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Research Article

Optimization and Control on High Frequency Resonance of Train-Network Coupling Systems

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With the wide application of the electrified railway, high frequency resonances in train-network coupling system often occur, which not only affect the normal operation of trains but also seriously restrict the rapid development of electrified railway. However, the resonance mechanism has not been fully grasped and needs to be solved. First of all, this paper analyzed the new harmonic characteristics of mixed running of alternating current (AC) and direct current (DC) trains in electrified railway. Then, the mechanism of high frequency resonance and the burnout failure reason of the resistor connected with capacitance (RC) branch in the dc train were analyzed. Simulations were carried out to find some basic conclusions and rules. Subsequently, this paper reproduced and analyzed a real case of high frequency resonance accidents of train-network, burnout accident of RC branch of the dc train. Based on the above researches, optimized control methods to avoid the high frequency resonance were introduced. The achievement of this paper provides an important reference and theoretical basis for the comprehensive optimization and control of train-network coupling systems and the resonance suppression measures.

1. Introduction

At present, the electrified railway of China has entered a stage of rapid development [1]. According to the "long-term railway network planning" promulgated by the State Council, by the year of 2020, Chinese total railway mileage will reach 12 million kilometers, and the electrified railway will reach 7.2 million kilometers [2].

DC trains as the mainstream had been used for a long time in China, but with the maturity and popularization of ac drive technology, ac trains began to run on a large scale [1, 3, 4]. At present, the number and application density of ac trains in China rank first in the world. So the condition of mixed running of AC and DC trains will be continued for a period of time [5, 6].

With the wide application of the AC trains, huge economic benefits were gotten, but some new problems appeared [7–9], such as low frequency and high frequency oscillations of train-network coupling systems, which would hazard the lightning arrester, high voltage transformer, substation DC screen, and the RC branches of DC trains.

In order to find out the possible resonance points of train-network coupling systems, measuring and analyzing large amount of data from actual tests is commonly used. Nevertheless, this method costs much time and manpower, and the result accuracy is hardly guaranteed.

At present, the research on harmonic model of traction power supply system and the harmonic characteristic of ac trains are widely investigated, but the high frequency resonance mechanism of train-network coupling systems has not been fully grasped. So there is no comprehensive guidance on how to avoid the resonance of the train-network coupling systems [10–13]. Therefore, the research and optimization on high frequency resonance of train-network coupling systems are imperative [14–17].

Reference [14] built the model of traction power supply system. But the train model was an ideal current source, which could not reflect the actual harmonic characteristic of AC trains. Reference [15] studied various kinds of harmonic characteristics of AC trains by simulation, but the trains were connected with an ideal voltage source, which could not reflect the harmonic character of the traction power supply system. The condition of mixed running of AC and DC trains exists in China, but few articles studied the corresponding harmonic character. In [16, 17], the impedance-frequency

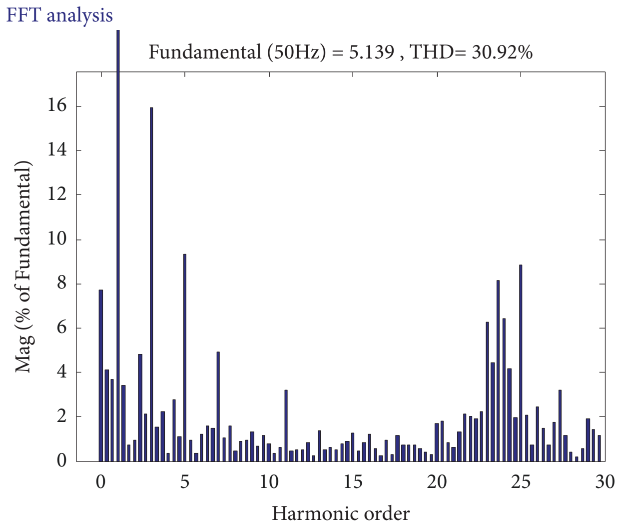


Figure 1: Current harmonic distribution of Lanzhou traction substation mixed with AC and DC trains.

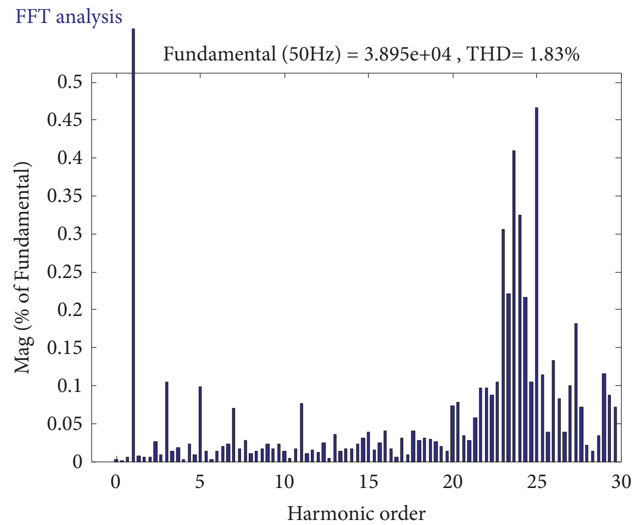


Figure 2: Voltage harmonic distribution of Lanzhou traction substation mixed with AC and DC trains.

characteristic of train-network coupling systems was analyzed, but the coupling mechanism of train-network systems needs to be further researched.

This paper analyzed the new harmonic characteristics of mixed running of AC and DC trains firstly. Then, the mechanism of high frequency resonance and the reason of the RC branch burnout failure in the DC train were analyzed, and simulations were carried out to uncover some meaningful conclusions. Thirdly, this paper reproduced and analyzed the actual high frequency resonance phenomena of train-network coupling system and the burnout failures of the RC branch of DC trains. Based on the above researches, some advices of optimized controls on trains or network to avoid the high frequency resonance were introduced.

