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Research Article

Dynamical Analysis and Optimal Harvesting Strategy for a Stochastic Delayed Predator-Prey Competitive System with Lévy Jumps

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This paper develops a theoretical framework to investigate optimal harvesting control for stochastic delay differential systems. We first propose a novel stochastic two-predator and one-prey competitive system subject to time delays and Lévy jumps. Then we obtain sufficient conditions for persistence in mean and extinction of three species by using the stochastic qualitative analysis method. Finally, the optimal harvesting effort and the maximum of expectation of sustainable yield (ESY) are derived from Hessian matrix method and optimal harvesting theory of delay differential equations. Moreover, some numerical simulations are given to illustrate the theoretical results.

1. Introduction

Optimal control problem in the field of biological mathematics has been widely concerned by researchers. Resource exploitation always aims to maximum sustainable yield (MSY) or the profit associated with the maximum economic yield (MEY) [1]. How to obtain MSY or MEY involves the optimal harvesting control problem. Therefore, it is interesting and meaningful to investigate optimal harvesting control strategies for biological population models, especially stochastic population models. A number of researchers have investigated single-species or two-species population models [2–4]. However, the above two classes of models can not describe some natural phenomena completely and it is believed that models with three or more species can explain the dynamical behaviors of the population accurately [5–8]. Predator-prey models are arguably known as the most fundamental building blocks of the biosystems and ecosystems as the biomasses are grown out of their resource, which have been widely investigated [9–17]. As we all know, after a predator catches and feeds on a prey, the number of the predators will not increase at once, which needs the processes of digestion and absorption. Therefore, the time delays were considered in many differential systems [18–21], especially stochastic delay models [22–29].

The basic model we consider is based on the delay Lotka-Volterra model with two competitive predators and one prey. We propose the following model by assuming that the random perturbations of intrinsic growth rate are subjected to Gaussian white noise [30]:

$$\begin{aligned}
\mathrm{d}x(t) &= x(t)\left[\bar{r}_1 - a_{11}x(t) - a_{12}y_1(t-\tau_{12})\right.\\
&\quad \left. - a_{13}y_2(t-\tau_{13})\right]\mathrm{d}t + \bar{\sigma}_1 x(t)\,\mathrm{d}B_1(t),\\
\mathrm{d}y_1(t) &= y_1(t)\left[\bar{r}_2 - a_{21}e^{-d_{21}\tau_{21}}x(t-\tau_{21}) - a_{22}y_1(t)\right.\\
&\quad \left. - a_{23}e^{-d_{23}\tau_{23}}y_2(t-\tau_{23})\right]\mathrm{d}t + \bar{\sigma}_2 y_1(t)\,\mathrm{d}B_2(t),\\
\mathrm{d}y_2(t) &= y_2(t)\left[\bar{r}_3 - a_{31}e^{-d_{31}\tau_{31}}x(t-\tau_{31})\right.\\
&\quad \left. - a_{32}e^{-d_{32}\tau_{32}}y_1(t-\tau_{32}) - a_{33}y_2(t)\right]\mathrm{d}t\\
&\quad + \bar{\sigma}_3 y_2(t)\,\mathrm{d}B_3(t),
\end{aligned} \tag{1}$$

with initial value

$$x(\theta) = \varphi_0(\theta),$$

$$
\begin{aligned}
&y_1(\theta) = \varphi_1(\theta),\\
&y_2(\theta) = \varphi_2(\theta),\\
&\qquad\qquad \theta \in [-\tau_0, 0],\\
&\tau_0 = \max_{i,j=1,2,3}\{\tau_{ij}\},
\end{aligned}
\tag{2}
$$

where $x(t)$ denotes the number of the prey species at time t and $y_i(t)$ (i=1, 2) denote the predator species at time t. $\overline{r}_1 > 0$ stands for the growth rate of prey. $\overline{r}_2 < 0$ and $\overline{r}_3 < 0$ stand for the death rates of the two predators, respectively. $a_{ii} > 0$ is the interspecific competition rate of species i, $i = 1, 2, 3$, and $a_{23}e^{-d_{23}\tau_{23}} > 0$ and $a_{32}e^{-d_{32}\tau_{32}} > 0$ are the competition rates of two predators. a_{12} and a_{13} are positive representing capture rates, and $a_{21}e^{-d_{21}\tau_{21}}$ and $a_{31}e^{-d_{31}\tau_{31}}$ are negative representing the growth rates from prey relatively. $\tau_{ij} \geq 0$ signifies the time delay, and $\varphi_i(\theta) > 0$ ($i = 0, 1, 2$) are continuous functions on $[-\tau_0, 0]$. $B_i(t)$ (i=1,2,3) are mutually independent Brownian motion with $B_i(0) = 0$ defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ that satisfies that it is right continuous and increasing with $\mathcal{F}_0$ that contains all $\mathcal{P}$-null sets.

Various harvesting models have been used to investigate the optimal harvesting policies of renewable resources (e.g., Beddington and May [30], Neubert [31]). Recently, many authors explored the optimal harvesting models [32–37]. According to the catch-per-unit-effort (CPUE) hypothesis, we consider that predators y_1 and y_2 are subject to exploitation with harvesting effort rates $h_1 > 0$ and $h_2 > 0$, respectively.

However, in some cases, models just perturbed by the Gaussian white noise can not effectively and efficiently describe the circumstance when the species suffer sudden catastrophic disturbance in nature. The sudden environmental change can affect the dynamical behavior of the species significantly. Therefore, it is necessary to use the discontinuous stochastic process (e.g., Lévy jump) to model the abrupt nature phenomenon in ecosystem [38–43].

To introduce the Lévy jump into the underlying stochastic model (1), we first give some facts about the Lévy jump [33]. Generally, a Lévy process $L_i(t)$ can be decomposed into the sum of a linear drift, a Brownian motion $B_i(t)$, and a superposition of centered Poisson processes with different jump sizes $\lambda(\mathrm{d}v)$ which is the rate of arrival of the Poisson process with jump of size v. According to the Lévy decomposition theorem [44], we know that

$$
L_i(t) = \widetilde{a}_i t + \widetilde{\sigma}_i B_i(t) + \int_{\mathbb{Y}} v\widetilde{N}(t, \mathrm{d}v), \quad i = 1, 2, \ldots, n, \tag{3}
$$

where $\widetilde{a}_i \in R$, $\widetilde{\sigma}_i > 0$, $\widetilde{N}(t, \mathrm{d}v) = N(t, dv) - \lambda(\mathrm{d}v)t$ is a compensated Poisson process, and N is a Poisson measure with characteristic measure λ on a measurable subset $\mathbb{Y}$ of $(0, +\infty)$ with $\lambda(\mathbb{Y}) < \infty$. The distribution of a Lévy process $L_i(t)$ has the property of infinite divisibility and is characterized by its characteristic function $\phi_{L_i}(t)$, which is given by the following Lévy-Khintchine formula [45]:

$$
\begin{aligned}
\phi_{L_i}(t) &:= \mathbb{E}\left[e^{\iota t L_i}\right]\\
&= \exp\left\{\widetilde{a}_i \iota t - \frac{1}{2}\widetilde{\sigma}_i^2 t^2 + \int_{\mathbb{Y}}\left[e^{\iota t v} - 1 - \iota t v I_{|v|<1}\right]\lambda(\mathrm{d}v)\right\},
\end{aligned}
\tag{4}
$$

where I_G is the indicator function of set G and ι is the imaginary unit. The distribution of Lévy jump $L_i(t)$ can be completely parameterized by $(\widetilde{a}_i, \widetilde{\sigma}_i, \lambda)$.

Motivated by the above discussion, we can assume that the intrinsic growth rate $\overline{r}_1$ and the death rates $\overline{r}_2$ and $\overline{r}_3$ of model (1) perturbed by the Lévy jump to signify the sudden climate change, $\overline{r}_i \longrightarrow \overline{r}_i + \overline{\gamma}_i \mathrm{d}L_i(t)$ [42, 46], and then we can obtain the following stochastic model incorporating Lévy jump:

$$
\begin{aligned}
\mathrm{d}x(t) &= x(t)\left[r_1 - a_{11}x(t) - a_{12}y_1(t - \tau_{12})\right.\\
&\quad \left.- a_{13}y_2(t - \tau_{13})\right]\mathrm{d}t + \sigma_1 x(t)\,\mathrm{d}B_1(t) + x(t)\\
&\quad \cdot \int_{\mathbb{Y}} \gamma_1(v)\widetilde{N}(\mathrm{d}t, \mathrm{d}v),\\
\mathrm{d}x_2(t) &= y_1(t)\left[r_2 - h_1 - a_{21}\mathrm{e}^{-d_{21}\tau_{21}}x(t - \tau_{21}) - a_{22}y_1(t)\right.\\
&\quad \left.- a_{23}e^{-d_{23}\tau_{23}}y_2(t - \tau_{23})\right]\mathrm{d}t + \sigma_2 y_1(t)\,\mathrm{d}B_2(t) + y_1(t)\\
&\quad \cdot \int_{\mathbb{Y}} \gamma_2(v)\widetilde{N}(\mathrm{d}t, \mathrm{d}v),\\
\mathrm{d}x_3(t) &= y_2(t)\left[r_3 - h_2 - a_{31}e^{-d_{31}\tau_{31}}x(t - \tau_{31})\right.\\
&\quad \left.- a_{32}e^{-d_{32}\tau_{32}}y_1(t - \tau_{32}) - a_{33}y_2(t)\right]\mathrm{d}t\\
&\quad + \sigma_3 y_2(t)\,\mathrm{d}B_3(t) + y_2(t)\int_{\mathbb{Y}} \gamma_3(v)\widetilde{N}(\mathrm{d}t, \mathrm{d}v),
\end{aligned}
\tag{5}
$$

here $r_i = \overline{r}_i + \overline{\gamma}_i\widetilde{a}_i$, $\sigma_i = \overline{\sigma}_i + \overline{\gamma}_i\widetilde{\sigma}_i$, $\gamma_i(v) = \overline{\gamma}_i(v)$, $i = 1, 2, 3$, and with initial value (2).

