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Research Article

An ASVSF-SLAM Algorithm with Time-Varying Noise Statistics Based on MAP Creation and Weighted Exponent

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The probability-based filtering method has been extensively used for solving the simultaneous localization and mapping (SLAM) problem. Generally, the standard filter utilizes the system model and prior stochastic information to approximate the posterior state. However, in the real-time situation, the noise statistics properties are relatively unknown, and the system is inaccurately modeled. Thus the filter divergence might occur in the integration system. Moreover, the expected accuracy might be challenging to be reached due to the absence of the responsive time-varying of both the process and measurement noise statistic which naturally can enlarge the uncertainty in the continuous system. Consequently, the traditional strategy needs to be improved aiming to provide an ability to estimate those properties. In order to accomplish this issue, the new adaptive filter is proposed in this paper, termed as an adaptive smooth variable structure filter (ASVSF). Sequentially, the improved SVSF is derived and implemented; the process and measurement noise statistics are estimated by utilizing the maximum a posteriori (MAP) creation and the weighted exponent concept, and the covariance correction step is added based on the divergence suppression concept. In this paper the ASVSF is applied to overcome the SLAM problem of an autonomous mobile robot; henceforth it is abbreviated as an ASVSF-SLAM algorithm. It is simulated and compared to the classical algorithm. The simulation results demonstrated that the proposed algorithm has better performance, stability, and effectiveness.

1. Introduction

The position tracking is an unavoidable part of localization that needs to be noticed. In most cases of mobile robot navigation, a robot should have the ability to locate its position and gather certain information related to the features of the environment automatically. This task is well-known as simultaneous localization and mapping (SLAM) [1–15] which was first proposed by Smith, Self, and Cheeseman in 1988. The SLAM-based mobile robot navigation has intensively received attention because of some challenging factors that need to be solved such as wide uncertainty, system complexity, inaccurate system model, limited prior information, noise statistics of the process and measurement, computational cost, and filter divergence. Essentially, those reasons lead to the probability-based estimation [5, 8–12, 16, 17] that has been proposed in many cases of the robot navigation. The most popular method is the Extended Kalman Filter (EKF) [3, 5–12, 16–23] which is a nonlinear version of Kalman Filter (KF) [3, 5, 6, 9, 12, 17, 20, 24, 25]. Nevertheless, it has many incompatibilities and difficulties such as the deviant solution from the state trajectory, less optimal state estimation, and large estimation error due to the linearization process and computational cost [12, 19]. This limitation makes its practical application becomes limited nowadays.

In order to address these problems, many nonlinear estimation methods have been approached, such as Unscented Kalman Filter (UKF) [3, 4, 6, 26], Cubature Kalman Filter [3, 6, 17, 20, 27], and Smooth Variable Structure Filter (SVSF) [1, 2, 13, 18, 21–23, 25, 28–34]. Essentially, these methods have been reducing the EKF problem in various solutions. For this reason, many researchers have switched to use one of

those methods rather than using EKF. The stable performance has also been demonstrated in some cases of solving SLAM-based navigation problem as presented in [1–13, 21, 22]. Most of the mentioned filters above require the accurate system model and known stochastic prior information. However, in the real-case application, the system is inaccurately modeled, and the prior knowledge of the noise statistics is unknown or partially known. An inaccuracy of modeling the system might enlarge the estimation error [4, 26, 35]. The internal and external uncertainties also might affect the change of the statistical characteristic which undoubtedly leads to the divergences of the filter performance [4, 24, 26, 35].

Regarding these possibilities, the error estimation has been approached for providing the responsive online approximation to the change of noise statistics. This strategy is well-known as the adaptive filter algorithm which modifies the conventional method with the specific attempt and combination. Mehra et al. classified the adaptive approaches into four categories: Bayesian, Maximum-Likelihood, Correlation, and Covariance Matching. And the similar types are utilized as well in this paper called MAP estimation and Covariance Correction. It is approached to enhance SVSF capability. Relatively, the SVSF is a new predictor-corrector estimator based on the sliding mode concept [2, 18, 21, 28]. Referring to some pieces of literature, it has been experiencing fast and significant development. The SVSF was first initiated in 2007 [1, 21, 28] which was a derived form referring to its successor termed a Variable Structure Filter (VSF) [34, 36] and Extended Variable Structure Filter (EVSF) [1, 2, 34]. Then we proceed with a presence of new form that revises the original SVSF by adding the error covariance matrix without affecting its accuracy and stability [1, 25, 28, 32]. As a common form of the filtering technique, it was then enhanced by involving the time-varying boundary layer width to replace its previous characteristic [23, 28, 32, 37], and even now there has been existing new second-order SVSF which satisfies both first and second sliding condition [28, 38]. Thus, it is not surprising that the SVSF has been regarded for having a better and robust performance to model uncertainty compared to other existing filters nowadays. Furthermore, the effectiveness of SVSF has also been variously shown in different applications on either the linear [25, 28] or nonlinear system [2, 18, 22, 28–32, 38] such as for the state and parameter estimation [18, 28], signal processing [28], fault detection and diagnosis [28], and target tracking [25, 28, 30]. Additionally, the characteristic of SVSF also allows it to be combined with a certain filtering method as the effort to obtain the optimal solution [34, 37, 39–41].

For these reasons, the SVSF is used in this experiment. However, as a standard filter, the SVSF is originally not designed with the ability to estimate the noise statistics properties which easily changes due to the uncertainties in the integration system. Therefore, the SVSF estimator was modified and improved in this paper. First, the improved SVSF was derived based on the one-step smoothing technique [24, 42, 43]. Then by referring to the prior knowledge, the noise statistics parameter of SVSF was estimated based on the maximum a posteriori (MAP) creation [4, 14, 15, 24, 26, 27, 35] and weighted exponent concept [4, 24, 26, 27]. It aims to produce the time-varying of those parameters.

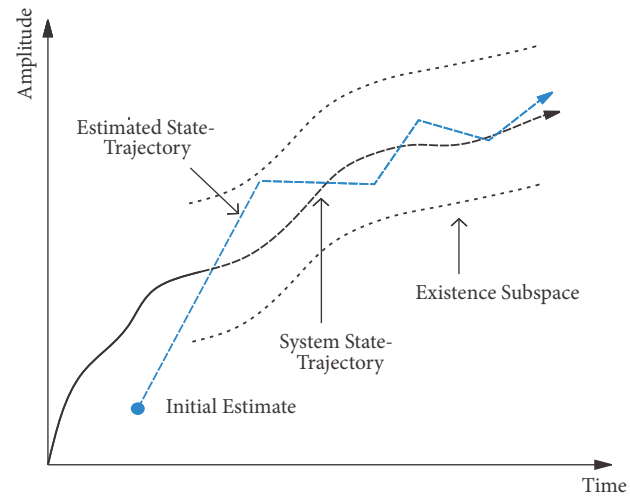

Figure 1: Smooth variable structure filter concept.

