Research Article

Nonlinear Compound Control for Nonsinusoidal Vibration of the Mold Driven by Servo Motor with Variable Speed

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Received 24 April 2019; Revised 3 September 2019; Accepted 10 September 2019; Published 22 October 2019

Academic Editor: Francisc Pozo

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In this paper, a fuzzy PI control method based on nonlinear feedforward compensation is proposed for the nonsinusoidal vibration system of mold driven by servo motor, rotated in single direction with variable speed. During controller design, there are mainly two issues to consider: (i) nonlinear relationship (approximate periodic function) between mold displacement and servo motor speed and (ii) uncertainties caused by backlash due to motor variable speed. So, firstly, the relationship between mold displacement and motor rotation speed is built directly based on the rotation vector method. Then, an observer is designed to estimate the uncertainties and feedforward compensation. Secondly, as the motor rotates in single direction with variable speed, a fuzzy control with bidirectional parameter adjustment is adopted to improve rapidity and stability based on the traditional PI method. Finally, some simulation results show the effectiveness of the proposed control method.

1. Introduction

Nonsinusoidal vibration of continuous casting mold is one of the key technologies to develop efficient continuous casting [1, 2]. It is a new drive mode that servo motor, rotated in single direction with variable speed, drives continuous casting mold vibrated in nonsinusoidal waveform [3]. Compared with the existing modes, it has a simplified and compact structure, long service life, and energy-saving property and is of convenient maintenance. In this system, there are mainly three parts: a servo motor as an actuator, transmission mechanism (including a gear reducer, an eccentric shaft, and a connector), and a mold. Through the transmission mechanism, when the servo motor rotates at a constant speed, the mold vibrates in sinusoidal form. When the servo motor rotates at variable speed, the mold vibrates in a nonsinusoidal manner. This paper mainly considers the controller design in case of mold nonsinusoidal vibration. The tracking control of mold vibration displacement is realized by the control of servo motor speed. To develop the corresponding controller, two major issues need to be

addressed: The first issue is nonlinear relationship [4] (approximate sine periodic function and the inverse mapping is not unique) between servo motor speed and mold displacement. Second, the transmission mechanism is realized by gear meshing, and there is backlash between the gears to avoid being stuck due to the heating and expansion caused by friction. It is necessary to compensate the uncertainties caused by backlash, while the servo motor rotated in single direction with variable speed.

Since the mold displacement is in the periodic form, if the expected mold displacement is taken as the given signal directly and the closed-loop control is carried out, the servo motor rotation speed will change alternately positive and negative, which cannot meet the technological requirements for the motor. As to that problem, the piecewise mapping function [5, 6] is used to convert the mold displacement to motor angular displacement by using the arcsine function to keep the mapping unique. In fact, the tracking error of mold displacement is mitigated by adjusting the motor speed. If the relationship between motor speed and mold displacement can be established directly, the tracking control of

mold vibration displacement can be converted to servo motor speed control. In this paper, a new mathematical algorithm based on the rotation vector method is proposed to build the relationship directly.

On the other hand, under the integral action of the eccentric shaft, the backlash may cause the accumulation disturbance to mold displacement. Generally, when the backlash's width is known, the optimal control [7] or adaptive control method [8, 9] could be adopted. Actually, the backlash is nonlinear and difficult to establish a model. In [10–12], the backlash is seen as a black box or bounded disturbance and compensated by robust terms. In [13, 14], the adaptive fuzzy and neural estimated inverse control method are proposed to mitigate the hysteresis nonlinearities for actuator and effective to promote tracking performance. Although the backlash exists in transmission mechanism (not the actuator), the inverse model of the transmission mechanism could be established based on the mathematical algorithm as an observer to estimate the effect of backlash on mold vibration displacement and compensated by adjusting the motor speed.

For the motor speed closed-loop control, the PI control method has advantages of simple structure and easy implementation to the motor control. As to the fixed parameters in traditional PI, the combination of fuzzy control and PI has strong robustness to system parameter perturbation and disturbance [15–17]. In view of the mold nonsinusoidal vibration, the servo motor rotates in single direction and variable speed, which means the motor needs to continuously accelerate and decelerate. However, in traditional fuzzy rules, the fuzzy gain adjustment direction is only one way when the speed error is positive or negative, which makes it only applicable to one-way speed regulation (or only applicable to the accelerating, the decelerating will produce large overshoot). So, it needs the fuzzy gain has two-way speed regulation to improve rapidity and stability.

