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Research Article

A New Mathematical Method to Study the Singularity of 3-RSR Multimode Mobile Parallel Mechanism

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In the process of parallel mechanism design, it is difficult to avoid the singularity, especially in the mobile parallel mechanism. Therefore, a new mathematical method to study the singularity of multimode mobile parallel mechanism is proposed. In this paper, the singularity of 3-RSR parallel mechanism (PM) is analyzed by using reciprocal screw methods and linear geometry theory from two aspects of fixed mode and all-attitude multiple motion modes. Specifically, the complete Jacobian matrix of the PM is obtained by using the screw theory, and the reciprocal screw of each branch is expressed with algebraic method and geometric drawing method. Furthermore, the singularity of the PM can be obtained by analyzing the reciprocal screw correlation and using the spatial linear geometry theory. Finally, we analyze the singular configuration of the PM under various modes, which provide theoretical guidance for the gait planning of the multimode mobile PM and will be useful for the selection of mechanism drive and time-sharing control.

1. Introduction

Parallel mechanisms with high load bearing capacity and precision have been widely used in processing and manufacturing, testing, logistics, and many other applications [1]. Recently, the parallel mechanisms have been used in mobile applications and developed into the 'mobile parallel mechanism', such as the multiple branches of the parallel mechanism which are represented directly as the legs of the mobile mechanism or the mobile robot [2, 3]; for example, these mechanisms are shown in [4–6]. The other is the 'all-attitude mode mobile parallel mechanism' which is represented by Yao [7, 8]. However, it is difficult to avoid singularities due to the structural characteristics of parallel mechanisms. In the process of mechanism design, many scholars have proposed a variety of methods to study singular configurations in [9–17] and tried to avoid singular configurations [18], but most of these methods use the calculation method to determine whether the value of the Jacobian matrix determinant is zero, or whether the Jacobian matrix is downgraded to judge whether the mechanism is singular. For the more complex Jacobian matrix, these methods can be used to study the singularity, but the calculation is cumbersome and the workload is large.

Therefore, we proposed a new mathematical method to study the singularity of parallel mechanism. In the reference of [19], the singularity of a 3-RSR in the fixed mode is only analyzed by screw theory. Different motion modes of 3-RSR mechanism with all-attitude multiple motion modes movement are proposed by Tian [20]. However, the driving control choices and avoidance of singularities in realizing these motion modes are unclear. This paper analyzes the singularity of 3-RSR multimode mobile parallel mechanism by means of the reciprocal screw theory and the spatial geometric theory from two points of view: the fixed mode and the all-attitude multiple motion modes, and the driving selection and control of mechanism in the moving mode are further studied. In the following, Section 3 uses algebraic methods to deduce the complete screw Jacobian matrix of the PM, and Section 4 analyzes the linear correlations of the constrained reciprocal and driving reciprocal screws and discusses the singularities of the PM under five different

configurations. In Section 5, on the basis of the theoretical methods proposed in Sections 3 and 4, the driving selection and the avoidance and utilization of singularities of 3-RSR parallel mechanism in realizing all-attitude multiple motion modes are analyzed. Compared with the previous analytical approaches, this method offers an intuitive and simple way to calculate the singularity. And more important, we established the relationship between the topology of a parallel mechanism and singular configurations. Through proper topology optimization, the singularity can be avoided.

2. Introduction to Reciprocal Screw Theory

A unit screw can be defined as a 6×1 matrix form [21]:

$$\widehat{\$}_i = \begin{pmatrix} s_i \\ r_i \times s_i + \lambda s_i \end{pmatrix} \quad (1)$$

where s_i is the unit vector representing the direction of the screw axis and r_i is the position vector of any points on the screw axis in terms of a reference coordinate system. The pitch of a screw is λ, and $r_i \times s_i$ is defined as the motion along the spiral direction.

For revolute joint, $\lambda = 0$. The screw form can be expressed as

$$\widehat{\$}_i = \begin{pmatrix} s_i & r_i \times s_i \end{pmatrix}^T \quad (2)$$

For prismatic joint, $\lambda = \infty$. The screw form can be expressed as

$$\widehat{\$}_i = \begin{pmatrix} 0 & s_i \end{pmatrix}^T \quad (3)$$

When the two screws are opposite to each other, the reciprocal product of two screws is zero and can be given by

$$\widehat{\$}_r \circ \widehat{\$}_i = 0 \quad (4)$$

The following conditions must be satisfied:

$$\widehat{\$}_r^T \cdot \widehat{\$}_i = 0 \quad (5)$$

The indices range is i=1,2,3,4,5,6-n, r=1,2,....n.

The transposition of a screw can be defined as

$$\widehat{\$}_r^T = \begin{bmatrix} s_{r4} & s_{r5} & s_{r6} & s_{r1} & s_{r2} & s_{r3} \end{bmatrix} \quad (6)$$

Equation (5) can be expressed as

$$\widehat{\$}_r^T \circ \widehat{\$}_i = s_{r4}s_1 + s_{r5}s_2 + s_{r6}s_3 + s_{r1}s_4 + s_{r2}s_5 + s_{r3}s_6 \quad (7)$$

where s_{ri} (i=1~6) is the *ith* component of the *rth* unit screw, s_i is (i=1...6), the reciprocal screw form of s_r.

3. Mechanism Description and Mobility Analysis

3.1. The Configuration of the Mechanism. The parallel mechanism is composed of two scaling platforms and three RSR chains, as shown in Figure 1. In this mechanism, the chains are symmetrical about the central plane, and the upper and the lower platform are composed of three 'V'-shaped rods through the prismatic joints to connect. All the revolute joints in each chain are always parallel with each other. When the mechanism is in the fixed mode, the lower platform becomes the base platform, and the upper platform becomes the moving platform. For the parallel mechanism with multiple motion modes, the upper and lower platforms are in contact with the ground alternately to form an 'all-attitude multiple modes mobile parallel mechanism'.

