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Research Article

Model Updating of a Stitched Sandwich Panel Based on Multistage Parameter Selection

Zhifu Cao,[1] Qingguo Fei,[1] Dong Jiang,[1,2] Shaoqing Wu,[1,3] and Zhiruo Fan[1]

[1]*Institute of Aerospace Machinery and Dynamics, Southeast University, Nanjing 211189, China*
[2]*School of Mechanical and Electronic Engineering, Nanjing Forestry University, Nanjing 210037, China*
[3]*Department of Engineering Mechanics, Southeast University, Nanjing 210096, China*

Correspondence should be addressed to Qingguo Fei; qgfei@seu.edu.cn

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The effective numerical model of stitched sandwich composite plays crucial role in dynamics analysis and structural design. A sensitivity-based multistage model updating method is proposed for modeling of the stitched sandwich panel using the experimental modal frequencies. Based on applying the periodic boundary condition, the initial equivalent finite element model of a stitched sandwich composite is constructed; the measured frequencies are obtained from the modal test. According to relative sensitivity analysis of modal frequencies with respect to updating parameters, the different well-conditioned groups of parameters are selected to be updated. This method is applied to a stitched sandwich panel with established configuration of the stitches. Results show that the proposed method with the smallest condition number of relative sensitivity matrix has a better performance than the multistage method using type-based parameter group selection and the traditional model updating approach.

1. Introduction

The stitched sandwich composites have widely been applied in engineering because of the brilliant property in through-the-thickness direction [1], which is designed to achieve high interlaminar strength and the high ratio of stiffness to weight [2]. Numerical model with high accuracy plays an important role in composite structural dynamics analysis. The mechanical properties of the stitched sandwich composite depend on many factors, and these properties are difficult to be determined.

The effective material parameters of the stitched sandwich composite can be determined from the experimental, theoretical, and numerical methods. Singh and Saponara [3] conducted a standard test on the change in flexural, core shear and compression with respect to variable stitch angle. The numerical methods are convenient and effective to obtain and investigate the mechanical properties and behaviors of the composites. Wang et al. [4] investigated the flexural properties of stitched sandwich structure using the experimental and numerical methods and, furthermore, analyzed the effects of stitching density and core thickness. Ai et al. [5–7] modeled the refined finite element model (FEM) of the stitched sandwich panel and discussed the effects of stitches, such as the stitching angle and step, on the mechanical and thermodynamic properties. The experimental, theoretical, and numerical comparisons of the mechanical behaviors of the stitched sandwich panel can also be found in Refs [4, 8–13]. The authors also make efforts on experimental and numerical methods to predict and verify structural behaviors of other types of composites [14–18]. These studies obtained effective material properties and further predicted structural responses of the complex composites. However, the test data were always used as the postvalidation data to verify the accuracy of the numerical model, and these studies without combination of the testing and simulation still leaded to discrepancies between real and numerical models. The accurate model parameters of the FEM should be identified and verified using corresponding test data.

In general, model updating is adopted to reduce the error in the parameter estimations of composites [19–21]. Modal properties reflect the macromechanical performance of the global structures. The modal parameters are applied to assess the accuracy of the equivalent dynamical model. Recently, model updating approaches [22, 23] were developed to estimate the elastic properties of composite materials by minimizing the discrepancies of modal frequencies between experimental and the computational results. Tam et al. [24] reviewed the material properties identification methods using the nondestructive vibrational evaluation. Li et al. [25] used the vibration test data to the identification of the stitched sandwich panel.

Without loss of generality, the parameter identification and model updating algorithms for other composites can also be applied to the stitched sandwich composites. In order to identify the material properties of the individual layer of laminate plates, Lauwagie et al. [26] proposed a multimodel updating routine by using a number of natural frequencies with different plate configurations. Cunha and Piranda [27] identified the stiffness properties of composite tubes from dynamic tests. Important factors including potential energy evaluation, placement of sensors, and multiple eigenvalue problems were discussed in model updating for the tubes. Ayorinde and Yu [28] applied the model updating method to identify properties of a composite rectangular plate, and the main factors affecting the vibration-based parameter identification method were discussed as well. Experiment on specimens of epoxy reinforced with carbon fibers and aluminum alloy was conducted by Antunes et al. [29], and the accuracy of the material properties obtained using the model updating method was studied. Considering the uncertainties in the experiment and equivalent parameter of composite materials, Jiang et al. [30, 31] proposed the combined initial parameter estimation and stochastic model updating-based parameter identification approach. Missoum et al. [32] proposed a numerical parameter identification process for the skin properties in a foam core sandwich panel.

Mottershead et al. [33] pointed out that the convergent solution is difficult to be obtained in the classical weighted least squares method because of an ill-conditioned sensitivity matrix. Hansen [34] and Vogel [35] proposed regularization methods for obtaining a solution of the inverse problem. Li and Law [36] proposed an adaptive Tikhonov regularization method for giving a stable and convergent solution. The treatment of ill-conditioned equations is a central problem to finite element model updating. The sensitivity-based model updating methods gave an efficient way to update the equivalent composite parameters using the test data and numerical model. The selection of the sensitive parameter with low condition number in inverse problem will improve the updating accuracy of the equivalent FEM of the stitched sandwich composite.

A relative sensitivity-based multistage model updating method is proposed to update the equivalent properties of the stitched sandwich panel. In Section 2, the equivalent modelling and modal testing for the stitched sandwich panel are presented. The sensitivity-based multistage model updating method is developed in Section 3. In Section 4, the application results of the proposed method on a stitched sandwich panel with established configuration of stitches are presented and compared with the traditional schemes. Conclusions are drawn in Section 5.

2. Equivalent Modelling and Modal Test

2.1. Homogenized Equivalent Modelling for Stitched Sandwich. The stitched sandwich specimen investigated in this study is shown in Figure 1. The geometric dimensions of the specimen are length $a = 400$ mm, width $b = 200$ mm, and height $h = 11.5$ mm. The stitched sandwich specimen consists of the foam core, two thin layers of face sheet, and stitching fibers. The thickness of the face sheet and the foam core are 1 mm (t_1), 0.5 mm (t_2), and 10 mm (t_3), respectively. As shown in Figure 1, the stitching threads are used for combining the skins and foam core together to reinforce through-the-thickness property. The cross-sectional diameter of the stitching thread is 1 mm, and the distance between two stitching fibers and the stitching step are same to 15 mm [5]. To investigate the dynamic properties of the stitched sandwich specimen, a numerical analysis is a convenient way to the structural analysis.

Due to the diversity in the consisted components, a refined numerical model of the stitched sandwich composite decreases the computation efficiency. Based on the previous study, the homogeneous equivalent modelling method [37] can be applied to the homogeneous equivalent modelling of the stitched sandwich specimen. In this modelling approach, a multicomponent composite material heterogeneous unit cell can be analyzed as the unified homogeneous orthotropic anisotropic model. The theory is introduced as follows.

The unit cell of the stitched sandwich specimen consisted of multicomponents as shown in Figure 1. The heterogeneous structure can be equivalent to the homogeneous orthotropic anisotropic structure. The constitutive relationship equation of the macroequivalent unit cell is given as

$$\begin{aligned}\boldsymbol{\sigma}^m &= \mathbf{C}\boldsymbol{\varepsilon}^m \\ &= \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ & C_{22} & C_{23} & & & \\ & & C_{33} & & & \\ & & & C_{44} & & \\ & \text{sym.} & & & C_{55} & \\ & & & & & C_{66} \end{bmatrix}\boldsymbol{\varepsilon}^m,\end{aligned} \tag{1}$$

where $\boldsymbol{\sigma}^m$ and $\boldsymbol{\varepsilon}^m$ are the homogenized stress and strain vector, respectively, and $\mathbf{C}$ is the stiffness matrix of composites. Inverse the stiffness matrix $\mathbf{C}$ to get the compliance matrix of composites $\mathbf{R}$: $\mathbf{R} = \mathbf{C}^{-1}$. By enforcing the uniform strain boundary conditions, the matrix can be obtained, and the equivalent elastic parameters can be determined from the compliance constants:

