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Research Article

Analytical Solution for One-Dimensional Consolidation of Soil with Exponentially Time-Growing Drainage Boundary under a Ramp Load

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By introducing the exponentially time-growing drainage boundary, this paper investigated the one-dimensional consolidation problem of soil under a ramp load. Firstly, the one-dimensional consolidation equations of soil are established when there is a ramp load acting on the soil surface. Then, the analytical solution of excess pore water pressure and consolidation degree is derived by means of the method of separation of variables and the integral transform technique. The rationality of this solution is also verified by comparing it with other existing analytical solutions. Finally, the consolidation behavior of soil is studied in detail for different interface parameters or loading scheme. The results show that the exponentially time-growing drainage boundary can reflect the phenomenon that the excess pore water pressure at the drainage boundaries dissipates smoothly rather than abruptly from its initial value to the value of zero. By adjusting the values of interface parameters b and c, the presented solution can be degraded to Schiffman's solution, which can compensate for the shortcoming that Terzaghi's drainage boundary can only consider the two extreme cases of fully pervious and impervious boundaries. The significant advantage of the exponentially time-growing drainage boundary is that it can be applied to describe the asymmetric drainage characteristics of the top and bottom drainage surfaces of the actual soil layer by choosing the appropriate interface parameters b and c.

1. Introduction

The consolidation problem of soil has always been one of the key problems of soil mechanics, for it has close relation with the deformation, strength, stability, and seepage of soil mechanics. Since Terzaghi [1] published his consolidation theory in 1925, noteworthy process has been achieved in consolidation theory. In order to accurately describe the soil properties, a considerable amount of scholars has made a great effort to build the governing equation of consolidation problem for soil according to different soil constitutive models, such as Terzaghi-Rendulic diffusion equation [2], three-dimensional Biot's consolidation equation [3], Maxwell model [4], Kelvin model [5, 6], and Merchant model [7]. Considering the significant influence of initial condition on the consolidation process of soil, a large number of investigators have also proposed various solutions for one-dimensional consolidation problem of soil under different loads, such as linear load [8], rectangular load [9], random load [10], cyclic loading [11–13], and ramp loading [13–15].

From the aforementioned literature review, it can be found that the drainage boundaries are normally treated as pervious or impervious, that is, Terzaghi's drainage boundary. Nevertheless, the drainage at the boundaries of consolidating soil is impeded for most engineering applications [16]. With regard to this aspect, Gray [16] first proposed an impeded drainage boundary which was later pursued by Schiffman and Stein [17] to investigate the consolidation problem of soil with variable permeability and compressibility. Following Gray's work, solutions of one-dimensional consolidation have been presented for two- [18] and multilayered soils [19, 20] with impeded drainage boundaries. However, Lee et al. [21] pointed out that the impeded drainage boundary conditions can be regarded as special cases of permeable, yet incompressible, top or bottom soil layer in multilayered system. Recently, Mei et al. [22] have put forward the exponentially time-growing drainage boundary condition (i.e., continuous drainage boundary as Mei's definition), including permeable and impermeable boundary conditions as two extremities, to consistently describe the all drainage boundary conditions of saturated soil. Soon afterwards, studies on consolidation theory with exponentially time-growing drainage boundary condition have been conducted for saturated soils with a single layer [23] and multilayers [24] and unsaturated soil with a single layer [25]. Since the exponentially time-growing drainage boundary can allow the excess pore water pressure to dissipate smoothly rather than abruptly from its initial value given by the initial conditions to the value of zero, it may have board application prospect in engineering.

The objective of this article is to develop a general analytical solution for the one-dimensional consolidation of soil with exponentially time-growing drainage boundary (abbreviated as ETGD boundary for convenience) under a ramp load. Moreover, the detailed solution is obtained for the excess pore water pressure and overall average consolidation degree in terms of excess pore water pressure. Then, a comparison is made between the present solution and Schiffman's solution [8] to verify the present solution. Compared with Conte's solution [13] and Olson's solution [14], it is also worth noting that the proposed solution draws closer to engineering reality for it can account for the variation of drainage boundaries with time during the consolidation process. In addition, the influence of the interface parameters of the ETGD boundary on the consolidation behavior of soil is discussed in detail based on the present solution.

2. Problem Description

The thickness, coefficient of permeability, and coefficient of compressibility of soil are denoted as $H = 2h$, k, and m_v, respectively. h is half the thickness of the soil. As shown in Figure 1, a ramp load $p(t)$ is applied on the top surface of soil layer, and the ultimate load and the loading time of the ramp load are represented as p_0 and t_0, respectively. For a single stage load with constant loading rate, it can be expressed as

$$p(t) = \begin{cases} mt & 0 \le t < t_0 \\ p_0 & t_0 \le t \end{cases} \tag{1}$$

where $m = p_0/t_0$ indicates the loading rate. For a constant ultimate load, the smaller t_0 means the larger loading rate.

According to Terzaghi's consolidation theory, the governing equation of one-dimensional consolidation problem of soil can be expressed as

$$\frac{\partial u}{\partial t} = C_v \frac{\partial^2 u}{\partial z^2} + \frac{\mathrm{d}p(t)}{\mathrm{d}t} \quad (0 \le z \le 2h) \tag{2}$$

where u represents the excess pore water pressure of soil; z is the downward vertical coordinate originated from the top boundary; t is the loading time. $C_v = k/\gamma_w m_v$ indicates the consolidation coefficient of soil layer and γ_w denotes the unit weight of water, respectively.

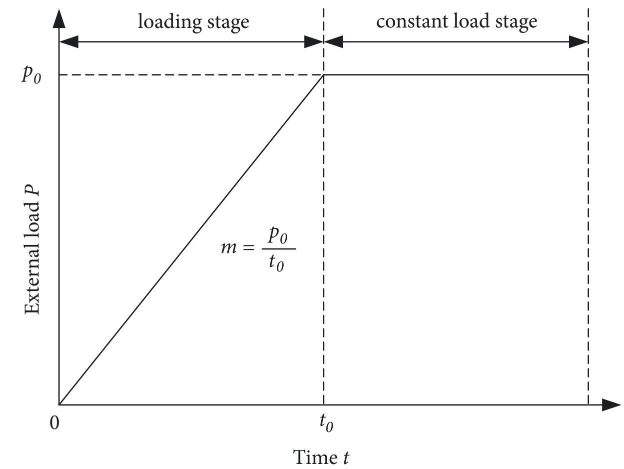

Figure 1: Relation of loading and time.