This paper provides an important reference and theoretical basis for the comprehensive optimization of train-network coupling systems and the resonance suppression measures.

2. Analysis on New Harmonic Characteristics of Trains

Considering the current situation of mixed running of AC and DC trains in China, this paper first analyzed the new harmonic characteristics of mixed running of AC and DC trains and then analyzed the new harmonic characteristics of DC trains under different conditions.

2.1. Analysis on the New Harmonic Characteristics of the Mixed Running of AC and DC Trains. Lanzhou traction substation whose power supply arms are connected with HXD1C AC trains, SS7E DC trains, etc. is located in the city of Lanzhou. The RC branches of DC trains were often burned out, and the low frequency resonance happened occasionally.

Figures 1 and 2, respectively, show the harmonic current and harmonic voltage data of traction substation which provides power to both AC and DC trains. It can be seen from the diagram that harmonic current and harmonic voltage mainly contain the 3rd, 5th, 7th, and 25th components.

Because of mixed running of AC and DC trains, the harmonic currents and voltages which are shown in Figures 1 and 2 have low order harmonics generated by DC trains and also high harmonics generated by AC trains. The harmonic spectrum distribution is broad.

2.2. Harmonic Characteristics of the AC Train under Different Working Conditions. For example, the current harmonics from Train HXD3B were analyzed under traction or braking conditions. The results such as the root mean square (RMS) and the Total Harmonic Distortion (THD) of current were summarized in Tables 1 and 2.

From Tables 1 and 2, we can see that the load ratio has influences on the harmonic current of the train. The RMS values of the harmonic current of the train under various load ratios are approximately equal. With the same load, the RMS value of the traction current is higher than that of the braking current. Figure 3 shows intuitively the train current THD curve with different power level according to Tables 1 and 2.

Seen from Figure 3, the current THD of Train HXD3B is decreased with the increase of the train power, whether under traction condition or braking condition. When the train runs with full load, the current THD value is small, which is no more than 5%; but when the train runs with light load or no load, the THD increases sharply.

3. Analysis and Simulation Research on High Frequency Resonance Mechanism of Train-Network Coupling Systems

In order to analyze the high frequency resonance mechanism of train-network coupling systems, this paper derived the formulas of resonance and harmonic propagation, and then the corresponding simulation researches were carried out.

Table 1: The current harmonics with load variation under traction condition in HXD3B.

Load ratio	RMS value of traction current (A)	THD value of traction current	RMS value of traction current harmonics (A)
100%	405.4	3.07%	12.44578
90%	364.1	3.35%	12.19735
80%	322.9	4.24%	13.69096
70%	283.8	4.54%	12.88452
60%	245.1	5.35%	13.11285
50%	205.6	6.12%	12.58272
40%	165.1	8.07%	13.32357
30%	127.8	10.61%	13.55958
20%	88.39	15.23%	13.461797
10%	51.61	25%	12.9025
8%	43.75	30.65%	13.409375
6%	38.73	37.65%	14.581845
4%	29.63	47.30%	14.01499
2%	26.58	47.93%	12.739794
1%	23.2	57.84%	13.41888

Table 2: The current harmonics with load variation under braking condition in HXD3B train.

Load ratio	RMS value of current fundamental wave (A)	THD value of braking current	RMS value of braking current harmonics (A)
100%	366	3.87%	14.1642
90%	330.1	3.99%	13.17099
80%	293.8	4.82%	14.16116
70%	257	5.15%	13.2355
60%	220.6	5.64%	12.44184
50%	183.3	7.04%	12.90432
40%	145.6	9.72%	14.15232
30%	109.1	12.36%	13.48476
20%	71.33	19.43%	13.859419
10%	41.25	30.80%	12.705
8%	35.06	37.57%	13.172042
6%	27.62	46.68%	12.893016
4%	24.75	52.02%	12.87495
2%	19.47	67.69%	13.179243
1%	17.41	73.30%	12.76153

Finally, the burnout mechanism of RC branches of DC train is analyzed theoretically as well.

3.1. Analysis on High Frequency Resonance Mechanism of Train-Network Coupling Systems. Traction power supply system contains many inductors and capacitors. Any network with inductors and capacitors has resonance points, but that does not mean the resonance must exist consequentially. Whether to trigger the resonance or not depends on the characteristic harmonic frequency. When the characteristic harmonic frequency of the system is equal to the resonant frequency of the network, or the two are close to each other, the harmonic resonance will occur.

Figure 4(a) is the traction power supply system schematic. Its equivalent circuit is shown in Figure 4(b) and the traction network length is L.

From Figure 4(b), we can get the following equations:

$$I_t = I_1 + I_2 \tag{1}$$

$$I_1 = I_t \bullet \frac{Z_2}{Z_1 + Z_2} \tag{2}$$

$$I_2 = I_t \bullet \frac{Z_1}{Z_1 + Z_2} \tag{3}$$

where I_t is the train current, I_1 is the traction network current in the direction of the traction substation (SS), I_2 is the traction network current in the direction of the division (SP), Z_1 is the traction network impedance in the direction of train position to the traction substation, Z_2 is the traction network impedance in the direction of the train position to the district, Z_{ss} is the equivalent impedance of traction substation system,