In this paper, a fuzzy PI control strategy based on nonlinear feedforward compensation is proposed for the mold vibration system. The main contributions are summarized as follows:

- (1) A mathematical algorithm is proposed to build the relationship between mold displacement and servo motor speed directly. Based on the algorithm, an observer is designed to estimate the effect of the backlash and other factors on mold displacement and feedforward compensate.
- (2) In comparison to [6], the tracking control of mold vibration displacement could be converted to servo motor speed control.
- (3) As to the mold nonsinusoidal vibration, the servo motor rotates in single direction and continuously accelerates and decelerates. Combined with the PI control method, a fuzzy control method with bidirectional parameter adjustment is adopted to improve rapidity and stability for the motor speed control.

The rest of this paper is organized as follows. The mathematical model of the mold vibration system is

analyzed in Section 2. The main results and theoretical analysis are given in Section 3. Simulation results are given in Section 4, followed by Section 5 that concludes the work.

2. Mathematical Model of the Mold Vibration System and Problem Statement

The continuous casting mold is driven by the servo motor through the coupling, reducer, eccentric shaft, and connecting rod. The device structure drawing is shown in Figure 1.

The model of servo motor is expressed as follows:

$$\begin{cases} \dot{n} = \frac{1.5p\psi_f}{J} \frac{60}{2\pi} i_q - \frac{B}{J} n - \frac{60}{2\pi} \frac{T_L}{J}, \\ i_q = -\frac{2\pi}{60} p n i_d - \frac{R_s}{L} i_q - \frac{p\psi_f}{L} \frac{2\pi}{60} n + \frac{u_q}{L}, \\ i_d = -\frac{R_s}{L} i_d + \frac{2\pi}{60} p n i_q + \frac{u_d}{L}, \end{cases} \quad (1)$$

where n is the rotate speed of the motor, i_d and i_q are stator d - and q -axes currents, u_d and u_q are the stator d - and q -axes voltages, p is the pole pair numbers, R_s is the stator resistance, L is the stator inductance, ψ_f is the flux linkage, J is the rotor inertia, B is the viscous friction coefficient, and T_L is the load torque.

In order to decouple the speed and currents, the vector control strategy of $i_d^* = 0$ is used. Here, two PI controllers, which are used to stabilize the d - q axes current errors, are adopted in the two current loops, respectively. In this paper, the speed loop controller is designed mainly.

The transmission mechanism mainly comprises a reducer, an eccentric shaft, and a connecting rod. The reducer realizes transmission through gear meshing. The backlash is essential to avoid being stuck caused by tooth friction, heating, and expansion. When the servo motor rotates at constant speed or accelerates in one direction, the backlash can be ignored and the reduction ratio is fixed. When the servo motor rotates in single direction with continuous acceleration and deceleration, the backlash may affect the speed regulation system just as a hysteresis disturbance. The impact is shown in Figure 2.

Under the integral action of the eccentric shaft, the backlash may cause the accumulation disturbance to the angle of the eccentric shaft. Meanwhile, the initial mechanical zero deviation will also cause the initial phase difference to the angle. So the equation of the mold vibration displacement can be expressed as [6]

$$S = h \sin\left(\frac{1}{i + \Delta i} \int_0^t \frac{2\pi n}{60} dt + d\right), \quad (2)$$

where S is the mold displacement, n is the actual motor speed, i is the transmission ratio, Δi is the uncertainty caused by the backlash and other factors, and d is the initial phase offset of the eccentric shaft as a constant.

Through the analysis of the mathematical model, the expected motor speed n^* is transmitted to the speed

FIGURE 1: Device structure drawing of the continuous casting mold vibration system driven by the servo motor.

FIGURE 2: The influence of backlash while the servo motor rotated with continuous acceleration and deceleration.

controller, which usually adopted the PI method with simple structure and easy implementation. The mold vibration displacement control is completed by mechanical transmission parts following the servo motor speed. The overall block diagram is shown in Figure 3.