Second, to provide the ability against the filter divergence, the ASVSF was completed by the ability to correct the error covariance referring to the concept of divergence suppression [4, 44, 45]. In this paper the proposed method is applied for solving the SLAM problem of a wheeled mobile robot; henceforth it is termed as the ASVSF-SLAM algorithm. Then, by simulating and comparing to the conventional of SVSF-SLAM algorithm, it can be noted that the proposed method has better stability and accuracy refers to the benchmark in terms of Root Mean Square Error (RMSE) of estimated path and map. In this case, RMSE is used as the best measure to know the residual or deviation value which represents the difference between the actual and estimated values [28]. Besides that, this validation can also be readily observed from the small gap between a reference trajectory and the predicted result represented graphically.

The remaining parts of this paper are organized as follows. Section 2 contains SVSF. Section 3 presents the adaptive SVSF with the derivation of the improved SVSF, suboptimal MAP estimator, modified SVSF, unbiased suboptimal MAP estimator and its time-varying, and the divergence suppression method for covariance correction step. Section 4 presents an ASVSF-SLAM algorithm which is expanded with the discussion of the motion model, direct point-based observation, and inverse point-based observation. Section 5 presents some number of comparative result and discussion. Section 6 presents the conclusion.

2. Smooth Variable Structure Filter

The SVSF is new estimator based on sliding mode concept which has been becoming increasingly popular due to its robustness and stability to disturbance and uncertainty. It utilizes a switching gain to converge the estimate within a boundary of the true state value [28, 34, 41]. The analogy of SVSF process can be seen in Figure 1.

Considering that it is applied to nonlinear system, the dynamic model is described as below:

$$x_k = f\left(x_{k-1}, u_k\right) + w_{k-1} \tag{1}$$

$$z_k = h\left(x_k\right) + v_k \tag{2}$$

where k is discrete time index, $x \in \mathbb{R}^n$ is the state vector, u is the control vector, $z \in \mathbb{R}^m$ is the measurement vector, $w \in \mathbb{R}^n$ and $v \in \mathbb{R}^m$ are small adaptive process and measurement noise, respectively, and $f(.)$ and $h(.)$ are the nonlinear function and measurement model, respectively. The SVSF predicts the state estimate $\hat{x}_{k|k-1}$ obtained using the previous estimate state $\hat{x}_{k-1|k-1}$ and control vector u_k

$$\hat{x}_{k|k-1} = \hat{f}\left(\hat{x}_{k-1|k-1}, u_k\right) \tag{3}$$

using the partial derivative of $\hat{f}(.)$ with respect to $\hat{x}_{k-1|k}$ which is denoted by F_x; the a priori state error covariance matrix expressed by $P_{k|k-1} \in \mathbb{R}^{nxn}$ can be calculated as follows:

$$P_{k|k-1} = F_x P_{k-1|k-1} {F_x}^T + Q_{k-1} \tag{4}$$

Next, by utilizing the predicted state estimate $\hat{x}_{k|k-1}$, the corresponding predicted measurement $\hat{z}_{k|k-1}$ and the measurement error $e_{z,k|k-1}$ can be calculated as follows:

$$\hat{z}_{k|k-1} = \hat{h}\left(\hat{x}_{k|k-1}\right) \tag{5}$$

$$e_{z,k|k-1} = z_k - \hat{z}_{k|k-1} \tag{6}$$

Referring to [34, 41], it is considered that

$$A = \left(\left|e_{z,k|k-1}\right|_{abs} + \gamma \left|e_{z,k-1|k-1}\right|_{abs}\right) \tag{7}$$

$$\psi_k = \left(\overline{\overline{A}}^{-1} H P_{k|k-1} H^T S_k^{-1}\right)^{-1} \tag{8}$$

$$S_k = H P_{k|k-1} H^T + R_k \tag{9}$$

$$sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right] = \begin{cases} 1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \geq 1 \\ \psi^{-1}\overline{\overline{e_{z,k|k-1}}} & if \ -1 < \psi^{-1}\overline{\overline{e_{z,k|k-1}}} < 1 \\ -1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \leq -1 \end{cases} \tag{10}$$

and then the SVSF gain K_k^{SVSF} is calculated as below:

$$K_k^{SVSF} = H^+ \left\{\overline{\overline{A}} \circ sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right]\right\} \left[\overline{\overline{e_{z,k|k-1}}}\right]^{-1} \tag{11}$$

where $\overline{\cdot}$ indicates the diagonal term, γ refers to the convergence rate with element $0 < \gamma_{ii} \leq 1$, ψ refers to the boundary layer width, R_k is the measurement error covariance matrix, and H is linear measurement matrix, whereas $.^+$ and $\circ$ refer to the pseudo-inverse and Schur matrix multiplication, respectively.

Utilizing the equation above, the updated state $\hat{x}_{k|k}$, updated state error covariance matrix $P_{k|k}$, and new corresponding measurement error $e_{z,k+1|k+1}$ are described as follows:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k^{SVSF} e_{z,k|k-1} \tag{12}$$

$$P_{k|k} = \left(I - K_k^{SVSF} H\right) P_{k|k-1} \left(I - K_k^{SVSF} H\right)^T + K_k^{SVSF} R_k {K_k^{SVSF}}^T \tag{13}$$

$$e_{z,k|k} = z_k - \hat{h}\left(\hat{x}_{k|k}\right) \tag{14}$$

Equations (1)–(14) summarize all steps of SVSF. Note that the estimation process is stable and converges to the existence subspace if the following condition is satisfied [28]:

$$\left|e_{z,k-1|k-1}\right|_{abs} > \left|e_{z,k|k}\right|_{abs} \tag{15}$$

3. Adaptive SVSF

As can be analysed from the SVSF presented above, the noise statistic parameter is always constant for all processes and considered well-known initially. However, in the real application, it is impossible to define those parameters accurately by means they are partially known or even unknown. It might degrade the filtering performance. For these reasons, the adaptive SVSF algorithm is proposed in this paper. The process was initially started by reconsidering the prior information of the dynamic nonlinear system modelled as described in (1) and (2). Secondly, it is considered that the process noise w_k, measurement noise v_j, and initial state vector x_0 are assumed to be mutually uncorrelated for any discrete time index j or k; then the mean $E[.]$ and covariance cov[.] of the process and measurement noise can be defined as follows:

$$\begin{aligned} E\left[w_k\right] &= q_k, \\ \operatorname{cov}\left[w_k, w_j\right] &= Q_k \delta_{kj} \\ E\left[v_k\right] &= r_k, \\ \operatorname{cov}\left[v_k, v_j\right] &= R_k \delta_{kj} \\ \operatorname{cov}\left[w_k, v_j\right] &= 0 \end{aligned} \tag{16}$$

where δ refers to Kronecker delta function.