In this paper, we devote our main attention to obtain the optimal harvesting control strategy of system (5). To this end, we firstly investigate the dynamical behavior of the three species including persistence in mean and extinction and asymptotically stable distribution. Then we explore how the time delay, sudden environmental shock expressed by Lévy jump, and other factors affect the optimal harvesting policy and the maximum expectation of sustainable yield.

This paper is organized as follows. We discuss the persistence in mean and extinction of the three species in Section 2. Based on the conclusion of Section 2, we consider the optimal harvesting policy in Section 3. Finally, we conclude our results by numerical simulations and discussions in the last section.

2. Persistence in Mean and Extinction

For convenience, we give some notations which will be used for analyzing our main results. Let $R_+^3 = \{a = (a_1, a_2, a_3) \in R^3 \mid a_i > 0, i = 1, 2, 3\}$ and let $C([-\tau_0, 0]; R_+^3)$ stand for all continuous functions from $[-\tau_0, 0]$ to R_+^3, where R^3 is a 3-dimensional Euclidean space and R_+^3 is the positive cone in R^3. Correspondingly, $R_+ = (0, +\infty)$. Denote $d_1 = r_1$, $d_2 =$

$r_2 - h_1$, and $d_3 = r_3 - h_2$. To make it more convenient, let $c_{11} = a_{11}$, $c_{12} = a_{12}$, $c_{13} = a_{13}$, $c_{21} = a_{21}e^{-d_{21}\tau_{21}}$, $c_{22} = a_{22}$, $c_{23} = a_{23}e^{-d_{23}\tau_{23}}$, $c_{31} = a_{31}e^{-d_{31}\tau_{31}}$, $c_{32} = a_{32}e^{-d_{32}\tau_{32}}$, and $c_{33} = a_{33}$. Additionally, denote $\alpha_1 = (c_{11}, c_{21}, c_{31})^{\mathrm{T}}$, $\alpha_2 = (c_{12}, c_{22}, c_{32})^{\mathrm{T}}$, $\alpha_3 = (c_{13}, c_{23}, c_{33})^{\mathrm{T}}$ and $\beta = (d_1, d_2, d_3)^{\mathrm{T}}$, and $\gamma = (1/2)(\sigma_1^2, \sigma_2^2, \sigma_3^2)^{\mathrm{T}}$. Accordingly, we define

$$\begin{aligned}
\Phi &= \det(\alpha_1, \alpha_2, \alpha_3),\\
\Upsilon &= \det(\alpha_1, \beta, \gamma),\\
\Phi_1 &= \det(\beta, \alpha_2, \alpha_3),\\
\widetilde{\Phi}_1 &= \det(\gamma, \alpha_2, \alpha_3),\\
\Phi_2 &= \det(\alpha_1, \beta, \alpha_3),\\
\widetilde{\Phi}_2 &= \det(\alpha_1, \gamma, \alpha_3),\\
\Phi_3 &= \det(\alpha_1, \alpha_2, \beta),\\
\widetilde{\Phi}_3 &= \det(\alpha_1, \alpha_2, \gamma),\\
\Delta_1 &= c_{22}d_1 - c_{12}d_2,\\
\Delta_2 &= c_{11}d_2 - c_{21}d_1,\\
\Delta_3 &= c_{11}d_3 - c_{31}d_1,\\
\widetilde{\Delta}_1 &= \frac{c_{22}\sigma_1^2}{2} - \frac{c_{12}\sigma_2^2}{2},\\
\widetilde{\Delta}_2 &= \frac{c_{11}\sigma_2^2}{2} - \frac{c_{21}\sigma_1^2}{2},\\
\widetilde{\Delta}_3 &= \frac{c_{11}\sigma_3^2}{2} - \frac{c_{31}\sigma_2^2}{2},\\
\langle f(t)\rangle &= t^{-1}\int_0^t f(s)\,\mathrm{d}s,\\
\langle f\rangle^* &= \limsup_{t\longrightarrow+\infty} t^{-1}\int_0^t f(s)\,\mathrm{d}s,\\
\langle f\rangle_* &= \liminf_{t\longrightarrow+\infty} t^{-1}\int_0^t f(s)\,\mathrm{d}s.
\end{aligned} \tag{6}$$

Φ_{ij} is the complement minor of c_{ij} in the deterministic Φ, $i, j = 1, 2, 3$. Now we give a fundamental assumption of Lévy jump.

Assumption 1. Assume that $\gamma_i(v) > -1$, and there exists a constant K such that

$$\int_{\mathbb{Y}} \ln(1 + \gamma_i(v))^2 \lambda(\mathrm{d}v) \le K < \infty. \tag{7}$$

Assumption 1 implies that the intensity of Lévy jump can not be sufficiently large.

Lemma 2. *For any given initial value $\varphi(\theta) = (\varphi_0(\theta), \varphi_1(\theta), \varphi_2(\theta))^{\mathrm{T}} \in C([-\tau_0, 0]; R_+^3)$, model (5) has a unique global positive solution $X(t) = (x(t), y_1(t), y_2(t))^{\mathrm{T}}$ on $t \ge 0$ a.s.*

Proof. Firstly, we consider the following stochastic model:

$$\begin{aligned}
\mathrm{d}\ln x(t) &= \Big[d_1 - \frac{1}{2}\sigma_1^2 - c_{11}x(t) - c_{12}y_1(t-\tau_{12})\\
&\quad - c_{13}y_2(t-\tau_{13}) - \int_{\mathbb{Y}}[\gamma_1 - \ln(1+\gamma_1)]\lambda(\mathrm{d}v)\Big]\mathrm{d}t\\
&\quad + \sigma_1\mathrm{d}B_1(t) + \int_{\mathbb{Y}}\ln(1+\gamma_1)\widetilde{N}(\mathrm{d}t, \mathrm{d}v),\\
\mathrm{d}\ln y_1(t) &= \Big[d_2 - \frac{1}{2}\sigma_2^2 - c_{21}x(t-\tau_{21}) - c_{22}y_1(t)\\
&\quad - c_{23}y_2(t-\tau_{23}) - \int_{\mathbb{Y}}[\gamma_2 - \ln(1+\gamma_2)]\lambda(\mathrm{d}v)\Big]\mathrm{d}t\\
&\quad + \sigma_2\mathrm{d}B_2(t) + \int_{\mathbb{Y}}\ln(1+\gamma_2)\widetilde{N}(\mathrm{d}t, \mathrm{d}v),\\
\mathrm{d}\ln y_2(t) &= \Big[d_3 - \frac{1}{2}\sigma_3^2 - c_{31}x(t-\tau_{31}) - c_{32}y_1(t-\tau_{32})\\
&\quad - c_{33}y_2(t) - \int_{\mathbb{Y}}[\gamma_3 - \ln(1+\gamma_3)]\lambda(\mathrm{d}v)\Big]\mathrm{d}t\\
&\quad + \sigma_1\mathrm{d}B_1(t) + \int_{\mathbb{Y}}\ln(1+\gamma_1)\widetilde{N}(\mathrm{d}t, \mathrm{d}v)
\end{aligned} \tag{8}$$

with initial value (2). It is easy to see that the coefficients of model (8) satisfy the local Lipschitz condition; therefore model (8) has a unique local solution $X(t)$ on $[0, \tau_e]$, where τ_e is the explosion time. According to Itô's formula, we can find that

$$X(t) = (x(t), y_1(t), y_2(t))^{\mathrm{T}} \tag{9}$$

is the unique positive local solution to model (5). Now let us prove $\tau_e = +\infty$. Thus, we introduce the following auxiliary model:

$$\begin{aligned}
\mathrm{d}u(t) &= u(t)[d_1 - c_{11}u(t)]\mathrm{d}t + \sigma_1 u(t)\mathrm{d}B_1(t)\\
&\quad + u(t)\int_{\mathbb{Y}}\gamma_1(v)\widetilde{N}(\mathrm{d}t, \mathrm{d}v),\\
\mathrm{d}v_1(t) &= v_1(t)[d_2 - c_{21}u(t-\tau_{21}) - c_{22}v_1(t)]\mathrm{d}t\\
&\quad + \sigma_2 v_1(t)\mathrm{d}B_2(t)\\
&\quad + v_1(t)\int_{\mathbb{Y}}\gamma_2(v)\widetilde{N}(\mathrm{d}t, \mathrm{d}v),\\
\mathrm{d}v_2(t) &= v_2(t)[d_3 - c_{31}u(t-\tau_{31}) - c_{33}v_2(t)]\mathrm{d}t\\
&\quad + \sigma_3 v_2(t)\mathrm{d}B_3(t) + v_2(t)\int_{\mathbb{Y}}\gamma_3(v)\widetilde{N}(\mathrm{d}t, \mathrm{d}v)
\end{aligned} \tag{10}$$

with initial value

$$\begin{aligned}
u(\theta) &= \varphi_0(\theta),\\
v_1(\theta) &= \varphi_1(\theta),\\
v_2(\theta) &= \varphi_2(\theta),\\
&\qquad \theta \in [-\tau_0, 0].
\end{aligned} \tag{11}$$

Taking advantage of the comparison theorem for stochastic equation [47] yields that, for $t \in [0, \tau_e]$,

$$\begin{aligned} &x(\theta) < u(\theta), \\ &y_i(\theta) < v_i(\theta) \quad a.s., \ i = 1, 2. \end{aligned} \tag{12}$$