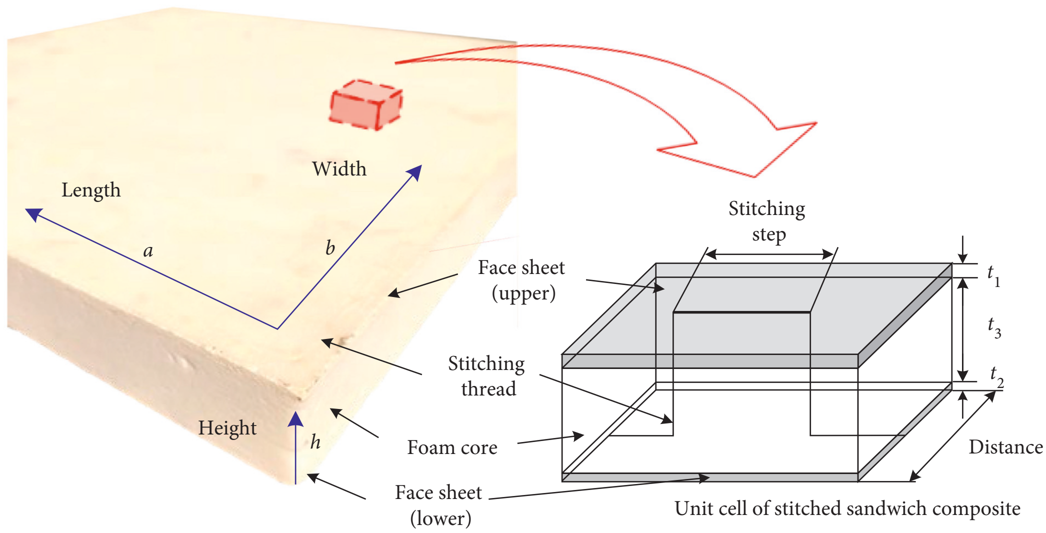


Figure 1: The stitched composite sandwich panel.

$$\begin{aligned}
\overline{E}_{11} &= \frac{1}{R_{11}},\\
\overline{E}_{22} &= \frac{1}{R_{22}},\\
\overline{E}_{33} &= \frac{1}{R_{33}},\\
\overline{G}_{12} &= \frac{1}{R_{44}},\\
\overline{G}_{23} &= \frac{1}{R_{55}},\\
\overline{G}_{31} &= \frac{1}{R_{66}},\\
\overline{\nu}_{12} &= \frac{R_{21}}{R_{11}},\\
\overline{\nu}_{23} &= \frac{R_{32}}{R_{22}},\\
\overline{\nu}_{31} &= \frac{R_{31}}{R_{33}},
\end{aligned} \tag{2}$$

where E is Young's modulus, G is the shear modulus, and ν is Poisson's ratio. Subscripts correspond to the different directions of the material. The symbol ($\bar{}$) means the equivalent value of the elastic properties.

Figure 2 shows the estimation procedure of the equivalent elastic properties of the stitched composite panel. If the unit cell in Figure 1 is subjected to unit macrostrain (i.e., $\varepsilon_{11} = 1$ and $\varepsilon_{22} = \varepsilon_{33} = \varepsilon_{23} = \varepsilon_{31} = \varepsilon_{12} = 0$), the stress vector is obtained by static analysis and equals to the first column of the stiffness matrix (i.e., $[C_{11}\ C_{21}\ C_{31}\ C_{41}\ C_{51}\ C_{61}]^{\mathrm{T}}$). Similarly, the retained five columns of the stiffness matrix are calculated by giving appropriate enforced unit macrostrain (loops from l_2 to l_6 in Figure 2). The macroscopic compliance matrix $\mathbf{S}$ then be obtained, and the equivalent elastic parameters finally be determined using equation (2). Table 1 gives the initial equivalent elastic properties of the stitched sandwich panel.

The refined and equivalent FEM of the stitched sandwich panel are shown in Figure 3. The refined FEM consists of shell elements (face sheet), solid elements (foam core), and bar elements (stitching thread). Based on the homogenized modeling method, the effect of the stitching thread can be considered in the equivalent FEM, which consists of shell elements (face sheet) and solid elements (foam core). The total number of the refined and equivalent FEM modes are 9963 and 462, respectively. The Lanczos method in Nastran is used for the numerical modal analysis, and the computation time of the equivalent model is 11.191 s, while the computation time of the refined model is 103.792 s. The basic properties of the computer are AMD Ryzen 5 2600X Six-Core Processor 3.6 GHz and 16 GB RAM. It is shown that the equivalent model has high efficiency for modal analysis.

According to the equivalent properties determined using the homogenized parameter estimation method, the orthotropic material properties of the shell and solid elements in Figure 3 are listed in Tables 2 and 3, respectively.

2.2. Experimental Modal Analysis. Experimental modal analysis (EMA) performs well on determining the natural frequencies and mode shapes of structures. The impact hammer testing has become widespread as an efficient and economical mean of finding the modal parameters in EMA [38]. The general procedure and test arrangement of modal testing using the impact hammer with one accelerometer is shown in Figure 4. The process involves mechanically exciting the test structure via impulse force and then measuring the resulting vibratory response. Assume that the accelerometer locates at #i, and the test structure is driven with the impact hammer moving from #1 to #n. The row of the frequency response function (FRF) matrix is computed

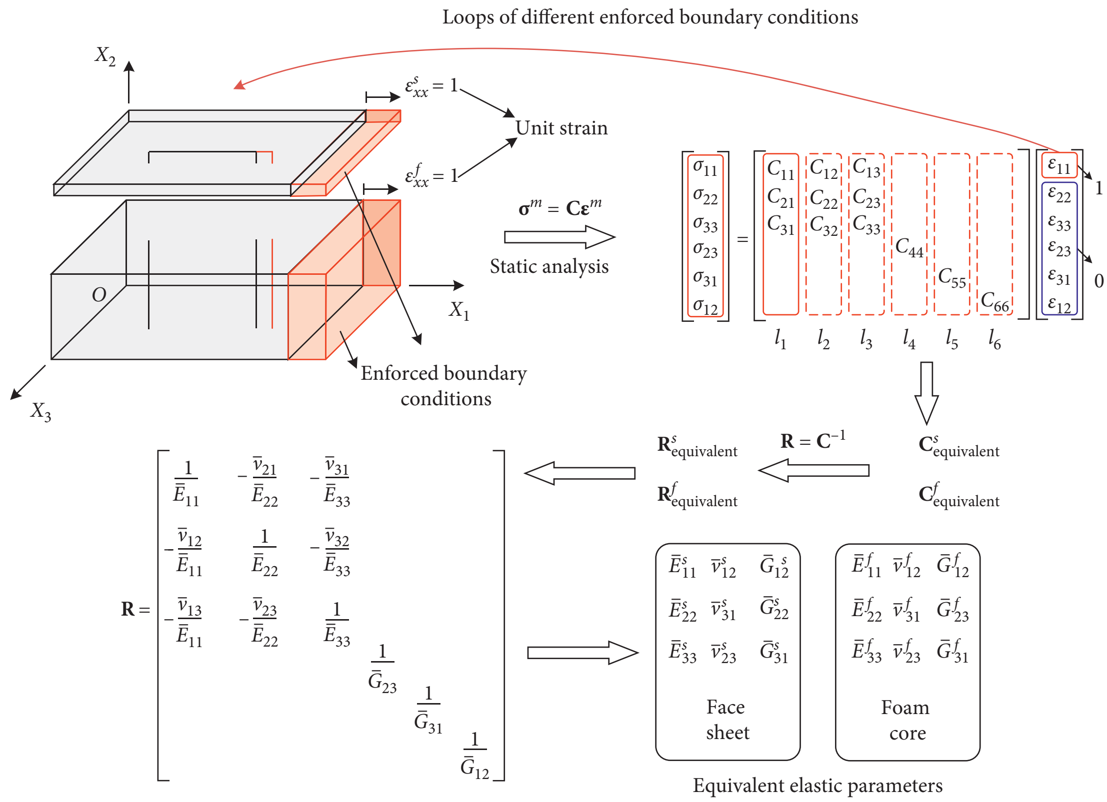


FIGURE 2: Estimation procedure of the equivalent elastic properties of the stitched sandwich composite changing the different forced boundary conditions in every loop.

TABLE 1: Equivalent material properties of skin and core of the stitched sandwich panel.