In order to realize the mold nonsinusoidal vibration, n^* can be calculated by expected mold displacement. For example, the mold nonsinusoidal vibration waveform uses the Demark nonsinusoidal equation:

$$S^* = h \sin(\omega t - A \sin(\omega t)). \quad (3)$$

The corresponding angular velocity of the eccentric shaft is

$$V(t) = \omega(1 - A \cos(\omega t)), \quad (4)$$

where $\omega = (2\pi/60)f$, in which f is the nonsinusoidal vibration frequency of the mold. $A = \pi\alpha/(2 \sin((\pi/2)(1 + \alpha)))$, where α is the wave slope, $\alpha \in [0, 1)$. Then, n^* can be expressed as

$$\begin{aligned} n^* &= \frac{60iV(t)}{(2\pi)} \\ &= \frac{60i\omega(1 - A \cos(\omega t))}{(2\pi)}. \end{aligned} \quad (5)$$

The expected mold nonsinusoidal vibration velocity waveform and the corresponding servo motor speed are shown in Figure 4.

According to Figure 4, the servo motor rotates in single direction ($n > 0$) and continuously accelerates and decelerates to realize the mold nonsinusoidal vibration.

However, the following two problems need to be considered during the controller design:

FIGURE 3: Diagram of the mold driven by servo motor.

- (i) The nonlinear links exist in the system: first, there are uncertainties such as backlash, initial phase deviation, and time-varying load disturbance. So the mechanical transmission part $N(\cdot)$ is nonlinear and unknown. Second, there is an essential nonlinear periodic function relationship between the servo motor speed n and the mold displacement S in the forward control channel.
- (ii) The uncertainties need to be compensated by adjusting the motor speed. It is a common control strategy to combine the fuzzy control with the PI method to improve control performance. However, the traditional fuzzy PI is mostly suitable for single-direction speed regulation and will produce large overshoot when the servo motor continuously accelerates and decelerates.

3. Design of the Nonlinear Compound Controller

The control purpose is to realize the tracking control of mold vibration displacement. But the elimination of vibration displacement tracking error is realized by adjusting the motor speed. In this paper, the servo motor is taken as the control object and the controller is mainly of two parts: one is the mathematical algorithm, which constructs the functional relationship between the mold displacement and the motor speed directly. Based on the algorithm, an observer is designed to estimate the uncertainties and feedforward compensation. The other is the fuzzy control with bidirectional parameter adjustment. It can improve rapidity and stability in case of continuous acceleration and deceleration for the servo motor. The block diagram is shown in Figure 5.

3.1. Nonlinear Mathematical Algorithm. Let θ represent the angular displacement of the eccentric shaft. Then, $\theta = (1/(i + \Delta i)) \int_0^t (2\pi n/60) dt + d$. So it follows that

$$\dot{\theta} = \omega_p = \frac{2\pi n_p}{60i}, \quad (6)$$

where ω_p is the angular velocity of the eccentric shaft corresponding to θ and n_p is defined as the motor speed corresponding to θ . In (6), it is a linear correspondence from n_p to θ . Formula (2) can be expressed as

$$S = h \sin(\theta). \quad (7)$$

According to the rotating vector method as shown in Figure 6, every point of the sine or cosine curve is one to one correspondence with the reference circle's position. So, it is

FIGURE 4: Expected mold nonsinusoidal vibration velocity waveform and the corresponding motor speed ($\alpha = 0.24$). (a) Mold nonsinusoidal vibration velocity waveform. (b) Servo motor speed waveform.

FIGURE 5: Block diagram of the compensation control scheme.

FIGURE 6: Corresponding relation between the sine (or cosine) function and the reference circle.

concluded that a point of the reference circle corresponds to a position of the mold, which can avoid the multsolution of sine or cosine function in the single cycle.