Then, this work introduces the ETGD boundary as follows [20]:

$$\begin{aligned} u(0,t) &= p(t)\, e^{-bt} \\ u(2h,t) &= p(t)\, e^{-ct} \end{aligned} \tag{3}$$

where b and c are interface parameters which reflect the drainage properties of the top and bottom drainage surfaces of soil layer, respectively. The two parameters can be obtained by experimental simulation or engineering measurement inversion.

The initial boundary is given as

$$u(z,0) = p(0) \tag{4}$$

3. Analytical Solution for This Model

Letting $u = v + (2h - z)p(t)e^{-bt}/2h + zp(t)e^{-ct}/2h$ and substituting it into (2) yield

$$\frac{\partial v}{\partial t} = C_v \frac{\partial^2 v}{\partial z^2} + f(z,t) \tag{5}$$

where

$$\begin{aligned} f(z,t) = {} & \frac{bp(t)(2h-z)e^{-bt}}{2h} + \frac{cp(t)ze^{-ct}}{2h} \\ & + \left(1 - \frac{(2h-z)e^{-bt} + ze^{-ct}}{2h}\right)\frac{\mathrm{d}p(t)}{\mathrm{d}t} \end{aligned} \tag{6}$$

Combining with the expression form of ramp load $p(t)$, $f(z,t)$ can be rewritten as

$$f(z,t) = \begin{cases} f_1(z,t) & 0 \le t < t_0 \\ f_2(z,t) & t_0 \le t \end{cases} \tag{7}$$

where

$$f_1(z,t) = \frac{m(2h-z)(bt-1)e^{-bt}}{2h} + \frac{mz(ct-1)e^{-ct}}{2h} + m \tag{8}$$

$$f_2(z,t) = \frac{bp_0(2h-z)e^{-bt}}{2h} + \frac{cp_0 z e^{-ct}}{2h} \tag{9}$$

Then, the boundary conditions and initial condition can be rewritten as

$$\begin{aligned} v(0,t) &= 0 \\ v(2h,t) &= 0 \end{aligned} \tag{10}$$

$$v(z,0) = 0 \tag{11}$$

The general solution of (5) can be set as

$$v(z,t) = \sum_{n=1}^{\infty} v_n(t)\sin\frac{n\pi z}{2h} \tag{12}$$

Substituting (12) into (5) gives

$$\begin{aligned} &\sum_{n=1}^{\infty} v_n'(t)\sin\frac{n\pi z}{2h} + C_v\sum_{n=1}^{\infty}\left(\frac{n\pi}{2h}\right)^2 v_n(t)\sin\frac{n\pi z}{2h} \\ &\quad = \sum_{n=1}^{\infty} f_n(t)\sin\frac{n\pi z}{2h} \end{aligned} \tag{13}$$

where $f_n(t)$ can be expressed according to Fourier series as

$$\begin{aligned} f_n(t) &= \frac{1}{h}\int_0^{2h} f(z,t)\sin\frac{n\pi z}{2h}\mathrm{d}z \\ &= \begin{cases} f_{n1}(t) & 0 \le t < t_0 \\ f_{n2}(t) & t_0 \le t \end{cases} \end{aligned} \tag{14}$$

where

$$\begin{aligned} f_{n1}(t) &= \frac{2m}{n\pi}\left[(bt-1)e^{-bt} - (-1)^n(ct-1)e^{-ct}\right] \\ &\quad + \frac{2m}{n\pi}\left[1-(-1)^n\right] \end{aligned} \tag{15}$$

$$f_{n2}(t) = \frac{2p_0}{n\pi}\left[be^{-bt} - (-1)^n ce^{-ct}\right] \tag{16}$$

In order to make (13) always true, the following equation can be derived:

$$v_n'(t) + C_v\left(\frac{n\pi}{2h}\right)^2 v_n(t) - f_n(t) = 0 \tag{17}$$

For a single stage load with constant loading rate, it can be determined that $u(z,0) = p(0) = 0$. Moreover, it can also obtain that $v_n(0) = 0$ because $v(z,0) = 0$.

Utilizing the Laplace transform technique to solve (17) yields

$$v_n(t) = \int_0^t f_n(\tau)e^{-D(t-\tau)}\mathrm{d}\tau \tag{18}$$

where

$$D = C_v\left(\frac{n\pi}{2h}\right)^2 \tag{19}$$

3.1. Solution of Loading Stage. For loading stage (i.e., $0 \le t < t_0$), one can derive

$$v_n(t) = \frac{2m}{n\pi}e^{-Dt}\int_0^t f_{n1}(\tau)e^{\tau}\mathrm{d}\tau \tag{20}$$

Substituting (20) into (12) gives

$$\begin{aligned} &v(z,t) \\ &\quad = \sum_{n=1}^{\infty}\frac{2m\left[(bt-1)e^{-bt}+e^{-Dt}\right]}{n\pi(D-b)}\sin\frac{n\pi z}{2h} \\ &\qquad - \sum_{n=1}^{\infty}\frac{2m(-1)^n\left[(ct-1)e^{-ct}+e^{-Dt}\right]}{n\pi(D-c)}\sin\frac{n\pi z}{2h} \\ &\qquad - \sum_{n=1}^{\infty}\frac{2mb\left(e^{-bt}-e^{-Dt}\right)}{n\pi(D-b)^2}\sin\frac{n\pi z}{2h} \\ &\qquad + \sum_{n=1}^{\infty}\frac{2mc(-1)^n\left(e^{-ct}-e^{-Dt}\right)}{n\pi(D-c)^2}\sin\frac{n\pi z}{2h} \\ &\qquad + \sum_{n=1}^{\infty}\frac{2m\left(1-(-1)^n\right)\left(1-e^{-Dt}\right)}{n\pi D}\sin\frac{n\pi z}{2h} \end{aligned} \tag{21}$$