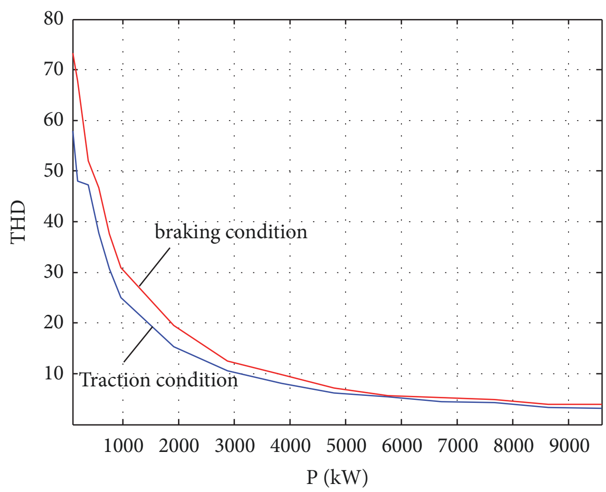
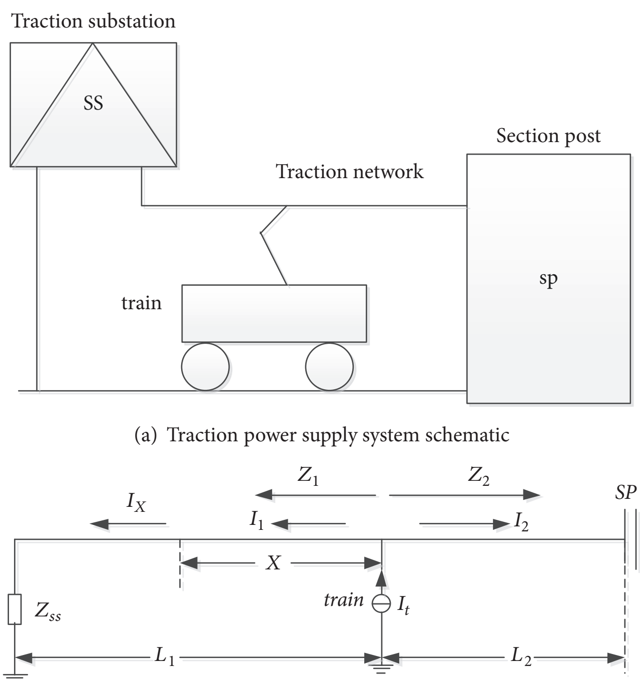


FIGURE 3: Relationship curve of THD with HXD3B train power.



(a) Traction power supply system schematic

(b) Equivalent circuit diagram

FIGURE 4: Principle of traction power supply system and its equivalent circuit diagram.

and I_x is the traction network current at the position x of the train. According to the steady state power transmission line equation, the traction power supply system on both sides of the train could be described by π equivalent circuit, as shown in Figure 5 [18, 19].

According to the equivalent model [20–22], we can get the following equations:

$$Z_{\pi 1} = Z_c \sinh \gamma L_1 = Zl_1 \frac{\sinh \gamma L_1}{\gamma L_1}$$
$$\frac{Y_{\pi 1}}{2} = \frac{\cosh L_1 - 1}{Z_c \sinh \gamma L_1} = \frac{Y}{2} L_1 \frac{\tanh \gamma L_1/2}{\gamma L_1/2}$$

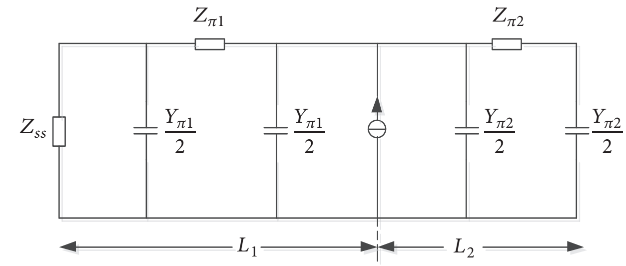


FIGURE 5: Equivalent model of traction power supply system.

$$Z_{\pi 2} = Z_c \sinh \gamma L_2 = ZL_2 \frac{\sinh \gamma L_2}{\gamma L_2}$$
$$\frac{Y_{\pi 1}}{2} = \frac{\cosh L_2 - 1}{Z_c \sinh \gamma L_2} = \frac{Y}{2} L_2 \frac{\tanh \gamma L_2/2}{\gamma L_2/2} \tag{4}$$

where Z_c is the characteristic impedance of the line, $Z_C = \sqrt{Z/Y}$, Z and Y are the equivalent impedance and admittance per unit length respectively, γ is the transmission coefficient of the line, and $\gamma = \sqrt{ZY}$.

Thus, we can get

$$Z_1 = Z_C \frac{Z_{SS} \cosh \gamma L_1 + Z_C \sinh \gamma L_1}{Z_{SS} \sinh \gamma L_1 + Z_C \cosh \gamma L_1} \tag{5}$$

$$Z_2 = Z_C \frac{\cosh \gamma L_2}{\sinh \gamma L_2} \tag{6}$$

From formula (2), we can derive

$$\begin{aligned} I_1 &= I_t \cdot \frac{Z_2}{Z_1 + Z_2} \\ &= I_t \cdot \frac{\cosh \gamma L_2 \left(Z_{SS} \sinh \gamma L_1 + Z_C \cosh \gamma L_1\right)}{Z_{SS} \sinh \gamma L + Z_C \cosh \gamma L} \end{aligned} \tag{7}$$

Based on the two-port network equation, the traction network current can be obtained as follows (in the direction of SS):