According to Theorem 4.2 in Jiang and Shi [48], the explicit solution of model (10) is

$$\begin{aligned} u(t) &= \frac{\exp\{b_1 t + \sigma_1 B_1(t) + M_1(t)\}}{u^{-1}(0) + c_{11}\int_0^t \exp\{b_1 s + \sigma_1 B_1(s) + M_1(s)\}\,\mathrm{d}s}, \\ v_1(t) &= \frac{\exp\left\{b_2 t + c_{21}\int_0^t u(t-\tau_{21})\,\mathrm{d}s + \sigma_2 B_2(t) + M_2(t)\right\}}{v_1^{-1}(0) + c_{22}\int_0^t \exp\left\{b_2 s - \int_0^s c_{21} u(\varsigma - \tau_{21})\,\mathrm{d}\varsigma + \sigma_2 B_2(s) + M_2(s)\right\}}, \\ v_2(t) &= \frac{\exp\left\{b_3 t + c_{31}\int_0^t u(t-\tau_{31})\,\mathrm{d}s + \sigma_3 B_3(t) + M_3(t)\right\}}{v_2^{-1}(0) + c_{33}\int_0^t \exp\left\{b_3 s - \int_0^s c_{31} u(\varsigma - \tau_{31})\,\mathrm{d}\varsigma + \sigma_3 B_3(s) + M_2(s)\right\}}, \end{aligned} \tag{13}$$

where $\eta_i = \int_{\mathbb{Y}}[\gamma_i(v) - \ln(1+\gamma_i(v))]\lambda(\mathrm{d}v)$, $b_i = d_i - (1/2)\sigma_i^2 - \eta_i$, and $M_i(t) = \int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_i(v))\widetilde{N}(\mathrm{d}s, \mathrm{d}v)$, $i = 1, 2, 3$ [33]. It is not difficult to see that $y_1(t)$, $y_2(t)$, and $y_3(t)$ are existent on $t \geq 0$; thereby $\tau_e = +\infty$. This completes the proof of Lemma 2. □

Lemma 3 (see [22]). *Suppose that $\phi(t) \in C[\Omega \times [0, +\infty); R_+]$.*

(i) If there exist three constants λ_0, λ, and $T \geq 0$, such that

$$\begin{aligned} \ln\phi(t) \leq{}& \lambda t - \lambda_0 \int_0^t \phi(s)\,\mathrm{d}s + \alpha\sum_{i=1}^{3} B_i(t) \\ &+ \sum_{i=1}^{3}\delta_i \int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_i(v))\widetilde{N}(\mathrm{d}s, \mathrm{d}v), \quad a.s., \end{aligned} \tag{14}$$

for all $t \geq T$, where α and δ_i are constants; then

$$\begin{aligned} &\langle\phi\rangle^* \leq \frac{\lambda}{\lambda_0} \quad a.s., \ if \ \lambda \geq 0, \\ &\lim_{t\to\infty}\phi(t) = 0 \quad a.s., \ if \ \lambda \leq 0. \end{aligned} \tag{15}$$

(ii) If there exist three constants λ_0, λ, and $T \geq 0$, such that

$$\begin{aligned} \ln\phi(t) \geq{}& \lambda t - \lambda_0 \int_0^t \phi(s)\,\mathrm{d}s + \alpha\sum_{i=1}^{3} B_i(t) \\ &+ \sum_{i=1}^{3}\delta_i \int_0^t\int_{\mathbb{Y}} \ln(1+\gamma_i(v))\widetilde{N}(\mathrm{d}s, \mathrm{d}v), \quad a.s., \end{aligned} \tag{16}$$

for all $t \geq T$; then $\langle\phi\rangle_* \geq \lambda/\lambda_0$ a.s.

Lemma 4. *For model (10), consider the following:*

(a) If $b_1 < 0$, then

$$\begin{aligned} &\lim_{t\to+\infty} u(t) = 0, \\ &\lim_{t\to+\infty} v_i(t) = 0, \quad a.s., \ i = 1, 2. \end{aligned} \tag{17}$$

(b) If $b_1 \geq 0$, $b_2 - c_{21}b_1/c_{11} \geq 0$, and $b_3 - c_{31}b_1/c_{11} \geq 0$, then

$$\begin{aligned} &\lim_{t\to+\infty}\langle u(t)\rangle = \frac{b_1}{c_{11}}, \\ &\lim_{t\to+\infty}\langle v_1(t)\rangle = \frac{b_2 - c_{21}b_1/c_{11}}{c_{22}}, \\ &\lim_{t\to+\infty}\langle v_2(t)\rangle = \frac{b_3 - c_{31}b_1/c_{11}}{c_{33}}, \\ &a.s.; \end{aligned} \tag{18}$$

(c) If $b_1 \geq 0$, $b_2 - c_{21}b_1/c_{11} < 0$, and $b_3 - c_{31}b_1/c_{11} < 0$, then

$$\begin{aligned} &\lim_{t\to+\infty}\langle u(t)\rangle = \frac{b_1}{c_{11}}, \\ &\lim_{t\to+\infty} v_1(t) = 0, \\ &\lim_{t\to+\infty} v_2(t) = 0, \\ &a.s.. \end{aligned} \tag{19}$$

(d) If $b_1 \geq 0$, $b_2 - c_{21}b_1/c_{11} \geq 0$, and $b_3 - c_{31}b_1/c_{11} < 0$, then

$$\begin{aligned} &\lim_{t\to+\infty}\langle u(t)\rangle = \frac{b_1}{c_{11}}, \\ &\lim_{t\to+\infty}\langle v_1(t)\rangle = \frac{b_2 - c_{21}b_1/c_{11}}{c_{22}}, \\ &\lim_{t\to+\infty} v_2(t) = 0, \\ &a.s.. \end{aligned} \tag{20}$$

(e) If $b_1 \geq 0$, $b_2 - c_{21}b_1/c_{11} < 0$, and $b_3 - c_{31}b_1/c_{11} \geq 0$, then

$$\begin{aligned} &\lim_{t\to+\infty}\langle u(t)\rangle = \frac{b_1}{c_{11}}, \\ &\lim_{t\to+\infty} v_1(t) = 0, \\ &\lim_{t\to+\infty}\langle v_2(t)\rangle = \frac{b_3 - c_{31}b_1/c_{11}}{c_{33}}, \\ &a.s. \end{aligned} \tag{21}$$

Proof. Firstly, we prove (a). Applying Itô's formula to model (10), we can get that

$$\begin{aligned} \ln u(t) - \ln u(0) ={}& b_1 t - c_{11}\int_0^t u(s)\,\mathrm{d}s + \sigma_1 B_1(t) \\ &+ M_1(t), \end{aligned} \tag{22}$$

$$\begin{aligned} \ln v_1(t) - \ln v_1(0) ={}& b_2 t - c_{21}\int_0^t u(s-\tau_{21})\,\mathrm{d}s \\ &- c_{22}\int_0^t v_1(s)\,\mathrm{d}s + \sigma_2 B_2(t) \\ &+ M_2(t), \end{aligned} \tag{23}$$

$$\begin{aligned}\ln v_2(t) - \ln v_2(0) = b_3 t - c_{31}\int_0^t u(s-\tau_{31})\,\mathrm{d}s \\ - c_{33}\int_0^t v_2(s)\,\mathrm{d}s + \sigma_3 B_3(t) \\ + M_3(t).\end{aligned} \quad (24)$$

It follows from (22) that

$$t^{-1}\ln\frac{u(t)}{u(0)} \le b_1 + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t}. \quad (25)$$

By Assumption 1, the quadratic variation of $M_1(t)$ is

$$\begin{aligned}\langle M_1(t), M_1(t)\rangle(t) &= \int_0^t\int_{\mathbb{Y}}(\ln(1+\gamma_1(v)))^2\lambda(\mathrm{d}v)\,\mathrm{d}s \\ &\le t\int_{\mathbb{Y}}(\ln(1+\gamma_1(v)))^2\lambda(\mathrm{d}v) < ct,\end{aligned} \quad (26)$$

where $\langle M_1(t), M_1(t)\rangle(t)$ is Meyer's angle bracket process. Utilizing the strong law of large numbers for local martingales gives that $\lim_{t\longrightarrow\infty}(B_1(t)/t) = 0$ and $\lim_{t\longrightarrow\infty}(M_1(t)/t) = 0$. Since $b_1 < 0$,

$$\lim_{t\longrightarrow+\infty} u(t) = 0, \quad a.s. \quad (27)$$

Substituting (27) into (23) gives that

$$\begin{aligned}\ln v_1(t) - \ln v_1(0) \le b_2 t + \varepsilon t - c_{22}\int_0^t v_1(s)\,\mathrm{d}s + \sigma_2 B_2(t) \\ + M_2(t),\end{aligned} \quad (28)$$

where ε is small enough satisfying $b_2 + \varepsilon < 0$. Applying (i) in Lemma 3 gives

$$\lim_{t\longrightarrow+\infty} v_1(t) = 0, \quad a.s. \quad (29)$$

Similarly, we can get that

$$\lim_{t\longrightarrow+\infty} v_2(t) = 0, \quad a.s. \quad (30)$$

Secondly, we prove (b). Since $b_1 \ge 0$, applying Lemma 3 to (22) yields that

$$\lim_{t\longrightarrow+\infty}\langle u(t)\rangle = \frac{b_1}{c_{11}}, \quad a.s. \quad (31)$$

We know that

$$t^{-1}\ln\frac{u(t)}{u(0)} = b_1 - c_{11}\langle u(t)\rangle + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t}, \quad (32)$$

$$\begin{aligned}t^{-1}\ln\frac{v_1(t)}{v_1(0)} - t^{-1}c_{21}\left(\int_{t-\tau_{21}}^t u(s)\,\mathrm{d}s - \int_{-\tau_{21}}^0 u(s)\,\mathrm{d}s\right) \\ = b_2 - c_{21}\langle u(t)\rangle - c_{22}\langle v_1(t)\rangle + \frac{\sigma_2 B_2(t)}{t} + \frac{M_2(t)}{t},\end{aligned} \quad (33)$$

$$\begin{aligned}t^{-1}\ln\frac{v_2(t)}{v_2(0)} - t^{-1}c_{31}\left(\int_{t-\tau_{31}}^t u(s)\,\mathrm{d}s - \int_{-\tau_{31}}^0 u(s)\,\mathrm{d}s\right) \\ = b_3 - c_{31}\langle u(t)\rangle - c_{33}\langle v_2(t)\rangle + \frac{\sigma_3 B_3(t)}{t} + \frac{M_3(t)}{t}.\end{aligned} \quad (34)$$