Theorem 1. As to (6) and (7), according to the rotating vector method, the functional relationship between the eccentric shaft angular velocity ω_p and the mold displacement S can be expressed as

$$\begin{aligned} \dot{\theta}_p &= \omega_p \\ &= \omega_{p0} + |\dot{S}| z_r^{-0.5} [\operatorname{sgn}(z_r) + z_r^{0.5} (1 - \operatorname{sgn}(z_r))] - \operatorname{sgn}(z_r) \omega_{p0}, \end{aligned} \quad (8)$$

where $z_r = h^2 - S^2$ and $\omega_{p0} = \omega_0 (1 + (\pi\alpha/2)\tan(\pi\alpha/2))$ is constant, $\alpha \in [0, 1)$.

Proof. $\square$

Case 1. $|S| \neq h$ and $\sin \theta \neq \pm 1$.

The time derivative of (7) is derived as

$$\dot{S} = h \dot{\theta} \cos(\theta). \quad (9)$$

Since the servo motor rotates continuously in one direction, then according to the rotation vector method, it can be calculated that

$$\dot{\theta} = \omega_p = \frac{\dot{S} \operatorname{sgn}(\dot{S})}{\sqrt{h^2 - S^2}}. \quad (10)$$

Case 2. $|S| = h$ and $\sin \theta = \pm 1$.

According to formula (4), the corresponding angular velocity can be obtained as follows:

$$\omega_{p0} = \omega (1 - A \cos(\omega t_0)), \quad (11)$$

where t_0 is the time when the nonsinusoidal vibration displacement curve reaches its peak and t_0 is not unique.

Considering the motor single-direction rotation, ω_{p0} is a positive constant. Only considering the time when the vibration displacement reaches the peak upward in this paper, it follows that

$$\omega t_0 - A \sin(\omega t_0) - \frac{\pi}{2} = 0. \quad (12)$$

The wave slope α is defined as

$$\alpha = \frac{4t_m}{T} = \frac{4(t_0 - (T/4))}{T} = \frac{4t_0}{T} - 1 = \frac{2\omega t_0}{\pi} - 1, \quad (13)$$

where T is the vibration cycle and t_m is the lagging time, which is the time difference to the peak displacement between nonsinusoidal waveform and sinusoidal waveform. Then, it follows that

$$A = \frac{\pi\alpha}{2 \sin((\pi/2)(1 + \alpha))} = \frac{\pi\alpha}{2 \cos(\pi\alpha/2)}. \quad (14)$$

From (13) and (14), one has

$$A \cos\left(\frac{\pi\alpha}{2}\right) = A \cos\left(\omega t_0 - \frac{\pi}{2}\right) = A \sin(\omega t_0) = \frac{\pi\alpha}{2}. \quad (15)$$

From (15), $\sin(\omega t_0) = (\pi\alpha/2A)$ and $\cos(\omega t_0) = -\sqrt{1 - (\pi\alpha/2A)^2}$. Then,

$$A \cos(\omega t_0) = -\sqrt{A^2 - \left(\frac{\pi\alpha}{2}\right)^2}. \quad (16)$$

Substituting (14) and (16) into (11), it follows that

$$\begin{aligned} \omega_{p0} &= \omega(1 - A \cos(\omega t_0)) = \omega \left(1 + \sqrt{A^2 - \frac{(\pi\alpha)^2}{4}}\right) \\ &= \omega \left(1 + \frac{\pi\alpha}{2} \sqrt{\frac{1}{(\cos^2(\pi\alpha/2)) - 1}}\right) \\ &= \omega \left(1 + \frac{\pi\alpha}{2} \tan\left(\frac{\pi\alpha}{2}\right)\right). \end{aligned} \quad (17)$$

Therefore, when S reaches the peak, ω_{p0} is related to ω and α , which are the same to the given signal.

From (10) and (17), it follows that

$$\dot{\theta}_p = \omega_p = \begin{cases} \frac{\dot{S} \operatorname{sgn}(\dot{S})}{\sqrt{h^2 - S^2}}, & |S| \neq h, \sin \theta_p \neq \pm 1, \\ \omega_{p0}, & |S| = h, \sin \theta_p = \pm 1. \end{cases} \quad (18)$$

Define $z_r = h^2 - S^2$ and convert equation (18) into an equation expressed as follows:

$$\begin{aligned} \dot{\theta}_p &= \omega_p \\ &= \omega_{p0} + |\dot{S}| z_r^{-0.5} [\operatorname{sgn}(z_r) + z_r^{0.5} (1 - \operatorname{sgn}(z_r))] - \operatorname{sgn}(z_r) \omega_{p0}, \end{aligned} \quad (19)$$

where $\omega_{p0} = \omega(1 + (\pi\alpha/2)\tan(\pi\alpha/2))$ is constant and only related to ω and α , $\alpha \in [0, 1]$. This completes the proof.