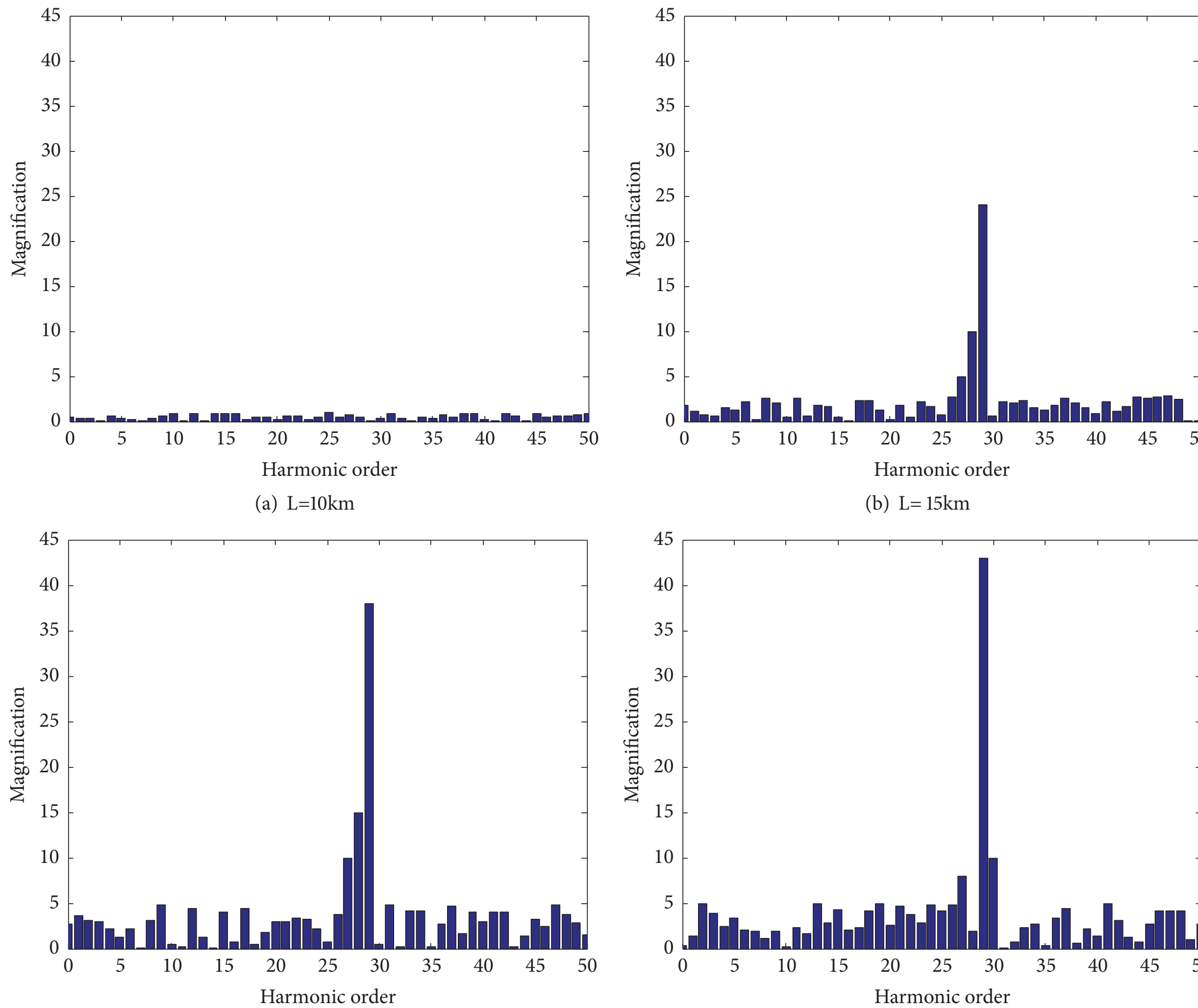
$$\begin{aligned} I_X &= I_t \\ &\cdot \frac{\cosh \gamma L_2 \left[Z_{SS} \sinh \gamma \left(L_1 - X\right) + Z_C \cosh \gamma \left(L_1 - X\right)\right]}{Z_{SS} \sinh \gamma L + Z_C \cosh \gamma L} \end{aligned} \tag{8}$$

The expression of the traction network current amplification factor K' is as follows:

$$\begin{aligned} K' &= \frac{I_X}{I_t} \\ &= \frac{\cosh \gamma L_2 \left[Z_{SS} \sinh \gamma \left(L_1 - X\right) + Z_C \cosh \gamma \left(L_1 - X\right)\right]}{Z_{SS} \sinh \gamma L + Z_C \cosh \gamma L} \end{aligned} \tag{9}$$

As can be seen from formula (9), the condition of the resonance of the traction network is

$$Z_{ss} \sin h\gamma L + Z_c \cos \gamma L = 0 \tag{10}$$



(a) L=10km
(b) L= 15km
(c) L= 20km
(d) L= 30km

Figure 6: The amplification of harmonic current in traction substation at different position.

Above expressions show that the resonance mechanism of train-network coupling systems is mainly affected by the length of the traction network, the system impedance (including the power supply impedance and the traction transformer impedance), the traction network distribution parameters, the train location, and so on.

3.2. Simulation Research on High Frequency Resonance Mechanism of Train-Network Coupling Systems

3.2.1. Simulation Research on Harmonic Current Amplification of Traction Substation. When the train is located at different positions of the traction network, the harmonic current amplification of the traction substation is shown below, where L is the distance from train to traction substation.

As shown in Figure 6, the resonant frequency of traction network is determined by its own electrical parameters, traction transformer, and power system impedance, which has nothing to do with the running position of the train. The harmonic amplification factor is related to the position of the train and the harmonic amplification factor of the traction substation increases with the distance from train to the traction substation. That means the train is not only a resonant excitation source, but also a resonant component.