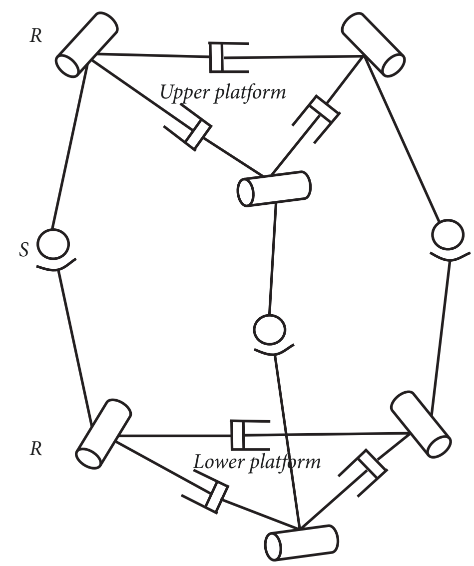


Figure 1: The 3-RSR parallel mechanism diagram.

3.2. Degree of Freedom Analysis in the Fixed Mode. The proposed mechanism is a novel 5-DoF parallel mechanism that is based on 3-RSR architecture. When the mechanism is in the fixed mode, the P of upper and lower platforms is locked, and the 3-RSR parallel mechanism is an underconstrained mechanism [22]. The DOF can be calculated by the following formula:

$$M = d(n - g - 1) + \sum_{i=1}^{g} f_i + \upsilon - \xi \quad (8)$$

where M is the mobility of the mechanism, d represents the order of the mechanism, n is the number of links, g is the number of kinematic pairs, f_i is the freedom of the *ith* pair, υ represents the parallel redundancy constraint, and ξ is the passive DOF, respectively. By (8), the DOF of the mechanism can be obtained and equal to 3.

The prismatic joints of the upper and lower platforms are synchronously scaled in the moving process, so that the total degree of freedom of the upper and lower platforms is 2. Thus, the DOF of the mechanism is 5.

4. Complete Screw Jacobian Matrix of 3-RSR Parallel Mechanism in the Fixed Mode

The singularity of the parallel mechanism is usually determined by solving the Jacobian matrix of the mechanism, which reflects the mapping relationship between the input and output of the mechanism. The complete Jacobian matrix consists of the constrained submatrix that reflects the force applied on each branch, and the driving submatrix that reflects the mapping between the linear velocity and angular velocity of the moving platform and the velocity of the driving joint. Therefore, the singularity of the mechanism can be determined by judging whether the Jacobian matrix of the mechanism is reduced in order.

In this section, the kinematic screw of the 3-RSR parallel mechanism is established by using screw theory. As shown in Figure 2, the spherical joint G_i can be equivalent to three intersecting noncoplanar revolute joints (axis $S_{2,i}$, $S_{3,i}$, $S_{4,i}$) and the axes of revolute joint at P_i point and Q_i point are represented by $S_{1,i}$ and $S_{5,i}$, respectively. Then each branch chain is composed of five single degrees of freedom revolute joints. The instantaneous kinematic screw of the moving platform $\$_c$ can be represented as a linear combination of five instantaneous kinematic screws.

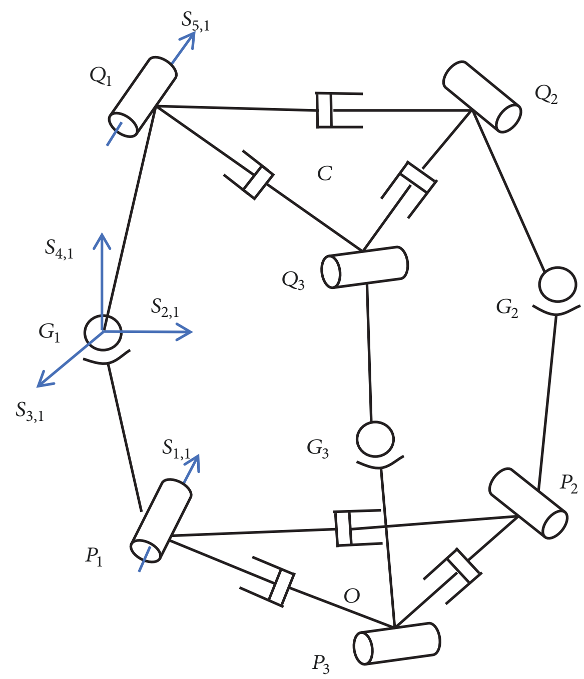


FIGURE 2: Kinematic screw of the 3-RSR parallel mechanism.

$$\$_c = \dot{\beta}_{1,i}\,\$_{1,i} + \dot{\beta}_{2,i}\,\$_{2,i} + \dot{\beta}_{3,i}\,\$_{3,i} + \dot{\beta}_{4,i}\,\$_{4,i} + \dot{\beta}_{5,i}\,\$_{5,i} \quad (i = 1 \sim 3) \tag{9}$$

where $\dot{\beta}_{j,i}$ $(j = 1 \sim 5,\ i = 1 \sim 3)$ represents the angular velocity of the *jth* revolute point of the *ith* branch chain, $\$_{j,i}$ $(i = 1 \sim 3,\ j = 1 \sim 5)$ represents a unit screws, $\$_C = [w_c; v_c]$, w_c represents the angular velocity of the moving platform relative to the base coordinate system, and v_c represents the linear velocity of the moving platform relative to the base coordinate system.