The prior information above is initially assumed to be not equal to zero. Additionally, Q_k and R_k are positive definite symmetric matrix; then the MAP estimates of q_k, Q_k, r_k, R_k, and X_k can be obtained by calculating the maximum value of the following conditional probability density function:

$$L = p\left[X_k, q, Q, r, R \mid Z_k\right] \tag{17}$$

where $X_k = [x_1, x_2, \ldots, x_k]$ and $Z_k = [z_1, z_2, \ldots, z_k]$. Next, applying the Bayes rule and referring to the property of the conditional probability, where L is proportional to $p[X_k, q, Q, r, R, Z_k]/p[Z_k]$ thus since its marginal likelihood $p[Z_k]$ plays no role in the optimization, then

$$L = p\left[Z_k \mid X_k, q, Q, r, R\right] p\left[X_k \mid q, Q, r, R\right] \cdot p\left[q, Q, r, R\right] \tag{18}$$

where $p[q, Q, r, R]$ is regarded to be constants obtained from the prior information. Then a posteriori distribution $p[X_k, q, Q, r, R \mid Z_k]$ can be calculated by multiplying $p[Z_k \mid X_k, q, Q, r, R]$ with $p[X_k, q, Q, r, R \mid Z_k]$ as derived below:

$$\begin{aligned}
p\left[X_k \mid q, Q, r, R\right] &= p\left[x_0\right] \prod_{i=1}^{k} p\left[x_i \mid x_{i-1}, q, Q\right] \\
&= \frac{1}{(2\pi)^{n/2} \left|P_0\right|^{1/2}} \exp\left[-\frac{1}{2}\left\|x_0 - \hat{x}_0\right\|^2_{P_0^{-1}}\right] \\
&\cdot \prod_{i=1}^{k} \frac{1}{(2\pi)^{n/2} |Q|^{1/2}} \exp\left[-\frac{1}{2}\left\|x_i - f\left(x_{i-1}\right) - q\right\|^2_{Q^{-1}}\right] \\
&= \frac{1}{(2\pi)^{n/2} \left|P_0\right|^{1/2}} \exp\left[-\frac{1}{2}\left\|x_0 - \hat{x}_0\right\|^2_{P_0^{-1}}\right] \\
&\times \frac{1}{(2\pi)^{nk/2} |Q|^{k/2}} \exp\left[-\frac{1}{2}\right. \\
&\left.\cdot \sum_{i=1}^{k} \left\|x_i - f\left(x_{i-1}\right) - q\right\|^2_{Q^{-1}}\right] \\
&= \frac{1}{(2\pi)^{n/2} (2\pi)^{nk/2}} \left|P_0\right|^{-1/2} |Q|^{-k/2} \\
&\cdot \exp\left\{-\frac{1}{2}\left[\left\|x_0 - \hat{x}_0\right\|^2_{P_0^{-1}}\right.\right. \\
&\left.\left.+ \sum_{i=1}^{k} \left\|x_i - f\left(x_{i-1}\right) - q\right\|^2_{Q^{-1}}\right]\right\} \\
&= \frac{1}{(2\pi)^{(nk+n)/2}} \left|P_0\right|^{-1/2} |Q|^{-k/2} \\
&\cdot \exp\left\{-\frac{1}{2}\left[\left\|x_0 - \hat{x}_0\right\|^2_{P_0^{-1}}\right.\right. \\
&\left.\left.+ \sum_{i=1}^{k} \left\|x_i - f\left(x_{i-1}\right) - q\right\|^2_{Q^{-1}}\right]\right\}
\end{aligned} \tag{19}$$

$$\begin{aligned}
p\left[Z_k \mid X_k, q, Q, r, R\right] &= \prod_{i=1}^{k} p\left[z_i \mid x_i, r, R\right] \\
&= \prod_{i=1}^{k} \frac{1}{(2\pi)^{m/2} |R|^{1/2}} \exp\left[-\frac{1}{2}\left\|z_i - h\left(x_i\right) - r\right\|^2_{R^{-1}}\right] \\
&= \frac{1}{(2\pi)^{mk/2}} |R|^{-k/2} \exp\left[-\frac{1}{2}\sum_{i=1}^{k}\left\|z_i - h\left(x_i\right)\right.\right. \\
&\left.\left.- r\right\|^2_{R^{-1}}\right]
\end{aligned} \tag{20}$$

$$\begin{aligned}
&p\left[Z_k \mid X_k, q, Q, r, R\right] p\left[X_k \mid q, Q, r, R\right] p\left[q, Q, r, R\right] \\
&== \frac{1}{(2\pi)^{mk/2}} |R|^{-k/2} \exp\left[-\frac{1}{2}\right. \\
&\left.\cdot \sum_{i=1}^{k} \left\|z_i - h\left(x_i\right) - r\right\|^2_{R^{-1}}\right] \times \frac{1}{(2\pi)^{(nk+n)/2}} \left|P_0\right|^{-1/2} \\
&\cdot |Q|^{-k/2} \exp\left\{-\frac{1}{2}\left[\left\|x_0 - \hat{x}_0\right\|^2_{P_0^{-1}}\right.\right. \\
&\left.\left.+ \sum_{i=1}^{k} \left\|x_i - f\left(x_{i-1}\right) - q\right\|^2_{Q^{-1}}\right]\right\} \times p\left[q, Q, r, R\right]
\end{aligned} \tag{21}$$

Now supposing that

$$C = \frac{1}{(2\pi)^{mk/2}} \frac{1}{(2\pi)^{(nk+n)/2}} \left|P_0\right|^{-1/2} p\left[q, Q, r, R\right] \cdot \exp\left[\frac{1}{2}\left\|x_0 - \hat{x}_0\right\|^2_{P_0^{-1}}\right] \tag{22}$$

then (21) can be simplified as follows:

$$L = C |R|^{-k/2} |Q|^{-k/2} \exp\left\{-\frac{1}{2}\left[\sum_{i=1}^{k}\left\|z_i - h\left(x_i\right) - r\right\|^2_{R^{-1}} + \sum_{i=1}^{k}\left\|x_i - f\left(x_{i-1}\right) - q\right\|^2_{Q^{-1}}\right]\right\} \tag{23}$$

Furthermore, to find the maximized parameter of the posterior distribution, firstly, take a logarithm as the monotonic function to simplify the calculation; secondly, find the first derivative of L with respect to q_k, Q_k, r_k, and R_k separately; then finally end it by equating it with zero. These steps are organized and derived as follows.