3.2.2. Simulation of Harmonic Current Amplification in Traction Network with Train at Fixed Position. When the position of train is fixed at the end of the traction network, the harmonic current amplification of the traction network is shown in Figure 7.

Figure 7 shows that when the position of the train is fixed, the trend of harmonic current amplification at different positions in traction network is basically the same, which indicates that the traction network location has nothing to do with the resonant frequency of traction network. But the harmonic current amplification factor varies with the position of the traction network. That is to say, when the position is close to the traction substation, the harmonic current amplification factor becomes large.

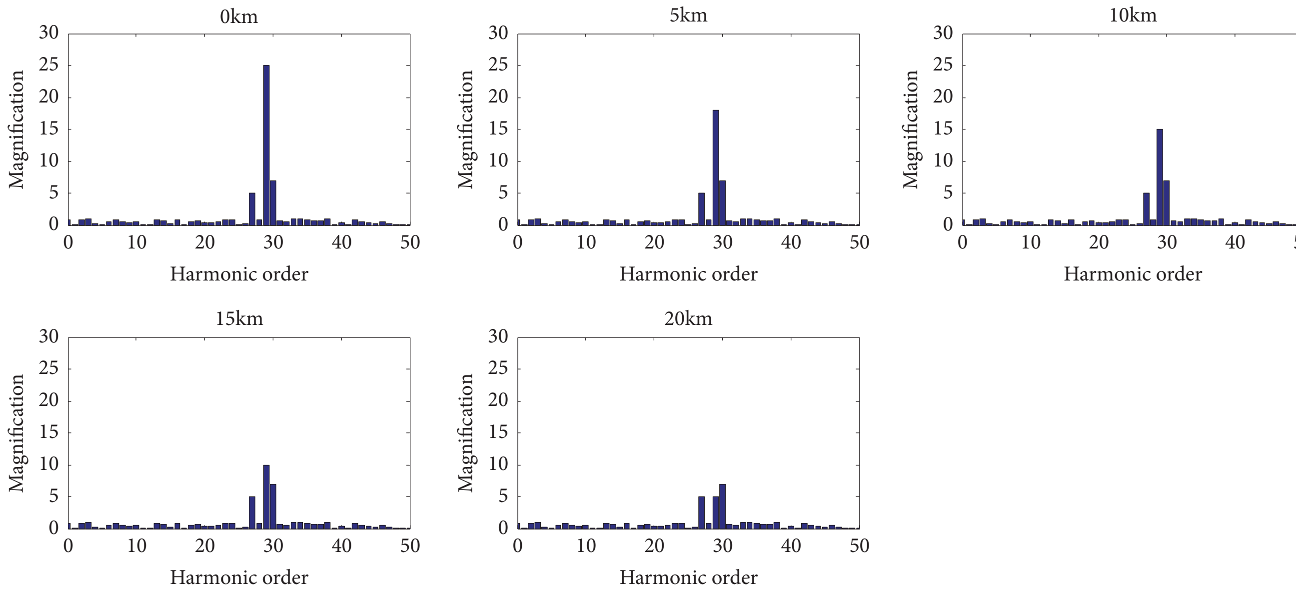


Figure 7: Amplification of harmonic current in traction network when the position of train is fixed.

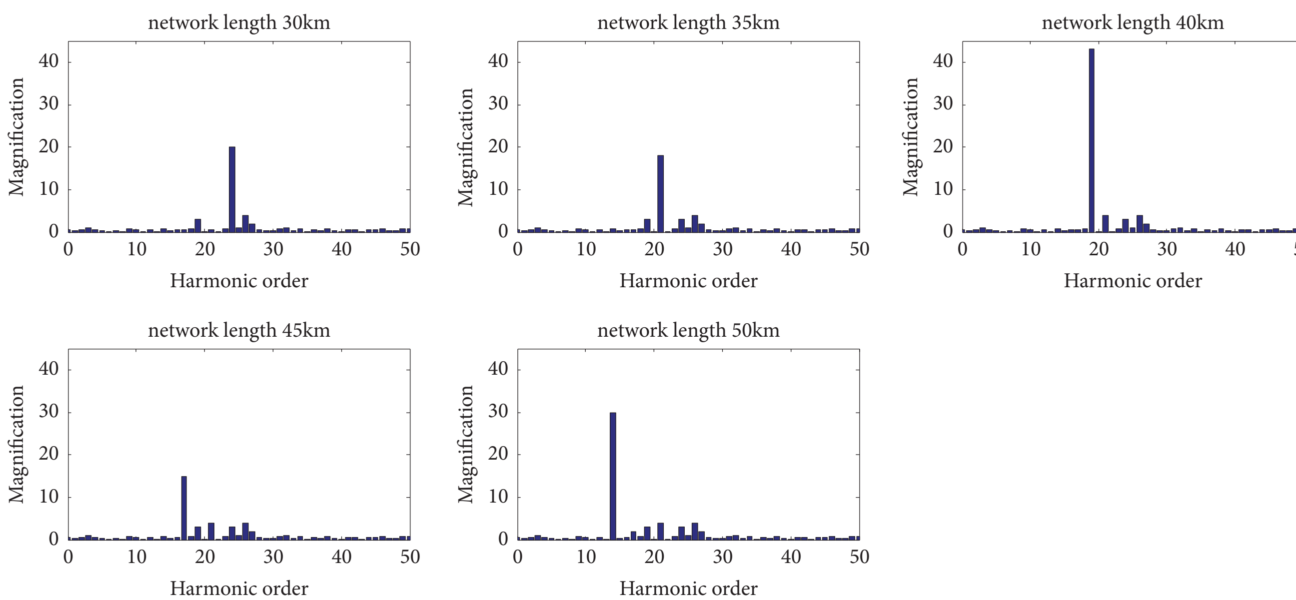


Figure 8: Harmonic current amplification of the traction substation with different length of traction network.