The screw of each branch chain can be expressed as

$$\begin{aligned}
\$_{1,i} &= \left[s_{1,i}; \overline{CP_i} \times s_{1,i}\right]\\
\$_{2,i} &= \left[s_{2,i}; \overline{CG_i} \times s_{2,i}\right]\\
\$_{3,i} &= \left[s_{3,i}; \overline{CG_i} \times s_{3,i}\right]\\
\$_{4,i} &= \left[s_{4,i}; \overline{CG_i} \times s_{4,i}\right]\\
\$_{5,i} &= \left[s_{5,i}; \overline{CQ_i} \times s_{5,i}\right]
\end{aligned} \tag{10}$$

$\$_{r,1,j}$ $(i = 1 \sim 3)$ is used to represent a reciprocal screw corresponding to the five-screw system $\$_{1,i}$, $\$_{2,i}$, $\$_{3,i}$, $\$_{4,i}$, $\$_{5,i}$ in the *ith* branch, and the dimension of the reciprocal screw is one. The reciprocal screw of each branch is obtained by finding the intersection of the two line vectors. In the following, the reciprocal screw of each branch can be obtained by the method of using the two-line vector intersection.

4.1. Constrained Submatrices. According to the principle that the two coplanar line vectors are reciprocal screw [20], the reciprocal screw can be obtained by finding the intersection point of the two line vectors. Thus, the specific method is to find a line vector passing through the spherical joint G_i and let this line vector intersect with the vector $\$_{1,i}$ of P_i point (the direction is $Unit(\overline{P_{i+2}P_{i+1}})$) and the vector $\$_{5,i}$ of Q_i point (the direction is $Unit(\overline{Q_{i+2}Q_{i+1}})$). According to reference [19], it can be seen that the line vector passing through the spherical joint G_i is $\overline{G_iD_{Di}}$.

Therefore, the reciprocal screw $\$_{r,1,i}$ can be derived as

$$\$_{r,1,i} = \left[Unit\left(\overline{G_iD_{Di}}\right); \overline{C_iG_i} \times Unit\left(\overline{G_iD_{Di}}\right)\right] \tag{11}$$

$\$_{r,1,i}$ is a line vector, the reciprocal product of $\$_{r,1,i}$ and (9) can be calculated and written as the following formula:

$$\$_{r,1,i} \circ \$_c = 0 \quad (i = 1 \sim 3) \tag{12}$$

According to reciprocal screw theory, (12) produces an equation for each branch chain, written as the form of a matrix:

$$J_A \cdot \$_C = 0 \tag{13}$$

where

$$J_A = \begin{bmatrix}
\left(\overline{CG_1} \times Unit\left(\overline{G_1D_{D1}}\right)\right)^T & Unit\left(\overline{G_1D_{D1}}\right)^T\\
\left(\overline{CG_2} \times Unit\left(\overline{G_2D_{D2}}\right)\right)^T & Unit\left(\overline{G_2D_{D2}}\right)^T\\
\left(\overline{CG_3} \times Unit\left(\overline{G_3D_{D3}}\right)\right)^T & Unit\left(\overline{G_3D_{D3}}\right)^T
\end{bmatrix} \tag{14}$$

The matrix J_A is a 3×6 constrained Jacobian matrices. Each row of J_A represents a unit constrained force screw applied by all joints of a branch chain.

4.2. Driving Submatrix. When the driving joint of each chain is locked, each branch chain contains only four single-degree-of-freedom revolute joint, the dimension of the reciprocal screw system is 2, and the $\$_{r,1,i}$ is a subset of the reciprocal screw system. The next step is to solve another reciprocal screw $\$_{r,2,i}$ ($i = 1 \sim 3$), which is called the driving force screw. The reciprocal product of $\$_{r,2,i}$ and $\$_{2,i}, \$_{3,i}, \$_{4,i}, \$_{5,i}$ is zero. However, the reciprocal product of $\$_{r,2,i}$ and $\$_{1,i}$ cannot be zero. Furthermore, the screw must pass the points G_i and Q_i; it can be derived as

$$\$_{r,2,i} = \left[Unit\left(\overline{G_iQ_i}\right); \overline{CG_i} \times Unit\left(\overline{G_iQ_i}\right)\right] \tag{15}$$

Equation (15) is a line vector, the reciprocal product of $\$_{r,2,i}$ and (9) can be calculated and written as the following formula:

$$\$_{r,2,i} \circ \$_c = \dot{\beta}_{1,i}\ \$_{r,2,i} \circ \$_{1,i} \tag{16}$$

Equation (16) produces an equation for each branch chain, written as the form of a matrix:

$$J_x \$_C = J_q \bullet \dot{q} \tag{17}$$

$$J_x = \begin{bmatrix} \left(\overline{CG_1} \times Unit\left(\overline{G_1Q_1}\right)\right)^T & Unit\left(\overline{G_1Q_1}\right)^T \\ \left(\overline{CG_2} \times Unit\left(\overline{G_2Q_2}\right)\right)^T & Unit\left(\overline{G_2Q_2}\right)^T \\ \left(\overline{CG_3} \times Unit\left(\overline{G_3Q_3}\right)\right)^T & Unit\left(\overline{G_3Q_3}\right)^T \end{bmatrix} \tag{18}$$

$$J_q = \begin{bmatrix} \$_{r,2,1} \circ \$_{1,1} & 0 & 0 \\ 0 & \$_{r,2,2} \circ \$_{1,2} & 0 \\ 0 & 0 & \$_{r,2,3} \circ \$_{1,3} \end{bmatrix} \tag{19}$$

$$\dot{q} = \begin{bmatrix} \dot{\beta}_{1,1} & \dot{\beta}_{1,2} & \dot{\beta}_{1,3} \end{bmatrix}^T \tag{20}$$

J_x is a 3×6 matrix. J_q is a 3×3 matrix.

