Hindawi
Mathematical Problems in Engineering
Volume 2018, Article ID 9042039, 18 pages
https://doi.org/10.1155/2018/9042039

Research Article

Iterative Solutions of Hirota Satsuma Coupled KDV and Modified Coupled KDV Systems

Muhammad Jibran,[1] Rashid Nawaz ,[2] Asfandyar Khan ,[2] and Sajjad Afzal[2]

[1]*Department of Computer Science and IT, Sarhad University of Science and Technology, Khayber Pakhtunkhwa, Pakistan*
[2]*Department of Mathematics, Abdul Wali Khan University, Mardan KP, Pakistan*

Correspondence should be addressed to Rashid Nawaz; rashid_uop@yahoo.com

Received 20 June 2018; Revised 24 October 2018; Accepted 31 October 2018; Published 3 December 2018

Academic Editor: Chris Goodrich

In this article the approximate solutions of nonlinear Hirota Satsuma coupled Korteweg De- Vries (KDV) and modified coupled KDV equations have been obtained by using reliable algorithm of New Iterative Method (NIM). The results obtained give higher accuracy than that of homotopy analysis method (HAM). The obtained solutions show that NIM is effective, simpler, easier, and explicit and gives a suitable way to control the convergence of the approximate solution.

1. Introduction

The nonlinear coupled system of partial differential equations has variety of applications in different fields of mechanics, biology, hydrodynamics, and plasma physics. The exact solution of nonlinear partial differential equations cannot be found easily. To find the approximate solutions of these types of equations many researchers have adopted different approaches. The well-known approaches for approximate solutions of coupled system of differential equations are numerical methods, iterative methods, perturbation methods, homotopy based methods, etc. Each approach has its own advantages and disadvantages. In numerical methods discretization is used which affect the accuracy. They also required most computational work and time. In the case of strong nonlinear problems the numerical methods do not give us more accurate result. Some numerical methods are collocation methods (CMs) [1, 2], finite difference method (FDM) [3, 4], finite element method (FEM) [5, 6], radial basis function (RBF) method [7, 8], etc. Perturbation techniques use perturbation quantities to transform a nonlinear problem into infinite number of linear subproblems. Perturbation methods have some limitations as well, such as nonavailability of perturbative quantity and strong nonlinearity. To avoid this difficulty, there do exist homotopy based methods as well [9–11]. Adomian Decomposition Method (ADM) [11] and Differential Transformation Method (DTM) [12] deal with the nonlinearity but their main drawback is that the convergence region of obtained solution is generally small. Iterative methods are mathematical techniques that produce a sequence of approximate solutions. They used initial guess to create sequential approximations. Since these methods involve repetition of the same process many times, computers can act well for finding solutions of equations iteratively. A definite way of operation of an iterative method is termination criteria. Iterative methods have important role in finding approximate solution of differential equations. Some of the well-known iterative methods are variational iteration method (VIM) [13, 14], modified variational iteration method (MVIM) [15, 16], optimal variational iteration method (OVIM) [17], etc.

Recently Daftardar-Gejji and Jafari [18] have proposed an iterative method for solving linear and nonlinear functional equations called the New Iterative Method (NIM). This method has proven useful for solving a variety of nonlinear equations such as algebraic equations, ODEs, PDEs, evolution equations, and system of nonlinear dynamical systems [19–33].

The KDV equation arises in the study of nonlinear dispersive waves [34]. It was derived by Korteweg and de Vries in 1895 for modeling of shallow water waves in canal [35]. This equation plays important role in diverse areas of engineering and scientific applications and, therefore, enormous amount of research work has been invested in the study of such equations. Wu et al. derived a new corresponding hierarchy of nonlinear evolution equations [35]. Fan worked on the extended tanh-function method and symbolic computation to obtain, respectively, four kinds of Soliton solutions for a new coupled MKDV and new generalized Hirota Satsuma coupled KDV equations [36]. Raslan et al. worked on the decomposition method to obtain the Soliton solutions for generalized Hirota Satsuma coupled KDV and coupled MKDV equations [37]. Zhong et al. applied the VIM to obtain approximate analytic solutions of coupled MKDV and generalized Hirota Satsuma coupled KDV equations [38]. Kangalgil et al. demonstrated the feasibility and validity of DTM for proposed equations [39]. Arife et al. presented the numerical solution of systems of Hirota Satsuma coupled KDV and coupled MKDV equations by means of HAM [9]. Ghoreishi et al. applied HAM for obtaining the approximate solution of MKDV system [40].

Our aim in this paper is to extend the application of NIM for finding the approximate solutions of nonlinear Hirota Satsuma coupled KDV and modified coupled KDV equations. The results obtained are in close agreement with the exact solutions than that of HAM. The accuracy of NIM can further be improved by increasing the number of iterations.

2. The Governing Coupled System

Recently, by introducing a 4 × 4 matrix spectral problem with three potentials, Wu et al. derived a new corresponding hierarchy of nonlinear evolution equations [35]. Two typical equations in the hierarchy are new generalized Hirota Satsuma coupled KDV and new coupled MKDV equations.

We considered the following new generalized Hirota Satsuma coupled KDV equation,

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx} - 3uu_x + 3vw_x + 3wv_x, \\ v_t &= -v_{xxx} + 3uv_x, \\ w_t &= -w_{xxx} + 3uw_x, \end{aligned} \tag{1}$$

and modified coupled KDV equation

$$\begin{aligned} u_t &= \frac{1}{2}u_{xxx} - 3u^2u_x + \frac{3}{2}v_{xx} + 3uv_x + 3vu_x - 3\lambda u_x, \\ v_t &= -v_{xxx} - 3vv_x - 3u_xv_x + 3u^2v_x + 3\lambda v_x. \end{aligned} \tag{2}$$

With $w = v^*$ (1)-(2) reduce to new complex coupled Hirota Satsuma KDV and modified coupled complex KDV equations, respectively [35, 41]. For $u = 0$, (2) reduces to generalized KDV equation and, for $v = 0$, it reduces to modified KDV equation.

2.1. Basic Idea of NIM. The basic mathematical theory of New Iterative Method is described as follows.

Let us consider the nonlinear equation of the form of

$$u(x,t) = f(x,t) + \mathbb{N}(u(x,t)). \tag{3}$$

where $f(x)$ is known function $x = x_1, x_2, x_3, \dots, x_n$ and $\mathbb{N}$ denotes nonlinear operator from a Banach space B⟶B. According to the basic idea of NIM solution of the above equation has the series form

$$u(x,t) = \sum_{k=0}^{\infty} u_k(x,t), \tag{4}$$

The nonlinear operator $\mathbb{N}$ can be decomposed as

$$\begin{aligned} \mathbb{N}\left(\sum_{k=0}^{\infty} u_k\right) &= \mathbb{N}(u_0) \\ &\quad + \sum_{k=1}^{\infty}\left\{\mathbb{N}\left(\sum_{j=0}^{k} u_j\right) - \mathbb{N}\left(\sum_{j=0}^{k-1} u_j\right)\right\}. \end{aligned} \tag{5}$$

Hence general equation of (1) can be written as

$$\begin{aligned} u(x) &= \sum_{k=0}^{\infty} u_k(x) \\ &= f + \mathbb{N}(u_0) \\ &\quad + \sum_{k=1}^{\infty}\left\{\mathbb{N}\left(\sum_{j=0}^{k} u_j\right) - \mathbb{N}\left(\sum_{j=0}^{k-1} u_j\right)\right\}, \end{aligned} \tag{6}$$

From (6) we have

$$\begin{aligned} u_0 &= f, \\ G_0 &= \mathbb{N}(u_0), \\ G_1 &= \mathbb{N}(u_0 + u_1) - \mathbb{N}(u_0), \\ G_2 &= \mathbb{N}(u_0 + u_1 + u_2) - \mathbb{N}(u_0 + u_1), \end{aligned} \tag{7}$$

and in general

$$G_m = \mathbb{N}(u_0 + u_1 + \dots + u_m) - \mathbb{N}(u_0 + u_1 + \dots u_{m-1}). \quad m = 1, 2, 3 \dots \tag{8}$$

The k-term approximate solution of the general equation (1) is

$$u = u_0 + u_1 + \dots + u_{k-1}. \tag{9}$$

2.2. Convergence Criteria of NIM

Theorem 1. *If $\mathbb{N}$ is C^{∞} in a neighborhood of u_0 and*

$$\begin{aligned} &\left\|\mathbb{N}^m(u_0)\right\| \\ &\quad = \sup\left\{\frac{\mathbb{N}^m(u_0)(k_1, k_2, \dots k_n)}{\left\|k_j\right\|} \le 1,\ 1 \le j \le m\right\} \\ &\le l, \end{aligned} \tag{10}$$

for any m and for some real $l > 0$ $\&\|u_j\| \leq M < (1/e)$ $j = 1, 2, \ldots\ldots.,$ *then the series* $\sum_{m=0}^{\infty} G_m$ *is absolutely convergent and*

$$\|u_m\| \leq lM^m e^{m-1}(e-1). \quad m = 1, 2, \ldots \tag{11}$$

Sufficient condition for convergence is as follows.

Theorem 2. *If* $\mathbb{N}$ *is* C^{∞} $\&\|\mathbb{N}^m(u_0)\| \leq M \leq e^{-1}$, $\forall m$, *then the series* $\sum_{m=0}^{\infty} G_m$ *is absolutely convergent.*

The reader is referred to [18, 22] to get more insight of convergence analysis.

3. Implementation of NIM to Hirota Satsuma Coupled KDV and Coupled MKDV Equations

Problem 3. Consider the coupled Hirota Satsuma KDV equation given by (1) together with following initial conditions [35].

$$\begin{aligned} u(x,0) &= \frac{1}{3}\left(\beta - 8k^2\right) + 4k^2\tanh^2(kx), \\ v(x,0) &= \frac{-4\left(3k^4F_0 - 2\beta k^2F_2 + 4k^4F_2\right)}{3F_2^2} \\ &\quad + \frac{4k^2}{F_2}\tanh^2(kx), \\ w(x,0) &= F_0 + F_2\tanh^2(kx), \end{aligned} \tag{12}$$

where $k = 0.1, \lambda = 1$. By using the initial conditions, the given equation (1) is equivalent to the following integral equations:

$$\begin{aligned} u(x,t) &= \frac{1}{3}\left(\beta - 8k^2\right) + 4k^2\tanh^2(kx) \\ &\quad + \int\left(\frac{1}{2}u_{xxx} - 3uu_x + 3vw_x + 3wv_x\right)dt, \\ v(x,t) &= \frac{-4\left(3k^4F_0 - 2\beta k^2F_2 + 4k^4F_2\right)}{3F_2^2} \\ &\quad + \frac{4k^2}{F_2}\tanh^2(kx) + \int\left(-v_{xxx} + 3uv_x\right)dt, \\ w(x,t) &= F_0 + F_2\tanh^2(kx) + \int\left(-w_{xxx} + 3uw_x\right)dt. \end{aligned} \tag{13}$$

Appling the basic idea of NIM and using the recursive relation (6) we have the following.