3.2.3. Simulation of Harmonic Current Amplification in Traction Substation with Different Traction Network Length. Changing the length of the traction network, the harmonic current amplifications of the traction substation are shown in Figure 8.

Figure 8 shows that, with the increase of the length of traction network, the resonance frequency is decreased. So the resonance frequency of the traction network is inversely proportional to the length of traction network.

The resonance frequency of traction network could be ascertained by distributed capacitance and system equivalent reactance of the traction network. Under the same suspension structure traction network condition, the longer the length of traction network is, the larger the distributed capacitance value of the traction network is, and therefore the lower the resonant frequency of the traction network is.

3.3. Analysis on the Burning Mechanism of RC Branches of DC Train. Due to the existence of the traction network impedance, voltage phase of AC trains are different at different position, and the phase difference of train's current harmonics is relevant to the distance between trains, traction network distribution parameters, and the train traction current.

When the DC trains and the AC trains stop in the centralized area, the distance between the trains is very close,

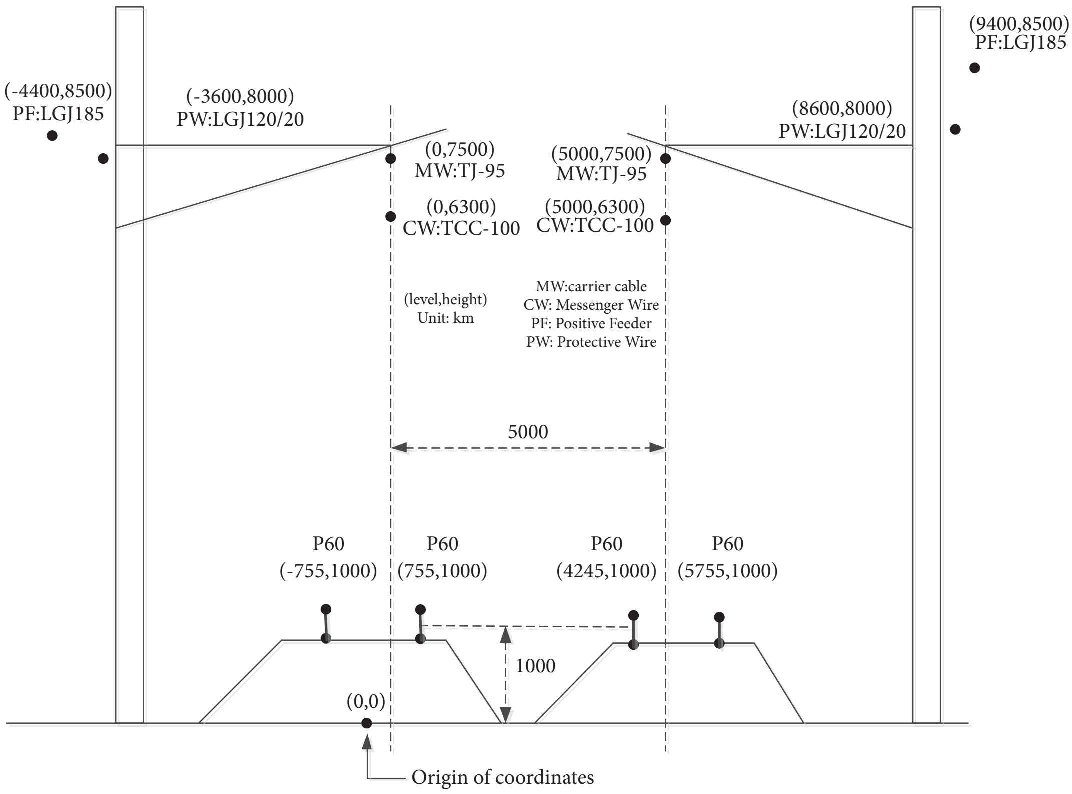


Figure 9: The structure of the network between Yanjiao and Qinhuangdao.

and the traction current is relatively small. In this case, the voltage phase of each train is completely consistent, and the harmonic current phase is also completely consistent. So the harmonic increases linearly. The RC branch current will contain a large amount of high frequency components caused by high frequency components of traction network's voltage. When the high frequency current amplitude reaches a certain degree, the fundamental wave will be submerged in the harmonics and the RC branches work in the heating state. Because the resistor is designed for a transient overvoltage process, the RC branches will be burned within a short time.

4. Replay and Analysis on Resonance Accident of Train-Network Coupling Systems

To study the resonance mechanism more effectively, based on train-network integration simulation platform, the actual high frequency resonance accident of AC trains and the burnout failure of RC branches of the DC train were replayed and analyzed.

4.1. Replay and Analysis on High Frequency Resonance of Ac Trains. The Jingha railway was built and put into operation in 1985. The railway section between Yanjiao and Qinhuangdao uses the autotransformer (AT) power supply mode and traction substations adopt Scott coupling method. The power supply arm length is 50 km and there is a AT station per 10 km. The leakage resistance of rail is 100 Ω•m. Its structure was shown in Figure 9 with the various types and the coordinates of the conductors [23].