Equation (17) is multiplied by J_q^{-1}; the $\dot{q}$ can be derived as

$$\dot{q} = J_a \$_C \tag{21}$$

where

$$\begin{aligned} J_a &= J_q^{-1} J_x \\ &= \begin{bmatrix} \left[\left(\overline{CG_1} \times Unit\left(\overline{G_1Q_1}\right)\right)^T \quad Unit\left(\overline{G_1Q_1}\right)^T\right] / \$_{r,2,1} \circ \$_{1,1} \\ \left[\left(\overline{CG_2} \times Unit\left(\overline{G_2Q_2}\right)\right)^T \quad Unit\left(\overline{G_2Q_2}\right)^T\right] / \$_{r,2,2} \circ \$_{1,2} \\ \left[\left(\overline{CG_3} \times Unit\left(\overline{G_3Q_3}\right)\right)^T \quad Unit\left(\overline{G_3Q_3}\right)^T\right] / \$_{r,2,3} \circ \$_{1,3} \end{bmatrix} \end{aligned} \tag{22}$$

For (22), we only need to know the direction of each vector in each row, regardless of the magnitude of each vector, so the above matrix can be simplified as

$$J_b = \begin{bmatrix} \left(\overline{CG_1} \times Unit\left(\overline{G_1Q_1}\right)\right)^T & Unit\left(\overline{G_1Q_1}\right)^T \\ \left(\overline{CG_2} \times Unit\left(\overline{G_2Q_2}\right)\right)^T & Unit\left(\overline{G_2Q_2}\right)^T \\ \left(\overline{CG_3} \times Unit\left(\overline{G_3Q_3}\right)\right)^T & Unit\left(\overline{G_3Q_3}\right)^T \end{bmatrix} \tag{23}$$

J_b is 3×6 driving Jacobian matrices, which is the mapping between the linear velocity and angular velocity of the moving platform and the velocity of the driving joint.

4.3. The Complete Jacobian Matrix. The constrained submatrix and the driving submatrix of the mechanism can be written as a complete 6×6 matrix; that is, the screw expression of the complete Jacobian matrix of the mechanism is as follows:

$$J_W = \begin{bmatrix} \left(\overline{CG_1} \times Unit\left(\overline{G_1D_{D1}}\right)\right)^T & Unit\left(\overline{G_1D_{D1}}\right)^T \\ \left(\overline{CG_2} \times Unit\left(\overline{G_2D_{D2}}\right)\right)^T & Unit\left(\overline{G_2D_{D2}}\right)^T \\ \left(\overline{CG_3} \times Unit\left(\overline{G_3D_{D3}}\right)\right)^T & Unit\left(\overline{G_3D_{D3}}\right)^T \\ \left(\overline{CG_1} \times Unit\left(\overline{G_1Q_1}\right)\right)^T & Unit\left(\overline{G_1Q_1}\right)^T \\ \left(\overline{CG_2} \times Unit\left(\overline{G_2Q_2}\right)\right)^T & Unit\left(\overline{G_2Q_2}\right)^T \\ \left(\overline{CG_3} \times Unit\left(\overline{G_3Q_3}\right)\right)^T & Unit\left(\overline{G_3Q_3}\right)^T \end{bmatrix} \tag{24}$$

The line geometry can be used to determine the linear correlation of the reciprocal screws in the matrix to determine whether the mechanism is full rank or not, so as to judge the singular configuration of the mechanism.

According to linear algebra, the correlation of each line vector is the same as the vector $\overline{G_iD_{Di}}$ and $\overline{G_iQ_i}$. In order to represent the linear correlation of the Jacobian matrix more easily, the constrained reciprocal screw and driving reciprocal screw are expressed by vector $\overline{G_iD_{Di}}$ and $\overline{G_iQ_i}$. Taking the first branch chain as an example, when the revolute axes of the upper and lower platforms are not parallel to each other, the geometric representations of constrained reciprocal screw and driving reciprocal screw of a branch chain are as shown in Figures 3 and 4

5. Study the Singularity of the 3-RSR PM

5.1. Singularity Analysis of 3-RSR PM in the Fixed Mode. The fundamental reason for the singularity of the parallel mechanism is that the force screw vectors of each branch of the mechanism acting at the end of the operation are linearly correlated, so that the required degree of freedom cannot be achieved. The screw expression of a Jacobian matrix is obtained by the analysis in Section 3. The theory of linear geometry which is a systematic theory for the study of linear correlation of space vectors [10], which can judge the correlation of the screw vectors in different positions of the mechanism. Therefore, according to the line geometry theory, the singularity of the mechanism can be analyzed, which makes the singularity of the mechanism more intuitive in space and avoids the cumbersome calculation process. At the same time, this method also provides a new idea for finding the singularity of the parallel mechanism with multiple moving modes during the movement.

From the analysis of (24), the algebraic expressions of the constrained reciprocal screw and the driving reciprocal screw of the mechanism can be obtained. At the same time, the

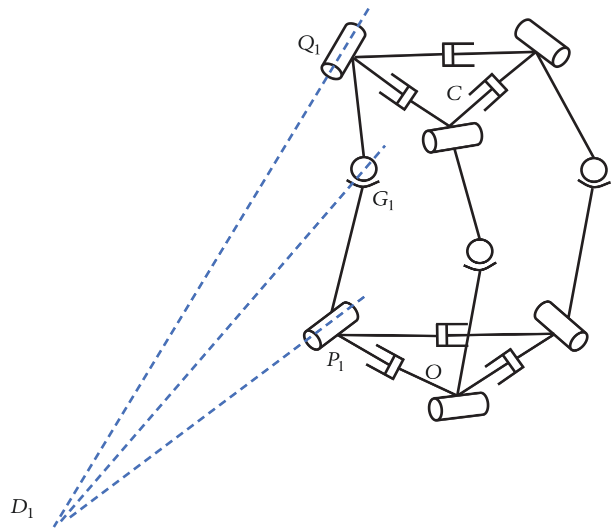


FIGURE 3: The geometric representation of the constrained reciprocal screw space.