Furthermore, the excess pore water pressure of soil during the loading stage can be derived as

$$\begin{aligned} u &= v + \frac{(2h-z)mte^{-bt}}{2h} + \frac{zmte^{-ct}}{2h} \\ &= \sum_{n=1}^{\infty}\frac{2m\left[(bt-1)e^{-bt}+e^{-Dt}\right]}{n\pi(D-b)}\sin\frac{n\pi z}{2h} \\ &\quad - \sum_{n=1}^{\infty}\frac{2m(-1)^n\left[(ct-1)e^{-ct}+e^{-Dt}\right]}{n\pi(D-c)}\sin\frac{n\pi z}{2h} \\ &\quad - \sum_{n=1}^{\infty}\frac{2mb\left(e^{-bt}-e^{-Dt}\right)}{n\pi(D-b)^2}\sin\frac{n\pi z}{2h} \\ &\quad + \sum_{n=1}^{\infty}\frac{2mc(-1)^n\left(e^{-ct}-e^{-Dt}\right)}{n\pi(D-c)^2}\sin\frac{n\pi z}{2h} \\ &\quad + \sum_{n=1}^{\infty}\frac{2m\left(1-(-1)^n\right)\left(1-e^{-Dt}\right)}{n\pi D}\sin\frac{n\pi z}{2h} \\ &\quad + \frac{(2h-z)mte^{-bt}}{2h} + \frac{zmte^{-ct}}{2h} \end{aligned} \tag{22}$$

3.2. Solution of Constant Load Stage. For constant load stage (i.e., $t_0 \le t$), one can obtain

$$v_n(t) = \frac{2m}{n\pi} e^{-Dt} \left[\int_0^{t_0} f_{n1}(\tau) e^{\tau} d\tau + \int_{t_0}^{t} f_{n2}(\tau) e^{\tau} d\tau \right] = \sum_{n=1}^{\infty} \frac{2p_0 b e^{-bt}}{n\pi (D-b)} \sin \frac{n\pi z}{2h} - \sum_{n=1}^{\infty} \frac{(-1)^n 2p_0 c e^{-ct}}{n\pi (D-c)} \sin \frac{n\pi z}{2h} + \sum_{n=1}^{\infty} \frac{2p_0 A_n e^{-Dt}}{n\pi} \sin \frac{n\pi z}{2h} \quad (23)$$

where

$$A_n = \frac{-e^{(D-b)t_0} + 1}{t_0 (D-b)} - \frac{(-1)^n \left[-e^{(D-c)t_0} + 1 \right]}{t_0 (D-c)} - \frac{b \left(e^{(D-b)t_0} - 1 \right)}{t_0 (D-b)^2} + \frac{c (-1)^n \left(e^{(D-c)t_0} - 1 \right)}{t_0 (D-c)^2} + \frac{\left(1 - (-1)^n\right) \left(e^{Dt_0} - 1 \right)}{t_0 D} \quad (24)$$

Then, the excess pore water pressure of soil during the constant load stage can be obtained as

$$u = v + \frac{p_0 (2h - z) e^{-bt}}{2h} + \frac{p_0 z e^{-ct}}{2h} = \sum_{n=1}^{\infty} \frac{2p_0 b e^{-bt}}{n\pi (D-b)} \sin \frac{n\pi z}{2h} - \sum_{n=1}^{\infty} \frac{(-1)^n 2p_0 c e^{-ct}}{n\pi (D-c)} \sin \frac{n\pi z}{2h} + \sum_{n=1}^{\infty} \frac{2p_0 A_n e^{-Dt}}{n\pi} \sin \frac{n\pi z}{2h} + \frac{p_0 (2h - z) e^{-bt}}{2h} + \frac{p_0 z e^{-ct}}{2h} \quad (25)$$

4. Verification and Parametric Study

4.1. Degeneration of Solution. In order to verify the effectiveness and accuracy of the presented solution, it is needed to degenerate the proposed solution and compare it with existing solutions. Therefore, this section conducts the verification for both loading stage and constant load stage.

4.1.1. Degeneration of Solution at Loading Stage. When the interface parameters b and c approach infinity, (22) can be degenerated as

$$u(z,t) = \sum_{n=1}^{\infty} \frac{2m \left(1 - (-1)^n\right) \left(1 - e^{-Dt}\right)}{n\pi D} \sin \frac{n\pi z}{2h} = \frac{16 p_0}{T_{v0} \pi^3} \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} \left(1 - e^{-n^2 \pi^2 T_v / 4}\right) \sin \frac{n\pi z}{2h} \quad (26)$$

4.1.2. Degeneration of Solution at Constant Load Stage. When the interface parameters b and c tend to infinity, (24) can be reduced as

$$A_n = \frac{\left(1 - (-1)^n\right) \left(e^{Dt_0} - 1 \right)}{t_0 D} \quad (27)$$

Then, (25) can be degenerated as

$$u = \sum_{n=1}^{\infty} \frac{2p_0 A_n e^{-Dt}}{n\pi} \sin \frac{n\pi z}{2h} = \frac{16 p_0}{T_{v0} \pi^3} \cdot \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} \left(1 - e^{-n^2 \pi^2 T_{v0} / 4}\right) e^{-n^2 \pi^2 (T_v - T_{v0})/4} \sin \frac{n\pi z}{2h} \quad (28)$$

Apparently, (26) and (28) are consistent with the corresponding equations proposed by Schiffman utilizing Terzaghi's drainage boundary [8].

Then, a relevant computer program is also developed and applied to compare the presented solution with Schiffman's solution [8]. For the degeneration of the ETGD boundary, the interface parameters, both b and c are set as sufficiently small values (i.e., $b = c = 0.001\text{d}^{-1}$) and large enough values (i.e., $b = c = 1000\text{d}^{-1}$) which can be regarded as the impervious boundary and pervious boundary, respectively. The unit weight, elastic modulus, Poisson's ratio, and permeability coefficient of soil are $18.1\text{kN} \cdot \text{m}^{-3}$, 10 MPa, 0.3, and $7.4 \times 10^{-3}\text{m} \cdot \text{d}^{-1}$, respectively. Lastly, the time factor, which is expressed as $T_v = C_v t / H^2$, is also introduced for convenient comparison, and T_{v0} is the corresponding time factor with respect to the final loading time.