Using (31) in (32) gives that

$$\lim_{t\longrightarrow+\infty} t^{-1}\ln u(0) = 0, \quad a.s. \quad (35)$$

Multiplying (32) and (33) by $-c_{21}$ and c_{11}, respectively, and, adding them, we can derive that

$$\begin{aligned}&t^{-1}c_{11}\ln\frac{v_1(t)}{v_1(0)} - t^{-1}c_{11}c_{21}\left(\int_{t-\tau_{21}}^t u(s)\,\mathrm{d}s - \int_{-\tau_{21}}^0 u(s)\,\mathrm{d}s\right) \\ &\quad - t^{-1}\ln\frac{u(t)}{u(0)} \\ &= b_2 c_{11} - c_{11}c_{22}\langle v_1(t)\rangle + t^{-1}c_{11}\sigma_2 B_2(t) - b_1 c_{21} \\ &\quad - t^{-1}c_{21}\sigma_1 B_1(t) - t^{-1}c_{21}M_1(t) + t^{-1}c_{11}M_2(t).\end{aligned} \quad (36)$$

An application of (31) gives that

$$\begin{aligned}&\lim_{t\longrightarrow+\infty} t^{-1}\int_{t-\tau_{21}}^t u(s)\,\mathrm{d}s \\ &= \lim_{t\longrightarrow+\infty}\left(t^{-1}\int_0^t u(s)\,\mathrm{d}s - t^{-1}\int_0^{t-\tau_{21}} u(s)\right)\mathrm{d}s = 0, \\ &\qquad a.s.\end{aligned} \quad (37)$$

Consequently, utilizing (35), (36), (37), and Lemma 3 yields that

$$\lim_{t\longrightarrow+\infty}\langle v_1(t)\rangle = \frac{b_2 - c_{21}b_1/c_{11}}{c_{22}}, \quad a.s. \quad (38)$$

Similarly, we can get when $b_3 - c_{31}b_1/c_{11} \ge 0$,

$$\lim_{t\longrightarrow+\infty}\langle v_2(t)\rangle = \frac{b_3 - c_{31}b_1/c_{11}}{c_{33}}, \quad a.s. \quad (39)$$

Thirdly, we prove (c). If $b_2 - c_{21}b_1/c_{11} < 0$, according to (35), (36), (37), and Lemma 3, one can obtain that

$$\lim_{t\longrightarrow+\infty} v_1(t) = 0, \quad a.s. \quad (40)$$

Similarly, if $b_3 - c_{31}b_1/c_{11} < 0$, we can obtain

$$\lim_{t\longrightarrow+\infty} v_2(t) = 0, \quad a.s. \quad (41)$$

The proofs of (d) and (e) are similar to that of (b) and (c), respectively, and hence are omitted. □

Lemma 5. *The solution of model (5) obeys*

$$\begin{aligned}&\limsup_{t\longrightarrow+\infty}\frac{\ln x(t)}{t} \le 0, \\ &\limsup_{t\longrightarrow+\infty}\frac{\ln y_i(t)}{t} \le 0, \quad a.s., \ i = 1, 2.\end{aligned} \quad (42)$$

Proof. Since $x(t) \le u(t)$, $y_i(t) \le v_i(t)$, $i = 1, 2$, we only need to prove

$$\begin{aligned}&\limsup_{t\longrightarrow+\infty}\frac{\ln u(t)}{t} \le 0, \\ &\limsup_{t\longrightarrow+\infty}\frac{\ln v_i(t)}{t} \le 0, \quad a.s., \ i = 1, 2.\end{aligned} \quad (43)$$

According to the proof of Lemma 4, with the same method of [22], we can get (43). Therefore, (42) is obtained. □

Theorem 6. *For system (5), if $\Phi > 0$, $\Phi_i > 0$, $i = 1, 2, 3$, and $\Upsilon > 0$, we set $\rho_1 = 2d_1/\sigma_1^2$, $\rho_2 = \Delta_2/\widetilde{\Delta}_2$, and $\rho_3 = \Phi_3/\widetilde{\Phi}_3$, and then $\rho_1 > \rho_2 > \rho_3$; moreover,*

(i) *if $\rho_1 < 1$, then prey species and predator species go to extinction a.s.; i.e.,*

$$\lim_{t\longrightarrow+\infty} x(t) = 0, \qquad \lim_{t\longrightarrow+\infty} y_i(t) = 0, \quad i = 1, 2, \ \text{a.s.}; \tag{44}$$

(ii) *if $\rho_1 > 1 > \rho_2$, then two predators go to extinction a.s., and prey is persistent in mean; i.e.,*

$$\lim_{t\longrightarrow+\infty} \langle x(t)\rangle = \frac{b_1}{c_{11}}, \quad \text{a.s.}; \tag{45}$$

(iii) *if $\rho_2 > 1 > \rho_3$, then predator species y_2 goes to extinction, while prey species x and predator species y_1 are persistent in mean; i.e.,*

$$\lim_{t\longrightarrow+\infty} \langle x(t)\rangle = \frac{\Delta_1 - \widetilde{\Delta}_1}{\Phi_{33}}, \qquad \lim_{t\longrightarrow+\infty} \langle y_1(t)\rangle = \frac{\Delta_2 - \widetilde{\Delta}_2}{\Phi_{33}} \quad \text{a.s.}; \tag{46}$$

(iv) *if $\rho_3 > 1$, then three species are persistent in mean; i.e.,*

$$\lim_{t\longrightarrow+\infty} \langle x(t)\rangle = \frac{\Phi_1 - \widetilde{\Phi}_1}{\Phi}, \qquad \lim_{t\longrightarrow+\infty} \langle y_1(t)\rangle = \frac{\Phi_2 - \widetilde{\Phi}_2}{\Phi}, \qquad \lim_{t\longrightarrow+\infty} \langle y_2(t)\rangle = \frac{\Phi_3 - \widetilde{\Phi}_3}{\Phi}, \quad \text{a.s.} \tag{47}$$

Proof. It is easy to prove $\rho_1 > \rho_2 > \rho_3$. Applying Itô's formula to model (5) yields that

$$\begin{aligned}\ln x(t) - \ln x(0) = {} & b_1 t - c_{11}\int_0^t x(s)\,ds - c_{12}\int_0^t y_1(s-\tau_{12})\,ds \\ & - c_{13}\int_0^t y_2(s-\tau_{13})\,ds + \sigma_1 B_1(t) + M_1(t),\end{aligned} \tag{48}$$

$$\begin{aligned}\ln y_1(t) - \ln y_1(0) = {} & b_2 t - c_{22}\int_0^t y_1(s)\,ds - c_{21}\int_0^t x(s-\tau_{21})\,ds \\ & - c_{23}\int_0^t y_2(s-\tau_{23})\,ds + \sigma_2 B_2(t) + M_2(t),\end{aligned} \tag{49}$$

$$\begin{aligned}\ln y_2(t) - \ln y_2(0) = {} & b_3 t - c_{33}\int_0^t y_2(s)\,ds - c_{31}\int_0^t x(s-\tau_{31})\,ds \\ & - c_{32}\int_0^t y_1(s-\tau_{32})\,ds + \sigma_3 B_3(t) + M_3(t).\end{aligned} \tag{50}$$

Firstly, we prove (i). Since c_{11}, c_{12}, and c_{13} are positive, we can get

$$t^{-1}\ln\frac{x(t)}{x(0)} \le b_1 + \frac{\sigma_1 B_1(t)}{t} + \frac{M_1(t)}{t}. \tag{51}$$

Note that $\rho_1 = 2d_1/\sigma_1^2 < 1$; then $b_1 < 0$. By Lemma 3, we get

$$\lim_{t\longrightarrow+\infty} x(t) = 0, \quad \text{a.s.} \tag{52}$$

Substituting this identity into (49), we can observe that, for sufficiently large t,

$$\begin{aligned}\ln y_1(t) - \ln y_1(0) \le {} & b_2 t + \varepsilon t - c_{22}\int_0^t y_1(s)\,ds + \sigma_2 B_2(t) \\ & + M_2(t),\end{aligned} \tag{53}$$

where ε is small enough such that $b_2 + \varepsilon < 0$. By Lemma 3, we can get

$$\lim_{t\longrightarrow+\infty} y_1(t) = 0, \quad \text{a.s.} \tag{54}$$

Similarly, we have

$$\lim_{t\longrightarrow+\infty} y_2(t) = 0, \quad \text{a.s.} \tag{55}$$

Secondly, we prove (ii). Since $1 > \rho_2 > \Delta_3/\widetilde{\Delta}_3$, we know

$$\frac{\Delta_3}{\widetilde{\Delta}_3} < 1, \qquad \text{i.e., } \frac{c_{11}d_3 - c_{31}d_1}{c_{11}\sigma_3^2/2 - c_{31}\sigma_1^2/2} < 1. \tag{56}$$

Simplifying the above inequality gives that $c_{11}b_3 < c_{31}b_1$, which means $b_3 - c_{31}b_1/c_{11} < 0$. Furthermore, from $\rho_2 < 1$, we have $b_2 - c_{21}b_1/c_{11} < 0$. According to (c) in Lemma 3 and (12), one can observe that

$$\lim_{t\longrightarrow+\infty} y_i(t) = 0, \quad \text{a.s.}, \ i = 1, 2. \tag{57}$$