For $\ln(L)$ is equal to

$$\begin{aligned}
&\ln(C) - \frac{k}{2}\ln|R| - \frac{k}{2}\ln|Q| \\
&- \frac{1}{2}\left[\sum_{i=1}^{k}\left\|z_i - h\left(x_i\right) - r_i\right\|^2_{R^{-1}}\right. \\
&\left.+ \sum_{i=1}^{k}\left\|x_i - f\left(x_{i-1}\right) - q_{i-1}\right\|^2_{Q^{-1}}\right]
\end{aligned} \tag{24}$$

Then $\hat{q}_k, \hat{r}_k, \hat{Q}_k, \hat{R}_k$ are

$$\hat{q}_k = \frac{\partial \ln(L)}{\partial q} = \frac{1}{k}\sum_{i=1}^{k}\left(\hat{x}_{i|k} - f\left(\hat{x}_{i-1|k}\right)\right) \tag{25}$$

$$\widehat{Q}_k = \frac{\partial \ln(L)}{\partial Q} = \frac{1}{k}\sum_{i=1}^{k}\left(\widehat{x}_{i|k} - f\left(\widehat{x}_{i-1|k}\right) - q_{i-1}\right) \cdot \left(\widehat{x}_{i|k} - f\left(\widehat{x}_{i-1|k}\right) - q_{i-1}\right)^T \tag{26}$$

$$\widehat{r}_k = \frac{\partial \ln(L)}{\partial r} = \frac{1}{k}\sum_{i=1}^{k}\left(z_i - h\left(\widehat{x}_{i|k}\right)\right) \tag{27}$$

$$\widehat{R}_k = \frac{\partial \ln(L)}{\partial R} = \frac{1}{k}\sum_{i=1}^{k}\left(z_i - h\left(\widehat{x}_{i|k}\right) - r_i\right) \cdot \left(z_i - h\left(\widehat{x}_{i|k}\right) - r_i\right)^T \tag{28}$$

3.1. Suboptimal MAP Noise Estimator. The complicated multistep smoothing term ($\widehat{x}_{i|k}$ and $\widehat{x}_{i-1|k}$) in (25)-(28) might cause inefficiency of the MAP estimate; therefore to find the conventional and efficient recursive form the simplification is needed. Note that the recursive update process only utilizes the estimate value at time $k-1$ and k; thus the simplification can be conducted by replacing $\widehat{x}_{i-1|k}$ with $\widehat{x}_{i-1|i}$ in (25) and (26) and $x_{i|k}$ with $x_{i|i}$ in (27)-(28). Therefore, the suboptimal of MAP noise estimator can be expressed as follows:

$$\widehat{q}_k = \frac{1}{k}\sum_{i=1}^{k}\left(\widehat{x}_{i|i} - f\left(\widehat{x}_{i-1|i}\right)\right) \tag{29}$$

$$\widehat{Q}_k = \frac{1}{k}\sum_{i=1}^{k}\left(\widehat{x}_{i|i} - f\left(\widehat{x}_{i-1|i}\right) - q_{i-1}\right) \cdot \left(\widehat{x}_{i|,i} - f\left(\widehat{x}_{i-1|i}\right) - q_{i-1}\right)^T \tag{30}$$

$$\widehat{r}_k = \frac{1}{k}\sum_{i=1}^{k}\left(z_i - h\left(\widehat{x}_{i|i}\right)\right) \tag{31}$$

$$\widehat{R}_k = \frac{1}{k}\sum_{i=1}^{k}\left(z_i - h\left(\widehat{x}_{i|i}\right) - r_i\right)\left(z_i - h\left(\widehat{x}_{i|i}\right) - r_i\right)^T \tag{32}$$

As can be analysed from the sequence equations above the estimate value of $x_{i-1|i}$ is not provided by the SVSF. Therefore, modifying the original form of SVSF is needed to compute the noise statistics estimator effectively

3.2. Modified SVSF. The process of modifying the SVSF can be done by calculating the one-step smoothing of the SVSF gain and its corresponding estimate value using the fixed point smoothing algorithm [24, 42, 43]. This process can be summarized as follows:

$$\widehat{x}_{k|k-1} = \widehat{f}\left(\widehat{x}_{k-1|k-1}\right) + q_{k-1} \tag{33}$$

$$P_{k|k-1} = F_x P_{k-1|k-1} {F_x}^T + Q_{k-1} \tag{34}$$

$$\widehat{z}_{k|k-1} = \widehat{h}\left(\widehat{x}_{k|k-1}\right) + r_k \tag{35}$$

$$e_{z,k|k-1} = z_k - \widehat{z}_{k|k-1} \tag{36}$$

$$A = \left(\left|e_{z,k|k-1}\right|_{abs} + \gamma\left|e_{z,k-1|k-1}\right|_{abs}\right) \tag{37}$$

$$S_k = HP_{k|k-1}H^T + R_k \tag{38}$$

$$\psi_k = \left(\overline{\overline{A}}^{-1} HP_{k|k-1}H^T S_k^{-1}\right)^{-1} \tag{39}$$

$$sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right] = \begin{cases} 1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \geq 1 \\ \psi^{-1}\overline{\overline{e_{z,k|k-1}}} & if\ -1 < \psi^{-1}\overline{\overline{e_{z,k|k-1}}} < 1 \\ -1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \leq -1 \end{cases} \tag{40}$$

$$K_k^{SVSF} = H^+\left\{\overline{\overline{A}} \circ sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right]\right\}\left[\overline{\overline{e_{z,k|k-1}}}\right]^{-1} \tag{41}$$

$$\widehat{x}_{k-1|k} = \widehat{x}_{k-1|k-1} + K_k^{SVSF} e_{z,k|k-1} \tag{42}$$