As shown in Figure 2, when the interface parameters b and c and the time factor T_v have smaller values, u/p_0 tends to 0.9 because the drainage velocity is small and the drainage time is very short. For the curve obtained by Schiffman's solution, there are mutations at $z = 0$ and $z = 2h$ which are different with the actual situations. When the interface parameters b and c and the time factor T_v have larger values, both the values of u/p_0 obtained by the presented solution and Schiffman's solution are consistent and in symmetrical distribution along depth. The values of u/p_0 tend to zero because the drainage velocity is large enough and the drainage time is long enough. The reason for that the values of u/p_0 are not zero is that the consolidation process is at loading stage and the increment of applied loading should be first borne by the excess pore water pressure. The results show that the solution obtained by Terzaghi's drainage boundary can not be reduced to the solution degenerated from the presented solution, and the curves obtained by the solution based on Terzaghi's drainage boundary are not smooth and have mutations at the top and bottom drainage surfaces. In other words, whatever the value of time factor is, the excess pore water pressure based on Terzaghi's drainage boundary at $z = 0$ and $z = 2h$ is always zero. Actually, the pore water

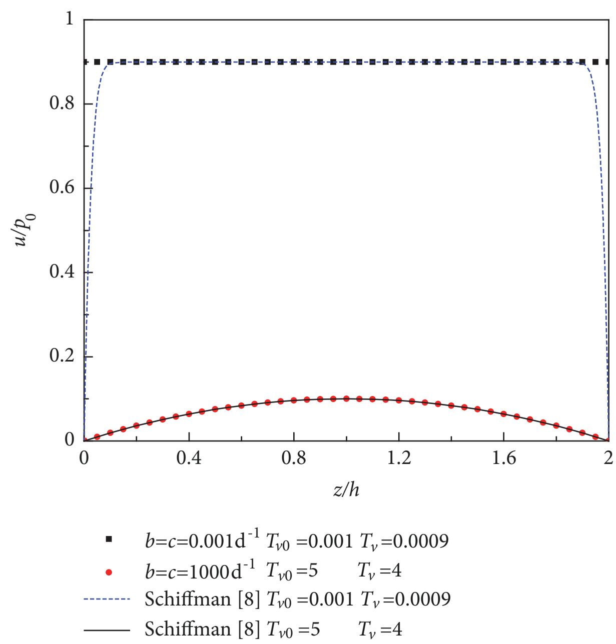

FIGURE 2: Comparison of presented solution with Schiffman's solution.

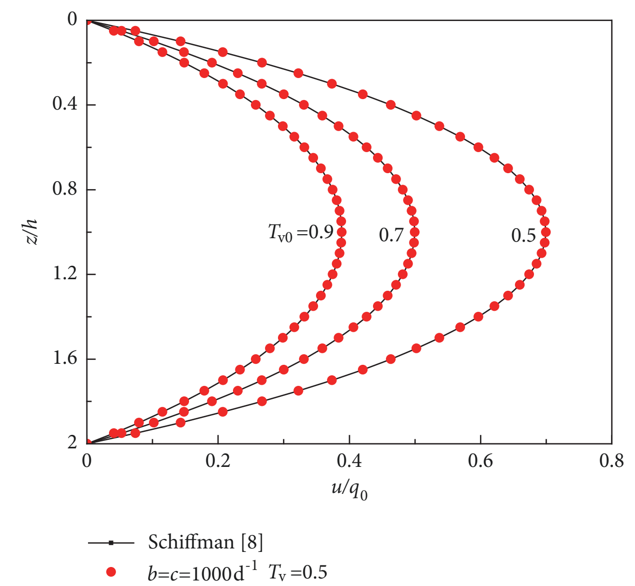

FIGURE 3: Influence of T_{v0} on excess pore water pressure at loading stage.

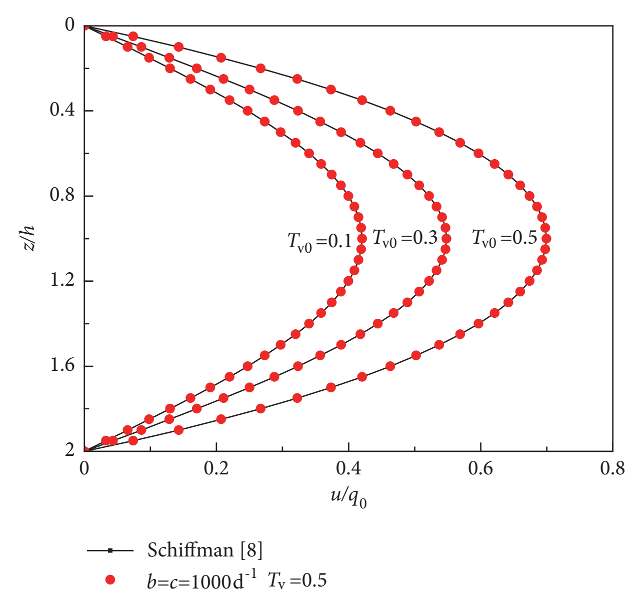

FIGURE 4: Influence of T_{v0} on excess pore water pressure at constant loading stage.

pressure can not be completely dissipated when the drainage time is zero or very small, and the exponentially time-growing drainage boundary can reflect this phenomenon that the excess pore water pressure at the drainage boundary dissipates smoothly rather than abruptly from its initial value given by the initial conditions to the value of zero.