Substituting the above identity into (48) and using Lemma 3, we can get

$$\lim_{t\longrightarrow+\infty} x(t) = \frac{b_1}{c_{11}}. \tag{58}$$

Thirdly, we prove (iii). Dividing (48), (49), and (50) by t, we can derive the following equations:

$$\begin{aligned} t^{-1}\ln\frac{x(t)}{x(0)} &= b_1 - c_{11}\langle x(t)\rangle - c_{12}\langle y_1(t)\rangle - c_{13}\langle y_2(t)\rangle \\ &\quad + t^{-1}c_{12}\left(\int_{t-\tau_{12}}^{t} y_1(s)\,\mathrm{d}s - \int_{-\tau_{12}}^{0} y_1(s)\,\mathrm{d}s\right) \\ &\quad + t^{-1}c_{13}\left(\int_{t-\tau_{13}}^{t} y_2(s)\,\mathrm{d}s - \int_{-\tau_{13}}^{0} y_2(s)\,\mathrm{d}s\right) \\ &\quad + t^{-1}\sigma_1 B_1(t) + t^{-1}M_1(t), \end{aligned} \tag{59}$$

$$\begin{aligned} t^{-1}\ln\frac{y_1(t)}{y_1(0)} &= b_2 - c_{22}\langle y_1(t)\rangle - c_{21}\langle x(t)\rangle - c_{23}\langle y_2(t)\rangle \\ &\quad + t^{-1}c_{21}\left(\int_{t-\tau_{21}}^{t} x(s)\,\mathrm{d}s - \int_{-\tau_{21}}^{0} x(s)\,\mathrm{d}s\right) \\ &\quad + t^{-1}c_{23}\left(\int_{t-\tau_{23}}^{t} y_2(s)\,\mathrm{d}s - \int_{-\tau_{23}}^{0} y_2(s)\,\mathrm{d}s\right) \\ &\quad + t^{-1}\sigma_2 B_2(t) + t^{-1}M_2(t), \end{aligned} \tag{60}$$

$$\begin{aligned} t^{-1}\ln\frac{y_2(t)}{y_2(0)} &= b_3 - c_{33}\langle y_2(t)\rangle - c_{31}\langle x(t)\rangle - c_{32}\langle y_1(t)\rangle \\ &\quad + t^{-1}c_{31}\left(\int_{t-\tau_{31}}^{t} x(s)\,\mathrm{d}s - \int_{-\tau_{31}}^{0} x(s)\,\mathrm{d}s\right) \\ &\quad + t^{-1}c_{32}\left(\int_{t-\tau_{32}}^{t} y_1(s)\,\mathrm{d}s - \int_{-\tau_{32}}^{0} y_1(s)\,\mathrm{d}s\right) \\ &\quad + t^{-1}\sigma_3 B_3(t) + t^{-1}M_3(t). \end{aligned} \tag{61}$$

Denote m, n as the solution of the following equations:

$$\begin{aligned} c_{11}m + c_{21}n &= c_{31}, \\ c_{12}m + c_{22}n &= c_{32}. \end{aligned} \tag{62}$$

Consequently,

$$\begin{aligned} m &= \frac{-\Phi_{13}}{\Phi_{33}} > 0, \\ n &= \frac{\Phi_{23}}{\Phi_{33}} > 0. \end{aligned} \tag{63}$$

By (12), (43), and Lemma 5, we have

$$\begin{aligned} &\limsup_{t\longrightarrow+\infty} \ln x(t) \le 0, \\ &\limsup_{t\longrightarrow+\infty} \ln y_i(t) \le 0, \quad i = 1, 2. \end{aligned} \tag{64}$$

In addition,

$$\begin{aligned} &\limsup_{t\longrightarrow+\infty} t^{-1}\int_{t-\tau_{21}}^{t} x(s)\,\mathrm{d}s = \limsup_{t\longrightarrow+\infty} t^{-1}\int_{t-\tau_{31}}^{t} x(s)\,\mathrm{d}s = 0, \\ &\limsup_{t\longrightarrow+\infty} t^{-1}\int_{t-\tau_{12}}^{t} y_1(s)\,\mathrm{d}s = \limsup_{t\longrightarrow+\infty} t^{-1}\int_{t-\tau_{32}}^{t} y_1(s)\,\mathrm{d}s \\ &\quad = 0, \\ &\limsup_{t\longrightarrow+\infty} t^{-1}\int_{t-\tau_{13}}^{t} y_2(s)\,\mathrm{d}s = \limsup_{t\longrightarrow+\infty} t^{-1}\int_{t-\tau_{23}}^{t} y_2(s)\,\mathrm{d}s \\ &\quad = 0. \end{aligned} \tag{65}$$

According to Lemma 5, for arbitrarily given $\varepsilon > 0$, there exists a $T_1 > 0$ such that when $t > T_1$

$$t^{-1}\left(m\ln\frac{x(t)}{x(0)} + n\ln\frac{y_1(t)}{y_1(0)}\right) \le \varepsilon. \tag{66}$$

Multiplying (59), (60), and (61) by $-m$, $-n$, and 1, respectively, and adding them, one can observe that, for sufficiently large t such that $t > T_1$,

$$\begin{aligned} &t^{-1}\ln\frac{y_2(t)}{y_2(0)} - t^{-1}\left(m\ln\frac{x(t)}{x(0)} + \ln\frac{y_1(t)}{y_1(0)}\right) \\ &\quad = \frac{\Phi_3 - \widetilde{\Phi}_3}{\Phi_{33}} - \frac{\Phi}{\Phi_{33}}\langle y_2(t)\rangle \\ &\qquad - mt^{-1}c_{12}\left(\int_{t-\tau_{12}}^{t} y_1(s)\,\mathrm{d}s - \int_{-\tau_{12}}^{0} y_1(s)\,\mathrm{d}s\right) \\ &\qquad - mt^{-1}c_{13}\left(\int_{t-\tau_{13}}^{t} y_2(s)\,\mathrm{d}s - \int_{-\tau_{13}}^{0} y_2(s)\,\mathrm{d}s\right) \\ &\qquad - nt^{-1}c_{21}\left(\int_{t-\tau_{21}}^{t} x(s)\,\mathrm{d}s - \int_{-\tau_{21}}^{0} x(s)\,\mathrm{d}s\right) \\ &\qquad - nt^{-1}c_{23}\left(\int_{t-\tau_{23}}^{t} y_2(s)\,\mathrm{d}s - \int_{-\tau_{23}}^{0} y_2(s)\,\mathrm{d}s\right) \\ &\qquad + t^{-1}c_{31}\left(\int_{t-\tau_{31}}^{t} x(s)\,\mathrm{d}s - \int_{-\tau_{31}}^{0} x(s)\,\mathrm{d}s\right) \\ &\qquad + t^{-1}c_{32}\left(\int_{t-\tau_{32}}^{t} y_1(s)\,\mathrm{d}s - \int_{-\tau_{32}}^{0} y_1(s)\,\mathrm{d}s\right) \\ &\qquad - t^{-1}\left(m\sigma_1 B_1(t) + n\sigma_2 B_2(t) - \sigma_3 B_3(t)\right) \\ &\qquad - t^{-1}\left(mM_1(t) + nM_2(t) - M_3(t)\right). \end{aligned} \tag{67}$$

Using (43) in (81) yields that

$$\begin{aligned} t^{-1}\ln\frac{y_2(t)}{y_2(0)} &\le \frac{\Phi_3 - \widetilde{\Phi}_3}{\Phi_{33}} + 2\varepsilon - \frac{\Phi}{\Phi_{33}}\langle y_2(t)\rangle \\ &\quad - t^{-1}\left(m\sigma_1 B_1(t) + n\sigma_2 B_2(t) - \sigma_3 B_3(t)\right) \\ &\quad - t^{-1}\left(mM_1(t) + nM_2(t) - M_3(t)\right). \end{aligned} \tag{68}$$

Since $\rho_3 = \Phi_3/\widetilde{\Phi}_3 < 1$, we can choose $\varepsilon > 0$ to be sufficiently small such that $(\Phi_3 - \widetilde{\Phi}(3))/\Phi_{33} + 2\varepsilon < 0$. Making use of the arbitrariness ε and Lemma 3 gives that

$$\lim_{t\longrightarrow+\infty} y_2(t) = 0, \quad a.s. \tag{69}$$

Consequently, model (5) reduces to the following model:

$$\begin{aligned} \mathrm{d}x(t) &= x(t)\left[d_1 - c_{11}x(t) - c_{12}y_1(t-\tau_{12})\right]\mathrm{d}t \\ &\quad + \sigma_1 x(t)\,\mathrm{d}B_1(t) + x(t)\int_{\mathbb{Y}}\gamma_1(v)\widetilde{N}(\mathrm{d}t,\mathrm{d}v), \\ \mathrm{d}y_1(t) &= y_1(t)\left[d_2 - c_{21}x(t-\tau_{21}) - c_{22}y_1(t)\right]\mathrm{d}t \\ &\quad + \sigma_2 y_1(t)\,\mathrm{d}B_2(t) \\ &\quad + y_1(t)\int_{\mathbb{Y}}\gamma_2(v)\widetilde{N}(\mathrm{d}t,\mathrm{d}v). \end{aligned} \tag{70}$$

For system (70), similarly to the proof of Theorem 1 in [49], the following identities can be derived:

$$\begin{aligned} \lim_{t\longrightarrow+\infty}\langle x(t)\rangle &= \frac{\Delta_1 - \widetilde{\Delta}_1}{\Phi_{33}}, \\ \lim_{t\longrightarrow+\infty}\langle y_1(t)\rangle &= \frac{\Delta_2 - \widetilde{\Delta}_2}{\Phi_{33}}, \\ &\quad a.s. \end{aligned} \tag{71}$$

Fourthly, we prove (iv). By (68), since $\rho_3 > 1$, we know from the arbitrariness of ε and Lemma 3 that

$$\langle y_2\rangle^* \le \frac{\Phi_3 - \widetilde{\Phi}_3}{\Phi}, \quad a.s. \tag{72}$$

Denote p, q as the solution of the following equations:

$$\begin{aligned} c_{11}p + c_{31}q &= c_{21}, \\ c_{13}p + c_{33}q &= c_{23}. \end{aligned} \tag{73}$$

Then we have

$$\begin{aligned} p &= \frac{-\Phi_{12}}{\Phi_{22}} > 0, \\ q &= \frac{\Phi_{32}}{\Phi_{22}} > 0. \end{aligned} \tag{74}$$

According to Lemma 5, for arbitrarily given $\varepsilon > 0$, there exists a $T_2 > 0$, such that

$$t^{-1}\left(p\ln\frac{x(t)}{x(0)} + q\ln\frac{y_2(t)}{y_2(0)}\right) \le \varepsilon. \tag{75}$$