Type	E_{11} (MPa)	E_{22} (MPa)	E_{33} (MPa)	μ_{12}	μ_{23}	μ_{31}	G_{12} (MPa)	G_{23} (MPa)	G_{31} (MPa)	ρ (kg·m^{-3})
Skin	5230	5230	5425	0.31	0.30	0.29	5630	2815	2815	1.63×10^3
Core	192.3	181.8	277.8	0.06	0.02	0.03	104	29	29	3.37×10^3

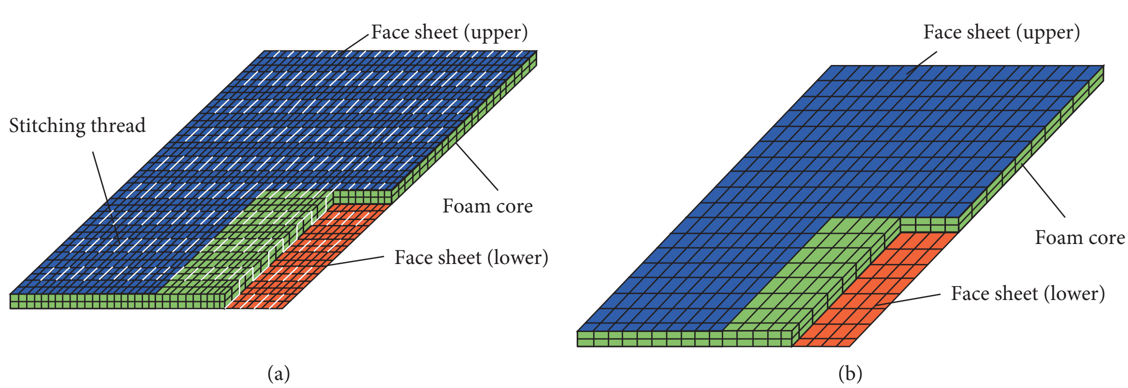


FIGURE 3: Refined (a) and equivalent (b) FEM of the stitched sandwich panel.

from the measuring vibratory response. The modal parameter estimation phase, which is referred to as curve fitting, is implemented using frequency-domain data in this study. The most common methods employ single-mode method and multiple-mode method. The estimation algorithm attempts to decompose measured data into the principle components that make up the measured data such as the frequency, damping, and mode shapes.

Figure 5 shows the modal testing setup of the stitched sandwich specimen in laboratory. As illustrated in Figure 5, the specimen is hung with soft ropes to simulate the free-free boundary condition. 45 excitation points are arranged on the

Table 2: Material properties of shell element in the equivalent finite element model.

Type	E_1 (MPa)	E_2 (MPa)	μ_{12}	G_{12} (MPa)	G_{23} (MPa)	G_{31} (MPa)	ρ (kg·m^3)
Skin	5230	5230	0.31	5630	2815	2815	1.63×10^3

Table 3: Material properties of solid elements in the equivalent finite element model.

Type	C_{11} (MPa)	C_{12} (MPa)	C_{13} (MPa)	C_{22} (MPa)	C_{23} (MPa)	C_{33} (MPa)	C_{44} (MPa)	C_{55} (MPa)	C_{66} (MPa)	ρ (kg·m^3)
Core	193.1	11.1	6.1	182.5	5.9	278.2	104.0	29.0	29.0	3.37×10^3

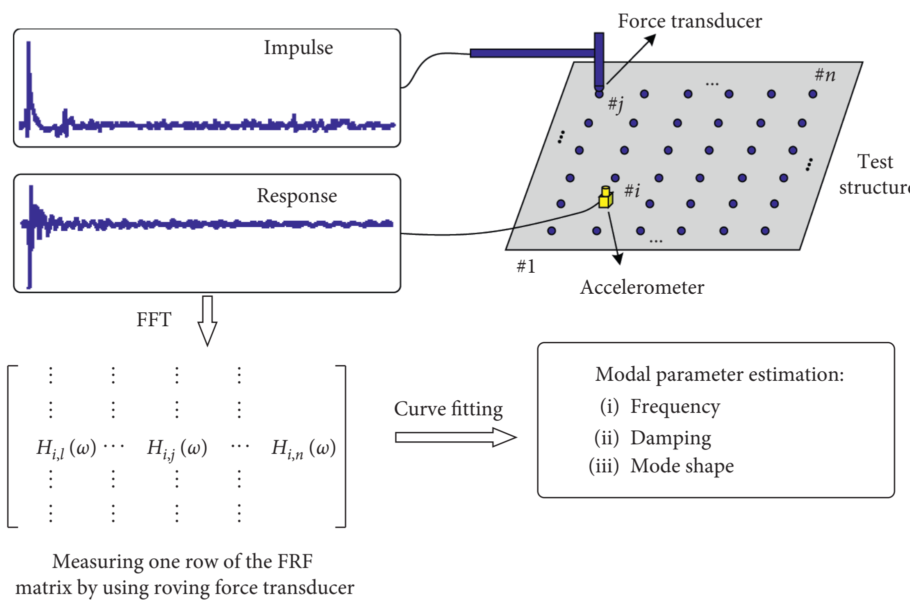


Figure 4: Procedure of the typical modal test with impact hammer.

panel, and the fixed accelerometer (sensitivity 42.8 pc/g) is located at point 39. The impact hammer (sensitivity 3.71 mV/N), driving from point 1 to 45, is applied to measure the impulse force. The analysis frequency is 1000 Hz.

Figure 6 illustrates several measured FRFs at points #12, #14, and #22. The curve fitting is applied to estimate the modal parameters of the test panel; the summation FRF (blue curve) is also shown in Figure 6. Based on the measured data, the estimated modal parameters, modal frequency, and mode shape are listed in Table 4 and Figure 7, respectively. Results illustrate that the equivalent model can represent the refined model as the low differences of the natural frequencies. However, the equivalent model still has discrepancies between experimental and numerical model; the equivalent FEM should be updated using the measured vibration data.

3. Model Updating Based on Multistage Parameter Selection

3.1. Sensitivity Analysis and Parameter Estimation. The purpose of model updating is to modify the property parameters of the numerical model and improve the accuracy of the analysis results. The model updating method is essentially an optimization which minimizes the discrepancies between the experiment and analytical outputs [33]. The objective function and the constraint are defined as

$$\begin{cases} \min \quad J(\mathbf{p}) = \mathbf{r}^{\mathrm{T}}\mathbf{W}\mathbf{r} = \left\| \mathbf{W}^{1/2}\left(\mathbf{z}^{\mathrm{e}} - \mathbf{z}^{\mathrm{a}}(\mathbf{p})\right)\right\|_2^2, \\ \text{s.t.} \quad \mathbf{p}_l \le \mathbf{p} \le \mathbf{p}_u, \end{cases} \tag{3}$$

where $\mathbf{p} \in \mathbb{R}^{M\times 1}$ is a vector of the parameters in the numerical model and $\mathbf{z}^{\mathrm{e}}$ and $\mathbf{z}^{\mathrm{a}}(\mathbf{p}) \in \mathbb{R}^{N\times 1}$ are the experimental and analytical modal data, respectively. r is the error vector of analytical modal data. $\mathbf{W}$ is a diagonal weighting matrix which represents the relative confidence in the experimental data in various modes. Iterations are taken to find the appropriate value $\mathbf{p}$ in the domain of definition $[\mathbf{p}_l, \mathbf{p}_u]$, which have a minimized value of the residual error in the objection function.