Moreover, further comparison of the proposed solution with Schiffman's solution is conducted by analyzing the influence of T_{v0} on excess pore water pressure for both loading stage and constant load stage. The smaller T_{v0} means the shorter loading time from loading stage to constant load stage and also implies the faster loading rate. As can be observed in Figure 3, at the loading stage ($T_v < T_{v0}$), the excess pore water pressure increases with the increasing loading rate for the external load increases with the increase of loading rate at a given time. However, at the constant load stage ($T_v > T_{v0}$) as shown in Figure 4, the excess pore water pressure at a given time decreases as the loading rate increases, for there is a longer time to dissipate the excess pore water pressure if the loading process is completed within a short period of time. In addition, it can also be seen from the diagrams that when the interface parameters b and c equal to 1000d^{-1}, the pore pressure curve obtained by the ETGD boundary coincides with the pore pressure curve obtained by Schiffman's solution. This indicates that the solution obtained by the ETGD boundary can be reduced to Schiffman's solution when b and c value are large enough, which further verifies the correctness of the present solution.

4.2. Influence of Number of Term in Series on Calculated Results. Figure 5 indicates the influence of number of terms in series on calculated results. For the parameters of $b = c = 1\text{d}^{-1}, T_{v0} = 1, z = h$, it can be seen that the excess pore water pressure of loading stage is larger than that of constant load stage. The excess pore water pressure increases as the time factor increases during the loading stage. But for the constant load stage, the excess pore water pressure decreases with the increase of time factor because the pore water pressure dissipates gradually. When the time factor has smaller value, it needs more number of terms in series to meet the convergence of solution. However, if the time factor is large enough, the fluctuation of the curves is small and the convergence of solution can be satisfied by only taking two or three terms. Therefore, unless otherwise specified, the

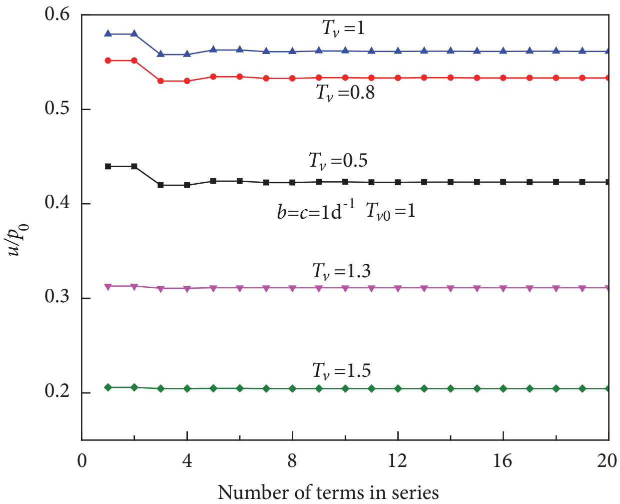

Figure 5: Influence of number of terms in series on calculated results.

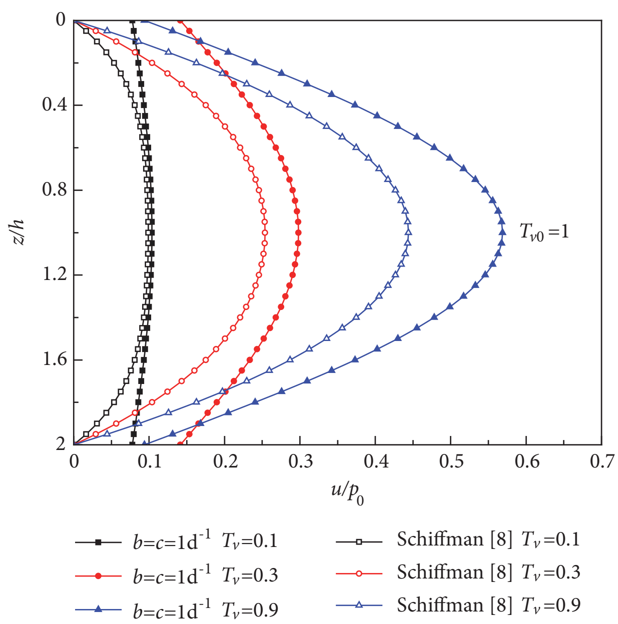

Figure 6: The excess pore water pressure distribution curves at loading stage while $b=c$.

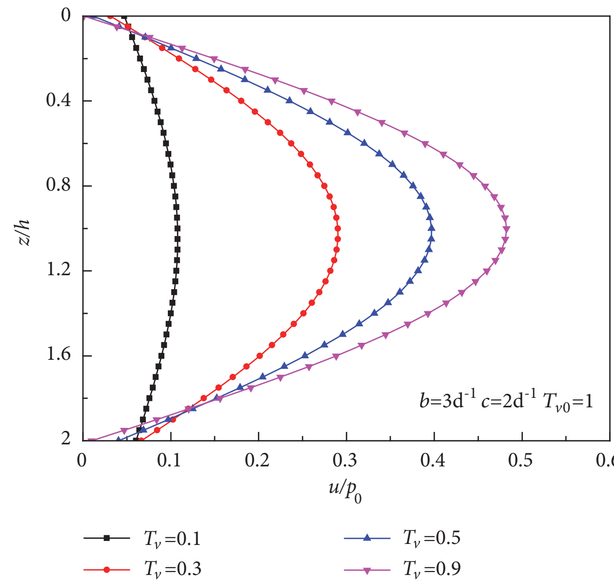

Figure 7: The excess pore water pressure distribution curves at loading stage while $b \neq c$.

number of terms in series is set as five terms for efficient calculation in the following analysis.

4.3. Influence of Interface Parameters b and c on the Excess Pore Water Pressure of Soil. Figure 6 illustrates the excess pore water pressure distribution curves at loading stage while $b=c$. When b is equal to c, the top and bottom drainage surfaces of soil exhibit the same permeability which results in the fact that the excess pore water pressure curves are in symmetrical distribution about $z = h$. With the gradual increase of time factor T_v, the excess pore water pressures at the drainage surfaces and interior of soil gradually dissipate, instead of that the excess pore water pressures obtained by Schiffman's solution are always zero at the drainage surfaces. It can be also observed from Figure 6 that the excess pore water pressure obtained by the presented solution will increase first and then decrease at the drainage surfaces. The reason for the abovementioned phenomena may be that at the beginning the excess pore water pressure at the drainage surfaces increases with the increase of the applied loading; after that the rate of dissipation becomes greater than the rate of loading increment. It is also worth noting that the solution based on Terzaghi's drainage boundary can not reflect the above-mentioned phenomena.