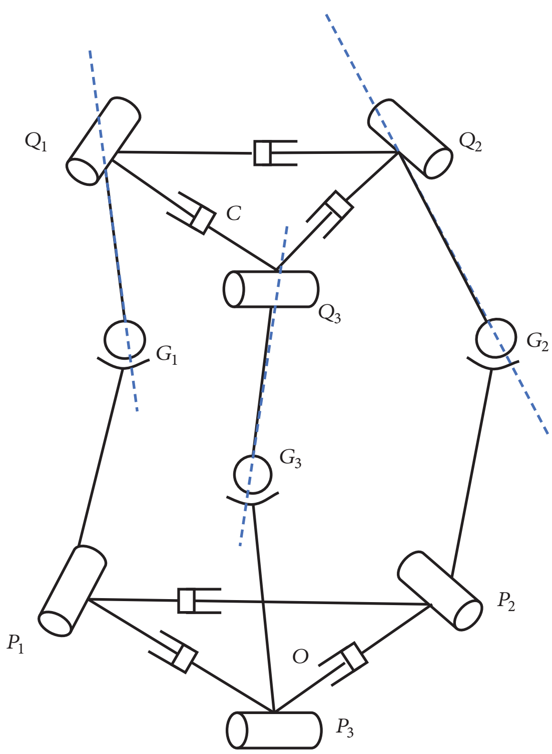


FIGURE 4: The geometric representation of the driving reciprocal screw space.

corresponding vector representation of the constrained reciprocal screw and the driving reciprocal screw in space can be drawn. The constrained reciprocal screw can be regarded as a line vector with pitch h=0, which represents the constrained force on the mechanism, and the driving reciprocal screw can be regarded as a couple of pitch $h = \infty$, which represents the constrained couple of the mechanism. According to the conclusion of the correlation and reversibility of the screw [21], we can get the general position of the 3-RSR mechanism in the traditional fixed mode, the three spherical joints are completely coincident, the connecting rod and the upper platform are in the same plane, the connecting rods are parallel in space, the connecting rod and the driving rod axis are coincident, etc. In this case, we have the correction between the constrained reciprocal screw and the driving reciprocal screw of the mechanism, that is, the constrained singularity and the driving singularity of the mechanism.

Thus, the correlation and expression of spatial vectors for the mechanism's constrained reciprocal screw and the driving reciprocal screw in various cases are shown in Table 1.

From Table 1, the singular configurations of 3-RSR in fixed mode can be as follows.

Case 1. When the mechanism is in a general position, according to the theory of linear geometry, the maximum linear independent number of the space line vector is 3, and the maximum linear independent number of the couple in any case is 3. It can be seen that in the general mode, the constrained reciprocal screw of the mechanism is linearly independent of the driving reciprocal screw, and there is no constrained singularity or structural singularity of the mechanism.

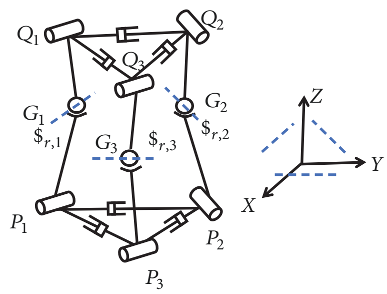
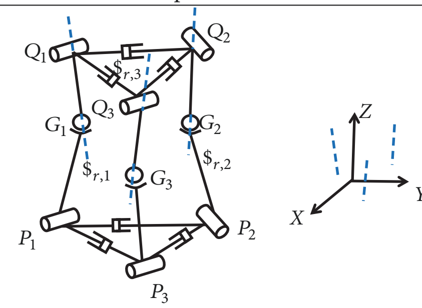
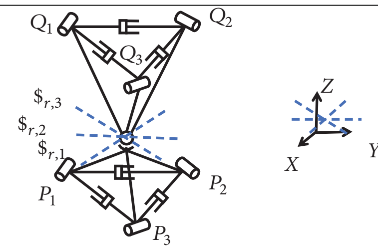
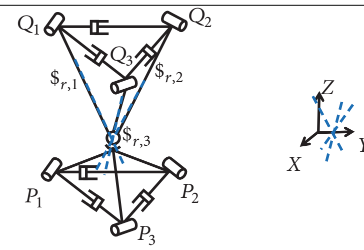
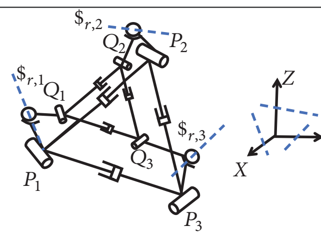
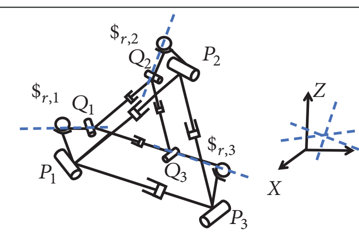
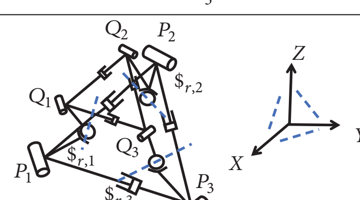
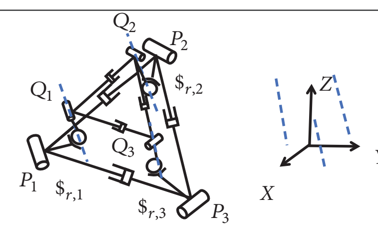
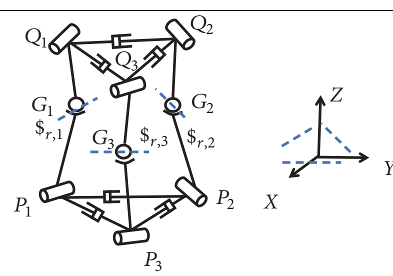
Case 2. When the three spherical joints of the mechanism are completely coincided, according to the theory of linear geometry, the three line vectors are coplanar and merged at one point. At this time, the maximum linear irrelevant number is two, and the three line vectors are linearly related, so that the mechanism generates constrained singularities. However, the three-driving reciprocal screws are couple, and the maximum linear independent number is three, no structural singularity.