Multiplying (59), (60), and (61) by $-p$, 1, and $-q$, respectively, and, adding them, we can obtain that, for $t > T_2$,

$$\begin{aligned} &t^{-1}\ln\frac{y_1(t)}{y_1(0)} - t^{-1}\left(p\ln\frac{x(t)}{x(0)} + q\ln\frac{y_2(t)}{y_2(0)}\right) \\ &= \frac{\Phi_2 - \widetilde{\Phi}_2}{\Phi_{22}} - \frac{\Phi}{\Phi_{22}}\langle y_1(t)\rangle \\ &\quad - pt^{-1}c_{12}\left(\int_{t-\tau_{12}}^{t} y_1(s)\,\mathrm{d}s - \int_{-\tau_{12}}^{0} y_1(s)\,\mathrm{d}s\right) \\ &\quad - pt^{-1}c_{13}\left(\int_{t-\tau_{13}}^{t} y_2(s)\,\mathrm{d}s - \int_{-\tau_{13}}^{0} y_2(s)\,\mathrm{d}s\right) \\ &\quad - qt^{-1}c_{31}\left(\int_{t-\tau_{31}}^{t} x(s)\,\mathrm{d}s - \int_{-\tau_{31}}^{0} x(s)\,\mathrm{d}s\right) \\ &\quad - qt^{-1}c_{32}\left(\int_{t-\tau_{32}}^{t} y_1(s)\,\mathrm{d}s - \int_{-\tau_{32}}^{0} y_1(s)\,\mathrm{d}s\right) \\ &\quad + t^{-1}c_{23}\left(\int_{t-\tau_{23}}^{t} y_2(s)\,\mathrm{d}s - \int_{-\tau_{23}}^{0} y_2(s)\,\mathrm{d}s\right) \\ &\quad + t^{-1}c_{21}\left(\int_{t-\tau_{21}}^{t} x(s)\,\mathrm{d}s - \int_{-\tau_{21}}^{0} x(s)\,\mathrm{d}s\right) \\ &\quad - t^{-1}\left(p\sigma_1 B_1(t) - \sigma_2 B_2(t) + q\sigma_3 B_3(t)\right) \\ &\quad - t^{-1}\left(pM_1(t) - M_2(t) + qM_3(t)\right). \end{aligned} \tag{76}$$

Using (43) in (76) yields that

$$\begin{aligned} t^{-1}\ln\frac{y_1(t)}{y_1(0)} &\le \frac{\Phi_2 - \widetilde{\Phi}_2}{\Phi_{22}} + 2\varepsilon - \frac{\Phi}{\Phi_{22}}\langle x_2(t)\rangle \\ &\quad - t^{-1}\left(p\sigma_1 B_1(t) - \sigma_2 B_2(t) + q\sigma_3 B_3(t)\right) \\ &\quad - t^{-1}\left(pM_1(t) - M_2(t) + qM_3(t)\right). \end{aligned} \tag{77}$$

Note that $\Phi_2/\widetilde{\Phi}_2 > \Phi_3/\widetilde{\Phi}_3 > 1$. According to the arbitrariness of ε and Lemma 3, we have

$$\langle y_1\rangle^* \le \frac{\Phi_2 - \widetilde{\Phi}_2}{\Phi}, \quad a.s. \tag{78}$$

It follows that, for any sufficiently small ε, there exist T_3 and T_4 such that

$$\begin{aligned} c_{12}\langle y_1(t)\rangle &\le c_{12}\langle y_1\rangle^* + \varepsilon \le \frac{c_{12}\left(\Phi_2 - \widetilde{\Phi}_2\right)}{\Phi} + \varepsilon, \\ &\qquad t > T_3, \\ c_{13}\langle y_2(t)\rangle &\le c_{13}\langle y_2\rangle^* + \varepsilon \le \frac{c_{13}\left(\Phi_3 - \widetilde{\Phi}_3\right)}{\Phi} + \varepsilon, \\ &\qquad t > T_4. \end{aligned} \tag{79}$$

Substituting (79) into (59) results in that, for sufficiently large t,

$$\begin{aligned} t^{-1}\ln\frac{x(t)}{x(0)} &\ge b_1 - c_{11}\langle x(t)\rangle - \frac{c_{12}\left(\Phi_2 - \widetilde{\Phi}_2\right)}{\Phi} \\ &\quad - \frac{c_{13}\left(\Phi_3 - \widetilde{\Phi}_3\right)}{\Phi} - 3\varepsilon + t^{-1}\sigma_1 B_1(t) \\ &\quad + t^{-1}M_1(t) \\ &= \frac{c_{11}\left(\Phi_1 - \widetilde{\Phi}_1\right)}{\Phi} - c_{11}\langle x(t)\rangle - 3\varepsilon \\ &\quad + t^{-1}\sigma_1 B_1(t) + t^{-1}M_1(t). \end{aligned} \tag{80}$$

According to the arbitrariness of ε and Lemma 3, we have

$$\langle x\rangle_* \geq \frac{\Phi_1 - \widetilde{\Phi}_1}{\Phi}, \quad a.s. \tag{81}$$

By $c_{31} < 0, c_{32} > 0$, one can observe that, for every sufficiently small ε, there exist T_5 and T_6, such that

$$\begin{aligned} c_{31}\langle x(t)\rangle &\leq c_{31}\langle x\rangle_* + \varepsilon \leq \frac{c_{31}\left(\Phi_1 - \widetilde{\Phi}_1\right)}{\Phi} + \varepsilon, \quad t > T_5, \\ c_{32}\left\langle y_1(t)\right\rangle &\leq c_{32}\left\langle y_1\right\rangle^* + \varepsilon \leq \frac{c_{32}\left(\Phi_3 - \widetilde{\Phi}_3\right)}{\Phi} + \varepsilon, \\ &\qquad t > T_6. \end{aligned} \tag{82}$$

Using (82) in (61) yields for t large enough

$$\begin{aligned} t^{-1}\ln\frac{y_2(t)}{y_2(0)} &\geq b_3 - c_{33}\left\langle y_2(t)\right\rangle - \frac{c_{31}\left(\Phi_1 - \widetilde{\Phi}_1\right)}{\Phi} \\ &\quad - \frac{c_{32}\left(\Phi_2 - \widetilde{\Phi}_2\right)}{\Phi} - 3\varepsilon + t^{-1}\sigma_3 B_3(t) \\ &\quad + t^{-1}M_3(t) \\ &= \frac{c_{33}\left(\Phi_3 - \widetilde{\Phi}_3\right)}{\Phi} - c_{33}\left\langle y_2(t)\right\rangle - 3\varepsilon \\ &\quad + t^{-1}\sigma_3 B_3(t) + t^{-1}M_3(t). \end{aligned} \tag{83}$$

According to the arbitrariness of ε and Lemma 3, we have

$$\left\langle y_2\right\rangle_* \geq \frac{\Phi_3 - \widetilde{\Phi}_3}{\Phi}, \quad a.s. \tag{84}$$

Combining (72) with (84), one can observe that

$$\lim_{t\longrightarrow+\infty}\left\langle y_2(t)\right\rangle = \frac{\Phi_3 - \widetilde{\Phi}_3}{\Phi}, \quad a.s. \tag{85}$$

In the similar way, using (72), (81), and (60) and then combining them with (78) yield that

$$\lim_{t\longrightarrow+\infty}\left\langle y_1(t)\right\rangle = \frac{\Phi_2 - \widetilde{\Phi}_2}{\Phi}, \quad a.s. \tag{86}$$

Subsequently, we can observe that

$$\lim_{t\longrightarrow+\infty}\langle x(t)\rangle = \frac{\Phi_1 - \widetilde{\Phi}_1}{\Phi}, \quad a.s. \tag{87}$$

This completes the proof of Theorem 6. □

3. Optimal Harvesting

Lemma 7. *For any $p > 1$, there exists a constant $K = K(p)$ which makes the solution $X(t)$ of model (5) satisfy the property that*

$$\begin{aligned} &\limsup_{t\longrightarrow+\infty}\mathbb{E}\left[x^p(t)\right] \leq K, \\ &\limsup_{t\longrightarrow+\infty}\mathbb{E}\left[y_i^p(t)\right] \leq K, \quad i = 1, 2. \end{aligned} \tag{88}$$

Proof. The proof is rather standard and hence is omitted. □

From Lemma 7, there is a $T > 0$ such that, for $t \geq T$, $\mathbb{E}[x^p(t)] \leq 2K$ and $\mathbb{E}[y_i^p(t)] \leq 2K$. Note that $\mathbb{E}[x(t)]$ and $\mathbb{E}[y_i(t)]$ $(i = 1, 2)$ are continuous; thus there is a constant $K_1 > 0$ such that $\mathbb{E}[x^p(t)] < K_1$ and $\mathbb{E}[y_i^p(t)] < K_1$ when $-\tau_0 \leq t < T$. Denote $L = \max\{2K, K_1\}$; then we have

$$\begin{aligned} &\mathbb{E}\left[x^p(t)\right] \leq L = L(p) \\ &\text{and } \mathbb{E}\left[y_i^p(t)\right] \leq L = L(p), \\ &\qquad t \geq \tau_0,\ p > 0,\ i = 1, 2. \end{aligned} \tag{89}$$

Lemma 8. *If $a_{11} > a_{12} + a_{13}, a_{22} > -a_{21}e^{-d_{21}\tau_{21}} + a_{23}e^{-d_{23}\tau_{23}}$, and $a_{33} > -a_{31}e^{-d_{31}\tau_{31}} + a_{32}e^{-d_{32}\tau_{32}}$, then model (5) will be asymptotically stable in distribution; i.e., when $t \longrightarrow +\infty$, there is a unique probability measure $\mu(\cdot)$ such that the transition probability density $p(t, \xi, \cdot)$ of $X(t)$ converges weakly to $\mu(\cdot)$ with any given initial value $\xi(\theta) \in C([-\tau, 0]; R_+^3)$.*

Proof. Since the proof of Lemma 8 is rather standard and hence is omitted. The similar proof can be found in [22]. □

We give the following extra notions to get the optimal harvesting policy:

$$\begin{aligned} P &= \begin{pmatrix} a_{22} & a_{23}e^{-d_{23}\tau_{23}} \\ a_{32}e^{-d_{32}\tau_{32}} & a_{33} \end{pmatrix}, \\ \Lambda &= \left(\lambda_1, \lambda_2\right)^{\mathrm{T}} = \left[P\left(P^{-1}\right)^{\mathrm{T}} + I\right]^{-1} Q, \end{aligned} \tag{90}$$

where $Q = (Q_1, Q_2)^{\mathrm{T}}, Q_i = r_{i+1} - (1/2)\sigma_{i+1}^2 - \eta_{i+1}, i = 1, 2$, and I is the unit matrix.