Then considering that the prior state $\widehat{x}_{k-1|k}$ replaces the term of $\widehat{x}_{k-1|k-1}$ in the normal SVSF, the remaining part of modified SVSF is chained as follows:

$$\widehat{x}_{k|k-1} = \widehat{f}\left(\widehat{x}_{k-1|k}\right) + q_{k-1} \tag{43}$$

$$\widehat{z}_{k|k-1} = \widehat{h}\left(\widehat{x}_{k|k-1}\right) + r_k \tag{44}$$

$$e_{z,k|k-1} = z_k - \widehat{z}_{k|k-1} \tag{45}$$

$$S_k = HP_{k|k-1}H^T + R_k \tag{46}$$

$$A = \left(\left|e_{z,k|k-1}\right|_{abs} + \gamma\left|e_{z,k-1|k-1}\right|_{abs}\right) \tag{47}$$

$$\psi_k = \left(\overline{\overline{A}}^{-1} HP_{k|k-1}H^T S_k^{-1}\right)^{-1} \tag{48}$$

$$sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right] = \begin{cases} 1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \geq 1 \\ \psi^{-1}\overline{\overline{e_{z,k|k-1}}} & if\ -1 < \psi^{-1}\overline{\overline{e_{z,k|k-1}}} < 1 \\ -1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \leq -1 \end{cases} \tag{49}$$

$$K_k^{SVSF} = H^+\left\{\overline{\overline{A}} \circ sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right]\right\}\left[\overline{\overline{e_{z,k|k-1}}}\right]^{-1} \tag{50}$$

$$\widehat{x}_{k|k} = \widehat{x}_{k|k-1} + K_k^{SVSF} e_{z,k|k-1} \tag{51}$$

$$P_{k|k} = \left(I - K_k^{SVSF}H\right)P_{k|k-1}\left(I - K_k^{SVSF}H\right)^T + K_k^{SVSF}R_k{K_k^{SVSF}}^T \tag{52}$$

$$e_{z,k|k} = z_k - \widehat{h}\left(\widehat{x}_{k|k}\right) \tag{53}$$

Now, the estimate values $\widehat{x}_{i|i}$ and $\widehat{x}_{i-1|i}$ can be adopted from (51) and (42), respectively

3.3. Unbiased Suboptimal MAP Noise Estimator. Next, to guarantee that the recursive process and measurement noise

statistics are unbiased, the modified suboptimal MAP noise estimators are then derived referring to unbiased estimation.

First, by substituting (43) into (51), the general term $\hat{x}_{i|i} - f(\hat{x}_{i-1|i})$ in (29) and (30) can be rewritten as follows:

$$\hat{x}_{i|i} - f(\hat{x}_{i-1|i}) = K_k^{SVSF} e_{z,i} + q_{i-1} \tag{54}$$

Similarly, replacing $\hat{x}_{i|i}$ with (51), the general term $z_i - h(\hat{x}_{i|i})$ in (31) and (32) can be written as follows:

$$z_i - h(\hat{x}_{i|i}) = z_i - h\left(\hat{x}_{i|i-1} + K_i^{SVSF} e_{z,i}\right) \tag{55}$$

Note that $e_{z,i} = e_{z,i|i-1}$, and the suboptimal MAP estimation (28)-(31) can be rearranged as follows:

$$\hat{q}_k = \frac{1}{k}\sum_{i=1}^{k}\left(K_k^{SVSF} e_{z,i} + q_{i-1}\right) \tag{56}$$

$$\hat{Q}_k = \frac{1}{k}\sum_{i=1}^{k}\left(K_k^{SVSF} e_{z,i} e_{z,i}{}^T K_k^{SVSF^T}\right) \tag{57}$$

$$\hat{r}_k = \frac{1}{k}\sum_{i=1}^{k}\left(I - HK_i^{SVSF}\right) e_{z,i} + r_i \tag{58}$$

$$\hat{R}_k = \frac{1}{k}\sum_{i=1}^{k}\left(I - HK_i^{SVSF}\right) e_{z,i} e_{z,i}{}^T \left(I - HK_i^{SVSF}\right)^T \tag{59}$$

Since the innovation e_z and its covariance $e_z e_z{}^T$ are contained in the process and measurement noise estimator, therefore,

$$e_{z,i} = h(\tilde{x}_{i|i-1}) + v_i - r_i \tag{60}$$

and referring to state error covariance in (52), we have

$$\begin{aligned} K_i^{SVSF} e_{z,i} e_{z,i}{}^T K_i^{SVSF^T} &= P_{i|i} - P_{i|i-1} + P_{i|i-1} H^T K_i{}^{SVSF^T} \\ &\quad + HK_i^{SVSF} P_{i|i-1} \end{aligned} \tag{61}$$

Next, considering that the expectations are

$$E\left[e_{z,i}\right] = 0 \tag{62}$$

$$E\left[e_{z,i} e_{z,i}{}^T\right] = HP_{i|i-1} H^T + R_i \tag{63}$$

then by substituting (60)-(63) into (56)-(59), we have

$$E\left[\hat{q}_k\right] = q_k \tag{64}$$

$$E\left[\hat{r}_k\right] = r_k \tag{65}$$

$$\begin{aligned} E\left[\hat{Q}_k\right] = -Q_k + E\Bigg[\frac{1}{k}\sum_{i=1}^{k}\Big(&P_{i|i} + P_{i|i-1} H^T K_i{}^{SVSF^T} \\ &+ HK_i^{SVSF} P_{i|i-1} - F_{i-1} P_{i-1|i-1} F_{i-1}{}^T\Big)\Bigg] \end{aligned} \tag{66}$$

$$\begin{aligned} E\left[\hat{R}_k\right] = R_k + E\Bigg[\frac{1}{k}\sum_{i=1}^{k}\Big(&HK_i^{SVSF} e_{z,i} e_{z,i}{}^T H^T K_i{}^{SVSF^T} \\ &- HK_i^{SVSF} e_{z,i} e_{z,i}{}^T \\ &- e_{z,i} e_{z,i}{}^T H^T K_i{}^{SVSF^T} HP_{i|i-1} H^T\Big)\Bigg] \end{aligned} \tag{67}$$