Based on Theorem 1 and formula (6), n_p can be calculated as

$$\begin{aligned} n_p &= N^{-1}(S) \\ &= \frac{60i}{2\pi} \left\{ \omega_{p0} + |\dot{S}| z_r^{-0.5} [\operatorname{sgn}(z_r) + z_r^{0.5} (1 - \operatorname{sgn}(z_r))] - \operatorname{sgn}(z_r) \omega_{p0} \right\}. \end{aligned} \quad (20)$$

Remark 1. If S and $\dot{S}$ are known, the corresponding servo motor speed can be calculated based on equation (20).

To mitigate the influence of disturbance in the transmission mechanism part, a disturbance observer is designed based on the mathematical algorithm as shown in Figure 7. n_p is the motor speed corresponding to the actual mold displacement S . The difference between n_p and n can be regarded as the effect of the backlash and other factors on the speed regulation system and can be compensated through feedforward. Q is a low-pass filter. The forward channel is a high gain to suppress the influence of disturbance on the output. Q is adopted as

$$Q(s) = \frac{3(\lambda s + 1)}{(\lambda s)^3 + 3(\lambda s)^2 + 3\lambda s + 1}, \quad (21)$$

where λ is the time constant and s is the complex parameter.

FIGURE 7: Block diagram of the disturbance observer.

FIGURE 8: Structure diagram of the fuzzy controller.

3.2. Fuzzy PI Controller. As the servo motor rotates in a single direction and continuously accelerates and decelerates to realize the mold nonsinusoidal vibration, new fuzzy rules with bidirectional speed regulation are adopted for the PI self-tuning to improve rapidity and stability of the servo motor speed control. The structure diagram of the fuzzy controller is shown in Figure 8.

The inputs of fuzzy controller are speed error e and error rate ec , and the outputs are the adjustment of PI control parameters ΔK_p and ΔK_i . The final PI parameters are

$$\begin{cases} K_p = K'_p + \Delta K_p, \\ K_i = K'_i + \Delta K_i, \end{cases} \quad (22)$$

where K'_p and K'_i are initial setting values of proportion and integral coefficient, respectively.

As to input variables and output values, seven linguistic variables in its domain are taken: NB, NM, NS, Z, PS, PM, and PB, which mean negative big, negative medium, negative small, zero, positive small, positive medium, and positive big. The normalized domain corresponds to $\{-3, -2, -1, 0, 1, 2, \text{ and } 3\}$.

The amplitude of the mold displacement is 3 mm, and the error e is mainly in $[-3, 3]$. So the range of e is set as $[-3, 3]$ (if the actual value exceeds the range, take the limiting value). According to the mold vibration model, the ec range can be measured as $[-80, 80]$. The quantitative factors of e and ec are 1 and 0.0375, respectively.

According to the experience of PI parameter setting, it should be adjusted quickly when the error is large and it requires fine adjustment when the error is small. Therefore, the membership function should be close to the equilibrium point and sparse on both sides of the fuzzy domain, which will increase the amount of program implemented [18, 19]. In this paper, the gain coefficient [20] is introduced to adjust the parameters based on the symmetric membership function.

The gain coefficients of ΔK_p and ΔK_i are β_1 and β_2 , respectively, and the setting principle is as follows:

- (1) When $|e|$ is large, take larger β_1 and β_2 to speed up the response speed.
- (2) When $|e|$ is medium large, reduce β_1 and β_2 to make the change of proportion and integral coefficient not be dramatic.
- (3) When $|e|$ is small, in the case of ec larger, appropriately increase β_1 and reduce β_2 to prevent K_I from increasing too fast to produce overshoot. In the case of ec being medium or small, increase β_2 to improve the integral effect.