Theorem 9. *Suppose that $a_{22} > a_{23}e^{-d_{23}\tau_{23}}, a_{33} > a_{32}e^{-d_{32}\tau_{32}}$, $\Phi > 0$, and $P^{-1} + (P^{-1})^{\mathrm{T}}$ is positive definite.*

(i) *If $\lambda_i \geq 0$ and when $h_i = \lambda_i, i = 1, 2$, we have $\Gamma_2 > 0$, $\Gamma_3 > 0$, and then the optimal harvesting effort is $H^* = \Lambda = [P(P^{-1})^{\mathrm{T}} + I]^{-1}Q$ and the maximum of ESY is*

$$Y^* = \Lambda^{\mathrm{T}} P^{-1}(Q - \Lambda), \tag{91}$$

where $\Gamma_i = \Phi_i - \widetilde{\Phi}_i$, $i = 2, 3$.

(ii) *When $h_i = \lambda_i, i = 1, 2$, there is a $\Gamma_i \leq 0$ or $\lambda_i < 0$, and then the optimal harvesting strategy does not exist.*

Proof. Define $G = \{H = (h_1, h_2)^{\mathrm{T}} \in R^2 \mid \Gamma_{i+1} > 0,\ h_i > 0,\ i = 1, 2\}$. Therefore, the harvesting effort $H \in G$. If the optimal harvesting effort H^* exists, it must belong to G.

Firstly, we prove (i). Obviously, G is not empty, since $\Lambda \in G$. By (iv) of Theorem 6, for any $H \in G$, we have

$$\begin{aligned} \lim_{t\longrightarrow+\infty} t^{-1}\int_0^t H^{\mathrm{T}} X(s)\,ds &= \sum_{i=1}^{2} h_i \lim_{t\longrightarrow+\infty} t^{-1}\int_0^t y_i(s)\,ds \\ &= H^{\mathrm{T}} P^{-1}(Q - H). \end{aligned} \tag{92}$$

According to Lemma 8, we obtain that model (5) has a unique invariant measure $\mu(\cdot)$ which is strong mixing and ergodic by Corollary 3.4.3 and Theorem 3.2.6 in [50], respectively. Hence, it can be derived from (3.3.2) in [50] that

$$\lim_{t\longrightarrow+\infty} t^{-1}\int_0^t H^{\mathrm{T}}X(s)\,\mathrm{d}s = \int_{R_+^2} H^{\mathrm{T}}X\mu(\mathrm{d}X). \tag{93}$$

Let $\varrho(X)$ represent the stationary probability density of model (5). Since the invariant measure of model (5) is unique, and then, by the one-to-one correspondence between $\varrho(x)$ and its corresponding invariant measure $\mu(\cdot)$, we obtain

$$\begin{aligned} Y(H) &= \lim_{t\longrightarrow+\infty}\sum_{i=1}^{2}\mathbb{E}\left(h_i y_i(t)\right) = \lim_{t\longrightarrow+\infty}\mathbb{E}\left(H^{\mathrm{T}}X(t)\right) \\ &= \int_{R_+^2} H^{\mathrm{T}}X\varrho(X)\,\mathrm{d}X = \int_{R_+^2} H^{\mathrm{T}}X\mu(\mathrm{d}X). \end{aligned} \tag{94}$$

Combining (92) with (94), we can get

$$Y(H) = H^{\mathrm{T}}P^{-1}(Q-H). \tag{95}$$

Let $\Lambda = (\lambda_1,\lambda_2)^{\mathrm{T}}$ be the unique solution of the following equation:

$$\begin{aligned} \frac{\mathrm{d}Y(H)}{\mathrm{d}H} &= \frac{\mathrm{d}H^{\mathrm{T}}}{\mathrm{d}H}P^{-1}(Q-H) \\ &\quad + \frac{\mathrm{d}}{\mathrm{d}H}\left[(Q-H)^{\mathrm{T}}\left(P^{-1}\right)^{\mathrm{T}}\right]H \\ &= P^{-1}Q - \left[P^{-1}+\left(P^{-1}\right)^{\mathrm{T}}\right]H = 0. \end{aligned} \tag{96}$$

Hence $\Lambda = [P(P^{-1})^{\mathrm{T}} + I]^{-1}Q$. Obviously, the following Hessian matrix

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}H^{\mathrm{T}}}\left[\frac{\mathrm{d}Y(H)}{\mathrm{d}H}\right] &= \left(\frac{\mathrm{d}}{\mathrm{d}H}\left[\left(\frac{\mathrm{d}Y(H)}{\mathrm{d}H}\right)^{\mathrm{T}}\right]\right)^{\mathrm{T}} \\ &= \left(\frac{\mathrm{d}}{\mathrm{d}H}\left[\mathrm{Q}^{\mathrm{T}}\left(\mathrm{P}^{-1}\right)^{\mathrm{T}} - H^{\mathrm{T}}\left[P^{-1}+\left(P^{-1}\right)^{\mathrm{T}}\right]\right]\right)^{\mathrm{T}} \\ &= -P^{-1}-\left(P^{-1}\right)^{\mathrm{T}} \end{aligned} \tag{97}$$

is negative definite, so Λ is the global maximum point of $Y(H)$. In other words, if $\Lambda \in G$, i.e., $\lambda_i \ge 0$ and $\Gamma_{i+1} > 0$, $i = 1,2$, then the optimal harvesting effort is $H^* = \Lambda$ and Y^* is the maximum value of ESY.

Secondly, we prove (ii). Obviously, if there is an i ($i = 1,2$) such that $h_i < 0$, the optimal harvesting strategy does not exist. Then we suppose that the optimal harvesting effort $\overline{H}^* = (\overline{h}_1^*,\overline{h}_2^*)^{\mathrm{T}}$ exists. So $\overline{H}^* \in G$, i.e., $\Gamma_{i+1}|_{h_i=\overline{h}_i^*} > 0$, $\overline{h}_i^* \ge 0$, $i = 1,2$. That is to say, $\overline{H}^*$ is the unique solution of (96). On the other hand, $\Lambda = (\lambda_1,\lambda_2)^{\mathrm{T}}$ is also the solution of (96). Hence, $\lambda_i = \overline{h}_i^* \ge 0$, and $\Gamma_{i+1}|_{h_i=\lambda i} = \Gamma_{i+1}|_{h_i=\overline{h}_i^*} > 0$, i=1,2. It is in contradiction with the condition, which implies that the optimal harvesting strategy does not exist.

This completes the proof of Theorem 9. □

4. Numerical Simulations and Discussions

It is imperative to understand the influence of environmental perturbations on the coexistence and extinction of species. In this paper, we consider a stochastic competitive delay model of two predators and one prey, taking both Gaussian white noise and Lévy jump into account. Compared with [24], we consider a model where the prey species predated by two different predators and also we introduce harvesting efforts into the two predator species, which is novel and more realistic. This relationship can be found in Yangtze river. There are two kinds of fishes, sturgeon and siniperca chuatsi, which feed on some shrimps. The two fishes are harvested on spring and our model reflects this phenomenon. Additionally, our main purpose is not only to investigate the dynamical behavior, but also to obtain the optimal harvesting strategy of model (5). How to deal with time delay and jump are two key points. The work [51] used the graph-theoretic approach to deal with the delay and jump in network. In this paper, we utilize variable substitution to eliminate the delay. Quardratic variation and the strong law of large numbers for local martingales are applied to deal with the jump.

Theorem 6 describes sufficient conditions for persistence in mean and extinction of three species, which are derived from the comparison theorem of stochastic differential equations and limit superior theory. When it comes to the harvesting of predators, it is essential to consider the optimal harvesting policy and the maximum expectation of sustainable yield (ESY). Theorem 9 gives the optimal harvesting effort and the maximum of ESY by using Hessian Matrix method and optimal harvesting theory of differential equations.

The authors of [33] have investigated the influence of competition of two species on the optimal harvesting strategy. In contrast to [33], the dynamical behaviors of a three species model are analyzed more sufficiently. Not only do we consider the competition of the species, we also take the predation into account. Our results show that the capture rates can affect the persistence in mean and extinction of the species. Correspondingly, the capture rates also influence the optimal harvesting strategy and the maximum expectation of sustainable yield. To sum up, our main results are summarized as follows:

(I) Extinction and persistence

Define $\rho_1 = 2d_1/\sigma_1^2$, $\rho_2 = \Delta_2/\widetilde{\Delta}_2$, and $\rho_3 = \Phi_3/\widetilde{\Phi}_3$.

(i) If $\rho_1 < 1$, then three species go to extinction a.s.; i.e.,

$$\begin{aligned} &\lim_{t\longrightarrow+\infty} x(t) = 0, \\ &\lim_{t\longrightarrow+\infty} y_i(t) = 0, \quad i = 1,2, \ \textit{a.s.}. \end{aligned} \tag{98}$$

(ii) If $\rho_1 > 1 > \rho_2$, then two predators go to extinction a.s., and prey is persistent in mean; i.e.,

$$\lim_{t\longrightarrow+\infty}\langle x(t)\rangle = \frac{b_1}{c_{11}}, \quad \textit{a.s.}. \tag{99}$$