Case 3. When the 3-RSR connecting rod and the upper platform are in the same plane, according to the theory of linear geometry, the maximum linear independent number of the space line vector is 3, and the maximum linear independent number of the couple in the same plane is 2. At this time, it can be seen that the driving reciprocal screws of the three branches are linearly related. Thus, the mechanism has no constrained singularities, but it has structural singularities.

Case 4. When the connecting rods of the mechanism are parallel in space, according to the theory of linear geometry, the three couples in space are parallel to each other and the maximum linear independent number is 1. At this time, it can be seen that the driving reciprocal screws of the three branches are linearly related. Thus, the mechanism has no constrained singularities, but it has structural singularities.

Case 5. When the axes of the driving rods and the connecting rods of the mechanism coincide, according to the theory of linear geometry, the maximum linear independent number of the spatial line vector is 3. At this time, the mechanism has no constrained singularities, the reciprocal screws of the mechanism's branches do not exist, but the mechanism has structural singularities.

Table 1: Study the singularity of the 3-RSR PM in the fixed mode.

Order	Geometric characteristics	Mechanism constrained reciprocal screws sptial representation	Mechanism driven reciprocal screw spatial representation
1	General position		
2	Complete coincidence of three spherical joints		
3	The connecting rods and the upper platform are in the same plane		
4	The connecting rods are parallel in space		
5	The axes of the connecting rods coincide with the axes of the driving rods		Currently, the mechanism driving reciprocal screws do not exist, and the mechanism has structural singularity.

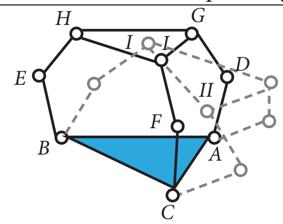
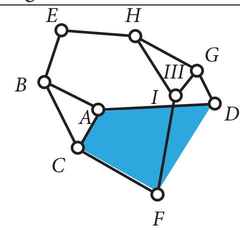
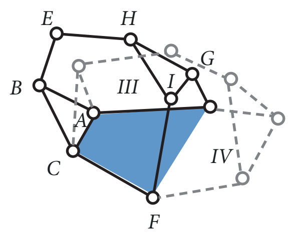
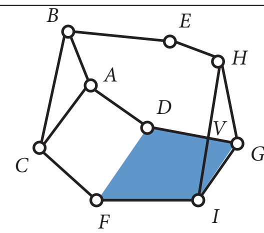
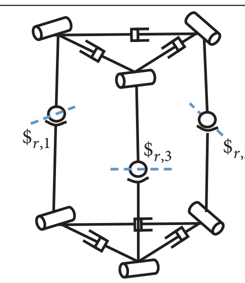
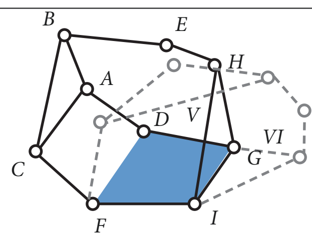
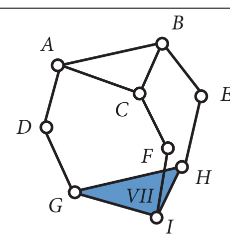
6. Singularity Analysis of 3-RSR with Full Attitude and Multiple Motion Modes

Parallel mechanisms have unique closed-loop characteristics. In the design of mobile mechanisms and mobile robots, we make full use of kinematic relationship between the parallel mechanism platforms and the branch chains and combine the idea of multiple operating modes of the parallel platform [23, 24] to integrate multiple motion modes [25] on a mobile parallel mechanism and let the branch chains and platforms participate in rolling, walking, self-traversing, and so on. Finally, all-attitude multiple modes mobile parallel mechanism can be formed.

In this paper, the 3-RSR mobile PM can realize multi-direction rolling and self-traversing modes due to its own mechanism characteristics [20]. However, the driving choices and avoidance of singularity in realizing these motion modes are unclear.

In the process of realizing rolling motion and self-traversing movement, when the gait changes, the surface formed by the rods connected with the ground can be regarded as a base platform, and the other rods can be

Table 2: Study the singularity of the 3-RSR PM in the rolling mode.

Order	Gait planning of rolling mode		Study the singularity in each gait
step1 to step2	 step 1	 step 2	According to Table 1, from step1 to step2, the singular configurations shown in Table 1 do not appear.
step 2 to step3	 step 2	 step 3	During this process, the singular configuration that may occur according to Table 1 is the constrained singular configuration 
Step3 to step4	 step 3	 step 4	According to Table 1, from step3 to step4, the singular configurations shown in Table 1 donot appear.

regarded as the moving platform. That is to say, the moving platform and the base platform will change with the change of each gait.