Note that q_k, r_k, Q_k, R_k are the representation of the suboptimal MAP estimation in (56)-(59); thus the unbiased MAP estimation can be summarized as follows:

$$\hat{q}_k = \frac{1}{k}\sum_{i=1}^{k}\left(K_i^{SVSF} e_{z,i} + q_{i-1}\right) \tag{68}$$

$$\hat{r}_k = \frac{1}{k}\sum_{i=1}^{k}\left(I - HK_i^{SVSF}\right) e_{z,i} + r_i \tag{69}$$

$$\hat{Q}_k = \frac{1}{k}\sum_{i=1}^{k}\left(P_{i|i-1} - F_{i-1} P_{i-1|i-1} F_{i-1}{}^T\right) \tag{70}$$

$$\begin{aligned} \hat{R}_k = \frac{1}{k}\sum_{i=1}^{k}\Big[&2\left(\left(I - HK_i^{SVSF}\right) e_{z,i} e_{z,i}{}^T \left(I - HK_i^{SVSF}\right)^T\right) \\ &- e_{z,i} e_{z,i}{}^T + HP_{i|i-1} H^T\Big] \end{aligned} \tag{71}$$

3.4. Time-Varying Unbiased Noise Estimator and Weighting Exponentially. The time-varying noise estimator is proposed in this paper. According to the unbiased suboptimal MAP estimator calculated in (68)-(71), then the time-varying unbiased noise estimators are derived as follows:

$$\hat{q}_k = \hat{q}_{k-1} + \frac{1}{k}\left(K_k^{SVSF} e_{z,k}\right) \tag{72}$$

$$\hat{r}_k = \hat{r}_{k-1} + \frac{1}{k}\left[\left(I - HK_k^{SVSF}\right) e_{z,k}\right] \tag{73}$$

$$\hat{Q}_k = \frac{k-1}{k}\hat{Q}_{k-1} + \frac{1}{k}\left[P_{k|k-1} - F_{k-1} P_{k-1|k-1} F_{k-1}{}^T\right] \tag{74}$$

$$\begin{aligned} \hat{R}_k = &\frac{k-1}{k}\hat{R}_{k-1} \\ &+ \frac{1}{k}\Big[2\left(\left(I - HK_k^{SVSF}\right) e_{z,k} e_{z,k}{}^T \left(I - HK_k^{SVSF}\right)^T\right) \\ &- e_{z,k} e_{z,k}{}^T + HP_{k|k-1} H^T\Big] \end{aligned} \tag{75}$$

The following alternative forms (76)-(79) are regarded as the modified form of the time-varying unbiased noise statistics estimator (72)-(75). It is calculated by referring to the fading

memory weighted exponent method and by utilizing the weighting coefficient d_k to replace the exponential $1/k$.

$$\widehat{q}_k = \widehat{q}_{k-1} + d_k \left(K_k^{SVSF} e_{z,k}\right) \tag{76}$$

$$\widehat{r}_k = \widehat{r}_{k-1} + d_k \left[\left(I - HK_k^{SVSF}\right) e_{z,k}\right] \tag{77}$$

$$\widehat{Q}_k = (1 - d_k)\, \widehat{Q}_{k-1} + d_k \left[P_{k|k-1} - F_{k-1} P_{k-1|k-1} F_{k-1}{}^T\right] \tag{78}$$

$$\begin{aligned} \widehat{R}_k &= (1 - d_k)\, \widehat{R}_{k-1} \\ &\quad + d_k \left[2\left(\left(I - HK_k^{SVSF}\right) e_{z,k} e_{z,k}{}^T \left(I - HK_k^{SVSF}\right)^T\right)\right. \\ &\quad \left. - e_{z,k} e_{z,k}{}^T + HP_{k|k-1} H^T\right] \end{aligned} \tag{79}$$

where the residual measurement $e_{z,k} = e_{z,k|k-1}$ and weighting coefficient can be written.

$$\begin{aligned} \beta_i &= d_k b^{i-1}, \quad i = 1, \ldots, n \\ d_k &= \frac{(1-b)}{(1-b^k)} \end{aligned} \tag{80}$$

b is fading factor satisfying $0 < b < 1$ and β_i is the i-th weighting factor, which is defined as $\beta_i = \beta_{i-1} b$ and satisfied $\sum_{i=1}^{k} \beta_i$.

3.5. Addition of Divergence Suppression Method. Next, to prevent the occurrence of filter divergence, the covariance correction based on the divergence suppression concept is involved [4]. First, the convergence condition is derived referring to the covariance matching creation as described below:

$$v_k^T v_k \le S.tr\left[E\left(v_k^T v_k\right)\right] \tag{81}$$

where S is an adjustable coefficient presetting which is satisfied ($S \ge 1$) and the residual sequence $v_k = z_k - h(\widehat{x}_{k,k-1})$. Executing (81), if the convergence condition is satisfied, (76)-(79) are applied; otherwise, the correction method of the covariance $P_{k|k-1}$ is suggested against the divergence occurrence.

$$P_{k|k-1} = \lambda_k . P_{k|k-1} \tag{82}$$

where λ_k is known as the adaptive weighting coefficient which is calculated based on the fading factor formula [44, 45] as summarized as follows:

$$C_{0,k} = \begin{cases} v_k^T v_k & k = 1 \\ \dfrac{\rho C_{0,k} + v_k^T v_k}{1+\rho} & k > 1 \end{cases} \tag{83}$$

$$N_k = tr\left(C_{0,k} - R\right)^T \tag{84}$$

$$M_k = tr\left(P_{k,k-1}\right) \tag{85}$$

$$\lambda_0 = tr \frac{[N_k]}{tr\,[M_k]} \tag{86}$$

$$\lambda_k = \begin{cases} \lambda_0 & \lambda_0 \ge 1 \\ 1 & \lambda_0 < 1 \end{cases} \tag{87}$$

where $tr(.)$ refers to the matrix trace and ρ is the forgetful factor satisfied $0 < \rho \le 1$ (typically to be set 0.95). Note that increasing the factor will create a smaller proportion of the information before time k [4]. It causes the residual vector effect to become prominent so that the filter tracking ability will increase.

Up to this point, the adaptive SVSF can be summarized as follows.