Zeroth Order Solutions

$$u_0(x,t) = \frac{1}{3}\left(\beta - 8k^2\right) + 4k^2\tanh^2(kx), \tag{14}$$

$$\begin{aligned} v_0(x,t) &= \frac{-4\left(3k^4F_0 - 2\beta k^2F_2 + 4k^4F_2\right)}{3F_2^2} \\ &\quad + \frac{4k^2}{F_2}\tanh^2(kx), \end{aligned} \tag{15}$$

$$w_0(x,t) = F_0 + F_2\tanh^2(kx). \tag{16}$$

First Order Solutions

$$\begin{aligned} u_1(x,t) = \Bigg\{ t\Bigg(&-24k^3\operatorname{sech}^2(kx)\tanh(kx)\left(\frac{1}{3}\left(-8k^2+\beta\right) + 4k^2\tanh^2(kx)\right) \\ &+ 6k\operatorname{sech}^2(kx)F_2\tanh(kx)\left(-\frac{4\left(3k^4F_0 + 4k^4F_2 - 2k^2\beta F_2\right)}{3F_2^2} + \frac{4k^2\tanh^2(kx)}{F_2}\right) \\ &+ \frac{24k^3\operatorname{sech}^2(kx)\tanh(kx)\left(F_0 + F_2\tanh^2(kx)\right)}{F_2} + 2k^2\left(-16k^3\operatorname{sech}^4(kx)\tanh(kx) + 8k^3\operatorname{sech}^2(kx)\tanh^3(kx)\right)\Bigg), \Bigg\} \end{aligned} \tag{17}$$

$$\begin{aligned} v_1(x,t) = \Bigg\{ t\Bigg(&\frac{24k^3\operatorname{sech}^2(kx)\tanh(kx)\left((1/3)\left(-8k^2+\beta\right) + 4k^2\tanh^2(kx)\right)}{F_2} - \frac{4k^2\left(-16k^3\operatorname{sech}^4(kx)\tanh(kx)\right.}{F_2} \\ &+ \frac{\left.8k^3\operatorname{sech}^2(kx)\tanh^3(kx)\right)}{F_2}\Bigg), \Bigg\} \end{aligned} \tag{18}$$

$$\begin{aligned} w_1(x,t) = \Bigg\{ t\Bigg(&6k\operatorname{sech}^2(kx)F_2\tanh(kx)\left(\frac{1}{3}\left(-8k^2+\beta\right) + 4k^2\tanh^2(kx)\right) \\ &- F_2\left(-16k^3\operatorname{sech}^4(kx)\tanh(kx) + 8k^3\operatorname{sech}^2(kx)^2\tanh^3(kx)\right)\Bigg). \Bigg\} \end{aligned} \tag{19}$$

Second Order Solutions

$$u_2(x,t) = \Big\{\frac{1}{F_2^2} 8k^5 t^2 \operatorname{sech}^2(kx)\Big(72k^2\left(-1+k^2\right)^2 t(-2+\cosh(2kx))\operatorname{sech}^4(kx) F_0^2 \tanh(kx)$$
$$+3k\left(-1+k^2\right)F_0F_2\Big(2+\operatorname{sech}^2(kx)\Big(32kt\left(-6+6k^2-\beta\right)$$
$$+48kt\operatorname{sech}^2(kx)\Big(12-12k^2+\beta+8\left(-1+k^2\right)\operatorname{sech}^2(kx)\Big)-15\tanh(kx)\Big)\tanh(kx)\Big)$$
$$+F_2^2\Big(2k\left(-6+6k^2-\beta\right)+\operatorname{sech}^2(kx)\Big(3k\left(62-62k^2+5\beta\right)$$
$$+15k\operatorname{sech}^2(kx)\Big(-32+32k^2-\beta-21\left(-1+k^2\right)\operatorname{sech}^2(kx)\Big) \tag{20}$$
$$-4t\Big(2\Big(-72k^6+\beta^2+24k^4(6+\beta)-2k^2(6+\beta)^2\Big)$$
$$+3\operatorname{sech}^2(kx)\Big(216k^6-\beta^2-48k^4(9+\beta)+2k^2(6+\beta)(18+\beta)$$
$$+8k^2\left(-1+k^2\right)\operatorname{sech}^2(kx)\ \Big(4\left(9-9k^2+\beta\right)+15\left(-1+k^2\right)\operatorname{sech}^2(kx)\Big)\Big)\Big)$$
$$\cdot\ \tanh(kx)\Big)\Big)\Big),\Big\}$$

$$v_2(x,t) = \Big\{\frac{1}{F_2^2} 4k^4 t^2 \operatorname{sech}^2(kx)\ \Big(8k^2\beta\operatorname{sech}^4(kx)(-4kt\beta(-2+\cosh(2kx))+3\sinh(2kx))\tanh(kx)$$
$$+F_2\Big(-2\beta^2+3\left(-48k^4-16k^2(-3+\beta)+\beta^2\right)\operatorname{sech}^2(kx)\ +8k^2\operatorname{sech}^6(kx)$$
$$\cdot\Big(3\left(\beta+\left(-6+6k^2+\beta\right)\cosh(2kx)\right) \tag{21}$$
$$+kt\beta(-2+\cosh(2kx))\ \left(6-6k^2+\beta+\left(-6+6k^2+\beta\right)\cosh(2kx)\right)\ \tanh(kx)\Big)\Big)\Big),\Big\}$$

$$w_2(x,t) = \Big\{-\frac{1}{32} kt\operatorname{sech}^9(kx)\Big(-384k^3\left(-1+k^2\right)t\cosh^2(kx)\sinh(kx)\ (-8kt\beta+4kt\beta\cosh(2kx)$$
$$-\ 3\sinh(2kx)\,)\,F_0-\Big(kt\left(576k^2-576k^4+25\beta^2\right)\cosh(kx)+9kt\left(-96k^2+96k^4+\beta^2\right)\cosh(3kx)$$
$$+288k^3t\cosh(5kx)-288k^5t\cosh(5kx)-kt\beta^2\cosh(5kx)-kt\beta^2\cosh(7kx)$$
$$+5\beta\sinh(kx)-6144k^4t^2\beta\sinh(kx)+6144k^6t^2\beta\sinh(kx)+256k^4t^2\beta^2\sinh(kx)$$
$$+9\beta\sinh(3kx)+2688k^4t^2\beta\sinh(3kx)-2688k^6t^2\beta\sinh(3kx)+192k^4t^2\beta^2\sinh(3kx) \tag{22}$$
$$+5\beta\sinh(5kx)-384k^4t^2\beta\sinh(5kx)+384k^6t^2\beta\sinh(5kx)-64k^4t^2\beta^2\sinh(5kx)$$
$$+\ \beta\sinh(7kx)\,\Big)\,c_2\Big)-t\Big(6k\operatorname{sech}^2(kx)F_2\tanh(kx)\ \Big(\frac{1}{3}\left(-8k^2+\beta\right)+4k^2\tanh^2(kx)\Big)$$
$$-F_2\ \left(-16k^3\operatorname{sech}^4(kx)\tanh(kx)+8k^3\operatorname{sech}^2(kx)\tanh^3(kx)\right)\Big).\Big\}$$

The NIM yields the 2nd order solutions $u(x,t), v(x,t), w(x,t)$ as follows:

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots, \\ v(x,t) &= v_0(x,t) + v_1(x,t) + v_2(x,t) + \cdots, \\ w(x,t) &= w_0(x,t) + w_1(x,t) + w_2(x,t) + \cdots. \end{aligned} \tag{23}$$

$$\begin{aligned} u(x,t) = \Bigg\{ & \frac{1}{3}\left(-8k^2+\beta\right) + 4k^2\tanh^2(kx) + t\Bigg(-24k^3\operatorname{sech}^2(kx)\tanh(kx)\left(\frac{1}{3}\left(-8k^2+\beta\right)+4k^2\tanh^2(kx)\right) \\ & + 6k\operatorname{sech}^2(kx)F_2\tanh(kx)\left(-\frac{4\left(3k^4F_0+4k^4F_2-2k^2\beta F_2\right)}{3F_2^2}+\frac{4k^2\tanh^2(kx)}{F_2}\right) \\ & + \frac{24k^3\operatorname{sech}^2(kx)}{F_2} + \frac{\tanh(kx)}{F_2}\left(F_0+F_2\tanh^2(kx)\right) + 2k^2\left(-16k^3\operatorname{sech}^4(kx)\tanh(kx)\right. \\ & \left. + 8k^3\operatorname{sech}^2(kx)\tanh^3(kx)\right)\Bigg) + \frac{1}{F_2^2}8k^5t^2\operatorname{sech}^2(kx)\left(72k^2\left(-1+k^2\right)^2t\left(-2+\cosh(2kx)\right)\operatorname{sech}^4(kx)F_0^2\tanh(kx)\right. \\ & + 3k\left(-1+k^2\right)F_0F_2\left(2+\operatorname{sech}^2(kx)\left(32kt\left(-6+6k^2-\beta\right)+48kt\operatorname{sech}^2(kx)\left(12-12k^2\right.\right.\right. \\ & \left.\left.\left. + \beta + 8\left(-1+k^2\right)\operatorname{sech}^2(kx)\right) - 15\tanh(kx)\right)\tanh(kx)\right) + F_2^2\left(2k\left(-6+6k^2-\beta\right)\right. \\ & + \operatorname{sech}^2(kx)\left(3k\left(62-62k^2+5\beta\right)+15k\operatorname{sech}^2(kx)\left(-32+32k^2-\beta-21\left(-1+k^2\right)\operatorname{sech}^2(kx)\right)\right. \\ & - 4t\left(2\left(-72k^6+\beta^2+24k^4(6+\beta)-2k^2(6+\beta)^2\right)+3\operatorname{sech}^2(kx)\left(216k^6-\beta^2-48k^4(9+\beta)\right.\right. \\ & + 2k^2(6+\beta)(18+\beta) + 8k^2\left(-1+k^2\right)\operatorname{sech}^2(kx)\left(4\left(9-9k^2+\beta\right)\right. \\ & \left.\left.\left.\left.\left. + 15\left(-1+k^2\right)\operatorname{sech}^2(kx)\right)\right)\right)\tanh(kx)\right)\right)\right), \Bigg\} \end{aligned} \tag{24}$$

$$\begin{aligned} v(x,t) = \Bigg\{ & \frac{1}{3F_2^2}4k^2\left(-3k^2F_0 + \left(3-4k^2+2\beta+3\operatorname{sech}^2(kx)\left(-1-2k^2t^2\beta^2\right.\right.\right. \\ & \left.\left. + 3k^2t^2\operatorname{sech}^2(kx)\left(-48k^4-16k^2(-3+\beta)+\beta^2+8k^2\left(\beta+\left(-6+6k^2+\beta\right)\cosh(2kx)\right)\operatorname{sech}^4(kx)\right)\right)\right)F_2 \\ & + 6kt\beta\operatorname{sech}^2(kx)\tanh(kx)\left(\left(1+4k^4t^2\left(-2+\cosh(2kx)\right)\left(6-6k^2+\beta+\left(-6+6k^2+\beta\right)\cosh(2kx)\right)\right.\right. \\ & \left.\left. \times \operatorname{sech}^6(kx)\right)F_2 + 8k^3t\operatorname{sech}^2(kx)\left(2kt\beta\left(-2+3\operatorname{sech}^2(kx)\right)+3\tanh(kx)\right)\right)\right). \Bigg\} \end{aligned} \tag{25}$$

$$\begin{aligned} w(x,t) = \Bigg\{ & F_0 - \frac{1}{32}kt\operatorname{sech}^9(kx)\left(-384k^3\left(-1+k^2\right)t\cosh^2(kx)\sinh(kx)\left(-8kt\beta+4kt\beta\cosh(2kx)\right.\right. \\ & \left. -3\sinh(2kx)\right)F_0 - \left(kt\left(576k^2-576k^4+25\beta^2\right)\cosh(kx)\right. \\ & + 9kt\left(-96k^2+96k^4+\beta^2\right)\cosh(3kx) + 288k^3t\cosh(5kx) - 288k^5t\cosh(5kx) \\ & - kt\beta^2\cosh(5kx) - kt\beta^2\cosh(7kx) + 5\beta\sinh(kx) - 6144k^4t^2\beta\sinh(kx) \\ & + 6144k^6t^2\beta\sinh(kx) + 256k^4t^2\beta^2\sinh(kx) + 9\beta\sinh(3kx) + 2688k^4t^2\beta\sinh(3kx) \\ & - 2688k^6t^2\beta\sinh(3kx) + 192k^4t^2\beta^2\sinh(3kx) + 5\beta\sinh(5kx) - 384k^4t^2\beta\sinh(5kx) \\ & \left.\left. + 384k^6t^2\beta\sinh(5kx) - 64k^4t^2\beta^2\sinh(5kx) + \beta\sinh(7kx)\right)F_2\right) + F_2\tanh^2(kx), \Bigg\} \end{aligned} \tag{26}$$

Problem 4. Consider the modified coupled KDV equation given by (2) together with following initial conditions [35].