In this paper, the new fuzzy control rules with bi-directional output adjustment are adopted to improve the responsiveness. With the rotational speed error as the main criterion, K_p and K_I can be further refined as follows:

- (1) When $|e|$ is large, the tracking requirement can be achieved only through the proportional component due to the large error. Even if the integral is introduced, the response cannot be further improved, while the overshoot may appear. So K_p can be larger and K_I can be zero.
- (2) When $|e|$ is a medium value, take smaller K_p to reduce overshoot. At the same time, add K_I slowly to converge the static difference that cannot be eliminated by the proportional and avoid the integral saturation.
- (3) When $|e|$ is small, increasing K_p and K_I is beneficial to improve the system steady state performance and enhance the antidisturbance ability.

The effect of the time-varying load and the friction should be considered in the fuzzy control rules. The direction of the load and friction is always opposite to the motor. In the acceleration phase, the friction force and load torque will decrease $|ec|$, while in the deceleration phase it will increase $|ec|$. The larger $|ec|$ is, the easier it is to cause overshoot. Therefore, the fuzzy control rules are adjusted as follows:

- (1) Reducing the lower limit of K_p in the deceleration section could decrease the increasing rate of K_p . It could also reduce the torque of the motor when $|e|$ is a medium or small value, so as to reduce overshoot and make the control effect close to the acceleration section.
- (2) The rules for K_I are basically the same as that in the acceleration section. In the deceleration section, K_I increases from zero and the value after the system deceleration overshoot is much larger than that before the overshoot. So the integral output can quickly change from negative to positive and reach the required value when the speed is stable, thus reducing the overshoot.

According to the above rules and combined with the actual situation of the control system, ΔK_p and ΔK_I fuzzy rules are shown in Tables 1 and 2.

The weighted average method is often used in ambiguity resolution to obtain the modified parameter values ΔK_p and ΔK_I .

TABLE 1: Fuzzy control rules of ΔK_p .

ec	e						
	NB	NM	NS	ZO	PS	PM	PB
NB	PB	NS	ZO	PS	NM	NB	PS
NM	PM	NM	ZO	PS	NM	NB	PS
NS	PM	NM	ZO	PM	NS	NM	PM
ZO	PS	NB	NS	PB	NS	NS	PM
PS	PS	NB	NS	PM	ZO	ZO	PB
PM	PS	NB	NM	PS	ZO	PS	PB
PB	PS	NB	NM	PS	ZO	PS	PB

TABLE 2: Fuzzy control rules of ΔK_I .

ec	e						
	NB	NM	NS	ZO	PS	PM	PB
NB	NB	NM	ZO	PB	PB	ZO	NM
NM	NB	NM	ZO	PB	PM	ZO	NM
NS	NB	NS	PS	PB	PM	ZO	NB
ZO	NB	NS	PS	PB	PS	ZO	NB
PS	NB	ZO	PM	PB	PS	ZO	NB
PM	NB	ZO	PM	PB	ZO	NS	NB
PB	NB	ZO	PB	PB	ZO	NS	NB

TABLE 3: Parameters of the servo motor.

Parameters of the servo motor	Value
Rated power (kW)	20.4
Rated current (A)	45
Rated speed (r/min)	1500
Stator inductance (mH)	4.6
Stator resistance (Ω)	0.14
Number of pole pairs	3
Rotor flux linkage (Wb)	0.704
Rotor inertia ($\text{kg}\cdot\text{m}^2$)	0.1110

4. Simulation Research

Some simulations are used to illustrate the effectiveness of the proposed control method.

The mechanical transmission part parameters are as follows: reduction ratio: $i = 5$; amplitude of the mold: $h = 3\text{ mm}$. The nominal values of the servo motor parameters (Table 3) are selected as follows [4].

The servo motor's speed loop and current loop use the same set of parameters: $K_{pn} = 3.894\text{ A}\cdot\text{s/rad}$, $\tau_n = 43.2\text{ ms}$, $K_{piq} = 12.982\text{ V/A}$, $\tau_{iq} = 2\text{ ms}$, $K_{pid} = 12.982\text{ V/A}$, and $\tau_{id} = 2\text{ ms}$. In filter Q , $\lambda = 0.002$.