ASVSF Algorithm

(1) Modified SVSF

$$\begin{aligned} \widehat{x}_{k|k-1} &= \widehat{f}\left(\widehat{x}_{k-1|k-1}\right) + q_{k-1} \\ P_{k|k-1} &= F_x P_{k-1|k-1} F_x{}^T + Q_{k-1} \end{aligned} \tag{88}$$

(2) Covariance Correction

$$\begin{aligned} C_{0,k} &= \begin{cases} v_k^T v_k \\ \dfrac{\rho C_{0,k} + v_k^T v_k}{1+\rho} \end{cases} \\ N_k &= tr\left(C_{0,k} - R\right)^T \\ M_k &= tr\left(P_{k,k-1}\right) \\ \lambda_0 &= tr \frac{[N_k]}{tr\,[M_k]} \\ \lambda_k &= \begin{cases} \lambda_0 & \lambda_0 \ge 1 \\ 1 & \lambda_0 < 1 \end{cases} \\ P_{k|k-1} &= \lambda_k . P_{k|k-1} \end{aligned} \tag{89}$$

(3) Modified SVSF

$$\begin{aligned} \widehat{z}_{k|k-1} &= \widehat{h}\left(\widehat{x}_{k|k-1}\right) + r_k \\ \breve{e}_{z,k|k-1} &= z_k - \widehat{z}_{k|k-1} \\ A &= \left(\left|\breve{e}_{z,k|k-1}\right|_{abs} + \gamma \left|e_{z,k-1|k-1}\right|_{abs}\right) \\ S_k &= HP_{k|k-1} H^T + R_k \\ \psi_k &= \left(\overline{\overline{A}}^{-1} HP_{k|k-1} H^T S_k^{-1}\right)^{-1} \end{aligned}$$

$$
\begin{aligned}
&sat\left[\psi^{-1}\overline{\overline{\breve{e}_{z,k|k-1}}}\right] \\
&\quad = \begin{cases} 1 & \psi^{-1}\overline{\overline{\breve{e}_{z,k|k-1}}} \geq 1 \\ \psi^{-1}\overline{\overline{\breve{e}_{z,k|k-1}}} & if \ -1 < \psi^{-1}\overline{\overline{\breve{e}_{z,k|k-1}}} < 1 \\ -1 & \psi^{-1}\overline{\overline{\breve{e}_{z,k|k-1}}} \leq -1 \end{cases} \\
&K_k^{SVSF} = H^{+}\left\{\overline{\overline{A}} \circ sat\left[\psi^{-1}\overline{\overline{\breve{e}_{z,k|k-1}}}\right]\right\}\left[\overline{\overline{\breve{e}_{z,k|k-1}}}\right]^{-1} \\
&\widehat{x}_{k-1|k} = \widehat{x}_{k-1|k-1} + K_k^{SVSF}\breve{e}_{z,k|k-1}
\end{aligned} \tag{90}
$$

(4) Improved SVSF

$$
\begin{aligned}
&\widehat{x}_{k|k-1} = \widehat{f}\left(\widehat{x}_{k-1|k}\right) + q_{k-1} \\
&\widehat{z}_{k|k-1} = \widehat{h}\left(\widehat{x}_{k|k-1}\right) + r_k \\
&e_{z,k|k-1} = z_k - \widehat{z}_{k|k-1} \\
&S_k = HP_{k|k-1}H^T + R_k \\
&A = \left(\left|e_{z,k|k-1}\right|_{abs} + \gamma\left|e_{z,k-1|k-1}\right|_{abs}\right) \\
&\psi_k = \left(\overline{\overline{A}}^{-1}HP_{k|k-1}H^TS_k^{-1}\right)^{-1} \\
&sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right] \\
&\quad = \begin{cases} 1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \geq 1 \\ \psi^{-1}\overline{\overline{e_{z,k|k-1}}} & if \ -1 < \psi^{-1}\overline{\overline{e_{z,k|k-1}}} < 1 \\ -1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \leq -1 \end{cases} \\
&K_k^{SVSF} = H^{+}\left\{\overline{\overline{A}} \circ sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right]\right\}\left[\overline{\overline{e_{z,k|k-1}}}\right]^{-1} \\
&\widehat{x}_{k|k} = \widehat{x}_{k|k-1} + K_k^{SVSF}e_{z,k|k-1} \\
&P_{k|k} \\
&\quad = \left(I - K_k^{SVSF}H\right)P_{k|k-1}\left(I - K_k^{SVSF}H\right)^T \\
&\qquad + K_k^{SVSF}R_kK_k^{SVSF^T} \\
&e_{z,k|k} = z_k - \widehat{h}\left(\widehat{x}_{k|k}\right)
\end{aligned} \tag{91}
$$

(5) Time-Varying Noise Statistic

$$
\begin{aligned}
&\widehat{q}_k = \widehat{q}_{k-1} + d_k\left(K_k^{SVSF}e_{z,k}\right) \\
&\widehat{r}_k = \widehat{r}_{k-1} + d_k\left[\left(I - HK_k^{SVSF}\right)e_{z,k}\right] \\
&\widehat{Q}_k = \left(1 - d_k\right)\widehat{Q}_{k-1} + d_k\left[P_{k|k-1} - F_{k-1}P_{k-1|k-1}F_{k-1}^T\right] \\
&\widehat{R}_k = \left(1 - d_k\right)\widehat{R}_{k-1} \\
&\quad + d_k\left[2\left(\left(I - HK_k^{SVSF}\right)e_{z,k}e_{z,k}^T\left(I - HK_k^{SVSF}\right)^T\right)\right. \\
&\quad \left. - e_{z,k}e_{z,k}^T + HP_{k|k-1}H^T\right]
\end{aligned} \tag{92}
$$

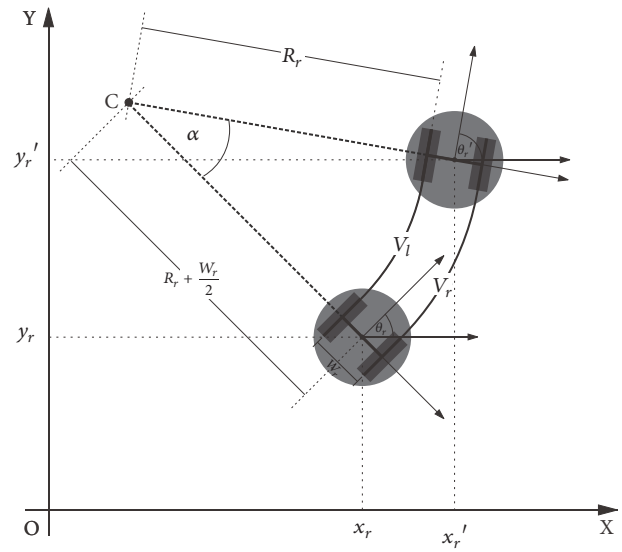

Figure 2: Kinematic configuration of wheeled mobile robot.