$$u(x,0) = k\tanh(kx), \quad v(x,0) = \frac{1}{2}\left(4k^2+\lambda\right) - 2k^2\tanh^2(kx). \tag{27}$$

Zeroth Order Solutions

$$u_0(x,t) = k\tanh(kx), \tag{28}$$

$$v_0(t,x) = \frac{1}{2}\left(4k^2+\lambda\right) - 2k^2\tanh^2(kx). \tag{29}$$

First Order Solutions

$$\begin{aligned} u_1(x,t) = \Big\{ & t\Big(-3k^2\lambda\operatorname{sech}^2(kx) - 15k^4\operatorname{sech}^2(kx) \\ & \cdot\tanh^2(kx) + 3k^2\operatorname{sech}^2(kx)\Big(\frac{1}{2}\left(4k^2+\lambda\right) \\ & - 2k^2\tanh^2(kx)\Big) - 3k^2\left(2k^2\operatorname{sech}^4(kx)\right. \\ & \left.- 4k^2\operatorname{sech}^2(kx)\tanh^2(kx)\right) + \frac{1}{2} \\ & \cdot k\left(-2k^3\operatorname{sech}^4(kx)\right. \\ & \left.+ 4k^3\operatorname{sech}^2(kx)\tanh^2(kx)\right)\Big),\Big\} \end{aligned} \tag{30}$$

$$\begin{aligned} v_1(x,t) = \Big\{ & t\Big(-12k^3\lambda\operatorname{sech}^2(kx)\tanh(kx) + 12k^5 \\ & \cdot\operatorname{sech}^4(kx)\tanh(kx) - 12k^5\operatorname{sech}^2(kx)\tanh^3(kx) \\ & + 12k^3\operatorname{sech}^2(kx)\tanh(kx)\Big(\frac{1}{2}\left(4k^2+\lambda\right) \\ & - 2k^2\tanh^2(kx)\Big) \\ & + 2k^2\left(-16k^3\operatorname{sech}^4(kx)\tanh(kx)\right. \\ & \left.+ 8k^3\operatorname{sech}^2(kx)\tanh^3(kx)\right)\Big).\Big\} \end{aligned} \tag{31}$$

Second Order Solutions

$$\begin{aligned} u_2(x,t) = \Big\{ & -\frac{1}{64}k^2\operatorname{sech}^8(kx)\Big(20k^2t - 2752k^8t^3 \\ & + 30t\lambda - 4416k^6t^3\lambda - 432k^4t^3\lambda^2 + 15t\left(2k^2+3\lambda\right) \\ & \cdot\cosh(2kx) + 8k^4t^3\left(154k^2-9\lambda\right)\left(2k^2+3\lambda\right) \\ & \cdot\cosh(2kx) + 6t\left(2k^2+3\lambda\right)\cosh(4kx) \\ & - 8k^4t^3\left(10k^2-9\lambda\right)\left(2k^2+3\lambda\right)\cosh(4kx) + 2k^2t \\ & \cdot\cosh(6kx) + 3t\lambda\cosh(6kx) - 4022k^5t^2 \\ & \cdot\sinh(2kx) + 48k^{11}t^4\sinh(2kx) - 546k^3t^2\lambda \\ & \cdot\sinh(2kx) + 216k^9t^4\lambda\sinh(2kx) + \frac{45}{2}kt^2\lambda^2 \\ & \cdot\sinh(2kx) + 324k^7t^4\lambda^2\sinh(2kx) + 162k^5t^4\lambda^3 \\ & \cdot\sinh(2kx) + 584k^5t^2\sinh(4kx) - 264k^3t^2\lambda \\ & \cdot\sinh(4kx) + 18kt^2\lambda^2\sinh(4kx) + 2k^5t^2 \\ & \cdot\sinh(6kx) + 6k^3t^2\lambda\sinh(6kx) + \frac{9}{2}kt^2\lambda^2 \\ & \cdot\sinh(6kx)\Big) - t\Big(-3k^2\lambda\operatorname{sech}^2(kx) - 15k^4 \\ & \cdot\operatorname{sech}^2(kx)\tanh^2(kx) + 3k^2\operatorname{sech}^2(kx) \\ & \cdot\Big(\frac{1}{2}\left(4k^2+\lambda\right) - 2k^2\tanh^2(kx)\Big) \\ & - 3k^2\left(2k^2\operatorname{sech}^4(kx) - 4k^2\operatorname{sech}^2(kx)\tanh^2(kx)\right) \\ & + \frac{1}{2}k\left(-2k^3\operatorname{sech}^4(kx)\right. \\ & \left.+ 4k^3\operatorname{sech}^2(kx)\tanh^2(kx)\right)\Big),\Big\} \end{aligned} \tag{32}$$

$$\begin{aligned} v_2(x,t) = \Big\{ & \frac{1}{8}k^4t^2\operatorname{sech}^2(kx)\Big(8\left(2k^2-3\lambda\right)^2 - 90k^6t \\ & \cdot\operatorname{sech}^8(kx)\left(t\left(2k^2+3\lambda\right)^2 + 64k\tanh(kx)\right) + 3k^4 \\ & \cdot\operatorname{sech}^6(kx) \\ & \cdot\Big(3\left(-736 + t^2\left(10k^2-3\lambda\right)\left(2k^2+3\lambda\right)^2\right) \\ & + 64kt\left(50k^2-9\lambda\right)\tanh(kx)\Big) + 4\operatorname{sech}^2(kx) \\ & \cdot\Big(-3\left(212k^4 - 84k^2\lambda + 9\lambda^2\right) \\ & + 64k^5t\left(2k^2-3\lambda\right)\tanh(kx)\Big) + 2k^2\operatorname{sech}^4(kx) \\ & \cdot\Big(-468\lambda \\ & + 3k^2\left(1480 - t^2\left(2k^2-3\lambda\right)\left(2k^2+3\lambda\right)^2\right) \\ & - 4kt\left(580k^4 - 276k^2\lambda + 9\lambda^2\right)\tanh(kx)\Big)\Big),\Big\} \end{aligned} \tag{33}$$

the NIM yields the solutions $u(x,t)$, $v(x,t)$ as follows:

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots,$$
$$v(x,t) = v_0(x,t) + v_1(x,t) + v_2(x,t) + \cdots. \tag{34}$$

$$\begin{aligned} u(x,t) = \Big\{ & -\frac{1}{64}k^2\operatorname{sech}^8(kx)\Big(20k^2t - 2752k^8t^3 + 30t\lambda - 4416k^6t^3\lambda - 432k^4t^3\lambda^2 + 15t\left(2k^2+3\lambda\right)\cosh(2kx) \\ & + 8k^4t^3\left(154k^2 - 9\lambda\right)\left(2k^2+3\lambda\right)\cosh(2kx) + 6t\left(2k^2+3\lambda\right)\cosh(4kx) - 8k^4t^3\left(10k^2-9\lambda\right)\left(2k^2+3\lambda\right)\cosh(4kx) \\ & + 2k^2t\cosh(6kx) + 3t\lambda\cosh(6kx) - 4022k^5t^2\sinh(2kx) + 48k^{11}t^4\sinh(2kx) - 546k^3t^2\lambda\sinh(2kx) + 216k^9t^4\lambda \\ & \cdot\sinh(2kx) + \frac{45}{2}kt^2\lambda^2\sinh(2kx) + 324k^7t^4\lambda^2\sinh(2kx) + 162k^5t^4\lambda^3\sinh(2kx) + 584k^5t^2\sinh(4kx) - 264k^3t^2\lambda \\ & \cdot\sinh(4kx) + 18kt^2\lambda^2\sinh(4kx) + 2k^5t^2\sinh(6kx) + 6k^3t^2\lambda\sinh(6kx) + \frac{9}{2}kt^2\lambda^2\sinh(6kx)\Big) + k\tanh(kx), \Big\} \end{aligned} \tag{35}$$

$$\begin{aligned} v(x,t) = \Big\{ & \frac{1}{8}(4\lambda + k^2\operatorname{sech}^2(kx)\Big(8\left(2 + k^2t^2\left(2k^2-3\lambda\right)^2\right) + kt\operatorname{sech}(kx)\Big(16\left(2k^2-3\lambda\right)\sinh(kx) \\ & - 90k^7t^2\operatorname{sech}^7(kx)\left(t\left(2k^2+3\lambda\right)^2 + 64k\tanh(kx)\right) \\ & + 4k\operatorname{sech}(kx)\left(-3t\left(212k^4 - 84k^2\lambda + 9\lambda^2\right) + 8k\left(-3 + 8k^4t^2\left(2k^2-3\lambda\right)\right)\tanh(kx)\right) \\ & + 3k^5t\operatorname{sech}^5(kx)\left(3\left(-736 + t^2\left(10k^2-3\lambda\right)\left(2k^2+3\lambda\right)^2\right) + 64kt\left(50k^2-9\lambda\right)\tanh(kx)\right) \\ & - 2k^3t\operatorname{sech}^3(kx)\Big(468\lambda + 3k^2\left(-1480 + t^2\left(2k^2-3\lambda\right)\left(2k^2+3\right)^2\lambda\right) + 4kt\left(580k^4 - 276k^2\lambda + 9\lambda^2\right)\tanh(kx)\Big)\Big)\Big). \Big\} \end{aligned} \tag{36}$$

Problem 5. Consider the coupled Hirota Satsuma KDV equation given by (1) together with following initial conditions [35].

$$\begin{aligned} u(x,0) &= \frac{1}{3}\left(\beta - 2k^2\right) + 2k^2\tanh^2(kx), \\ v(x,0) &= \frac{-4k^2F_0\left(\beta+k^2\right)}{3F_2^2} + \frac{4k^2\left(\beta+k^2\right)}{3F_2}\tanh(kx), \\ w(x,0) &= F_0 + F_2\tanh(kx). \end{aligned} \tag{37}$$

Zeroth Order Solutions

$$u_0(x,t) = \frac{1}{3}\left(\beta - 2k^2\right) + 2k^2\tanh^2(kx), \tag{38}$$

$$v_0(x,t) = \frac{-4k^2F_0\left(\beta+k^2\right)}{3F_2^2} + \frac{4k^2\left(\beta+k^2\right)}{3F_2}\tanh(kx), \tag{39}$$

$$w_0(x,t) = F_0 + F_2\tanh(kx). \tag{40}$$

First Order Solutions

$$\begin{aligned} u_1(x,t) = \Big\{ & t\Big(3k\operatorname{sech}^2(kx)F_2\left(-\frac{4k^2\left(k^2+\beta\right)F_0}{3F_2^2} + \frac{4k^2\left(k^2+\beta\right)\tanh(kx)}{3F_2}\right) \\ & + \frac{4k^3\left(k^2+\beta\right)\operatorname{sech}^2(kx)\left(F_0 + F_2\tanh(kx)\right)}{F_2} - 12k^3\operatorname{sech}^2(kx)\tanh(kx)\left(\frac{1}{3}\left(-2k^2+\beta\right) + 2k^2\tanh^2(kx)\right) \\ & + k^2\left(-16k^3\operatorname{sech}^4(kx)\tanh(kx) + 8k^3\operatorname{sech}^2(kx)\tanh^3(kx)\right)\Big), \Big\} \end{aligned} \tag{41}$$

$$v_1(x,t) = \left\{ t \left(\frac{4k^3 \left(k^2+\beta\right) \operatorname{sech}^2(kx) \left(\frac{1}{3}\left(-2k^2+\beta\right) + 2k^2 \tanh^2(kx) \right)}{F_2} - \frac{4k^2\left(k^2+\beta\right)\left(-2k^3 \operatorname{sech}^4(kx)\right.}{3F_2} + \frac{4k^3 \operatorname{sech}^2(kx) \tanh^2(kx))}{3F_2} \right) . \right\} \tag{42}$$

$$w_1(x,t) = t\left(3k \operatorname{sech}^2(kx) F_2 \left(\frac{1}{3}\left(-2k^2+\beta\right) + 2k^2\tanh^2(kx)\right) - F_2\left(-2k^3\operatorname{sech}^4(kx) + 4k^3\operatorname{sech}^2(kx)\tanh^2(kx)\right)\right). \tag{43}$$