The control purpose is to realize the tracking control of mold vibration displacement, and the servo motor is taken as the controller object. The expected signal for the mold is taken as Demark nonsinusoidal function: $S^* = h \sin(\omega t - A \sin(\omega t))$. The corresponding given servo motor speed is $n^*(t) = 60i\omega(1 - A \cos(\omega t))/(2\pi)$, where $\omega = 2\pi f/60$. In this paper, $h = 3\text{ mm}$, $\alpha = 0.24$, and $f = 130\text{ times/min}$.

The load disturbance is adopted as follows [6]:

$$T_L = (5.1335 + 6.4985 \sin(\omega t - A \sin \omega t)) \text{ Nm}. \quad (23)$$

FIGURE 9: (a) Tracking performance with and without compensation. (b) Tracking errors with and without compensation.

FIGURE 10: Mapping function between motor speed and mold vibration displacement.

FIGURE 12: Comparison of servo motor speed tracking errors of the proposed fuzzy PI control and the traditional PI control.

FIGURE 11: Curves of servo motor speed with feedforward compensation.

According to common requirements of machining precision $\pm 3\%$, it takes the worst situation: $\Delta i = 3\%i$. The initial zero offset of the eccentric shaft is $d = -0.2$ rad.

To show the effectiveness of the proposed controller, the simulation is conducted under two cases: with and without nonlinear feedforward compensation. The simulation results are illustrated in Figures 9–12. Figure 9 clearly shows that the tracking error when the nonlinear feedforward compensation applied is much better than the case without the compensation. It should be emphasized that Figure 9 sufficiently illustrates the validity of the proposed control scheme because of the precisely tracking performance and small tracking error.

Based on the nonlinear mathematical algorithm shown as equation (20), the mapping between the mold displacement and the servo motor speed is constructed to meet the

technological requirements for the motor rotated in single direction and unique mapping. Figure 10 is the mapping curve and illustrates the algorithm effective.

Figure 11 shows the given motor speed and the actual motor speed with feedforward compensation, respectively. The difference between the actual motor speed and the given speed is used to compensate the uncertainties caused by the backlash and other factors in mechanical transmission parts.

Figure 12 illustrates the servo motor speed tracking errors of the proposed fuzzy PI control scheme and the traditional PI control. It should be noted that the traditional PI method has a large overshoot than the proposed control scheme when the motor changes direction.

5. Conclusion

In this paper, a fuzzy PI control strategy based on nonlinear feedforward compensation is proposed for the mold displacement system. Firstly, a mathematical algorithm is proposed to build the relationship between mold displacement and servo motor speed. Based on the algorithm, an observer is designed to estimate the disturbance and feedforward compensation. Secondly, a fuzzy control method with bidirectional parameter adjustment is adopted compound with the PI control method to improve rapidity and stability of the servo motor speed control. Finally, the simulation results show the proposed method is effective.

The mathematical algorithm is mainly based on the Demark nonsinusoidal function in this paper. Future works will focus on the algorithm applying to other nonsinusoidal vibration displacement waveforms, such as piecewise waveform and composite waveform.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no competing interests regarding the publication of this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (no. 61873226), the National Science Foundation of Hebei Province (no. F2017203304), and the Hebei province Talent Project Training Funds of Hebei Province (no. A2016015002).