Resorting to the Taylor expansion, the optimization problem in equation (3) can be rewritten into the following equation in the jth iteration step in the scheme:

$$\Delta\mathbf{z}_j = \mathbf{W}^{1/2}\left(\mathbf{z}^{\mathrm{e}} - \mathbf{z}_j^{\mathrm{a}}\right) = \mathbf{S}_j\left(\mathbf{p}_{j+1} - \mathbf{p}_j\right), \quad j = 1, 2, 3, \ldots, \tag{4}$$

where $\mathbf{S}_j = \mathbf{W}^{1/2}\partial\mathbf{z}_j/\partial\mathbf{p}_j$ is the weighed sensitivity matrix of the modal data with respect to the selected parameters. The sensitivity matrix $\mathbf{S}$ is computed at the current value of the

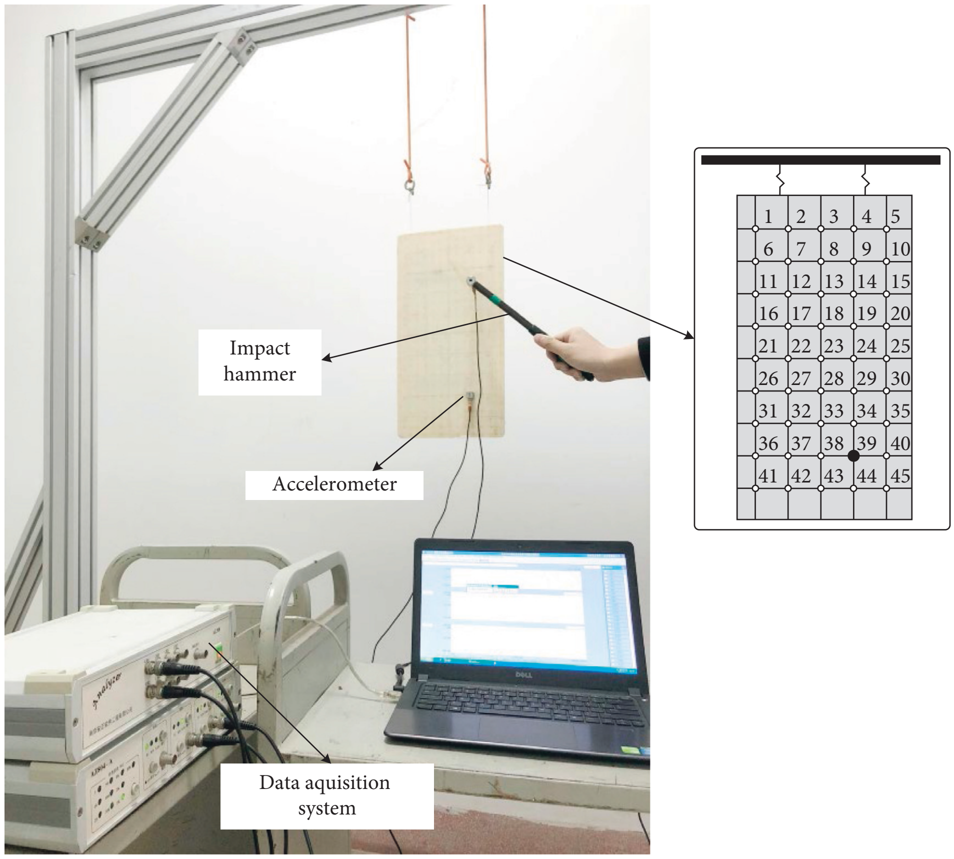


FIGURE 5: Modal test setup for the stitched sandwich panel.

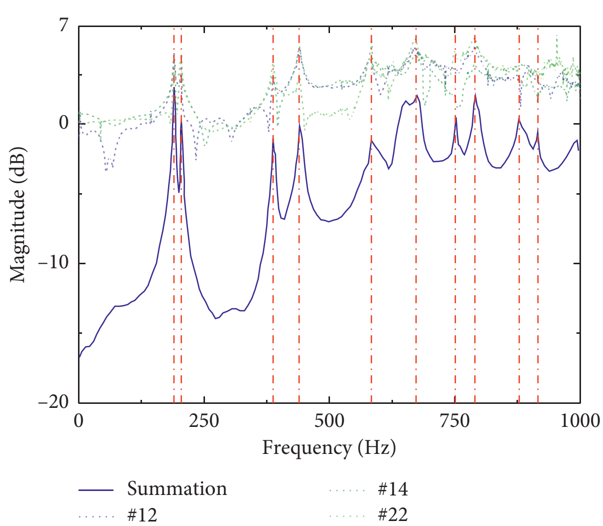


FIGURE 6: Measured summation frequency response function estimated from all measured data and three measured data at three points.

complete vector of parameters $\mathbf{p} = \mathbf{p}_j$. The derivative of the modal parameters with respect to parameters is calculated by

$$\frac{\partial \mathbf{z}}{\partial \mathbf{p}} = \begin{bmatrix} \dfrac{\partial z_1}{\partial p_1} & \cdots & \dfrac{\partial z_1}{\partial p_M} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial z_N}{\partial p_1} & \cdots & \dfrac{\partial z_N}{\partial p_M} \end{bmatrix}. \quad (5)$$

The outputs ($\mathbf{z}^e$ and $\mathbf{z}_j^a$) are typically eigenfrequencies, mode shapes, or complex frequency response functions of the structure. The residual $\Delta \mathbf{z}_j$ is assumed to be small if parameter $\mathbf{p}$ is in the vicinity of true value. The iteration format of parameters to be identified is obtained at each iteration:

$$\mathbf{p}_{j+1} = \mathbf{p}_j + \left(\mathbf{S}_j^{\mathrm{T}} \mathbf{S}_j\right)^{-1} \mathbf{S}_j^{\mathrm{T}} \mathbf{W}^{1/2} \left(\mathbf{z}^m - \mathbf{z}_j^{\mathrm{a}}\right). \quad (6)$$

This iteration procedure continues until consecutive estimates $\mathbf{p}_j$ and $\mathbf{p}_{j+1}$ are sufficiently converged. In equation (6), the solution $\mathbf{p}_{j+1}$ is affected by the inverse of the matrix ($\mathbf{S}_j^{\mathrm{T}} \mathbf{S}_j$). A large condition number of the matrix will lead to an ill-condition problem in which an inaccurate solution is occurred.

Table 4: Comparison of modal frequencies between refined and equivalent models.

Mode order	Experimental (Hz)	Numerical				
		Refined model (Hz)	Error (%)	Equivalent model (Hz)	Error* (%)	Diff. (%)
1	189.90	213.17	12.25	213.32	12.33	−0.02
2	204.59	125.05	−38.88	125.37	−38.72	0.26
3	387.64	414.38	6.90	415.23	7.12	0.21
4	439.56	322.22	−26.69	324.71	−26.13	0.77
5	583.81	609.20	4.35	610.97	4.65	0.29
6	672.74	578.64	−13.99	561.10	−16.59	−3.03

Error = $(f^{r} - f^{e})/f^{e} \times 100\%$; Error* = $(f^{eq} - f^{e})/f^{e} \times 100\%$; Diff. = $(f^{eq} - f^{r})/f^{r} \times 100\%$; f^{r} is the analytical frequency of the refined model; f^{eq} is the analytical frequency of the equivalent model; f^{e} is the experimental frequency.

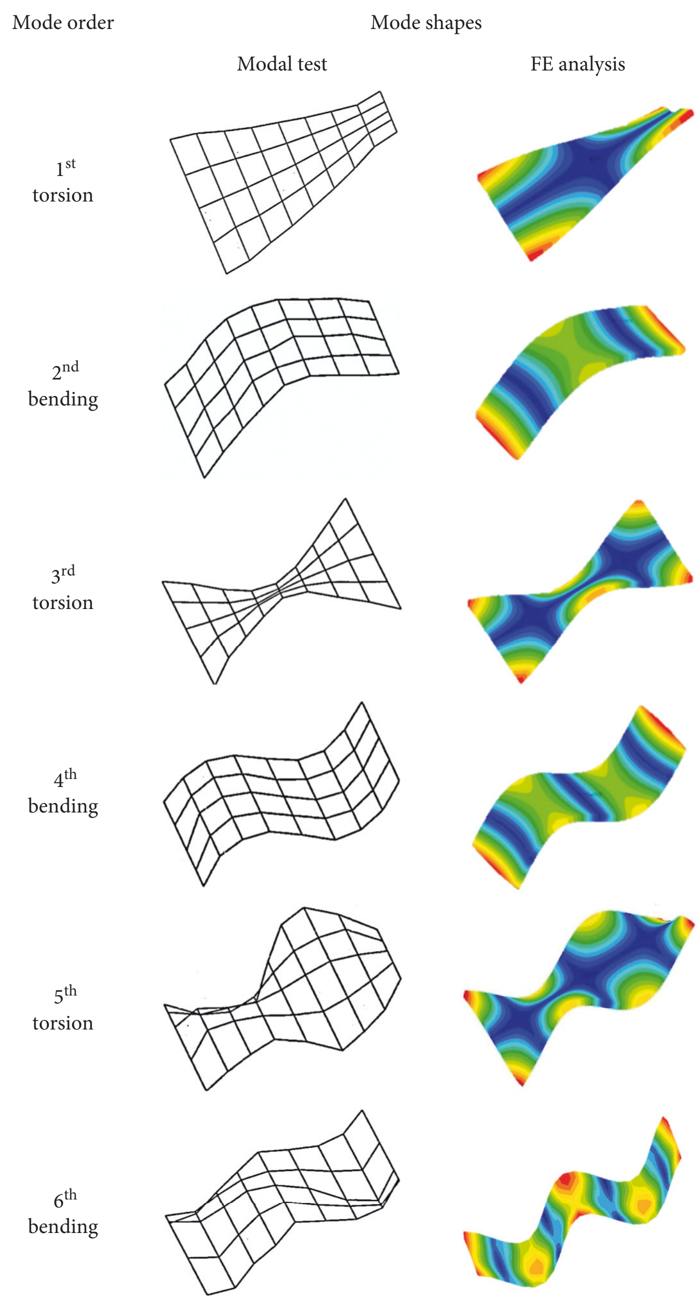


Figure 7: First six mode shapes obtained from experiment and FE analysis.