Second Order Solutions

$$\begin{aligned} u_2(x,t) = \Big\{ & -2k^5 \operatorname{sech}^6(kx) \Big(-3kt^2\beta + 2kt^2\beta\cosh(2kx) - kt^2\beta\cosh(4kx) - 4t\sinh(2kx) + \frac{4}{3}k^2 5t^3\beta^2\sinh(2kx) \\ & + \frac{4}{3}t^3\beta^3\sinh(2kx) + t\sinh(4kx)\Big) - t\Bigg(3k\operatorname{sech}^2(kx) F_2\left(-\frac{4k^2\left(k^2+\beta\right)F_0}{3F_2^2} + \frac{4k^2\left(k^2+\beta\right)\tanh(kx)}{3F_2}\right) \\ & + \frac{4k^3\left(k^2+\beta\right)\operatorname{sech}^2(kx)\left(F_0 + F_2\tanh(kx)\right)}{F_2} - 12k^3\operatorname{sech}^2(kx)\tanh(kx)\left(\frac{1}{3}\left(-2k^2+\beta\right) + 2k^2\tanh^2(kx)\right) \\ & + k^2\left(-16k^3\operatorname{sech}^4(kx)\tanh(kx) + 8k^3\operatorname{sech}^4(kx)\tanh^3(kx)\right)\Bigg), \Big\} \end{aligned} \tag{44}$$

$$\begin{aligned} v_2(x,t) = \Big\{ & -\frac{1}{6F_2}k^3\beta\left(k^2+\beta\right)\operatorname{sech}^6(kx)\big(-3t - 32k^4t^3\beta - 4t\cosh(2kx) + 32k^4t^3\beta\cosh(2kx) - t\cosh(4kx) \\ & + kt^2\beta\sinh(4kx)\big) - t\left(\frac{4k^3\left(k^2+\beta\right)\operatorname{sech}^2(kx)\left((1/3)\left(-2k^2+\beta\right) + 2k^2\tanh^2(kx)\right)}{F_2}\right. \\ & \left. - \frac{4k^2\left(k^2+\beta\right)\left(-2k^3\operatorname{sech}^4(kx) + 4k^3\operatorname{sech}^2(kx)\tanh^2(kx)\right)}{3F_2}\right) + 2kt^2\beta\sinh(2kx), \Big\} \end{aligned} \tag{45}$$

$$\begin{aligned} w_2(x,t) = \Big\{ & \frac{1}{8}k\beta\operatorname{sech}^6(kx)\big(3t + 32k^4t^3\beta + 4t\cosh(2kx) - 32k^4t^3\beta\cosh(2kx) + t\cosh(4kx) - 40k^3t^2\sinh(2kx) \\ & + 4k^3t^2\sinh(4kx)\big)F_2 - t\Big(3k\operatorname{sech}^2(kx) F_2\left(\frac{1}{3}\left(-2k^2+\beta\right) + 2k^2\tanh^2(kx)\right) \\ & - F_2\left(-2k^3\operatorname{sech}^4(kx) + 4k^3\operatorname{sech}^2(kx)\tanh^2(kx)\right)\Big). \Big\} \end{aligned} \tag{46}$$

The NIM yields the solutions $u(x,t), v(x,t), w(x,t)$ as follows:

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t), \\ v(x,t) &= v_0(x,t) + v_1(x,t) + v_2(x,t), \\ w(x,t) &= w_0(x,t) + w_1(x,t) + w_2(x,t). \end{aligned} \tag{47}$$

$$\begin{aligned} u(x,t) = \Big\{ & \frac{1}{3}\left(-2k^2+\beta\right) - 2k^5\operatorname{sech}^6(kx)\Big(-3kt^2\beta \\ & + 2kt^2\beta\cosh(2kx) - kt^2\beta\cosh(4kx) \\ & - 4t\sinh(2kx) + \frac{4}{3}k^2t^3\beta^2\sinh(2kx) \end{aligned}$$

$$+\frac{4}{3}t^3\beta^3\sinh(2kx)+t\sinh(4kx)\Big)+2k^2\cdot\tanh^2(kx)\Big),\Big\} \quad (48)$$

$$v(x,t)=\Big\{-\frac{4k^2(k^2+\beta)F_0}{3F_2^2}-\frac{1}{6F_2}k^3\beta(k^2+\beta)\cdot\operatorname{sech}^6(kx)\big(-3t-32k^4t^3\beta-4t\cosh(2kx)+32k^4t^3\beta\cosh(2kx)-t\cosh(4kx)+2kt^2\beta\sinh(2kx)+kt^2\beta\sinh(4kx)\big)+\frac{4k^2(k^2+\beta)\tanh(kx)}{3F_2},\Big\} \quad (49)$$

$$w(x,t)=\Big\{F_0-\frac{1}{8}k\beta\operatorname{sech}^6(kx)\big(-3t-32k^4t^3\beta-4t\cosh(2kx)+32k^4t^3\beta\cosh(2kx)-t\cosh(4kx)+2kt^2\beta\sinh(2kx)+kt^2\beta\sinh(4kx)\big)F_2+F_2\tanh(kx),\Big\} \quad (50)$$

Problem 6. Consider the coupled modified KDV equation given by (2) together with following initial condition [35].

$$u(x,0)=\frac{b_1}{2k}+k\tanh(kx),$$
$$v(x,0)=\frac{\lambda}{2}\left(1+\frac{k}{b_1}\right)+b_1\tanh(kx). \quad (51)$$

Zeroth Order Solutions

$$u_0(x,t)=\frac{b_1}{2k}+k\tanh(kx), \quad (52)$$

$$v_0(x,t)=\frac{\lambda}{2}\left(1+\frac{k}{b_1}\right)+b_1\tanh(kx). \quad (53)$$

First Order Solutions

$$u_1(x,t)=\Big\{t\Big(-3k^2\lambda\operatorname{sech}^2(kx)-3k^2\operatorname{sech}^2(kx)b_1\cdot\tanh(kx)+3k\operatorname{sech}^2(kx)b_1\left(\frac{b_1}{2k}+k\tanh(kx)\right)-3k^2\operatorname{sech}^2(kx)\left(\frac{b_1}{2k}+k\tanh(kx)\right)^2+3k^2\cdot\operatorname{sech}^2(kx)\left(\frac{1}{2}\lambda\left(1+\frac{k}{b_1}\right)+b_1\tanh(kx)\right)+\frac{1}{2}\cdot k\big(-2k^3\operatorname{sech}^4(kx)+4k^3\operatorname{sech}^2(kx)\tanh^2(kx)\big)\Big),\Big\} \quad (54)$$

$$v_1(x,t)=\Big\{t\Big(3k\lambda\operatorname{sech}^2(kx)b_1-3k^3\operatorname{sech}^4(kx)b_1+3k\operatorname{sech}^2(kx)b_1\left(\frac{b_1}{2k}+k\tanh(kx)\right)^2-3k\cdot\operatorname{sech}^2(kx)b_1\left(\frac{1}{2}\lambda\left(1+\frac{k}{b_1}\right)+b_1\tanh(kx)\right)-b_1\big(-2k^3\operatorname{sech}^4(kx)+4k^3\operatorname{sech}^2(kx)\tanh^2(kx)\big)\Big).\Big\} \quad (55)$$

Second Order Problem

$$u_2(x,t)=\Big\{\frac{1}{16b_1^2k}t^2\operatorname{sech}^2(kx)\Big(12b_1^2k^2\big(b_1(4k^4+6\lambda k^2)-3b_1^3-6\lambda k^3\big)-k^3t\big(-2b_1(2k^4+3\lambda k^2)+3b_1^3+6\lambda k^3\big)^2\cdot\operatorname{sech}^4(kx)-4b_1k\big(b_1(4k^4+6\lambda k^2)-3b_1^3-6\lambda k^3\big)\tanh(kx)\operatorname{sech}^2(kx)\big(b_1t(-3b_1^2+4k^4-6\lambda k^2)+3k^3(2\lambda t+1)\big)+12b_1^2k^2\big(-2b_1(2k^4+3\lambda k^2)+3b_1^3+6\lambda k^3\big)\operatorname{sech}^2(kx)+2\big(9b_1^3\lambda k^3-9b_1^4(\lambda-2)k^2-12b_1\lambda k^5(k^2-3(\lambda+1))+2b_1^2k^4(2k^2+3\lambda)(4k^2-3(\lambda+2))-9b_1^6-18\lambda^2k^6\big)\tanh(kx)\Big),\Big\} \quad (56)$$

$$v_2(x,t)=\bigg(\Big\{\frac{1}{128k^2b_1^2}t^2\operatorname{sech}(kx)^3\cdot\Big(24k^3t\operatorname{sech}(kx)^3b_1\big(14k^3\lambda+2(6k^4-7k^2\lambda)b_1-9b_1^3\big)\big(-6k^3\lambda+(4k^4+6k^2\lambda)b_1-3b_1^3\big)$$

$$- 8\sinh(kx)\, b_1 \left(-6k^3\lambda + \left(-4k^4 + 6k^2\lambda\right) b_1 + 3b_1^3\right)^2$$
$$- 64k^3 \operatorname{sech}(kx)\, b_1 \left(-36k^6 t\lambda^2 + b_1 \left(72k^5 t\lambda^2 + b_1 \left(16k^8 t - 9k^2\lambda - 36k^4 t\lambda^2 + 9k\lambda b_1 - 24k^4 t b_1^2 + 9t b_1^4\right)\right)\right)$$
$$+ 3t^2 \operatorname{sech}(kx)^3 \left(6k^3\lambda + \left(4k^4 - 6k^2\lambda\right) b_1 - 3b_1^3\right)\left(6k^4\lambda - 2\left(2k^5 + 3k^3\lambda\right) b_1 + 3kb_1^3\right)^2 \tanh(kx)$$
$$+ 192k^3\lambda \operatorname{sech}(kx)\left(k - b_1\right) b_1^2 \left(6k^3 (2 + t\lambda) + t b_1 \left(4k^4 - 6k^2\lambda - 3b_1^2\right)\right)\tanh(kx)\Big), \Big\} \tag{57}$$

NIM yields the solutions $u(x,t)$, $v(x,t)$ as follows:

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) + u_2(x,t) + \cdots, \\ v(x,t) &= v_0(x,t) + v_1(x,t) + v_2(x,t) + \cdots. \end{aligned} \tag{58}$$

$$\begin{aligned} u(x,t) = \Bigg\{ & t\Bigg(3k^2\operatorname{sech}^2(kx)\left(\frac{1}{2}\lambda\left(\frac{k}{b_1}+1\right) + b_1\tanh(kx)\right) - 3k^2\operatorname{sech}^2(kx)\left(\frac{b_1}{2k} + k\tanh(kx)\right)^2 - 3b_1k^2\tanh(kx) \\ & \cdot \operatorname{sech}^2(kx) + 3b_1 k\operatorname{sech}^2(kx)\left(\frac{b_1}{2k} + k\tanh(kx)\right) + \frac{1}{2}k\left(4k^3\tanh^2(kx)\operatorname{sech}^2(kx) - 2k^3\operatorname{sech}^4(kx)\right) - 3k^2\lambda \\ & \cdot \operatorname{sech}^2(kx)\Bigg) + \frac{1}{16b_1^2k} t^2\operatorname{sech}^2(kx)\Big(12b_1^2k^2\left(b_1\left(4k^4+6\lambda k^2\right) - 3b_1^3 - 6\lambda k^3\right) \\ & - k^3t\left(-2b_1\left(2k^4+3\lambda k^2\right) + 3b_1^3 + 6\lambda k^3\right)^2\operatorname{sech}^4(kx) - 4b_1k\left(b_1\left(4k^4+6\lambda k^2\right) - 3b_1^3 - 6\lambda k^3\right)\tanh(kx)\operatorname{sech}^2(kx) \\ & \cdot\left(b_1 t\left(-3b_1^2 + 4k^4 - 6\lambda k^2\right) + 3k^3(2\lambda t + 1)\right) + 12b_1^2k^2\left(-2b_1\left(2k^4+3\lambda k^2\right) + 3b_1^3 + 6\lambda k^3\right)\operatorname{sech}^2(kx) \\ & + 2\left(9b_1^3\lambda k^3 - 9b_1^4(\lambda-2)k^2 - 12b_1\lambda k^5\left(k^2 - 3(\lambda+1)\right) + 2b_1^2k^4\left(2k^2+3\lambda\right)\left(4k^2 - 3(\lambda+2)\right) - 9b_1^6 - 18\lambda^2k^6\right) \\ & \cdot\tanh(kx)\Big) + \frac{b_1}{2k} + k\tanh(kx), \Bigg\} \end{aligned} \tag{59}$$

