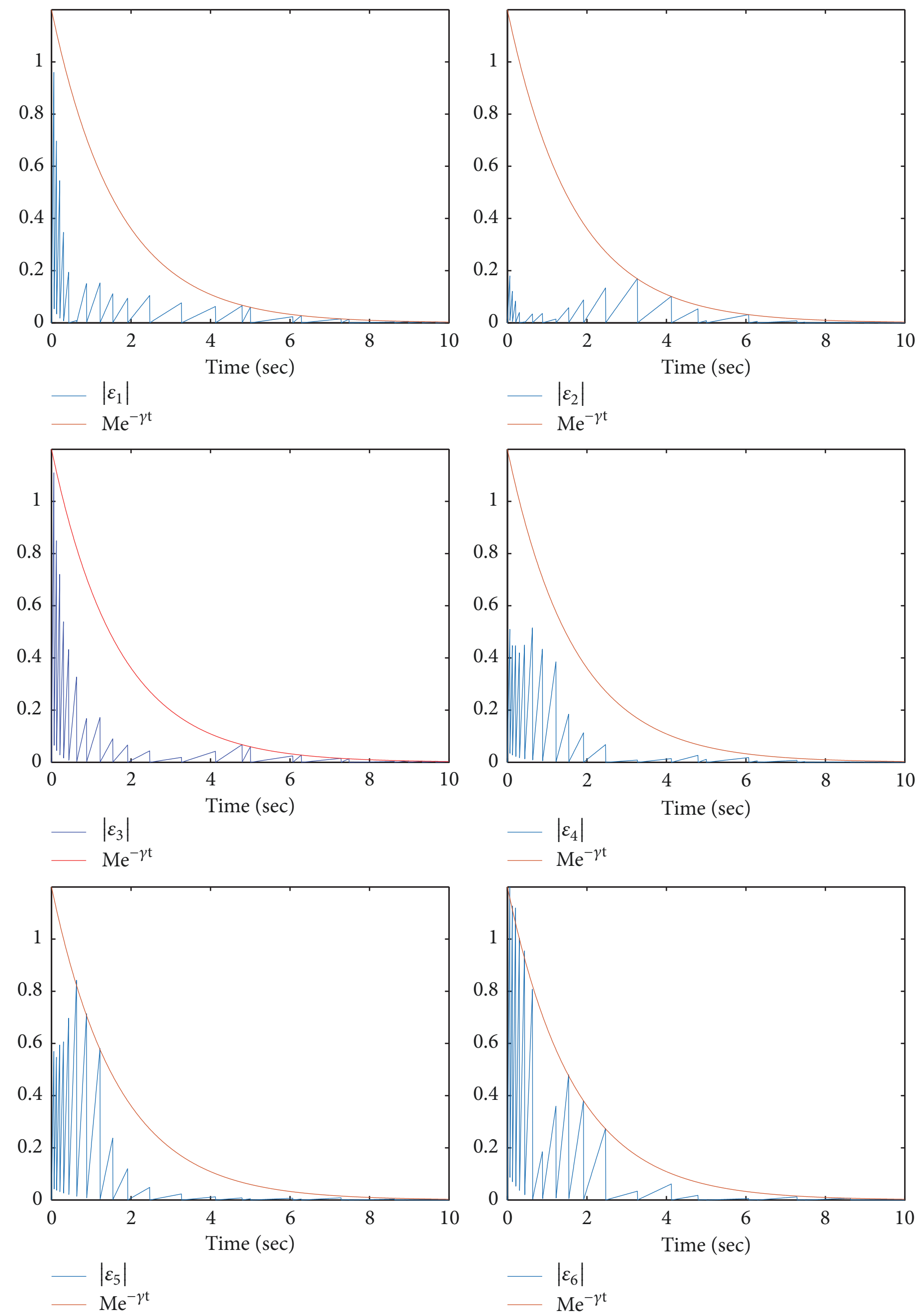

Figure 8: Six agents' error norms under $\mathscr{G}_2$.

References

[1] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[2] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[3] C. Q. Ma and J. F. Zhang, "Necessary and sufficient conditions for consensusability of linear multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 5, pp. 1263–1268, 2010.

[4] W. Sun, "Stabilization analysis of time-delay Hamiltonian systems in the presence of saturation," *Applied Mathematics and Computation*, vol. 217, no. 23, pp. 9625–9634, 2011.

[5] L. Gao, D. Wang, and G. Wang, "Further results on exponential stability for impulsive switched nonlinear time-delay systems with delayed impulse effects," *Applied Mathematics and Computation*, vol. 268, pp. 186–200, 2015.

[6] C. Q. Ma, T. Li, and J. F. Zhang, "Consensus control for leader-following multi-agent systems with measurement noises," *Journal of Systems Science and Complexity*, vol. 23, no. 1, pp. 35–49, 2010.

[7] H. Liu and F. Meng, "Interval oscillation criteria for second-order nonlinear forced differential equations involving variable exponent," *Advances in Difference Equations*, Paper No. 291, 14 pages, 2016.

[8] J. Guo, L. Y. Wang, G. Yin, Y. Zhao, and J. F. Zhang, "Identification of Wiener systems with quantized inputs and binary-valued output observations," *Automatica*, vol. 78, pp. 280–286, 2017.