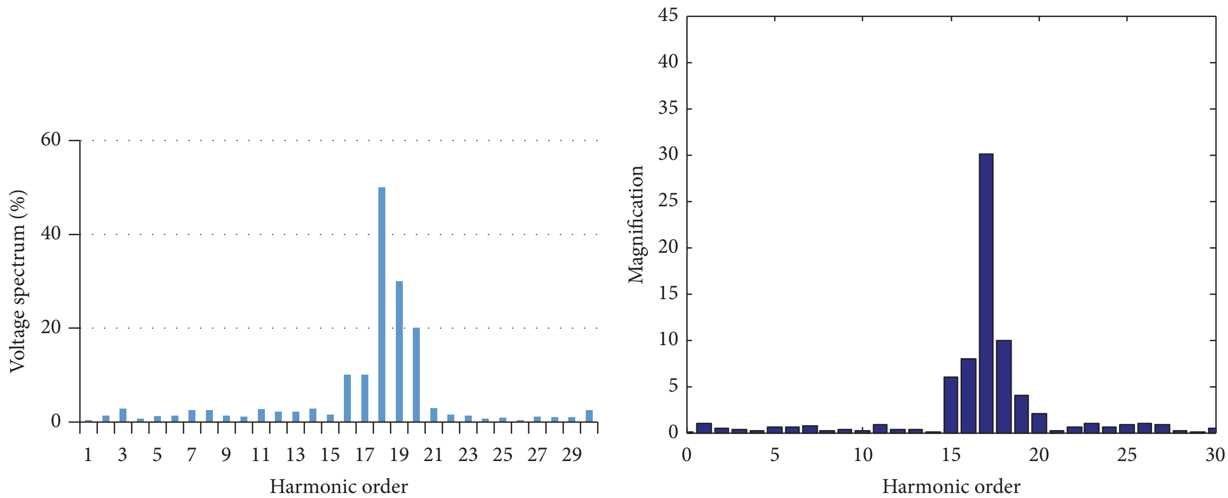
Since April 2007, capacitor banks connected in parallel to the traction busbar often trip in Jixian South substation. The monitoring system displays the voltage balance protection or overvoltage protection, and the bus voltage increases simultaneously.

Based on the AT traction power supply system parameters and the structure of the network shown in Figure 9, this paper established the corresponding train-network system's simulation model, and simulation results and experimental results were shown in Figure 10.

In Figure 10, the simulation replayed the high frequency resonant accident of Jixian South traction substation. Both measurement and simulation show that the resonance of Jixian South traction substation is produced by the high frequency harmonics. The traction network itself has a certain resonance frequency (the 18th harmonic). When network's nature frequency is equal or close to the resonant frequency of the AC trains, and the amplitude is large enough, or the damping resistance is small enough, the voltage of the frequency will be enlarged, which causes the bus voltage increased abnormally. As a result, it would trigger the protective devices.

DC trains generate mostly the 3rd, 5th, and 7th order harmonics, and the resonance frequency of the traction network is generally higher than 500Hz, so the high frequency resonance of DC trains had not appeared for many years.

4.2. Replay and Analysis on Burning Accident of RC Branches of DC Trains. When the DC trains and the AC trains stopped in the centralized area simultaneously and the main circuit



(a) The actual measured traction substation busbar's voltage spectrum

(b) The simulation's spectrum

Figure 10: Comparison of actual measured spectrum and simulation of traction substation busbar's voltage.

breaker was closed within 10 to 30 minutes, RC circuit resistance was burned. In extreme cases, the wires at the ends of the resistance may be on fire, resulting in security risks.

The test and simulation results of burning accident of RC branches of DC trains are shown in Figure 11.

The frequency of the RC branch maximum harmonic current is the 2750 Hz, and the peak value of branch current is over 100A. The fundamental component of the branch current has been overwhelmed by high order harmonics; that is to say, the RC branch current is a pure high order harmonic current. So it was burned in short time.

5. Optimization and Controls on Train-Network Coupling Systems

Based the above analyses of high frequency resonance mechanism of train-network systems, we can use controls on trains or network to avoid the high frequency resonance.

5.1. Optimizations on Trains

(1) Adjust the control method or parameter of converter of trains, so that the characteristic harmonic current frequency spectrum can avoid the resonance frequency of network.

(2) Add the high frequency filter on the main circuit of the trains.

(3) Amplify the absorbing resistance power or install the fan on DC trains.

(4) Coordinate the AC and DC trains, control the numbers of operation trains, and disperse them possibly.

5.2. Optimizations on Network

(1) Install the paralleling compensate element on the traction substation or section post to change the resonance frequency of network.

(2) Install the active power filter (APF) to restrain the low frequency harmonic component and compensate reactive power, and install the high pass filter (HPF) to restrain the high frequency harmonic component.

(3) Use the traction transformers with secondary winding independent connection such as SCOTT, V/V, etc., when the traction power supply system is designed or reformed.

(4) Adjust the traction the capacity of the substation or the length of traction network or short-circuit impedance of the traction transformers to change the resonance frequency of network.

(5) Install the managing devices with dynamic power compensation character such as static var compensator (SVC) and static var generator (SVG).

(6) Remove the customary stationary capacitance devices in the areas when the AC and DC trains are used together.

6. Conclusion

(1) In view of the special national conditions of China, the new harmonic characteristics of the mixed running of AC and DC electric trains were analyzed, and the harmonic characteristics of the AC train were also compared and analyzed under the traction and braking conditions along with the change of load ratio.