$$\begin{aligned} v(x,t) = \Bigg\{ & \frac{1}{128k^2b_1^2}\Big(64k^2\lambda b_1(k+b_1) - 32kt\operatorname{sech}^2(kx)\, b_1^2\left(6k^3\lambda + \left(4k^4 - 6k^2\lambda\right)b_1 - 3b_1^3\right) + 24k^3t^3\operatorname{sech}^6(kx) \\ & \cdot b_1\left(14k^3\lambda + 2\left(6k^4 - 7k^2\lambda\right)b_1 - 9b_1^3\right)\left(-6k^3\lambda + \left(4k^4 + 6k^2\lambda\right)b_1 - 3b_1^3\right) - 64k^3t^2\operatorname{sech}^4(kx) \\ & \cdot b_1\left(-36k^6t\lambda^2 + b_1\left(72k^5t\lambda^2 + b_1\left(16k^8t - 9k^2\lambda - 36k^4t\lambda^2 + 9k\lambda b_1 - 24k^4tb_1^2 + 9tb_1^4\right)\right)\right) + 128k^2b_1^3\tanh(kx) - 8t^2 \\ & \cdot\operatorname{sech}(kx)\, b_1\left(-6k^3\lambda + \left(-4k^4 + 6k^2\lambda\right)b_1 + 3b_1^3\right)^2\tanh(kx) + 3t^4\operatorname{sech}^6(kx)\left(6k^3\lambda + \left(4k^4 - 6k^2\lambda\right)b_1 - 3b_1^3\right) \\ & \cdot\left(6k^4\lambda - 2\left(2k^5 + 3k^3\lambda\right)b_1 + 3kb_1^3\right)^2\tanh(kx) + 192k^3t^2\lambda\operatorname{sech}^4(kx)\left(k - b_1\right) \\ & \cdot b_1^2\left(6k^3(2+t\lambda) + tb_1\left(4k^4 - 6k^2\lambda - 3b_1^2\right)\right)\tanh(kx)\Big). \Bigg\} \end{aligned} \tag{60}$$

4. Results and Discussion

NIM formulation is tested upon the nonlinear Hirota Satsuma coupled KDV and modified coupled KDV equations. We have used Mathematica 7 for most of our computational work. Table 1 shows the comparison of 2nd order NIM and HAM solution for $u(x,t)$, $v(x,t)$, and $w(x,t)$ components of Hirata Satsuma coupled KDV equation with initial condition given by (12), while Table 2 shows the absolute errors of NIM at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 1$.

TABLE 1: Comparison of 2nd order NIM solution with 2nd order HAM solution for nonlinear coupled Hirota Satsuma KDV equation using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 1$.

x	u_{exact}	u_{HAM} [9]	u_{NIM}	v_{exact}	v_{HAM} [9]	v_{NIM}	w_{exact}	w_{HAM} [9]	w_{NIM}
-50	0.346658	0.346659	0.346659	0.0657245	0.065726	0.0657245	1.99978	1.99982	1.99978
-40	0.346601	0.346613	0.346613	0.0656678	0.0656793	0.0656679	1.99836	1.99865	1.99836
-30	0.346185	0.346273	0.346269	0.0652518	0.0653356	0.0652524	1.98796	1.99006	1.98798
-20	0.343242	0.343845	0.343831	0.0623089	0.0628901	0.0623147	1.91439	1.92892	1.91457
-10	0.32719	0.329861	0.329974	0.0462566	0.0489609	0.0462879	1.51308	1.58069	1.51464
0	0.307064	0.306667	0.306682	0.0261307	0.0257347	0.0261333	1.00993	1.00003	1.01
10	0.332299	0.329874	0.329913	0.0513653	0.489077	0.0513998	1.6408	1.57936	1.64234
20	0.344338	0.343836	0.343839	0.0634044	0.0629245	0.0634037	1.94178	1.92978	1.94179
30	0.346343	0.346271	0.346269	0.0654099	0.0653417	0.0654095	1.99192	1.99021	1.9919
40	0.346623	0.346613	0.346613	0.0656894	0.0656801	0.0656893	1.9989	1.99867	1.9989
50	0.346661	0346659	0.346659	0.0657274	0.0657261	0.0657274	1.99985	1.99982	1.99985

Table 2: Absolute errors of 2nd order NIM solution for Hirota Satsuma KDV equation using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 1$.

x	u_{NIM}	v_{NIM}	w_{NIM}
50	1.55045×10^{-6}	1.02035×10^{-8}	2.55355×10^{-7}
40	1.14454×10^{-5}	7.60484×10^{-8}	1.91571×10^{-6}
30	8.39765×10^{-5}	5.96212×10^{-7}	1.56830×10^{-5}
20	5.88572×10^{-4}	5.77582×10^{-6}	1.81550×10^{-4}
10	2.78424×10^{-3}	3.12612×10^{-5}	1.55799×10^{-3}
0	3.81828×10^{-4}	2.65163×10^{-6}	6.62908×10^{-5}
-10	2.38551×10^{-3}	3.45628×10^{-5}	1.54605×10^{-3}
-20	4.98990×10^{-4}	6.68171×10^{-7}	1.22730×10^{-5}
-30	7.38904×10^{-5}	4.38566×10^{-7}	1.03679×10^{-5}
-40	1.01336×10^{-5}	6.69141×10^{-8}	1.66176×10^{-6}
-50	1.37393×10^{-6}	9.19754×10^{-9}	2.29735×10^{-7}

Table 3: Comparison of 2nd order NIM solution with 2nd order HAM solution for $u(x,t)$ part of nonlinear coupled modified KDV equation at $k = 0.1$ $\lambda = 1$, $t = 0.5$.

x	u_{exact}	u_{HAM} [9]	u_{NIM}	$u_{\text{Absolute errors}}$
-50	-0.0999922	-0.0999895	-0.0999922	5.01465×10^{-9}
-40	-0.0999423	-0.0999228	-0.0999423	3.68284×10^{-8}
-30	-0.0995746	-0.0994310	-0.0995744	2.59984×10^{-7}
-20	-0.0968991	-0.0958693	-0.0968978	1.31416×10^{-6}
-10	-0.0791524	-0.0729886	-0.0791603	7.82324×10^{-6}
0	-0.00753569	-0.0075500	-0.0075491	1.34504×10^{-5}
10	0.0728019	0.0793302	0.0728181	1.61449×10^{-5}
20	0.098286	0.0969362	0.0958309	2.28210×10^{-6}
30	0.0994251	0.0995800	0.0994254	2.97728×10^{-7}
40	0.099922	00999431	0.099922	4.00387×10^{-8}
50	0.0999894	0.0999923	0.0999894	5.41386×10^{-9}

Table 4: Comparison of 2nd order NIM solution with 2nd order HAM solution for $v(x,t)$ of nonlinear coupled modified KDV equation at $k = 0.1$ $\lambda = 1$, $t = 0.5$.

x	v_{exact}	v_{HAM} [9]	v_{NIM}	$v_{\text{Absolute errors}}$
-50	0.500003	0.50003	0.500004	1.04894×10^{-6}
-40	0.500023	0.500019	0.500031	8.34759×10^{-6}
-30	0.50017	0.500141	0.500229	5.89469×10^{-5}
-20	0.501221	0.501021	0.50163	4.09288×10^{-4}
-10	0.50747	0.506530	0.509398	1.92802×10^{-3}
0	0.519886	0.519889	0.519887	2.36659×10^{-7}
10	0.5094	0.510342	0.507475	1.92439×10^{-3}
20	0.501634	0.501833	0.501224	4.09834×10^{-4}
30	0.500229	0.500258	0.50017	5.90785×10^{-5}
40	0.500031	0.500035	0.500023	8.06431×10^{-6}
50	0.500004	0.500005	0.500003	1.09265×10^{-6}

Table 5 shows the comparison of 2nd order NIM and HAM solution for $u(x,t)$, $v(x,t)$, and $w(x,t)$ components of Hirata Satsuma coupled KDV equation with initial condition (37), while Table 6 shows the absolute errors of NIM at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $t = 2$, and $\beta = 1$.

Tables 3 and 4 show the comparison of 2nd order NIM solution with HAM solution for $u(x,t)$ and $v(x,t)$ components of modified coupled KDV equation with initial condition (27) at $k = 0.1$ $\lambda = 1$ and $t = 0.5$. Table 7 shows the comparison of 2nd order NIM with exact solution for $u(x,t)$ and $v(x,t)$ of coupled modified KDV equation with initial condition (51) at $k = 0.1$, $b_1 = 0.1$, $\lambda = 0.1$, and $t = 0.5$.

Figures 1, 3, and 5 show the 3D plots of 2nd order NIM solution, respectively, for $u(x,t)$, $v(x,t)$, and $w(x,t)$ parts of Hirata Satsuma coupled KDV equation with initial condition (12) at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 1$., while Figures

Table 5: Comparison of 2nd order NIM solution with the exact solution for $u(x,t)$, $v(x,t)$, and $w(x,t)$ of nonlinear coupled Hirota Satsuma KDV equation using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 2$.

x	u_{exact}	u_{NIM}	v_{exact}	v_{NIM}	w_{exact}	w_{NIM}
-50	0.346661	0.346662	-0.0269315	-0.0269315	0.00013	0.00013
-40	0.346627	0.346629	-0.0269199	-0.0269200	0.00100	0.00099
-30	0.346373	0.346391	-0.0268341	-0.0268349	0.00736	0.00731
-20	0.344595	0.344710	-0.0262170	-0.0262220	0.05319	0.05319
-10	0.335486	0.335776	-0.0224089	-0.0224203	0.33596	0.33596
0	0.327446	0.326635	-0.0108087	-0.0107733	1.19738	1.20000
10	0.340566	0.340779	-0.0022401	-0.0022525	1.83365	1.83273
20	0.345708	0.345795	-0.0003266	-0.0003308	1.97574	1.97543
30	0.346534	0.346548	-0.0000447	-0.0000453	1.99668	1.99664
40	0.346649	0.346651	$-6.05507\text{x}10^{-6}$	$-6.14233\text{x}10^{-6}$	1.99955	1.99954
50	0.346664	0.346664	$-8.19623\text{x}10^{-7}$	$-8.19623\text{x}10^{-7}$	1.99994	1.99994

Table 6: Absolute errors of 2nd order NIM solution of Hirota Satsuma KDV equation using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $t = 2$, and $\beta = 1$.

x	u_{NIM}	v_{NIM}	w_{NIM}
50	3.27577×10^{-7}	1.44533×10^{-8}	1.07327×10^{-6}
40	2.4149×10^{-6}	1.06534×10^{-7}	7.91091×10^{-6}
30	1.75413×10^{-5}	7.72958×10^{-7}	5.73978×10^{-5}
20	1.14092×10^{-4}	4.98263×10^{-6}	3.69997×10^{-4}
10	2.90545×10^{-4}	1.12311×10^{-5}	8.33992×10^{-4}
0	8.1114×10^{-4}	3.53457×10^{-5}	2.62468×10^{-3}
-10	2.14213×10^{-4}	1.24574×10^{-5}	9.25057×10^{-4}
-20	8.72541×10^{-5}	4.2114×10^{-6}	3.12728×10^{-4}
-30	1.33693×10^{-5}	6.35469×10^{-7}	4.71883×10^{-5}
-40	1.83949×10^{-6}	8.72611×10^{-8}	6.47979×10^{-6}
-50	2.49504×10^{-7}	1.18327×10^{-8}	8.78668×10^{-7}