[9] C. Q. Ma and J. F. Zhang, "On formability of linear continuous multi-agent systems," *Journal of Systems Science and Complexity*, vol. 25, no. 1, pp. 13–29, 2012.

[10] C. Altafini, "Consensus problems on networks with antagonistic interactions," *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 935–946, 2013.

[11] Z. Zheng, "Invariance of deficiency indices under perturbation for discrete Hamiltonian systems," *Journal of Difference Equations and Applications*, vol. 19, no. 8, pp. 1243–1250, 2013.

[12] J. Hu and W. X. Zheng, "Emergent collective behaviors on coopetition networks," *Physics Letters A*, vol. 378, no. 26-27, pp. 1787–1796, 2014.

[13] W. Sun and L. Peng, "Observer-based robust adaptive control for uncertain stochastic Hamiltonian systems with state and input delays," *Nonlinear Analysis: Modelling and Control*, vol. 19, no. 4, pp. 626–645, 2014.

[14] A. V. Proskurnikov, A. S. Matveev, and M. Cao, "Opinion dynamics in social networks with hostile camps: consensus vs. polarization," *IEEE Transactions on Automatic Control*, vol. 61, no. 6, pp. 1524–1536, 2016.

[15] C. Q. Ma and Z. Y. Qin, "Bipartite consensus on networks of agents with antagonistic interactions and measurement noises," *IET Control Theory & Applications*, vol. 10, no. 17, pp. 2306–2313, 2016.

[16] L. Zhang and Z. Zheng, "Lyapunov type inequalities for the Riemann-Liouville fractional differential equations of higher order," *Advances in Difference Equations*, no. 270, 20 pages, 2017.

[17] C. Q. Ma, Z. Y. Qin, and Y. B. Zhao, "Bipartite consensus of integrator multi-agent systems with measurement noise," *IET Control Theory & Applications*, vol. 11, no. 18, pp. 3313–3320, 2017.

[18] Z. Zheng and Q. Kong, "Friedrichs extensions for singular Hamiltonian operators with intermediate deficiency indices," *Journal of Mathematical Analysis and Applications*, vol. 461, no. 2, pp. 1672–1685, 2018.

[19] C. Q. Ma and L. Xie, "Necessary and sufficient conditions for leader-following bipartite consensus with measurement noise," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, pp. 1–6, 2018.

[20] C. Q. Ma, W. Zhao, and Y. B. Zhao, "Bipartite linear χ-consensus of double-integrator multi-agent systems with measurement noise," *Asian Journal of Control*, vol. 20, no. 1, pp. 577–584, 2018.

[21] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[22] J. Gu and F. Meng, "Some new nonlinear Volterra-Fredholm type dynamic integral inequalities on time scales," *Applied Mathematics and Computation*, vol. 245, pp. 235–242, 2014.

[23] Z. Liu and Z. Chen, "Reaching consensus in networks of agents via event-triggered control," *Journal of Information and Computational Science*, vol. 8, no. 3, pp. 393–402, 2011.

[24] W. Sun, Y. Wang, and R. Yang, "L2 disturbance attenuation for a class of time delay Hamiltonian systems," *Journal of Systems Science & Complexity*, vol. 24, no. 4, pp. 672–682, 2011.

[25] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2012.

[26] J. Guo, B. Mu, L. Y. Wang, G. Yin, and L. Xu, "Decision-based system identification and adaptive resource allocation," *IEEE Transactions on Automatic Control*, vol. 62, no. 5, pp. 2166–2179, 2017.

[27] J. Cai and Z. Zheng, "Inverse spectral problems for discontinuous Sturm-Liouville problems of Atkinson type," *Applied Mathematics and Computation*, vol. 327, pp. 22–34, 2018.

[28] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson, "Event-based broadcasting for multi-agent average consensus," *Automatica*, vol. 49, no. 1, pp. 245–252, 2013.

[29] Z. Zhang, F. Hao, L. Zhang, and L. Wang, "Consensus of linear multi-agent systems via event-triggered control," *International Journal of Control*, vol. 87, no. 6, pp. 1243–1251, 2014.

[30] B. Wang, X. Meng, and T. Chen, "Event based pulse-modulated control of linear stochastic systems," *IEEE Transactions on Automatic Control*, vol. 59, no. 8, pp. 2144–2150, 2014.

[31] D. Yang, W. Ren, X. Liu, and W. Chen, "Decentralized event-triggered consensus for linear multi-agent systems under general directed graphs," *Automatica*, vol. 69, pp. 242–249, 2016.

[32] Y. Zhou and J. Hu, "Event-based bipartite consensus on signed networks," in *Proceedings of the 3rd Annual IEEE International Conference on Cyber Technology in Automation, Control, and Intelligent Systems, IEEE-CYBER 2013*, pp. 326–330, China, May 2013.

[33] J. Zeng, F. Li, J. Qin, and W. X. Zheng, "Distributed event-triggered bipartite consensus for multiple agents over signed graph topology," in *Proceedings of the 2015 34th Chinese Control Conference (CCC)*, pp. 6930–6935, Hangzhou, China, July 2015.