3.2. Multistage Model Updating Procedure. To deal with the problem that the inaccurate inverse solution caused by large condition number of the sensitivity matrix and enhance the accuracy of the equivalent model updating, the method multistage parameter selection for model updating is developed in this section. In the proposed multistage model updating method, updating parameters are divided into several groups and only one group of parameter is adopted to be updating in one stage procedure. The modified procedure will be executed several times until every group of parameters is modified and the residual error between numerical and experimental data sufficiently converged to be minimum. Compared with the traditional method, the multistage method could decrease the condition number of the sensitivity matrix. The major step of the multistage updating procedure is parameter group selection, which is introduced as follows.

In the equivalent model of the stitched sandwich panel, updating parameters should be determined from the modal testing data, and the parameter groups $\mathbf{p}^1, \mathbf{p}^2, \ldots, \mathbf{p}^n$ have the relationship as

$$\begin{gathered} \mathbf{p}^1 \cup \mathbf{p}^2 \cdots \cup \mathbf{p}^n = \mathbf{p}, \\ \mathbf{p}^\alpha \cap \mathbf{p}^\beta = \varnothing, \quad \alpha, \beta = 1, 2, \ldots, n,\ \alpha \neq \beta, \end{gathered} \tag{7}$$

where n is the number of parameter groups.

Two methods of parameter group selection are introduced in this paper. A concise method is to divide parameters according to the type of parameters. In the case of the sandwich panel, parameters related to the tensile modulus can be put in the same one group, so are others related to shear modules. The advantage of the type-based method is easy and visualized to operate. In order to be quantitative, the second relative sensitivity-based grouping method is an alternative approach to multistage parameter grouping. The parameter sensitivity matrix in equation (4), representing the change of output data caused by the perturbation of model parameter, reflects the effects on the structural performance of the parameter. The advantage of relative sensitivity method is that it avoids the influence of the quantity or unit used for parameters. The formulation of relative sensitivity matrix is given by

$$\mathbf{S}_{\mathrm{r}} = \frac{\partial \mathbf{z}}{\partial \mathbf{p}^\alpha} \cdot \operatorname{diag}(\mathbf{p}^\alpha), \tag{8}$$

where $\mathbf{z}$ is the output modal data as in equation (5), $\mathbf{p}$ is the selected parameter vector of each group. The relative sensitivity matrix can also be used in equation (6). Based on the calculated relative sensitivity matrix of the frequencies with respect to updating parameters, the parameters that have the same magnitude of the relative sensitivity can be put into the same group. The group which has the heaviest magnitude should be set as the first group so that the parameters can be modified firstly, and the remaining ones then are updated. The checking rule of the grouping method is based on the condition number of the selected group sensitivity matrix.

The implementation process of the sensitivity-based multistage updating method, as shown in Figure 8, can be illustrated as follows:

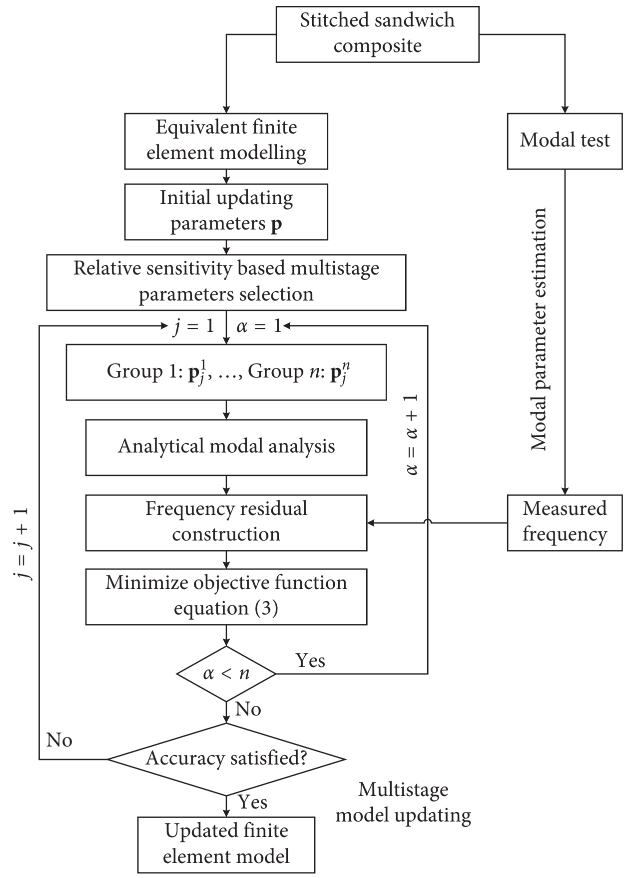


Figure 8: Flow chart of the multistage model updating procedure.

(1) Initial finite element model of the stitched sandwich panel is firstly established using the homogeneous equivalent method and estimate the initial equivalent parameters.

(2) Relative sensitivity analysis is used for the multistage parameters selection. The updating parameter vector $\mathbf{p}$ is selected into n groups: $\mathbf{p} = \{\mathbf{p}^1, \mathbf{p}^2, \ldots, \mathbf{p}^n\}$.

(3) Begining of multistage finite element model updating. Choose the α^{th} parameter group $\mathbf{p}^\alpha$ to be updating parameters, construct the frequency residual using the measured data, update the selected parameter group to achieve a new converged parameter set $\mathbf{p}^\alpha$, and finally renew the new group into updating parameter set $\mathbf{P}(\alpha < n)$.

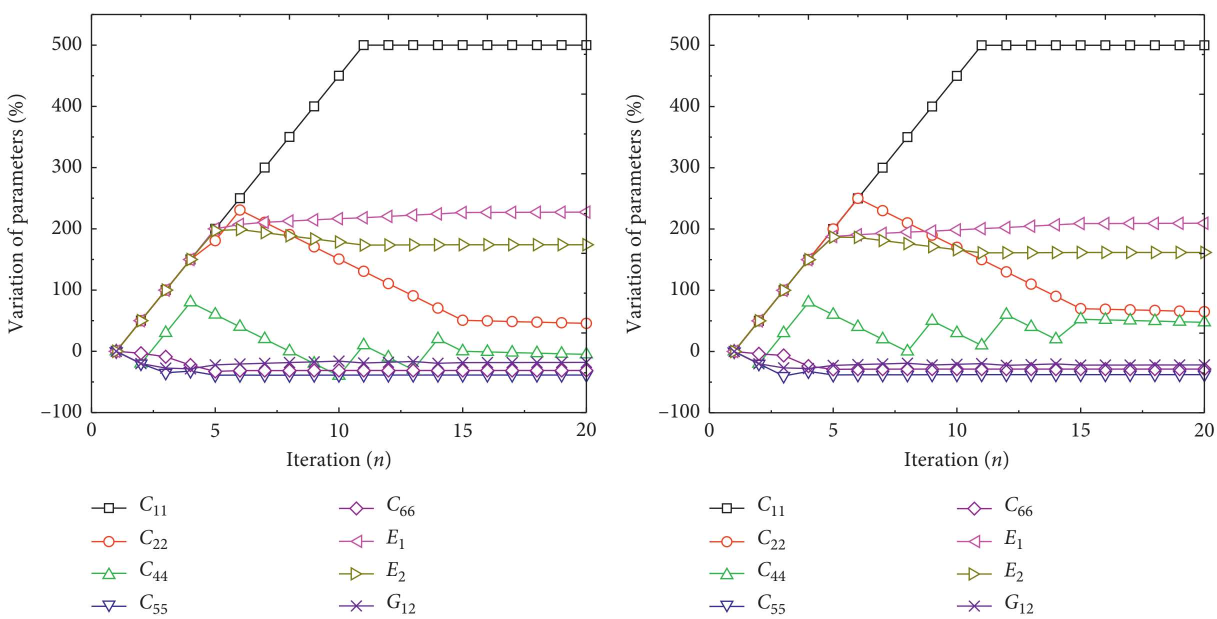
(4) When all the parameter groups are renewed and the accuracy is satisfied, go to step (5) and the updated finite element model is obtained; Otherwise, set $j = j + 1$, return to step (3); the converged criterion is $\|\Delta \mathbf{p}\| \leq 10^{-6}$ and $\|\Delta \mathbf{z}\| \leq 10^{-6}$.