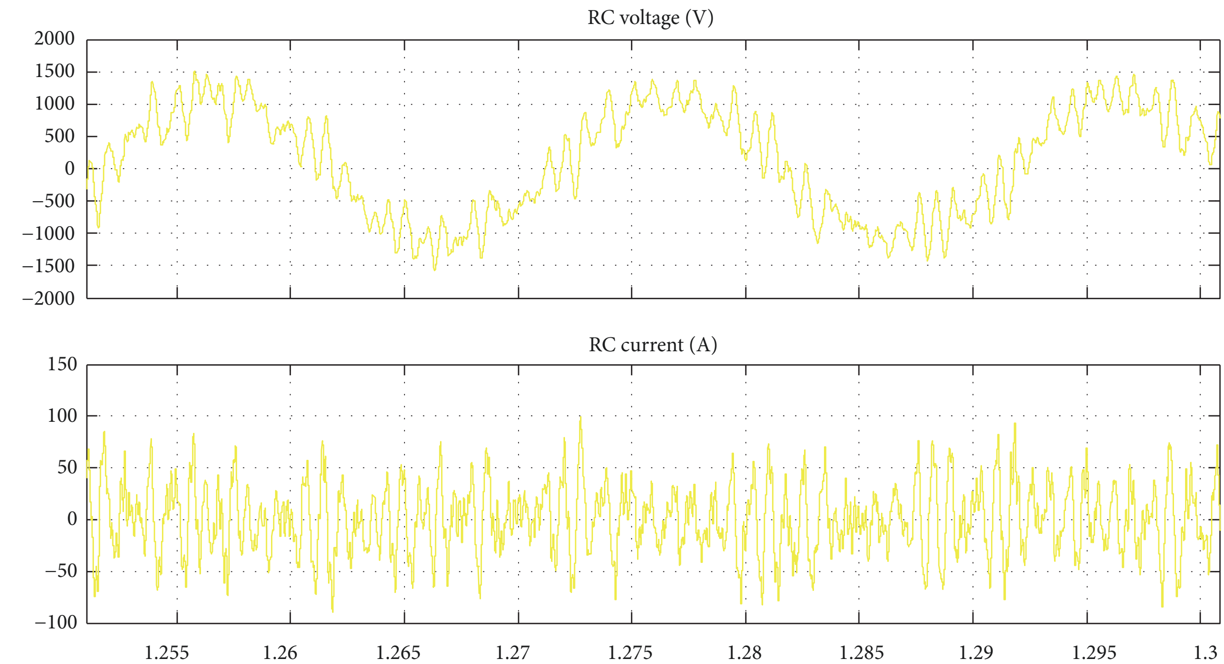
(2) The equivalent circuit of the train-network coupling systems was put forward, the conditional formula of high frequency resonance of train-network was deduced, and the corresponding simulation studies were carried out to summarize some basic conclusions and laws.

(3) The burnout mechanism of the RC branch of DC trains was analyzed in theory.

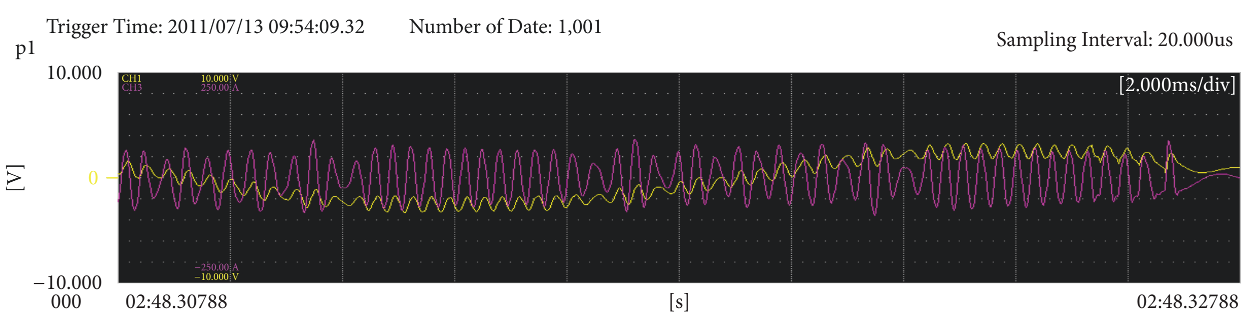
(4) High frequency resonance accident of train-network and burnout accident of the RC branch of DC trains were replayed and analyzed.

(5) Some advices of optimized controls on trains or network to avoid the high frequency resonance were introduced.

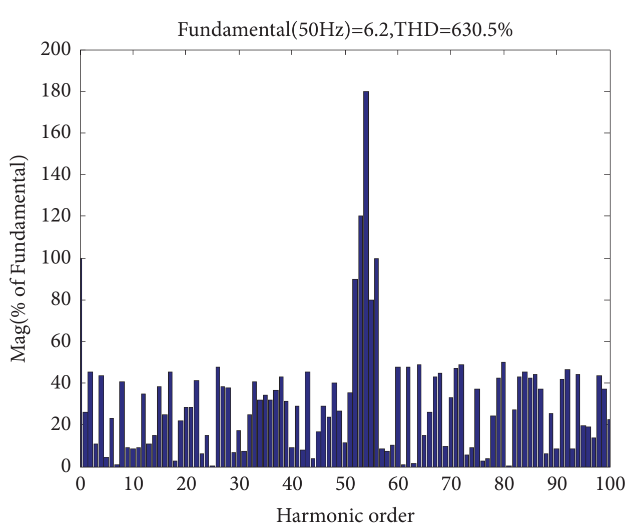
The achievement of this paper provides an important reference and theoretical basis for the comprehensive



(a) Voltage and current of RC branch by simulation



(b) Voltage and current of RC branch by actual measure



(c) RC branch current spectrum of DC trains by simulation

Figure 11: Replay on burning accident of RC branches of DC trains.

optimization and control of train-network coupling systems and the resonance suppression measures.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Disclosure

Jie Zhang (1980-), male, doctor, professor level senior engineer, is mainly engaged in the design of tractive drive system and research on control technology, e-mail:zhangjie1@csrzic.com.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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