References

- [1] X. K. Li, L. P. Zhang, L. D. Yang, and Y. F. Yao, "Dynamics study on new non-sinusoidal oscillation system of mold," *Journal of Iron and Steel Research International*, vol. 15, no. S1, pp. 517–522, 2008.
- [2] C. Zhou, X.-Z. Zhang, F. Wang, P.-F. Liu, and Y.-M. Fang, "Mechanism analysis of non-sinusoidal oscillation of continuous casting mold synchronously driven by double servomotors," *Journal of Iron and Steel Research International*, vol. 24, no. 3, pp. 251–257, 2017.
- [3] L. Liu, Y. R. Dun, Y. M. Fang, and G. Y. Li, "Modeling and verification of the nonlinear system of oscillation platform of continuous casting mold driven by servo motor," *Advances in Mechanical Engineering*, vol. 8, no. 7, pp. 1–9, 2016.
- [4] Y. M. Fang, G. Y. Li, J. X. Li, and L. Liu, "The model and analysis for displacement system of the continuous casting mold driven by servo motor," *Chinese Journal of Scientific Instrument*, vol. 35, no. 11, pp. 2615–2632, 2014.
- [5] H. C. Zheng, L. Liu, Y. M. Fang, and Q. Li, "Nonlinear auto disturbance rejection control for vibration displacement system of continuous cast mold driven by servo motor," in *Proceedings of the 35th Chinese Control Conference, CCC2016*, pp. 879–884, Chengdu, China, July 2016.
- [6] K. S. Kang, L. Liu, Y. M. Fang, and H. C. Zheng, "Backstepping sliding mode control for continuous cast mold oscillation displacement system driven by servo motor," *Control Theory & Applications*, vol. 33, no. 11, pp. 1442–1448, 2016.
- [7] G. Tao, X. Ma, and Y. Ling, "Optimal and nonlinear decoupling control of systems with sandwiched backlash," *Automatica*, vol. 37, no. 2, pp. 165–176, 2001.
- [8] A. Taware and G. Tao, "An adaptive dead-zone inverse controller for systems with sandwiched dead-zones," *International Journal of Control*, vol. 76, no. 8, pp. 755–769, 2003.
- [9] X. Sun and L. F. Zhou, "Discrete-time adaptive control of linear system with backlash nonlinearity," *Control Theory & Applications*, vol. 8, no. 1, pp. 96–100, 1991.
- [10] J. Guo, B. Yao, Q. W. Chen, and J. Jiang, "Adaptive robust control for nonlinear system with input backlash or backlash-like hysteresis," in *Proceedings of the IEEE International Conference on Control and Automation*, pp. 1962–1967, IEEE, Christchurch, New Zealand, December 2009.
- [11] C. Y. Su, M. Oya, and H. Hong, "Stable adaptive fuzzy control of nonlinear systems preceded by unknown backlash-like hysteresis," *IEEE Transactions on Fuzzy Systems*, vol. 11, no. 1, pp. 1–8, 2003.
- [12] C. Wen and J. Zhou, "Decentralized adaptive stabilization in the presence of unknown backlash-like hysteresis," *Automatica*, vol. 43, no. 3, pp. 426–440, 2007.
- [13] X. Y. Zhang, Y. Wang, X. K. Chen et al., "Decentralized adaptive neural approximated inverse control for a class of large-scale nonlinear hysteretic systems with time delays," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2018.
- [14] X. Y. Zhang, Y. Wang, C. L. Wang et al., "Adaptive estimated inverse output-feedback quantized control for piezoelectric positioning stage," *IEEE Transactions on Cybernetics*, vol. 49, no. 6, pp. 2016–2118, 2019.
- [15] A. Savran and G. Kahraman, "A fuzzy model based adaptive PID controller design for nonlinear and uncertain processes," *ISA Transactions*, vol. 53, no. 2, pp. 280–288, 2013.
- [16] M. Rosyadi, S. M. Mueen, R. Takahashi, and J. J. Tamura, "Fuzzy-PI controller design for PM wind generator to improve fault ride through of wind farm," *International Journal of Renewable Energy Research*, vol. 3, no. 2, pp. 308–314, 2013.
- [17] J.-X. Xu, C.-C. Hang, and C. Liu, "Parallel structure and tuning of a fuzzy PID controller," *Automatica*, vol. 36, no. 5, pp. 673–684, 2000.
- [18] P. Pei, Z. C. Pei, Z. Y. Tang, and H. Gu, "Position tracking control of PMSM based on fuzzy PID- variable structure adaptive control," *Mathematical Problems in Engineering*, vol. 2018, Article ID 5794067, 5 pages, 2018.

- [19] S. B. Cai, S. L. Wu, and F. J. Bao, "Position tracking control of PMSM based on fuzzy PID- variable structure adaptive control," *Mathematical Problems in Engineering*, vol. 2013, Article ID 375483, 2013.
- [20] H. Peng, J. Z. Wang, W. Shen, and D. Y. Li, "Double fuzzy control with compensating factor for electronic-hydraulic servovalve-controlled system," *Journal of Mechanical Engineering*, vol. 53, no. 24, pp. 184–192, 2017.

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