(5) The updated finite element model of stitched sandwich is obtained.

In brief, parameters in one group are modified at the same time in multistage model updating method, while parameters in the next group adopt the terminal value of the identification before and execute the modification in sequence. The multistage model updating method ensures that

Table 5: Relative sensitivity matrix of first four modes with respect to updating parameters.

Mode	C_{11}	C_{22}	C_{44}	C_{55}	C_{66}	E_1	E_2	G_{12}
1	1.05	16.96	0.02	37.93	2.14	158.56	15.51	0.28
2	0.62	12.98	2.54	46.54	40.75	121.22	12.19	43.96
3	18.68	0.79	0.13	2.41	87.07	−11.05	194.40	2.35
4	4.38	0.25	7.32	61.88	63.72	−1.62	43.20	126.43



(a) (b)

Figure 9: Convergence of selected parameters with different weighting matrices: (a) $\mathbf{W}$ and (b) $\acute{\mathbf{W}}$.

the experimental data are abundant at every step of model updating and the stability of every optimization iteration is enhanced. With the iteration times increasing of multistage updating, the parameter will converge at the true value.

4. Application to a Stitched Sandwich Panel

To verify the accuracy of the proposed method for the equivalent model updating, an established stitched sandwich panel as shown in Figure 1 is conducted in this paper. The equivalent finite element model of the stitched sandwich panel is shown in Figure 3. The experimental and numerical modal frequencies are listed in Table 4. As shown in Tables 2 and 3, 15 elastic parameters of the equivalent model should be updated to achieve the accurate model. The relative sensitivity analysis is conducted to the initial selection of these elastic parameters.

Table 5 gives the relative sensitivity of the sensitive parameters which will be updated using the measured data. It is noted that the number of the modes is larger than that of updating parameters, and solutions of the updating parameters can be executed to achieve the objective function of equation (3). The comparison application of the traditional and the proposed approach will be discussed as follows.

Table 6: Comparison of updated modal frequencies between experiment and analysis with different weighting matrices.

Mode order	Experiment (Hz)	Case I: $\mathbf{W}$		Case II: $\acute{\mathbf{W}}$	
		Numerical (Hz)	Error (%)	Numerical (Hz)	Error (%)
1	189.90	190.66	0.40	190.78	0.46
2	204.59	201.94	−1.29	199.57	−2.46
3	387.64	383.47	−1.08	384.03	−0.93
4	439.56	450.75	2.55	449.97	2.37
5	583.81	593.42	1.65	595.61	2.02
6	672.74	664.87	−1.17	661.87	−1.62
7	751.18	769.54	2.44	770.28	2.54
8	790.16	817.25	3.43	822.75	4.12
9	878.64	909.26	3.48	913.50	3.97
10	916.01	970.10	5.91	979.15	6.89
Mean error			2.34		2.74

Mean error means absolute average error, mean error $= \boldsymbol{\Sigma}^N$ absolute $(\text{Error}_i)/N$.

Table 7: Two cases of parameter groups.

Case number	$\mathbf{p}^1$	$\mathbf{p}^2$
1	C_{11}, C_{22}, E_1, E_2	C_{44}, C_{55}, C_{66}, G_{12}
2	C_{55}, C_{66}, E_1, E_2, G_{12}	C_{11}, C_{22}, C_{44}

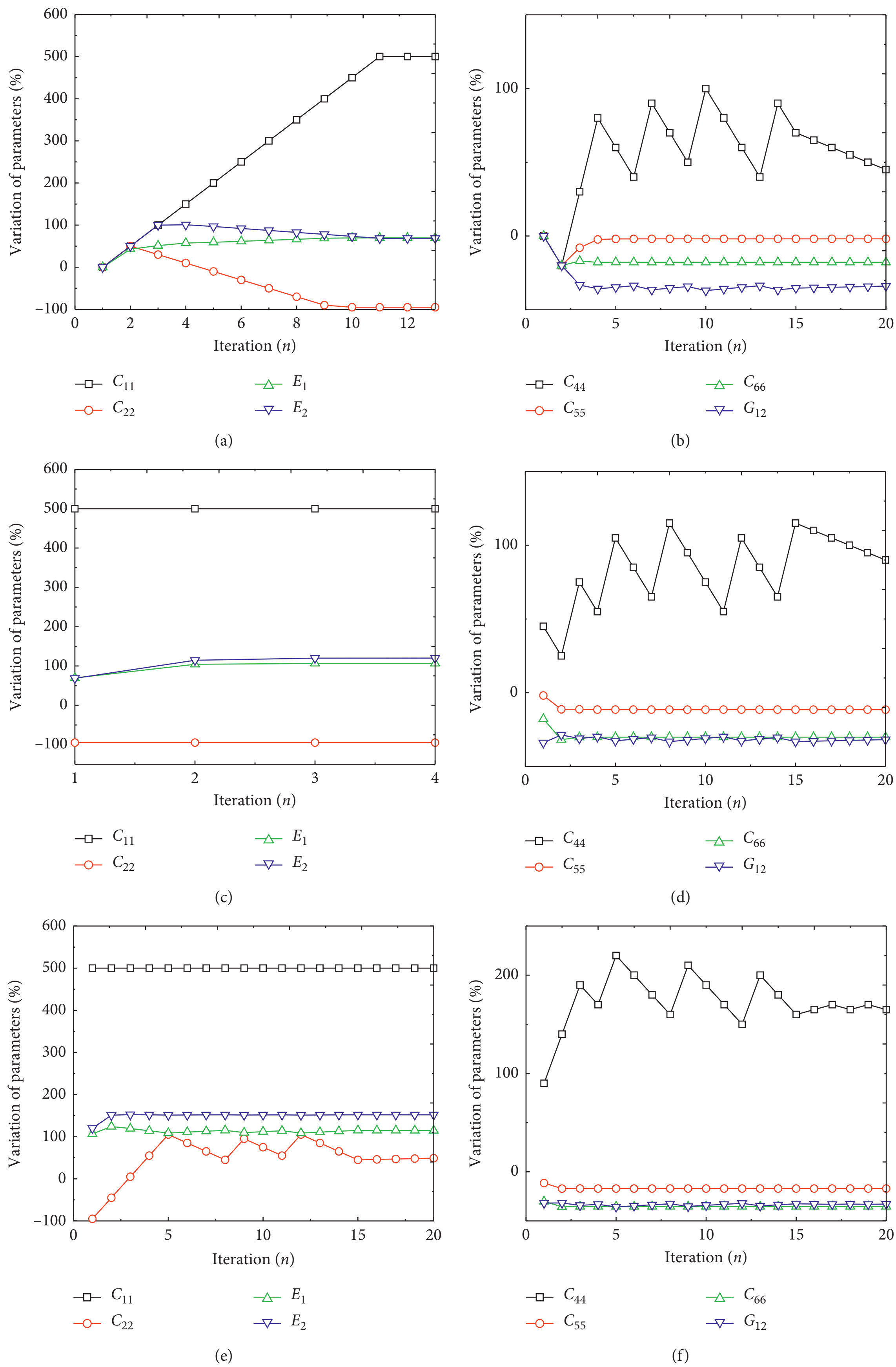


Figure 10: Continued.