Table 7: Comparison of 2nd order NIM solution with the exact solution for nonlinear coupled Hirota Satsuma MKDV equations with initial conditions (59) using $k = 0.1$, $b_1 = 0.1$, $\lambda = 0.1$, and $t = 0.5$.

x	Exact	u_{NIM}	$u_{\text{Absolute errors}}$	Exact	v_{NIM}	$v_{\text{Absolute errors}}$
-50	0.40001	0.40001	2.48113×10^{-8}	$9.77691\text{x}10^{-6}$	9.77629×10^{-6}	6.2454×10^{-10}
-40	0.400072	0.400072	1.83213×10^{-7}	0.0000722196	0.000072215	4.60471×10^{-9}
-30	0.400532	0.400531	1.34729×10^{-6}	0.000532406	0.000532373	3.34797×10^{-8}
-20	0.403868	0.403859	9.60358×10^{-6}	0.0038819	0.00386797	2.19196×10^{-7}
-10	0.425439	0.425385	5.33734×10^{-5}	0.0254388	0.0254382	5.70005×10^{-7}
0	0.503698	0.503672	2.60625×10^{-5}	0.103698	0.1037	1.75596×10^{-6}
10	0.57767	0.57774	6.95746×10^{-5}	0.17767	0.17767	5.80831×10^{-7}
20	0.596655	0.596671	1.61881×10^{-5}	0.196655	0.196655	2.12492×10^{-7}
30	0.599541	0.599543	2.36055×10^{-6}	0199541	0.199541	3.22892×10^{-8}
40	0.599938	0.599938	3.22719×10^{-7}	0.199938	0.199938	4.43792×10^{-9}
50	0.599992	0.599992	4.37352×10^{-8}	0.199992	0.199992	6.01862×10^{-10}

11, 13, and 15 show the 3D plots of approximate solution by NIM with initial condition (37) at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 2$. Figures 2, 4, and 6 show the comparison of 2D plots of 2nd order NIM solution with exact solution for $u(x,t)$, $v(x,t)$, and $w(x,t)$ parts of nonlinear Hirota Satsuma coupled KDV equation with initial condition given by (12) at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 1$, while Figures 12, 14, and 16 show the comparison of 2D plots of 2nd order approximate solution by NIM with exact solution for $u(x,t)$, $v(x,t)$, and $w(x,t)$ with initial condition given by (37) using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 2$.

The 3D plots for modified coupled KDV equation with initial condition (27) at $k = 0.1$, $\lambda = 1$ are given in Figures 7 and 9, while Figures 17 and 19 show the 3D graphs for the same equation with initial condition (42) at $k = 0.1$, $b_1 = 0.1$, and $\lambda = 0.1$. Figures 8 and 10 show the comparison of

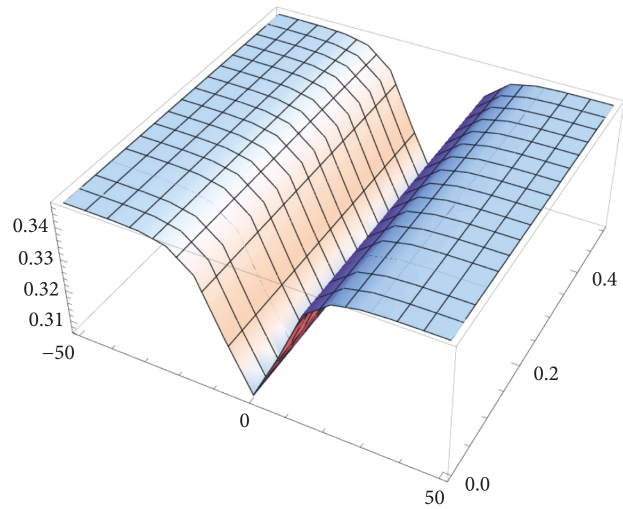

FIGURE 1: 3D graph for $u(x,t)$ of nonlinear coupled Hirota Satsuma KDV equation by NIM at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, and $\beta = 1$.

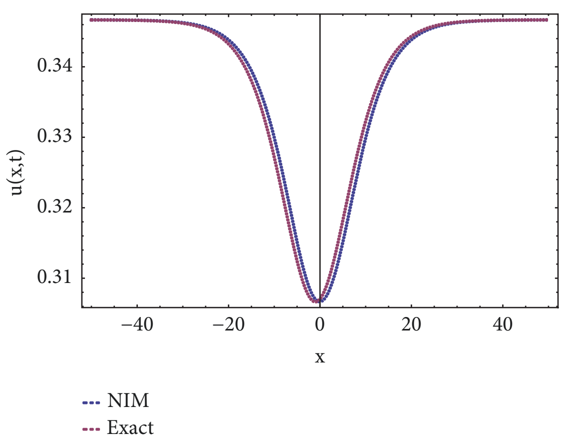

FIGURE 2: Comparison of NIM solution with exact solution for $u(x,t)$ of nonlinear coupled Hirota Satsuma KDV equation at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 1$.

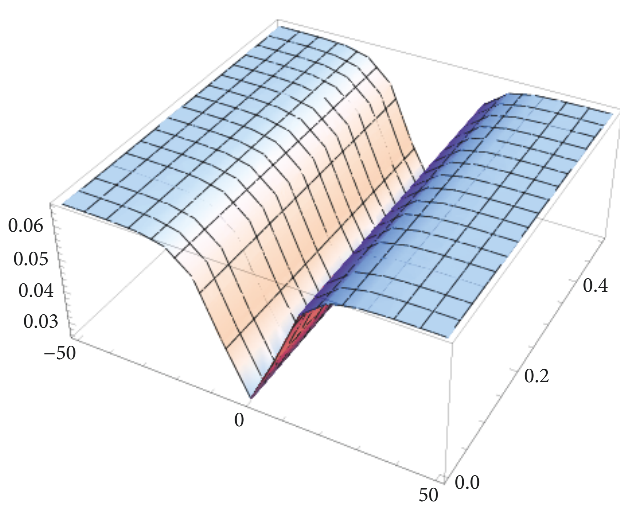

FIGURE 3: 3D graph for $v(x,t)$ of nonlinear coupled Hirota Satsuma KDV equation by NIM at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, and $\beta = 1$.

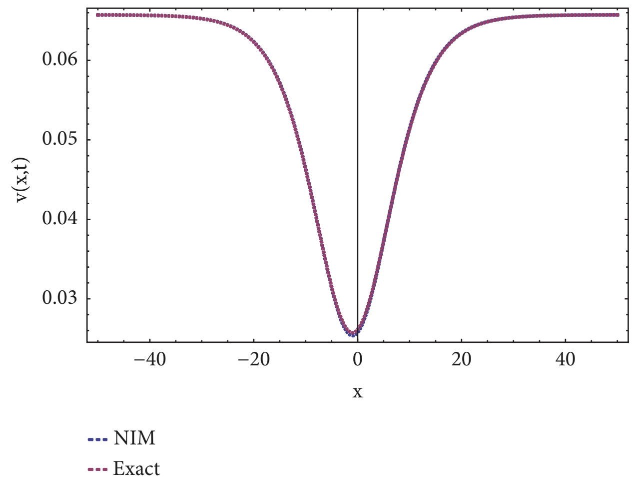

FIGURE 4: Comparison of NIM solution with exact solution for $v(x,t)$ nonlinear coupled Hirota Satsuma KDV equation at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 1$.

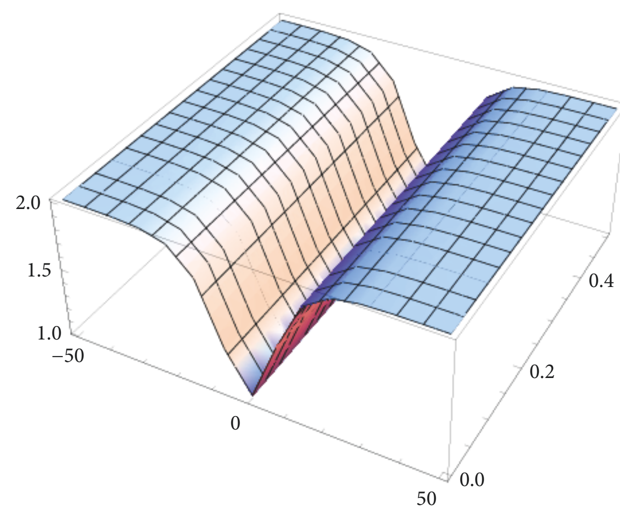

FIGURE 5: 3D graph for $w(x,t)$ of nonlinear coupled Hirota Satsuma KDV equation by NIM at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, and $\beta = 1$.

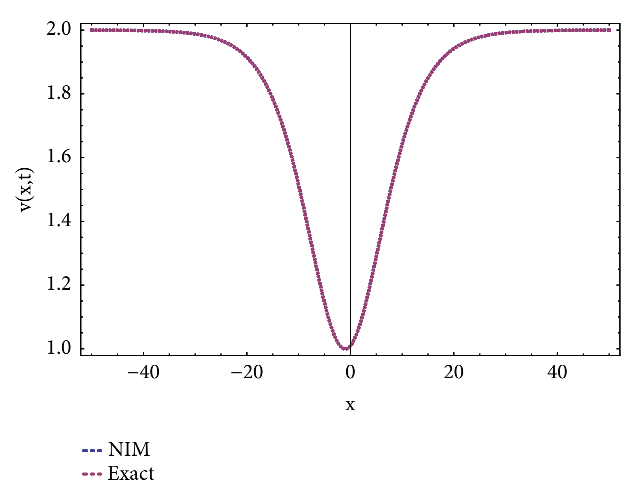

FIGURE 6: Comparison of NIM solution with exact solution for $w(x,t)$ of nonlinear coupled Hirota Satsuma KDV equation at $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$, and $t = 1$.

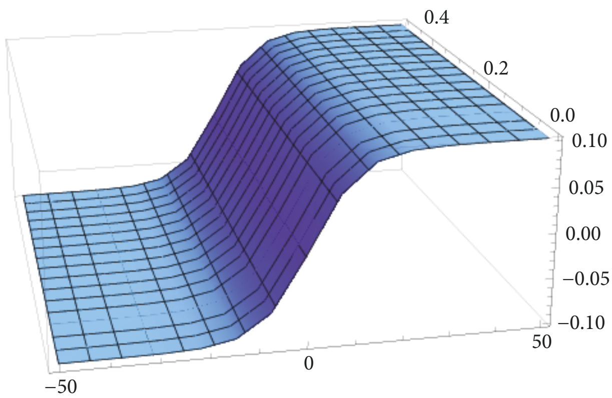

FIGURE 7: 3D graph for $u(x,t)$ of nonlinear coupled modified KDV equation by NIM using $k = 0.1$ and $\lambda = 1$.

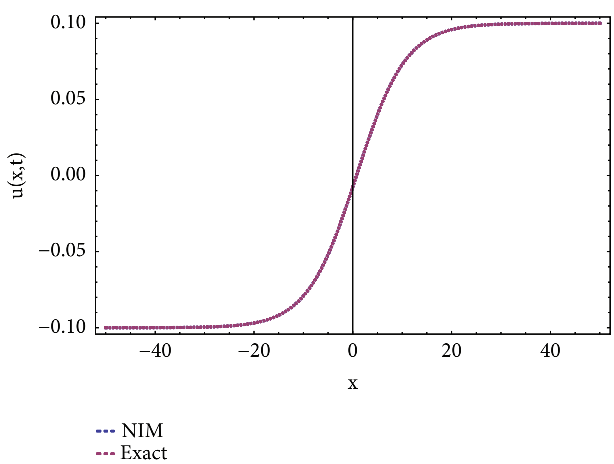

FIGURE 8: Comparison of NIM solution with exact solution for $u(x,t)$ of nonlinear coupled modified KDV equation at $k = 0.1$, $t = 0.5$, and $\lambda = 1$.