Based on this idea, the singularities of the 3-RSR parallel mechanism listed in Table 1 under the traditional fixed mode can provide a theoretical basis for each gait transformation of the mechanism in the all-attitude movement process, and for choosing the most suitable joint or driving rods when avoiding or utilizing the singularity.

6.1. Study the Singularity in Rolling Mode. A rolling cycle of the 3-RSR mobile PM in the rolling mode is shown in Figure 5. According to the position of the upper and lower platform in the rolling process, a rolling cycle can be divided into seven gaits. As shown in Table 2, the gait of *I*~*II*, *III*~*IV*, *V*~*VI* can be regarded as the movement of mechanism in the fixed mode. During the gait of *II*~*III*, *IV*~*V*, *VI*~*VII*, the mechanical structure remains unchanged; only the center of gravity moves. Therefore, there are no singularities in this process.

(i) Step 1 to Step 2. In this process, at the position *I* in Table 2, the plane formed by three points of A, B, and C can be taken as the fixed platform. By driving the rods AD and CF, the projection of center of gravity on the ground falls outside the area of plane ABC, so the mechanism can roll to position *II*. Then, by locking each rod of the mechanism, the mechanism moves around the AC axis to the position *III* under the action of gravity.

(ii) Step 2 to Step 3. From Step 2 to Step 3 in Table 2, the plane formed by the four points ADCF can be taken as the fixed platform when the mechanism is in position *III*. By driving the rods GD and IF, the projection of the center of gravity on the ground falls outside the area of plane ACFD, then reaching position *IV*. According to Table 1, the rod BE and the rod EH maybe collinear in this process, which makes the mechanism singular. Therefore, the motors of driving rod BE and EH rotate at a certain angle, so that the mechanism moves to the position *IV*. Then, each driving motor is locked; the mechanism rotates along the rod DF under the action of gravity to reach position *V*, and the singular configurations that may occur is the constrained singular configuration.

(iii) Step 3 to Step 4. At position *V* in Table 2, the mechanism is fixed on a plane formed by four-point DFIG. The motors of the driving rods AD and CF make the projection of the center of gravity of the mechanism on the ground outside the DFIG plan area. No single configuration can be found during this process as shown in Table 1. Under the action of gravity, the mechanism reaches position *V*.

From Table 2, it reveals that the rolling motion of full attitude multimode mobile 3-*RSR* parallel mechanism is based on ZMP principle. The implementation of each gait

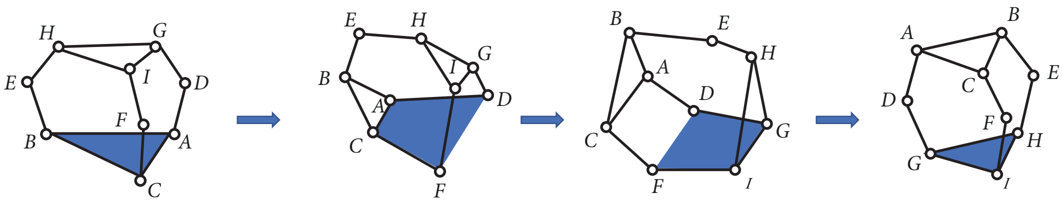


Figure 5: One cycle motion in the rolling mode.

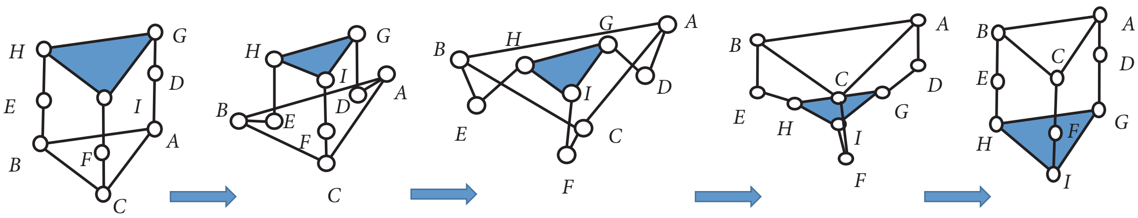


Figure 6: A cycle motion in self-traversing mode.

transition can be divided into two stages: the mechanism configuration change, center of gravity shift, and the roll forward. During the process of shifting the center of gravity, the motion pattern can be regarded as the movement under the fixed platform. The configuration remains variable as it rolls forward, but the center of gravity has not changed relative to the mechanism itself. Therefore, during the rolling process, it is only necessary to analyze the forward movement of the center of gravity.

6.2. Study the Singularity in Self-Traversing Mode. One cycle motion in the self-traversing mode is shown in Figure 6. The self-traversing process can be regarded as the process of alternation between upper and lower platforms. On the basis of the analysis, select the suitable joints to drive in order to avoid the possible singularity of the mechanism. As shown in Table 3, the singularities of the 3-RSR PM in self-traversing mode have been investigated.

(i) Step 1 to Step 2. According to Table 1, the mechanism is in a singular configuration at step 1 in Table 3. In this position, the three motors of the driving rods BE, CF, and AD are working, and the motor of the upper platform is also in use. The driving rods BE, CF, and AD are simultaneously contracting toward the center, and the other driving rods are locked; the mechanism can move from the state shown in step 1 to step 2.

(ii) Step 2 to Step 3. When the mechanism is in step 2, it can be seen from Table 1 that the mechanism is in a structurally singular state. At this time, the driving rods BE, CF, and AD are rotated by three motors, and the other motors are locked. Thus, the mechanism can reach step 3 state from step 2.