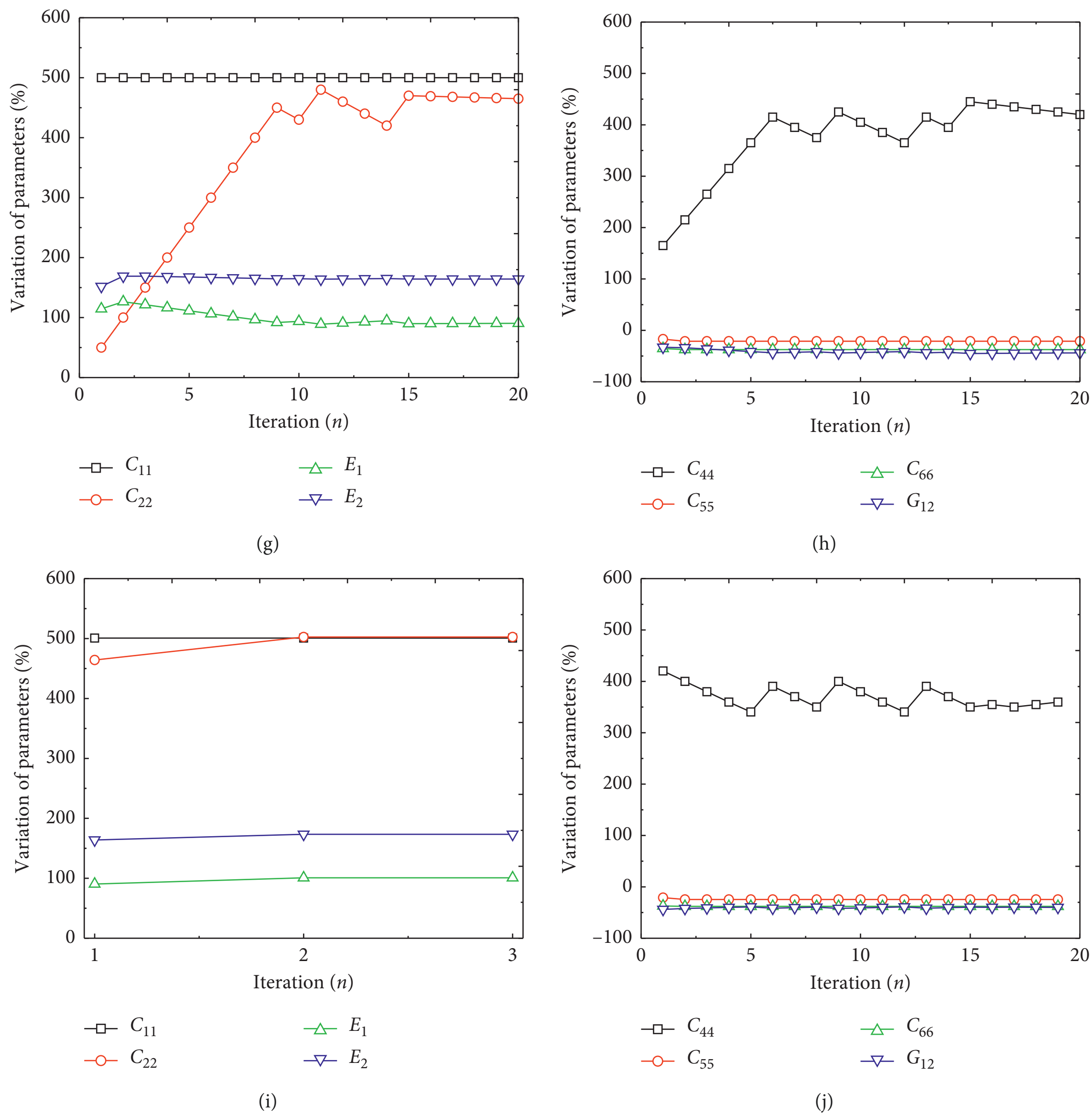


FIGURE 10: Convergence procedure of different parameter groups in each stage of case 1.

4.1. Traditional FE Model Updating. The elastic parameters listed in Table 5 are together to be updated using the traditional model updating procedure. As shown in Table 5, the magnitudes of the relative sensitivities are different, and two different weighting matrices are considered in the traditional method to the effect of the condition number. The two different weighting matrices are computed as $\mathbf{W} = \max(\mathbf{z}^e) \times \mathrm{diag}(\mathbf{z}^e)^{-1}$ and $\acute{\mathbf{W}} = \mathrm{round}(\mathbf{W})$, respectively. The symbol round$(^*)$ means to round the values $(^*)$, i.e., round $(3.4) = 3$. The former one $\mathbf{W}$ aims to equalize the effect of the magnitude of residual frequency. The other one $\acute{\mathbf{W}}$ is set up to investigate the stability of the algorithm. To ensure the parameter domain is wide enough to parameters searching, the upper and lower bounds are defined as $0.1\mathbf{p}_0 \leq \mathbf{p} \leq 6\mathbf{p}_0$. Once the parameters converge to the boundary of the range, the result would be regarded as unauthentic.

Figure 9 illustrates the convergent process of the selected parameters using two different weighting matrices. Table 6 indicates the comparison of updated frequencies between experiment and analysis. The maximum errors in different cases are 5.91% and 6.89%, and both of them occur at the 10^{th} mode. The frequency errors of the others are less than 5%; and the mean errors are 2.34% and 2.74% in case I and case II, respectively.

The results in Table 6 show that the sensitivity-based model updating algorithm is stable to the application from the two different weighting matrices. It seems that the updated numerical model can predict the accurate dynamic properties (modal frequency) using the traditional model updating method. However, some updated parameters touch the boundary of the range shown in Figure 9, which means that they might not be effective values after model updating. The condition numbers of the relative sensitivity

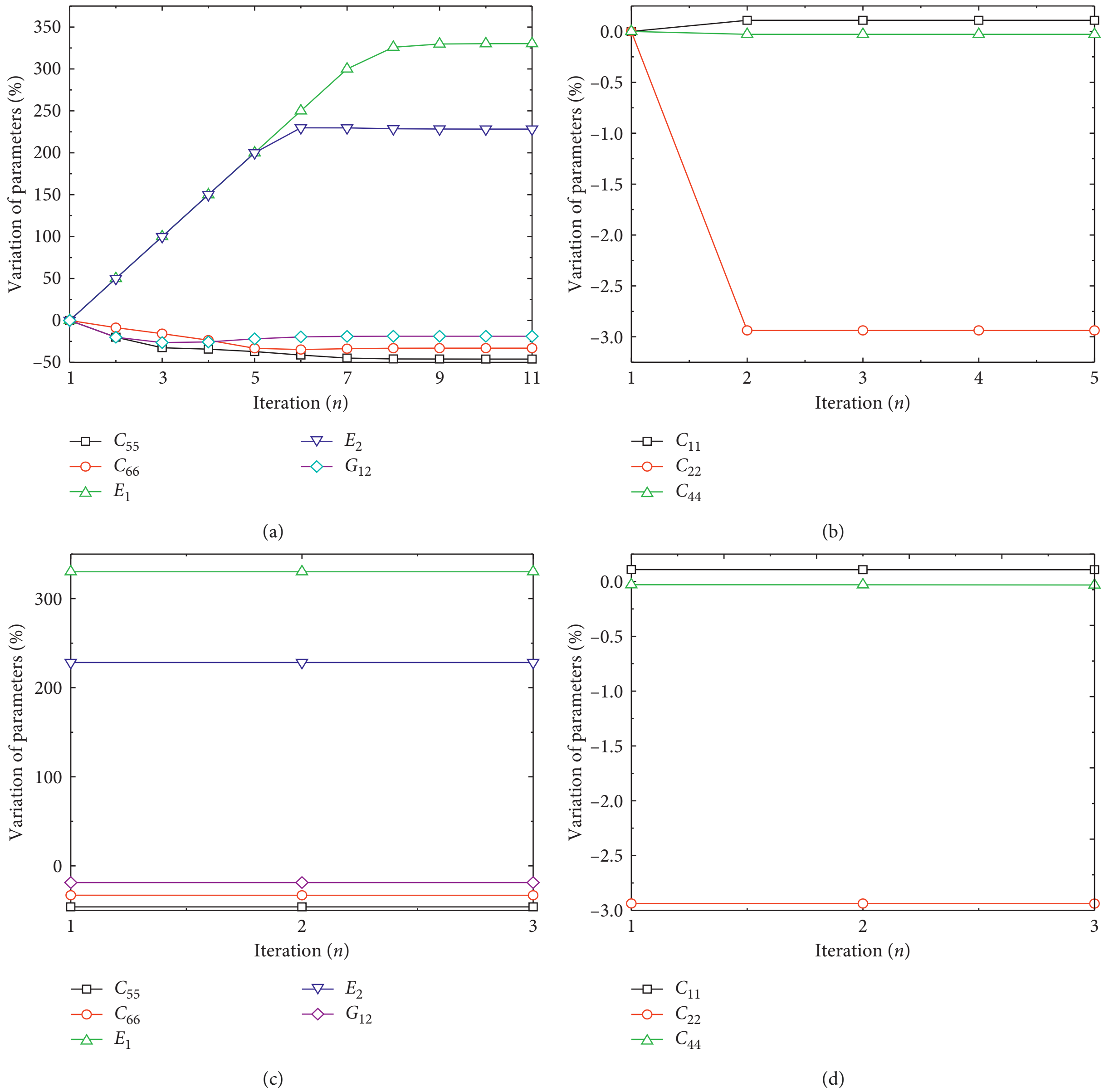


Figure 11: Convergence of groups of parameters in each stage in case 2.