As depicted in Figure 7, when b is not equal to c, for example, $b = 3d^{-1}, c = 2d^{-1}$, the upper part of the excess pore water pressure distribution curves at loading stage is obviously steeper than the lower part which means that the drainage velocity of the upper part of the soil is larger than that of the lower part of soil. Therefore, it can also be understood that the greater the value of the interface parameter, the larger the drainage velocity of the corresponding haft part of the soil. Terzaghi's drainage boundary can not consider the difference of the drainage capacity between the upper and lower parts of soil layer, for the excess pore water pressure distribution curves at loading stage based on Terzaghi's drainage boundary are always symmetrical about $z = h$ [8]. It can also be seen from Figure 7 that the significant advantage of the ETGD boundary is that it can be applied to approximately model the asymmetric drainage characteristics of the top and bottom drainage surfaces of actual soil layer by adjusting the interface parameters b and c.

As demonstrated in Figure 8, when b is equal to c, the top and bottom drainage surfaces of soil also exhibit the same permeability for the excess pore water pressure curves at constant load stage also distribute symmetrically about $z = h$. It is also found that the excess pore water

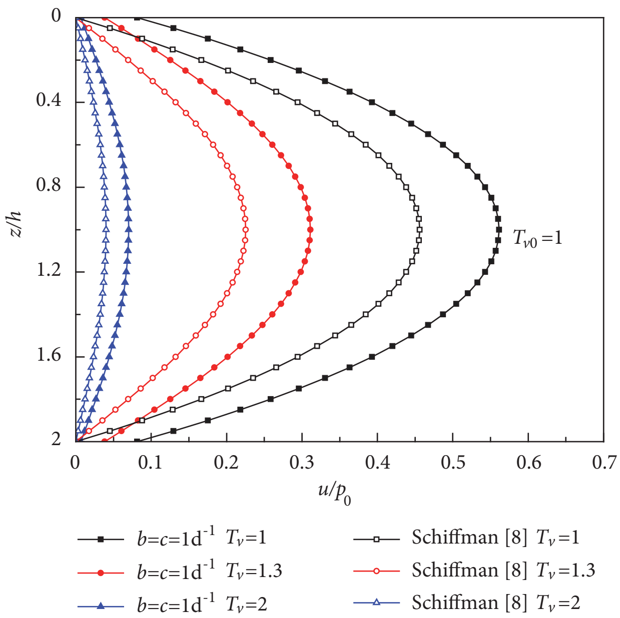

Figure 8: The excess pore water pressure distribution curves at constant load stage while $b=c$.

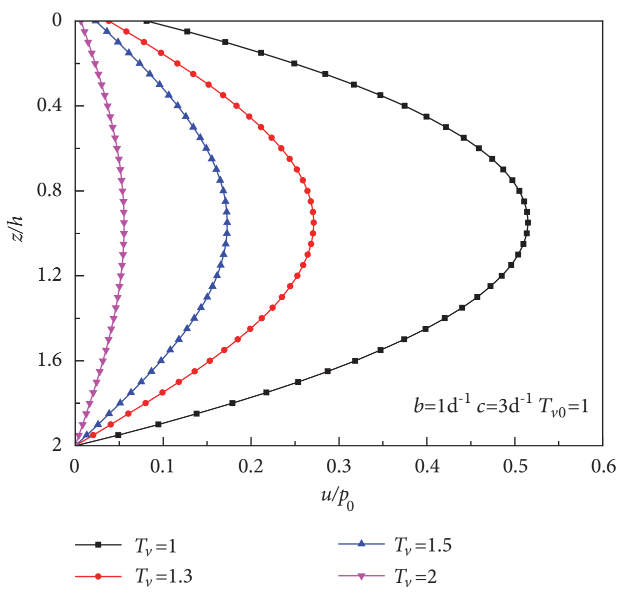

Figure 9: The excess pore water pressure distribution curves at constant load stage while $b \neq c$.

pressure at the drainage surfaces and interior of soil gradually dissipates as the time factor T_v increases, but the excess pore water pressures at constant load stage obtained by Schiffman's solution are also zero at the drainage surfaces.

As shown in Figure 9, when b is not equal to c, for example $b = 1\text{d}^{-1}, c = 3\text{d}^{-1}$, the lower part of the excess pore water pressure distribution curves at constant load stage is obviously steeper than the upper part which implies that the drainage velocity of the lower part of the soil is larger than that of the upper part of the soil. Meanwhile, the solution

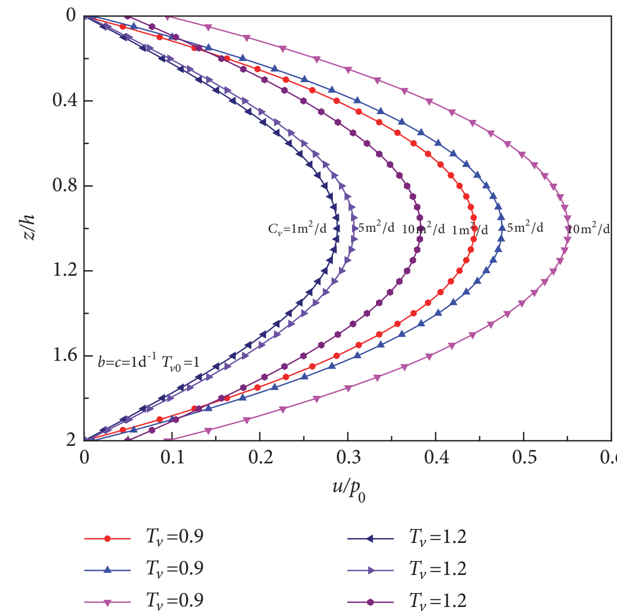

Figure 10: Influence of consolidation coefficient on excess pore water pressure distribution curves.

based on Terzaghi's single-drainage condition can not be extended to the soil layer with the upper and bottom surfaces having different drainage capacity, for the excess pore water pressure distribution curves at constant load stage based on Terzaghi's drainage boundary are always symmetrical about $z = h$.