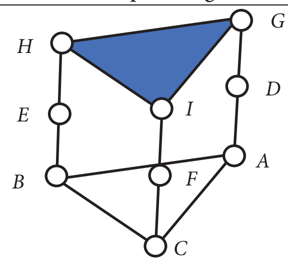
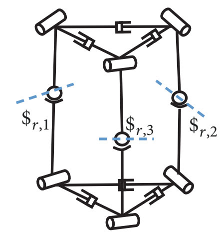
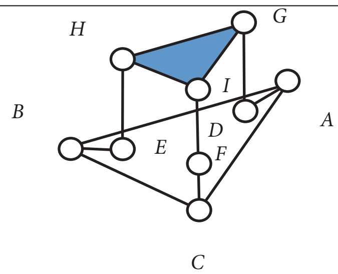
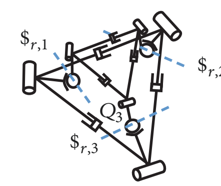
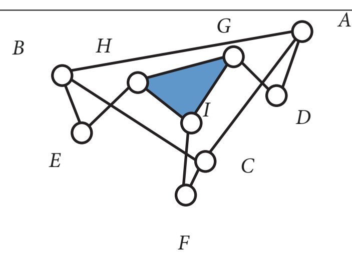
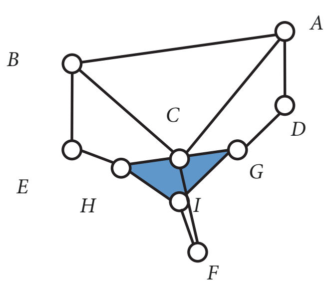
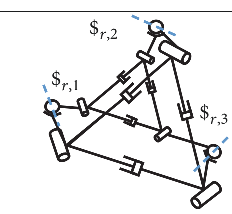
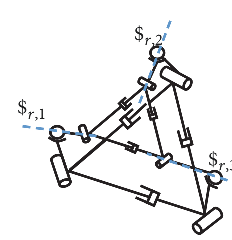
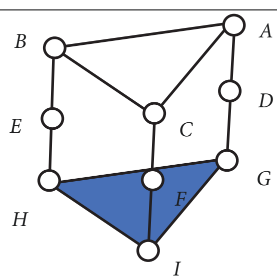
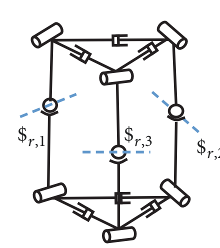
(iii) Step 3 to Step 4. In the process of step 3 to step 4, according to Table 1, it is known that the mechanism has no singular configuration, and the three motors that drive rods BE and BC are selected to work, and the motors of the upper and lower platforms and the connecting rod motors do not work.

(iv) Step 4 to Step 5. In the process of step 5 to step 6, when the mechanism moves to the state shown in step 5, the mechanism has no constrained singularity, but the mechanism has a driving singularity. Selecting the connecting rod EH, FI, GD three motors to work, the upper platform contracts to a minimum. The mechanism reaches the state indicated by step 5. When the mechanism is in the position of step 5, according to Table 1, the mechanism is in a singular configuration. All drivers need to be locked to avoid instability in this configuration. At this time, the three motors of the driving rod are locked, and the three motors of the connecting rods EH, FI, and GD are selected to work, and the upper platform is contracted to the minimum state. When the mechanism is in the position of step 5, according to Table 1, the mechanism is in a singular configuration. All drivers need to be locked to avoid instability in this configuration. Therefore, during the self-traversal mode, the possible singular configurations listed in Table 3 need to be considered in each gait. In addition, these singular configurations can be avoided or used by selecting the mechanism to drive and plan the motion gait.

7. Conclusions

In this paper, we offered a new mathematical method to study the singularity of parallel mechanism, the purpose is to select suitable driving motors for parallel mechanism in multiple motion modes to use and avoid singularity. This method is used to study the singularity of 3-RSR parallel mechanism in the fixed platform mode and all-attitude multimotion modes. Furthermore, the singularity diagrams of each movement mode are given respectively. Analysis results show that the method is feasible and practical compared with other research methods. In the near future, we would like to use this mathematical method to analyze the singularity

Table 3: Study the singularity of the 3-RSR PM in self-traversing mode.

Order	Gait planning	Study the singularity in each gait	
Step1		According to Table 1, the mechanism is in the constrained singular configuration at step 1.	
Step2		In step 2, Table 1 shows that the mechanism is in a structurally singular state.	
Step3		In the process of step 3 to step 4, the singular configurations shown in Table 1 do not appear.	
Step4		In the process of step 4 to step 5, Table 1 show that the mechanism is in the structure singular configuration and constrained singular configuration.	 
Step5		According to Table 1, the mechanism is in the constrained singular configuration in step 5.	

of multimode moving parallel mechanism. Also, multimode mobile parallel mechanism topology is the next plan. It also provides theoretical basis for the topology of multimode mobile parallel mechanism.

Data Availability

We propose a new mathematical method to study the singularity of multimode mobile parallel mechanism. The singularity of 3-RSR parallel mechanism (PM) is analyzed by using reciprocal screw methods and linear geometry theory from two aspects of fixed mode and full attitude multimotion mode. The complete Jacobian matrix of the PM is obtained by using the screw theory and the reciprocal screw of each branch is expressed with algebraic method and geometric drawing method. The singularity of the PM can be obtained by analyzing the reciprocal screw correlation and using the spatial line geometry theory. Finally we analyze the singular configuration of the PM under various modes, which provide theoretical guidance for the gait planning of the multimode mobile PM, and it is a theoretical derivation calculation. The formulas and graphs in the manuscript can completely derive the following conclusions, so no additional data can be uploaded.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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