Table 8: Comparison of modal frequencies between test and numerical models of cases 1 and 2.

Mode order	Test (Hz)	Numerical mode					
		Initial (Hz)	Error (%)	Case 1		Case 2	
				Updated (Hz)	Error (%)	Updated (Hz)	Error (%)
1	189.90	213.32	12.33	187.29	−1.37	189.16	−0.39
2	204.59	125.37	−38.72	201.34	−1.59	201.17	−1.67
3	387.64	415.25	7.12	380.19	−1.92	378.37	−2.39
4	439.56	324.71	−26.13	441.70	0.49	447.81	1.88
5	583.81	610.97	4.65	587.73	0.67	584.30	0.08
6	672.74	561.10	−16.59	655.02	−2.63	670.85	−0.28
7	751.18	768.72	2.33	767.95	2.23	763.80	1.68
8	790.16	817.67	3.48	803.54	1.69	805.00	1.88
9	878.64	875.59	−0.35	912.01	3.80	893.22	1.66
10	916.01	994.60	8.58	937.48	2.34	961.01	4.91
Mean error			12.03		1.87		1.68

Table 9: Condition number of the relative sensitivity matrix of two stages.

Iteration stage	Case 1	Case 2
One	12429	49.26
Two	8141	8.20

Table 10: Updated elastic parameters of equivalent finite element of stitched sandwich composite from case 2.

Foam core properties					Skin properties		
C_{11} (MPa)	C_{22} (MPa)	C_{44} (MPa)	C_{55} (MPa)	C_{66} (MPa)	E_1 (MPa)	E_2 (MPa)	G_{12} (MPa)
193.31	177.14	103.97	15.64	19.42	22500.41	17169.98	4572.30

matrix are 56555 and 60665, respectively. Therefore, the traditional method with two different weighting matrices does not completely avoid the ill-condition problem in the updating procedure, and it is essential to take afford to decrease the condition number of the relative sensitivity matrix. In this paper, the multistage model updating method with relative sensitivity-based parameter grouping is prior to being adopted to deal with the problem.

4.2. Multistage FE Model Updating. As shown in Figure 4, the major step of the process is the selection of the parameters for multistage finite element model updating. The proposed model updating method has advantages in the identification condition with many parameters and enhances the updating accuracy of the selected parameters. It ensures that the experimental data are abundant at every updated step and the condition number of sensitivity matrix is small enough to improve the inverse solution. Consequently, the parameters are divided into several groups with relative sensitivity analysis and updated using the multistage updating method.

To compare the effect of different parameter group methods, two cases as shown in Section 3.2 are investigated in this application. Using the type-based and sensitivity-based parameter group selection method, the two investigated cases are listed in Table 7. In case 1, parameters in the first group $\mathbf{p}^1$ are relative to the tensile property, while parameters in the second group $\mathbf{p}^2$ are corresponded to shear property. The two parameter groups in case 1 are $\mathbf{p}^1 = \{C_{11}, C_{22}, E_1, E_2\}$ and $\mathbf{p}^2 = \{C_{44}, C_{55}, C_{66}, G_{12}\}$. The parameter group selections in case 2 are based on the relative sensitivity analysis and their condition number. In each case, parameters are divided into two groups, and the number of parameters in each group is less than the number of measured frequency and appropriate to be updated. The multistage model updating process is executed separately according to the two cases. The weighting matrix in the application is the same as $\mathbf{W} = \max(\mathbf{z}^e) \times \operatorname{diag}(\mathbf{z}^e)^{-1}$.

Figures 10 and 11 show the convergent curves of the elastic parameters for the two different cases of parameter grouping. The equivalent parameters can be updated by the two means of the parameter grouping. As shown in Table 8, however, the mean errors for cases 1 and 2 are 1.87% and 1.68%, respectively. Results show that the accuracy of the updated modal frequencies is improved using the sensitivity-

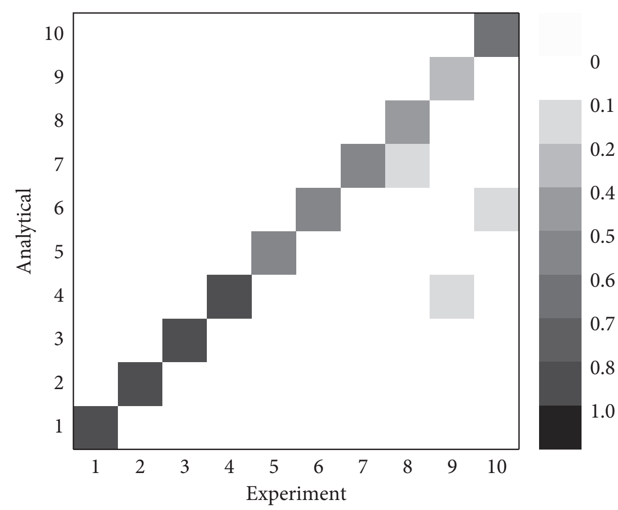


Figure 12: Modal assurance criterion (MAC) between experimental and analytical mode shapes.

based multistage model updating method. The condition numbers of the sensitivity matrix in one of the iterations are listed in Table 9. It is noted that the condition numbers of stages in case 1 (49.26, 8.20) are larger than that of case 2 (12429, 8141), which illustrates that the ill-condition problem could be occurred in the type-based multistage model updating method easily. The combination of parameters whose relative sensitivities are similar can reduce the condition number in the updating procedure, and the updated results will also be improved using the proposed method.

The final updated elastic parameters of the equivalent finite element model of the stitched sandwich panel are listed in Table 10, and the modal assurance criterion (MAC) between experimental and analytical mode shapes is shown in Figure 12. The MAC results show that the analytical mode shapes match well with the measured mode shapes.

5. Conclusions

A sensitivity-based multistage model updating method is proposed on a stitched sandwich panel using the experimental modal frequencies. In this method, the multistage parameter selections are based on the relative parameter sensitivity analysis, which avoids the large condition number of the sensitivity matrix in the updating procedure. The proposed method is used for an equivalent model of a stitched sandwich panel with established configuration of the stitches. The comparison between the proposed method and the traditional method utilizing two different weighting matrices is also conducted. The two types of weighting matrix have small impact on the decreasing condition number of the sensitivity matrix. Results show that the computational efficiency for modal analysis is improved using the equivalent model, and the equivalent model of the stitched sandwich panel can successfully be updated by the proposed method.

Nomenclature

$\boldsymbol{\sigma}^m$, $\boldsymbol{\varepsilon}^m$: Homogenized stress and strain vector
$\mathbf{C}$: Stiffness matrix of composites
$\mathbf{R}$: Compliance matrix of composites
E: Young's modulus
G: Shear modulus
ν: Poisson's ratio
$\mathbf{p}$: Vector of updating parameters
$\mathbf{W}$: Diagonal weighting matrix
$\mathbf{z}^e$, $\mathbf{z}^a$: Experimental and analytical modal data
$\mathbf{S}$, $\mathbf{S}_r$: Weighed sensitivity matrix and relative sensitivity matrix
$\max(*)$: Maximum elements of an array
$\mathrm{diag}(*)$: Diagonal elements of matrix
$\mathrm{round}(*)$: Round to nearest integer.

Data Availability

The data of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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