4.4. Influence of Consolidation Coefficient on the Excess Pore Water Pressure of Soil. Figure 10 shows the influence of consolidation coefficient on the excess pore water pressure of soil. For the loading stage, for example, $T_{v0} = 1, T_v = 0.9$, the excess pore water pressure of soil decreases as the consolidation coefficient decreases. For the constant load stage, for example, $T_{v0} = 1, T_v = 1.2$, the excess pore water pressure of soil also decreases with the decrease of consolidation coefficient. The reason for these phenomena is that, for the same time factor T_v, the smaller consolidation coefficient means the longer consolidation time, and the longer consolidation time will lead to more sufficient dissipation of the excess pore water pressure. It is also worth noting that the smaller consolidation coefficient means the weaker permeability of soil which results in smaller drainage velocity of soil. Therefore, the results show that the consolidation time has more significant influence on the excess pore water pressure than consolidation coefficient. As can be seen from Schiffman's results [8], the change of consolidation coefficient for constant time factor has very weak impact on the excess pore water pressure which means that Terzaghi's drainage boundary can not completely reflect the actual mechanism of consolidation process of soil influenced by time. It can also be observed from Figure 10 that the degree of the influence of consolidation coefficient on excess pore water pressure decreases with the decrease of consolidation coefficient.

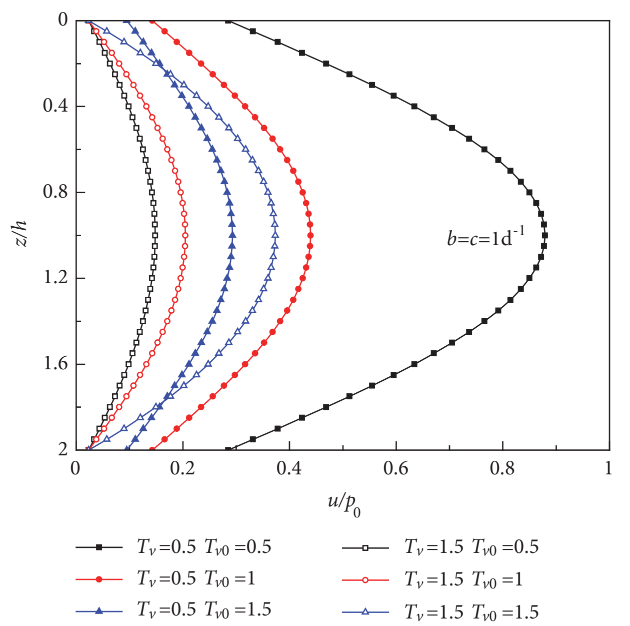

Figure 11: Influence of loading rate on excess pore water pressure distribution curves.

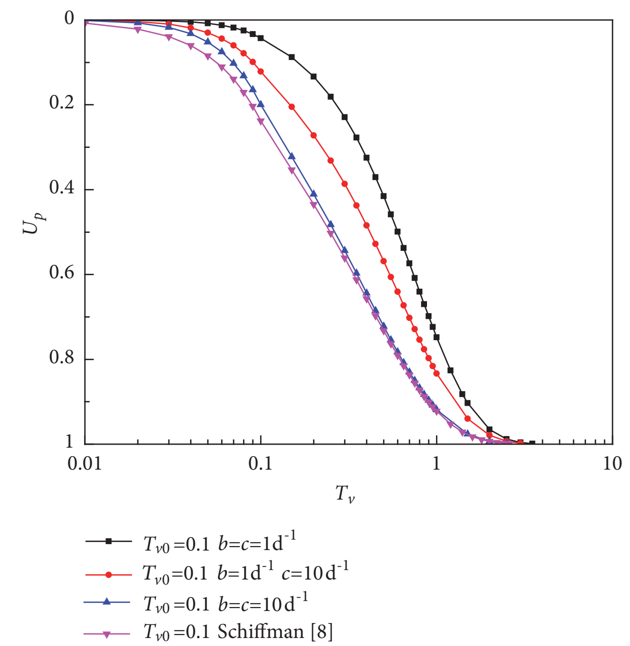

Figure 12: Influence of interface parameters b and c on the consolidation degree of soil.

4.5. Influence of Loading Rate on the Excess Pore Water Pressure of Soil. Figure 11 indicates the influence of loading rate on excess pore water pressure when the other parameters of soil remain constant. As can be observed in Figure 11, the excess pore water pressure increases with the increase of loading rate because the excess pore water pressure can not be sufficiently dissipated within the shorter consolidation time when the loading rate is too fast. For the constant load stage, the excess pore water pressure under the same time factor decreases as the loading rate increases, for there is a longer time to dissipate the excess pore water pressure if the loading process is completed within a short period of time.

5. Analysis of Consolidation Degree

According to Soil Mechanics, the overall average consolidation degree in terms of excess pore water pressure can be expressed as

$$U_P = \frac{\int_0^{2h} (p(t) - u)\,\mathrm{d}z}{2hp_0} \tag{29}$$

Then, by substituting (22) into (29), the overall average consolidation degree in terms of excess pore water pressure at loading stage can be obtained as

$$\begin{aligned} U_P &= \frac{\int_0^{2h} (p(t) - u)\,\mathrm{d}z}{2hp_0} = \frac{p(t)}{p_0} - \frac{\int_0^{2h} u\,\mathrm{d}z}{2hp_0} \\ &= \frac{T_v}{T_{v0}} - \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)\left[(bt-1)e^{-bt} + e^{-Dt}\right]}{(n\pi)^2 (D-b)\, t_0} \\ &\quad - \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)\left[(ct-1)e^{-ct} + e^{-Dt}\right]}{(n\pi)^2 (D-c)\, t_0} \\ &\quad + \sum_{n=1}^{\infty} \frac{2b(1-(-1)^n)\left(e^{-bt} - e^{-Dt}\right)}{(n\pi)^2 (D-b)^2\, t_0} \\ &\quad + \sum_{n=1}^{\infty} \frac{2c(1-(-1)^n)\left(e^{-ct} - e^{-Dt}\right)}{(n\pi)^2 (D-c)^2\, t_0} \\ &\quad - \sum_{n=1}^{\infty} \frac{4(1-(-1)^n)\left(1 - e^{-Dt}\right)}{(n\pi)^2 D t_0} - \frac{te^{-bt}}{2t_0} - \frac{te^{-ct}}{2t_0} \end{aligned} \tag{30}$$