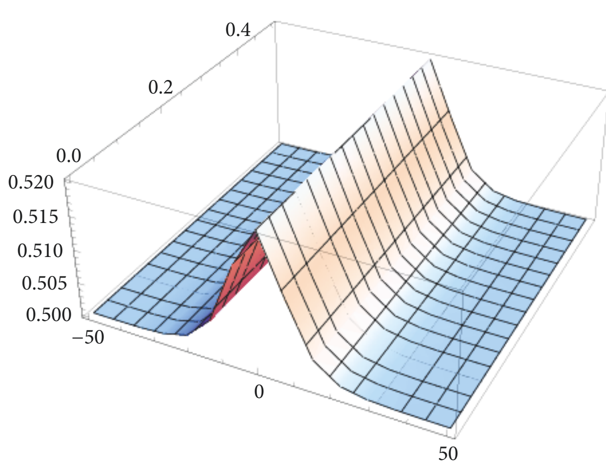

FIGURE 9: 3D graph for $v(x,t)$ of nonlinear coupled modified KDV equation by NIM using $k = 0.1$ and $\lambda = 1$.

2D plots of 2nd order NIM solution with exact solution for $u(x,t)$ and $v(x,t)$ parts of modified coupled KDV equation

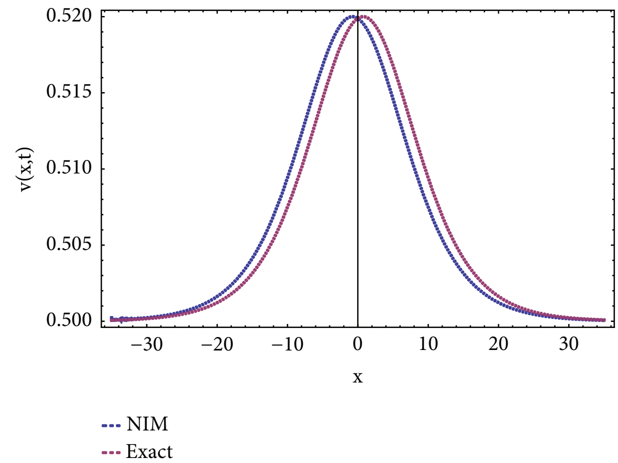

FIGURE 10: Comparison of NIM solution for $v(x,t)$ of nonlinear coupled modified KDV equation with exact solution using $k = 0.1$, $t = 0.5$, and $\lambda = 1$.

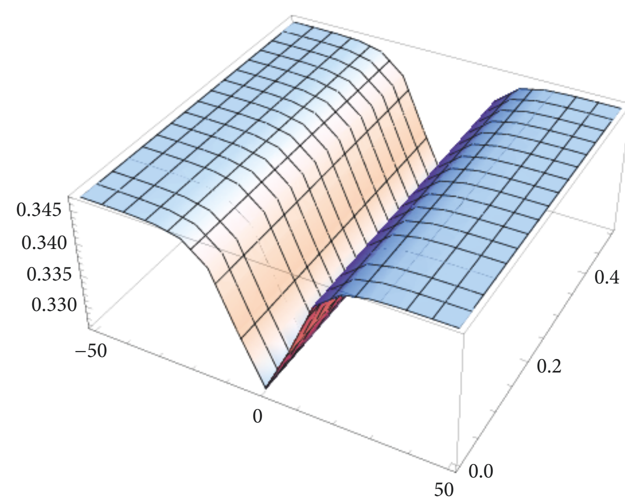

FIGURE 11: 3D graph for $u(x,t)$ part of Problem 5 by NIM using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $\beta = 1$.

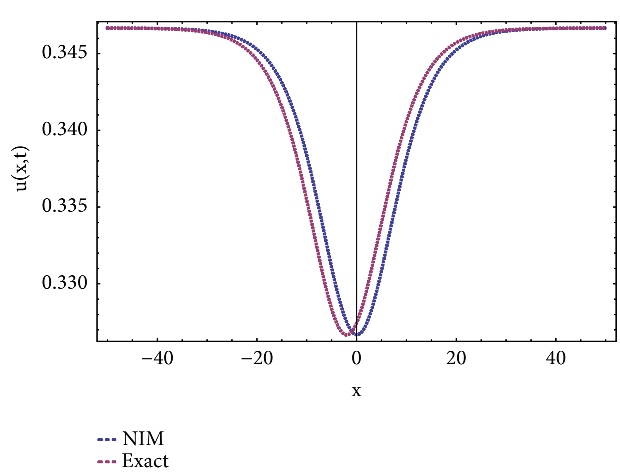

FIGURE 12: Comparing of numerical solution of $u(x,t)$ part of Problem 5 with exact solution by NIM using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $t = 2$, and $\beta = 1$.

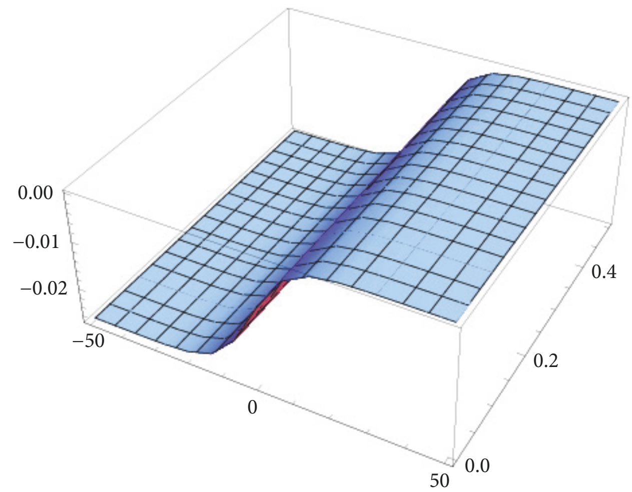

Figure 13: 3D graph for $v(x,t)$ part of Problem 5 by NIM using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, and $\beta = 1$.

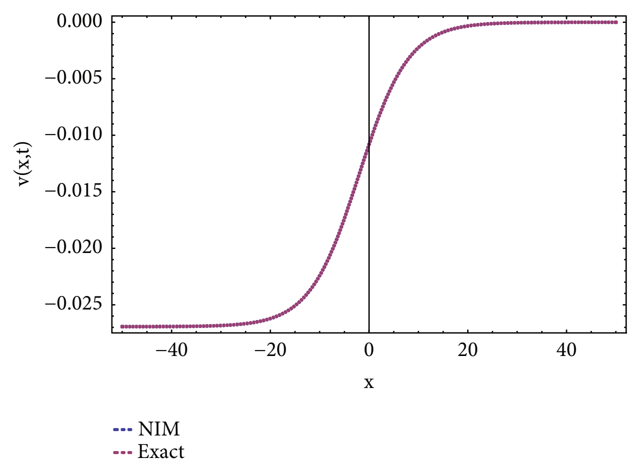

Figure 14: Comparison of NIM solution with exact solution for $v(x,t)$ part of Hirota Satsuma KDV equation using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $t = 2$, and $\beta = 1$.

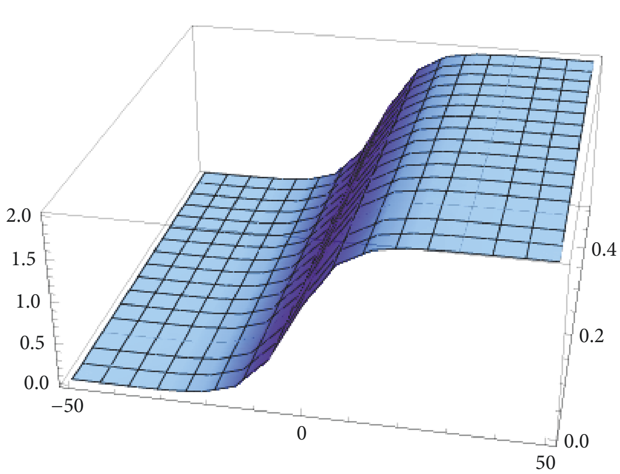

Figure 15: 3D graph for $w(x,t)$ part of Hirota Satsuma KDV equation by NIM using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, and $\beta = 1$.

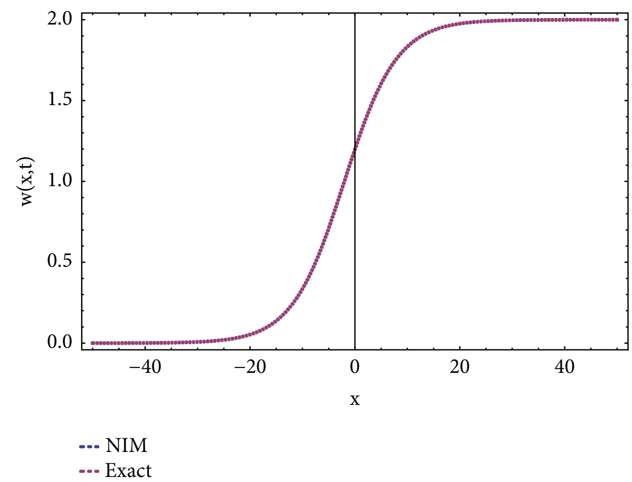

Figure 16: Comparison of NIM solution with exact solution for $w(x,t)$ part of Hirota Satsuma KDV equation using $F_0 = 1$, $F_2 = 1$, $k = 0.1$, $t = 2$, and $\beta = 1$.

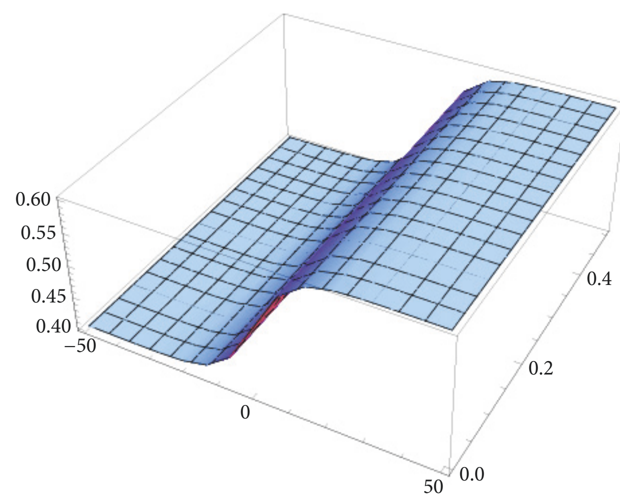

Figure 17: 3D graph for $u(x,t)$ part of coupled modified KDV equation by NIM using $k = 0.1$, $b_1 = 0.1$, and $\lambda = 0.1$.

of Problem 4 using $k = 0.1$, $t = 0.5$, $b_1 = 0.1$, and $\lambda = 1$, while Figures 18 and 20 show the comparison of 2D plots of 2nd order approximate solution by NIM with exact solution for $u(x,t)$, $v(x,t)$, and $w(x,t)$ parts modified coupled KDV equation using $k = 0.1$, $t = 0.5$, $b_1 = 0.1$, and $\lambda = 0.1$.

From the presented problems it is observed that in a few iterations the proposed method yields the approximate solutions which are in close agreement with exact solution.

5. Conclusion

NIM has been successfully implemented for finding the approximate solution of the nonlinear Hirota Satsuma coupled KDV equations and coupled MKDV equations. From obtained results it is concluded that NIM is very effective, simple, and fast convergent and is independent of the assumption of the unrealistic small parameters. The results

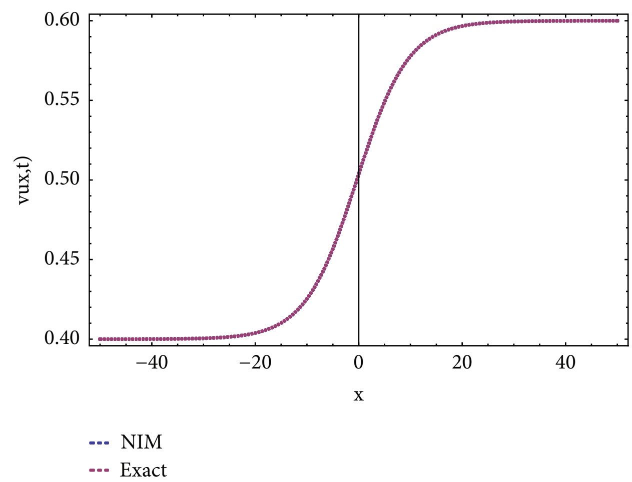

FIGURE 18: Comparison of NIM solution with exact solution for $u(x,t)$ part of coupled modified KDV equation using $k = 0.1$, $t = 0.5$, $b_1 = 0.1$, and $\lambda = 0.1$.