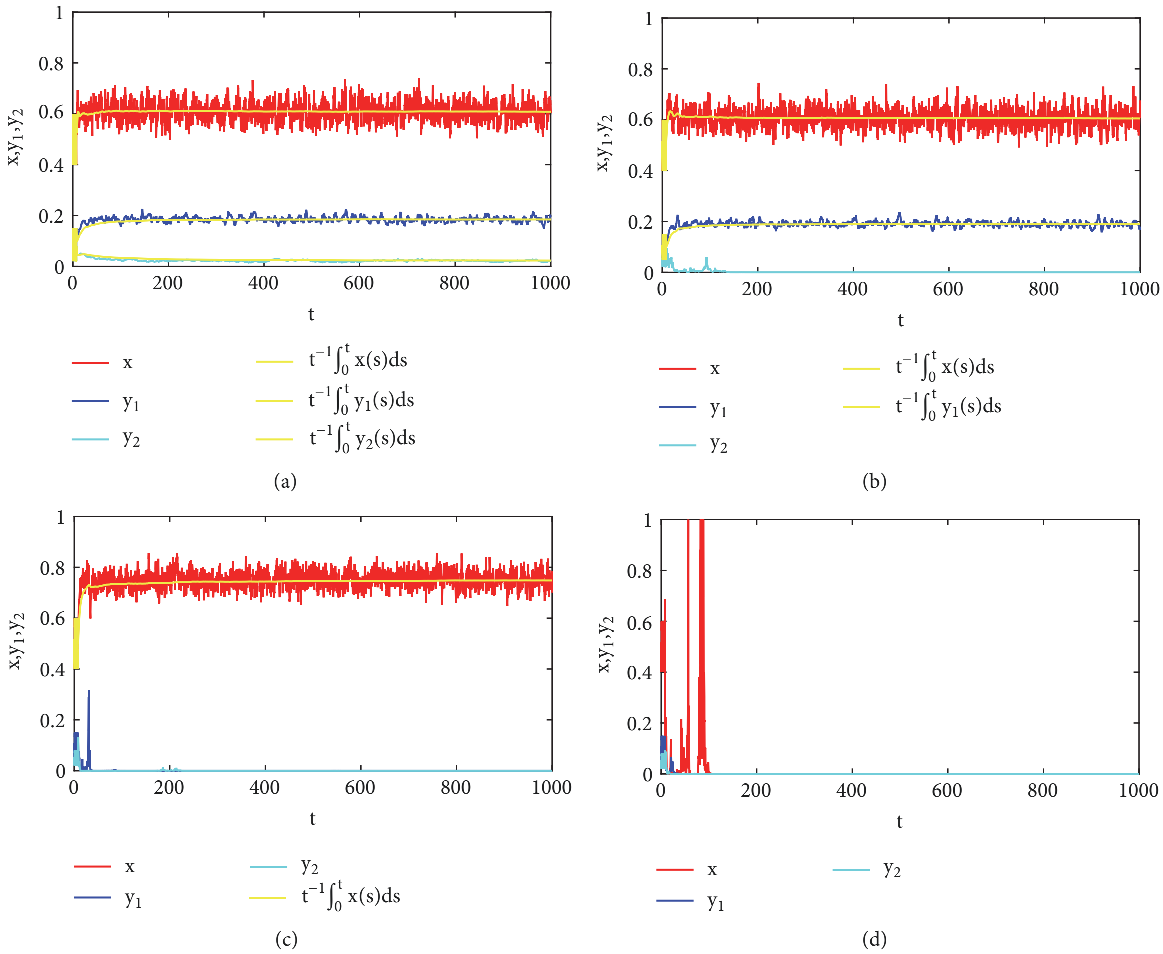

FIGURE 1: The persistence in mean and extinction of three species are given in Theorem 6. The densities of white noises are taken: (a) $\sigma_1 = 0.77$, $\sigma_2 = 0.32$, and $\sigma_3 = 0.32$, (b) $\sigma_1 = 0.77$, $\sigma_2 = 0.32$, and $\sigma_3 = 5$, (c) $\sigma_1 = 0.6$, $\sigma_2 = 10$, and $\sigma_3 = 8$, (d) $\sigma_1 = 15$, $\sigma_2 = 8$, and $\sigma_3 = 10$.

(iii) If $\rho_2 > 1 > \rho_3$, then predator species y_2 goes to extinction, while prey species x and predator species y_1 are persistent in mean; i.e.,

$$\begin{aligned} \lim_{t\longrightarrow+\infty} \langle x(t)\rangle &= \frac{\Delta_1 - \widetilde{\Delta}_1}{\Phi_{33}}, \\ \lim_{t\longrightarrow+\infty} \langle y_1(t)\rangle &= \frac{\Delta_2 - \widetilde{\Delta}_2}{\Phi_{33}} \end{aligned} \tag{100}$$

$$a.s..$$

(iv) If $\rho_3 > 1$, then three species are persistent in mean; i.e.,

$$\begin{aligned} \lim_{t\longrightarrow+\infty} \langle x(t)\rangle &= \frac{\Phi_1 - \widetilde{\Phi}_1}{\Phi}, \\ \lim_{t\longrightarrow+\infty} \langle y_1(t)\rangle &= \frac{\Phi_2 - \widetilde{\Phi}_2}{\Phi}, \\ \lim_{t\longrightarrow+\infty} \langle y_2(t)\rangle &= \frac{\Phi_3 - \widetilde{\Phi}_3}{\Phi}, \end{aligned} \tag{101}$$

$$a.s.$$

(II) Optimal harvesting strategy

Define $\Lambda = (\lambda_1, \lambda_2)^{\mathrm{T}} = [P(P^{-1})^{\mathrm{T}} + I]^{-1}Q$.

(i) If $\lambda_i \geq 0$ and when $h_i = \lambda_i$, $i = 1, 2$, we have $\Gamma_2 > 0, \Gamma_3 > 0$, then the optimal harvesting effort is $H^* = \Lambda = [P(P^{-1})^{\mathrm{T}} + I]^{-1}Q$, and the maximum of ESY is

$$Y^* = \Lambda^{\mathrm{T}} P^{-1} (Q - \Lambda), \tag{102}$$

where $\Gamma_i = \Phi_i - \widetilde{\Phi}_i$, $i = 2, 3$.

(ii) If $\lambda_i < 0$, then the optimal harvesting strategy does not exist.

Next, we give some numerical simulations to illustrate the biological significance of the results. We choose $r_1 = 1.2$, $r_2 = -0.05$, $r_3 = -0.005$, $h_1 = 0.1$, $h_2 = 0.005$, $a_{11} = 1.6$, $a_{12} = 1.2$, $a_{13} = 0.3$, $a_{21}e^{-d_{21}\tau_{21}} = -0.85$, $a_{22} = 1.9$, $a_{23}e^{-d_{23}\tau_{23}} = 0.6$, $a_{31}e^{-d_{31}\tau_{31}} = -0.4$, $a_{32}e^{-d_{32}\tau_{32}} = 1$, $a_{33} = 2.1$, $\tau_{12} = 3$, $\tau_{13} = 7$, $\tau_{21} = 1$, $\tau_{23} = 5$, $\tau_{31} = 4$, $\tau_{32} = 10$, and $\gamma_1 = \gamma_2 = \gamma_3 = 1$. Additionally, we denote $x(\theta) = 0.5 + 0.1\sin\theta$, $y_1(\theta) = 0.1 + 0.05\sin\theta$, and $y_2(\theta) = 0.05 + 0.03\sin\theta$. The densities of white noises will be given in Figures 1(a)–1(d).

Figure 1(a) shows that all three species are persistent in mean when $\rho_3 > 1$. Figure 1(b) shows that the prey and

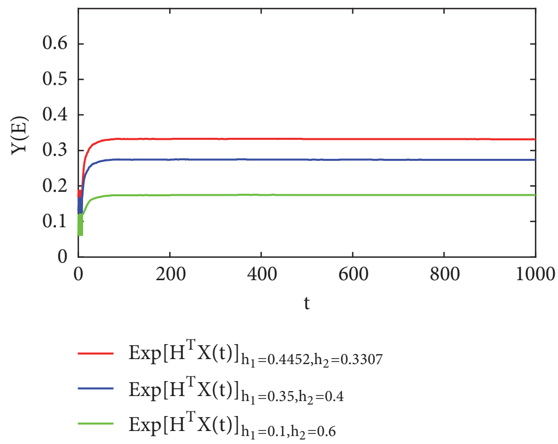

FIGURE 2: The optimal harvesting effort and the maximum of ESY are given in Theorem 9. Red line is with $h_1 = 0.4452$, $h_2 = 0.3307$, blue line is with $h_1 = 0.35$, $h_2 = 0.4$ and green line is with $h_1 = 0.1$, $h_2 = 0.6$.

one predator are persistent in mean while another predator is extinct when $\rho_2 > 1 > \rho_3$. We can find that only the prey is persistent in mean and the two predators are extinct when $\rho_1 > 1 > \rho_2$; see Figure 1(c). It is obvious that all three species are extinct when $\rho_1 < 1$; see Figure 1(d).

Regarding the optimal harvesting effort, when $a_{22} = 1.9 > a_{23}e^{-d_{23}\tau_{23}} = 0.6$, $a_{33} = 2.1 > a_{32}e^{-d_{32}\tau_{32}} = 1$, it is not difficult to estimate that $P^{-1} + (P^{-1})^{\mathrm{T}}$ is positive definite. Note that $\Lambda = [P(P^{-1})^{\mathrm{T}} + I]^{-1}Q$, and we can observe $\Lambda = (\lambda_1, \lambda_2)^{\mathrm{T}} = (0.4452, 0.3307)^{\mathrm{T}}$. Then we can find $\Gamma_2 > 0$, $\Gamma_3 > 0$. The conditions in Theorem 9 hold; therefore, we have $h_1 = \lambda_1 = 0.4452$, $h_2 = \lambda_2 = 0.3307$, and $Y^* = \Lambda^{\mathrm{T}}P^{-1}(Q - \Lambda) = 0.31$. Thus the optimal harvesting policy exists (see Figure 2).

In Figure 2, we illustrate not only the optimal harvesting policy but also another two harvesting policies. It is conspicuous that the optimal harvesting policy leads to the maximum of expectation of sustainable yield.

Based on the theoretical analysis and numerical simulations, we present the main biological and ecological meanings of our results.

(1) The perturbations we considered are not only Gaussian white noises but also Lévy jump. The results reveal that the Lévy jump may significantly affect the optimal harvesting effort and the maximum of ESY.

(2) Time delay is imperative in the ecological environment. It has significant relationship with the persistence in mean and optimal harvesting policy of model (5).

(3) We have investigated not only environmental disturbance on the species but harvesting effort affected by human and social factors. The results provide theoretical references for some modern fields, such as fishery management. It is beneficial for people to make a rational exploitation and derive maximum profit.

As a matter of fact, with the environmental pollution continually becoming worse, it is significant to consider the species in the polluted environment [4, 33]. There is no exaggeration that plenty of interesting topics deserve further investigations, for example, nonautonomous system, Markov process, impulsive effect, and partial differential system [40, 52–56]. We leave these for future investigations.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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