Furthermore, by substituting (25) into (29), the overall average consolidation degree defined by excess pore water pressure at constant load stage can be derived as

$$\begin{aligned} U_p &= \frac{\int_0^{2h} (p_0 - u)\,\mathrm{d}z}{2hp_0} = 1 - \frac{\int_0^{2h} u\,\mathrm{d}z}{2hp_0} \\ &= 1 - \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)\, be^{-bt}}{(n\pi)^2 (D-b)} \\ &\quad - \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)\, ce^{-ct}}{(n\pi)^2 (D-c)} \\ &\quad - \sum_{n=1}^{\infty} \frac{2(1-(-1)^n)\, A_n e^{-Dt}}{(n\pi)^2} - \frac{e^{-bt}}{2} - \frac{e^{-ct}}{2} \end{aligned} \tag{31}$$

Figure 12 depicts the influence of interface parameters b and c on the consolidation degree of soil. The consolidation degree obtained by the presented solution will approach that calculated by Schiffman's solution if the interface parameters b and c are large enough, for the boundary surfaces with increasing interface parameters will tend to be pervious boundary. In contrast, if the interface parameters b and c are becoming smaller, the consolidation degrees obtained by the proposed solution will gradually deviate from that calculated by Schiffman's solution. It can also be found that the difference between the consolidation degree obtained by the proposed solution and that calculated by Schiffman's solution increases with the decrease of time factor. It is also worth noting from Figure 12 that the early consolidation process of Terzaghi's drainage boundary is faster than that of the ETGD boundary, but the late consolidation process of the ETGD boundary is faster than that of Terzaghi's drainage boundary. Furthermore, if the interface parameters b and c are large enough, the consolidation curves obtained by the ETGD boundary will essentially coincide with those obtained by Terzaghi's drainage boundary at the late consolidation process. The results show that, for practical engineering, even though the early consolidation process of the soil with the boundary surfaces designed according to the ETGD boundary is slow, it can make more sufficient dissipation of the excess pore water pressure of the whole soil layer, and it will not lead to the slow drainage of the lower soil layer or even the discharge of water after the consolidation of the upper soil layer. At the same time, the ultimate consolidation time based on the ETGD boundary is almost the same as that based on Terzaghi's drainage boundary. When utilizing the ETGD boundary to design the actual drainage boundaries, there is no need to deal with the drainage surfaces as fully pervious boundaries. It means that the overall economy of the design based on the ETGD boundary is clearly superior to that based on Terzaghi's drainage boundary.

As shown in Figure 13, the smaller initial time factor T_{v0} implies the faster loading process of ramp load. It can be seen that the faster the loading process of ramp load, the faster the consolidation process. When the loading rate has larger value, the early consolidation process is faster but the late consolidation process is slower. In contrast, when the loading rate has slower value, the early consolidation process is slower but the late consolidation process is faster. For the ETGD boundary, when the consolidation degree is equal to 80%, the time factor is about 1.1, 1.3, and 8.8 when T_{v0} is equal to 0.1, 1, and 10, respectively. For Terzaghi's drainage boundary, when consolidation degree is equal to 80%, the time factor is about 0.63, 1.2, and 8.4 when T_{v0} is equal to 0.1, 1, and 10, respectively. The results imply that the ultimate consolidation time based on the ETGD boundary is almost the same as that based on Terzaghi's drainage boundary when the loading rate is slow enough.

6. Conclusions

In this article, an analytical solution for the one-dimensional consolidation of soil layer under a ramp load is derived by adopting the ETGD boundary. The proposed analytical solution is also verified by comparing it with Schiffman's solution [8]. Then, selective results are presented to illustrate

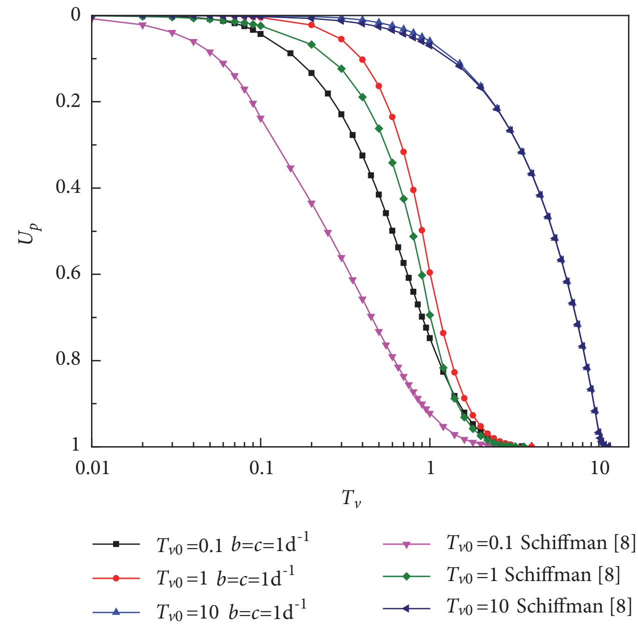

Figure 13: Influence of loading rate of ramp load on the consolidation degree of soil.

the consolidation behavior of soil with ETGD boundary. Schiffman's solution can be regarded as a special case of the present solution, and the excess pore water pressure and consolidation degree calculated by the presented solution will approach those calculated by Schiffman's solution if the interface parameters b and c are large enough. The important advantage of the ETGD boundary is that it can be utilized to describe the asymmetric drainage characteristics of the top and bottom drainage surfaces of the actual soil layer by choosing the appropriate interface parameters b and c. Significantly, the present solution is just for homogeneous soil, and the consolidation problem of multilayered soil will be investigated by the authors' team in near future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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