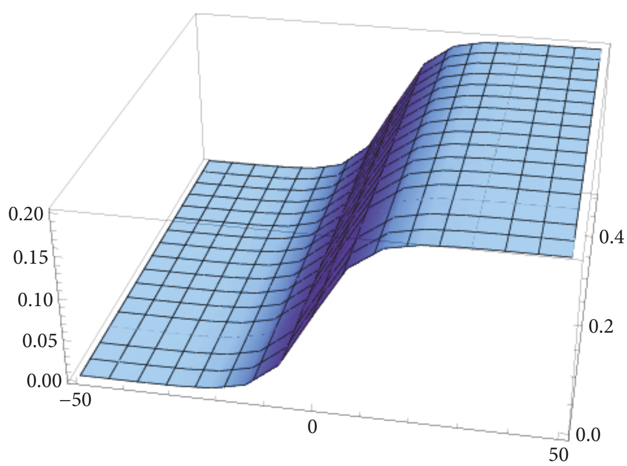

FIGURE 19: 3D graph for $v(x,t)$ part of coupled modified KDV equation by NIM using $k = 0.1$, $b_1 = 0.1$, and $\lambda = 0.1$.

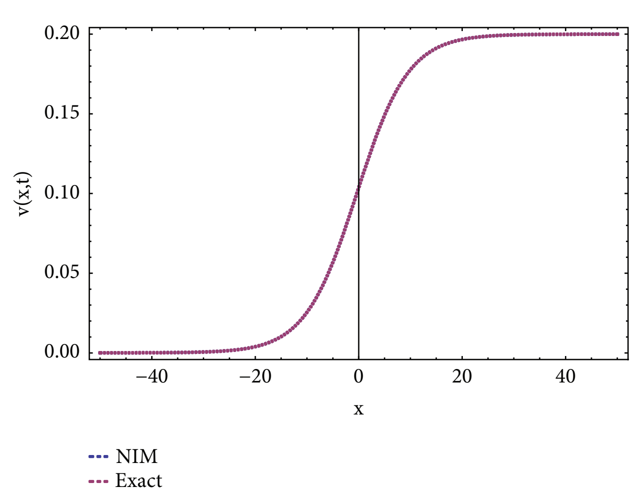

FIGURE 20: Comparison of NIM solution for $v(x,t)$ part of coupled modified KDV equation with exact solution using $k = 0.1$, $t = 0.5$, $b_1 = 0.1$, and $\lambda = 0.1$.

obtained by NIM are very consistent in comparison with HAM. NIM is easy to understand and easy to implement using computer packages such as Mathematica.

Data Availability

All data and related metadata underlying the findings are reported in the submitted manuscript. For symbolic computations Mathematica 7 has been used.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

We are thankful to Sarhad University of Science and Technology, Khyber Pakhtunkhwa, Pakistan, for contributing in the research charges to this work.

References

[1] R. Alt and J. Vignes, "Validation of results of collocation methods for ODEs with the CADNA library," *Applied Numerical Mathematics*, vol. 21, no. 2, pp. 119–139, 1996.

[2] H. Y. Hu and Z. C. Li, "Collocation methods for Poisson's equation," *Computer Methods in Applied Mechanics and Engineering*, vol. 195, no. 33, pp. 4139–4160, 2006.

[3] V. Thomée, "From finite differences to finite elements. A short history of numerical analysis of partial differential equations," *Journal of Computational and Applied Mathematics*, vol. 128, no. 1-2, pp. 1–54, 2001.

[4] T. Liszka and J. Orkisz, "The finite difference method at arbitrary irregular grids and its application in applied mechanics," *Computers & Structures*, vol. 11, no. 1-2, pp. 83–95, 1980.

[5] S. Roberts, "Finite element method for numerically solving PDEs," in *Instructional Workshop on Analysis and Geometry, Part I (Canberra, 1995)*, vol. 34, pp. 77–90, 1996.

[6] N. Moës, J. Dolbow, and T. Belytschko, "A finite element method for crack growth without remeshing," *International Journal for Numerical Methods in Engineering*, vol. 46, no. 1, pp. 131–150, 1999.

[7] M. Tatari and M. Dehghan, "On the solution of the non-local parabolic partial differential equations via radial basis functions," *Applied Mathematical Modelling*, vol. 33, no. 3, pp. 1729–1738, 2009.

[8] M. Dehghan and A. Shokri, "A numerical method for two-dimensional Schrödinger equation using collocation and radial basis functions," *Computers & Mathematics with Applications*, vol. 54, no. 1, pp. 136–146, 2007.

[9] A. S. Arife, S. Karimi Vanani, and A. Yildirim, "Numerical solution of hirota-satsuma couple Kdv and a coupled MKdv equation by means of homotopy analysis method," *World Applied Sciences Journal*, vol. 13, no. 11, pp. 2271–2276, 2011.

[10] M. Turkyilmazoglu, "Convergence accelerating in the homotopy analysis method: a new approach," *Advances in Applied Mathematics and Mechanics*, vol. 10, no. 4, pp. 925–947, 2018.

[11] M. Turkyilmazoglu, "Approximate analytical solution of the nonlinear system of differential equations having asymptotically stable equilibrium," *Filomat*, vol. 31, no. 9, pp. 2633–2641, 2017.

[12] M. Turkyilmazoglu, "Parametrized Adomian decomposition method with optimum convergence," *ACM Transactions on Modeling and Computer Simulation (TOMACS)*, vol. 27, no. 4, Art. 21, 22 pages, 2017.

[13] M. A. Noor and S. T. Mohyud-Din, "Variational iteration method for solving higher-order nonlinear boundary value problems using He's polynomials," in *Journal of Nonlinear Sciences and Numerical Simulation*, pp. 141–156, 2008.

[14] M. A. Noor and S. T. Mohyud-Din, "Variational iteration method for fifth-order boundary value problems using he's polynomials," *Mathematical Problems in Engineering*, vol. 2008, 11 pages, 2008.

[15] M. A. Noor and S. T. Mohyud-Din, "Modified variational iteration method for heat and wave-like equations," *Acta Applicandae Mathematicae*, vol. 104, no. 3, pp. 257–269, 2008.

[16] M. A. Noor and S. T. Mohyud-Din, "Modified variational iteration method for solving fourth-order boundary value problems," *Applied Mathematics and Computation*, vol. 29, no. 1-2, pp. 81–94, 2009.

[17] M. Turkyilmazoglu, "An optimal variational iteration method," *Applied Mathematics Letters*, vol. 24, no. 5, pp. 762–765, 2011.

[18] V. Daftardar-Gejji and H. Jafari, "An iterative method for solving nonlinear functional equations," *Journal of Mathematical Analysis and Applications*, vol. 316, no. 2, pp. 753–763, 2006.

[19] V. Daftardar-Gejji and S. Bhalekar, "Solving fractional boundary value problems with Dirichlet boundary conditions using a new iterative method," *Computers & Mathematics with Applications*, vol. 59, no. 5, pp. 1801–1809, 2010.

[20] V. Daftardar-Gejji and S. Bhalekar, "Solving fractional diffusion-wave equations using a new iterative method," *Fractional Calculus & Applied Analysis. An International Journal for Theory and Applications*, vol. 11, no. 2, pp. 193–202, 2008.

[21] S. Bhalekar and V. Daftardar-Gejji, "Solving fractional-order logistic equation using a new iterative method," *International Journal of Differential Equations*, vol. 2012, Article ID 975829, 12 pages, 2012.

[22] S. Bhalekar and V. Daftardar-Gejji, "Convergence of the new iterative method," *International Journal of Differential Equations*, vol. 2011, Article ID 989065, 10 pages, 2011.

[23] A. A. Hemeda, "New iterative method: application to nth-order integro-differential equations," *International Mathematical Forum. Journal for Theory and Applications*, vol. 7, no. 45-48, pp. 2317–2332, 2012.

[24] M. Yaseen and M. Samraiz, "The modified new iterative method for solving linear and nonlinear Klein-Gordon equations," *Applied Mathematical Sciences*, vol. 6, no. 57-60, pp. 2979–2987, 2012.

[25] J. Patade and S. Bhalekar, "Approximate analytical solutions of Newell-Whitehead-Segel equation using a new iterative method," *World Journal of Modelling and Simulation*, vol. 11, no. 2, pp. 94–103, 2015.

[26] V. Daftardar-Gejji, Y. Sukale, and S. Bhalekar, "A new predictor-corrector method for fractional differential equations," *Applied Mathematics and Computation*, vol. 244, pp. 158–182, 2014.

[27] K. I. Noor and M. A. Noor, "Iterative methods with fourth-order convergence for nonlinear equations," *Applied Mathematics and Computation*, vol. 189, no. 1, pp. 221–227, 2007.

[28] M. A. Noor, K. I. Noor, and M. Waseem, "A new decomposition technique for solving a system of linear equations," *Journal of the Association of Arab Universities for Basic and Applied Sciences*, vol. 16, pp. 27–33, 2014.

[29] F. A. Shah and M. A. Noor, "Some numerical methods for solving nonlinear equations by using decomposition technique," *Applied Mathematics and Computation*, vol. 251, pp. 378–386, 2015.

[30] M. A. Hafiz and M. S. Bahgat, "Modified of householder iterative method for solving nonlinear systems," *Journal of Mathematical and Computational Science*, vol. 2, no. 5, pp. 1200–1208, 2012.

[31] M. A. Noor, M. Waseem, K. Noor, and M. A. Ali, "New iterative technique for solving nonlinear equations," *Applied Mathematics and Computation*, vol. 265, Article ID 21250, pp. 1115–1125, 2015.

[32] F. A. Shah and M. A. Noor, "Higher order iterative schemes for nonlinear equations using decomposition technique," *Applied Mathematics and Computation*, vol. 266, Article ID 21175, pp. 414–423, 2015.

[33] M. B. Ghori, M. Usman, and S. T. Mohyud-Din, "Numerical studies for solving a predictive microbial growth model using a new iterative method," *International Journal of Modern Biology and Medicine*, vol. 5, no. 1, pp. 33–39, 2014.

[34] D. J. Korteweg and G. de Vries, "XLI. On the change of form of long waves advancing in a rectangular canal, and on a new type of long stationary waves," *Philosophical Magazine*, vol. 39, no. 240, pp. 422–443, 1895.

[35] Y. Wu, X. Geng, X. Hu, and S. Zhu, "A generalized Hirota-Satsuma coupled Korteweg-de Vries equation and Miura transformations," *Physics Letters A*, vol. 255, no. 4-6, pp. 259–264, 1999.

[36] E. G. Fan, "Soliton solutions for a generalized Hirota-Satsuma coupled KdV equation and a coupled MKdV equation," *Physics Letters A*, vol. 282, no. 1-2, pp. 18–22, 2001.

[37] K. R. Raslan, "The decomposition method for a Hirota-Satsuma coupled KdV Equation and a coupled MKdV Equation," *International Journal of Computer Mathematics*, vol. 81, no. 12, pp. 1497–1505, 2004.

[38] Z. Guo-Zhong, Y. Xi-Jun, X. Yun, Z. Jiang, and W. Di, "Approximate analytic solutions for a generalized HirotaSatsuma coupled KdV equation and a coupled MKDV equation," *Chinese Physics B*, vol. 19, no. 8, Article ID 080204, 2010.

[39] F. Kangalgil and F. Ayaz, "Solitary wave solutions for Hirota-Satsuma coupled KDV Equation and coupled mKDV Equation by differential transform method," *Arabian Journal for Science and Engineering. AJSE. Mathematics*, vol. 35, no. 2D, pp. 203–213, 2010.

[40] M. Ghoreishi, A. I. Ismail, and A. Rashid, "The solution of coupled modified KdV system by the homotopy analysis method," *TWMS Journal of Pure and Applied Mathematics*, vol. 3, no. 1, pp. 122–134, 2012.

[41] R. Hirota and J. Satsuma, "Soliton solutions of a coupled Korteweg-de Vries equation," *Physics Letters A*, vol. 85, no. 8-9, pp. 407-408, 1981.