The ASVF applies an improved/modified SVSF involving the divergence suppression method aiming to correct the state error covariance. Additionally, it is also completed by the time-varying unbiased MAP estimator so that ASVSF is able to estimate the noise statistics recursively.

4. Adaptive SVSF-Based SLAM Algorithm

The proposed method is approached to address the SLAM problem for the mobile robot. The used robot contains two driven wheels. By supposing this, it is simulated and initially located in certain planar environment. Then the kinematic configuration of the robot movement can be shown in Figure 2.

Assuming that x_r and y_r are the spatial position and θ_r is the orientation of the robot, then the robot state vector can be expressed as $x_R = [x_r, y_r, \theta_r]^T$.

The robot moves based on the odometry and differential steering system, therefore, since the control vector contains two control vectors represented by $u = [u_r, u_l]^T$ for right wheel u_r and left wheel velocity u_l, respectively, then the motion model is expressed as follows:

$$
x_{R,k} = \begin{cases} x_R^A, & v_r = v_l \\ x_R^B, & v_r \neq v_l \end{cases} \tag{93}
$$

$$
x_R^A = \begin{bmatrix} x_{r,k}^A \\ y_{r,k}^A \\ \theta_{r,k}^A \end{bmatrix} = \begin{bmatrix} x_{r,k-1}^A \\ y_{r,k-1}^A \\ \theta_{r,k-1}^A \end{bmatrix} + v\begin{bmatrix} \cos\left(\theta_{r,k-1}^A\right) \\ \sin\left(\theta_{r,k-1}^A\right) \\ 0 \end{bmatrix} \tag{94}
$$

$$x_R^B = \begin{bmatrix} x_{r,k}^B \\ y_{r,k}^B \\ \theta_{r,k}^B \end{bmatrix} = \begin{bmatrix} x_{r,k-1}^B \\ y_{r,k-1}^B \\ \theta_{r,k-1}^B \end{bmatrix} + \begin{bmatrix} \left(R_r + \frac{w_r}{2}\right)\left(\sin\left(\theta_{r,k-1}^B + \alpha\right) - \sin(\alpha)\right) \\ \left(R_r + \frac{w_r}{2}\right)\left(-\cos\left(\theta_{r,k-1}^B + \alpha\right) + \cos(\alpha)\right) \\ \alpha \end{bmatrix} \tag{95}$$

where k is time index, R_r is the radius caused by the motion with $R_r = v_l/\alpha$, w_r is width of the robot, and $\cdot^A$ and $\cdot^B$ are the different motion condition with respect to the measured velocity $v = [v_r, v_l]^T$. In fact, the robot motion is always followed by unavoidable small noise. Thus the measured right-wheel v_r and left-wheel velocity can be regarded as follows with existence of the small perturbation $n = [n_r, n_l]^T$:

$$v = u + n \tag{96}$$

$$\begin{aligned} n_r &= \zeta_1 u_r + \zeta_2 (u_l - u_r) \\ n_l &= \zeta_1 u_l + \zeta_2 (u_l - u_r) \end{aligned} \tag{97}$$

where ζ_1 is the move factor and ζ_2 is the turn factor.

4.1. Direct Point-Based Observation Model. The state vector is composed not only by robot state vector x_R but also by the landmark state represented by $x_L^i = [x_{L,x}^i, x_{L,y}^i]^T$ and the full state vector can be denoted by $x = [x_r, y_r, \theta_r, x_{L,x}^1, x_{L,y}^1, \ldots, \ldots, x_{L,x}^m, x_{L,y}^m]^T$, where $x_{L,x}^i$ and $x_{L,y}^i$ represent the i-th landmark position for $i = 1, 2, \ldots, m$.

Figure 3 illustrates that the robot is measuring the i-th detected landmark by utilizing the laser scanner. The position of the laser scanner is denoted by $x_{ls}^R = [x_{ls}, y_{ls}]^T$; then the direct point-based observation model can be calculated as follows:

$$\begin{bmatrix} x_{ls} \\ y_{ls} \end{bmatrix} = \begin{bmatrix} x_r \\ y_r \end{bmatrix} + d_{ls} \begin{bmatrix} \cos(\theta_r) \\ \sin(\theta_r) \end{bmatrix} \tag{98}$$

$$\begin{bmatrix} \delta_L^i \\ \beta_L^i \end{bmatrix} = \begin{bmatrix} \sqrt{\left(x_{L,x}^i - x_{ls}\right)^2 + \left(x_{L,y}^i - y_{ls}\right)^2} \\ \arctan\left(\dfrac{x_{L,y}^i - y_{ls}}{x_{L,x}^i - x_{ls}}\right) \end{bmatrix} \tag{99}$$

where d_{ls} is the displacement of laser scanner and δ_L^i and β_L^i are the distance and bearing sensed by laser scanner. Up to this point, we have satisfied the measurement function $h(.)$. Similarly, by considering that the measurement is followed by small perturbation denoted by $r = [r_\delta, r_\beta]^T$, we have

$$z_i = \begin{bmatrix} \delta_L^i \\ \beta_L^i \end{bmatrix} + \begin{bmatrix} r_\delta \\ r_\beta \end{bmatrix} \tag{100}$$

Then, since (93)-(100) are derived completely, both state transition in (33) and measurement function in (35) are satisfied. Next, H^+ can be calculated as follows:

$$F_{ps,i} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \underbrace{\cdots}_{2i-2} & 0 & 0 & 1 & 0 & \underbrace{\cdots}_{2m-2i} & 0 \end{bmatrix} \tag{101}$$

$$H_{ps,i} = H_{rpsx,i} \begin{cases} H_{rpsx,i} = H^T\left(HH^T\right)^{-1}, & \text{rows of } H \text{ are linearly independent} \\ H_{rpsx,i} = \left(HH^T\right)^{-1} H^T, & \text{columns of } H \text{ are linearly independent} \end{cases} \tag{102}$$

$$H^+ = {F_{ps,i}}^T H_{ps,i} \tag{103}$$

where H is the Jacobian matrix of $h(.)$ with respect to the predicted state in (33)-(53).

4.2. Inverse Point-Based Observation. Generally, new observed landmark must be initialized and added to the state vector. In this experiment, an inverse point-based observation model initiates the mapping process by utilizing the information of the current robot and landmark position which can be written as follows:

$$x_L^{new} = h^{-1}\left(x_{R,k}, z_i\right) \tag{104}$$

$$\begin{bmatrix} x_{L,x}^{new} \\ x_{L,y}^{new} \end{bmatrix}$$

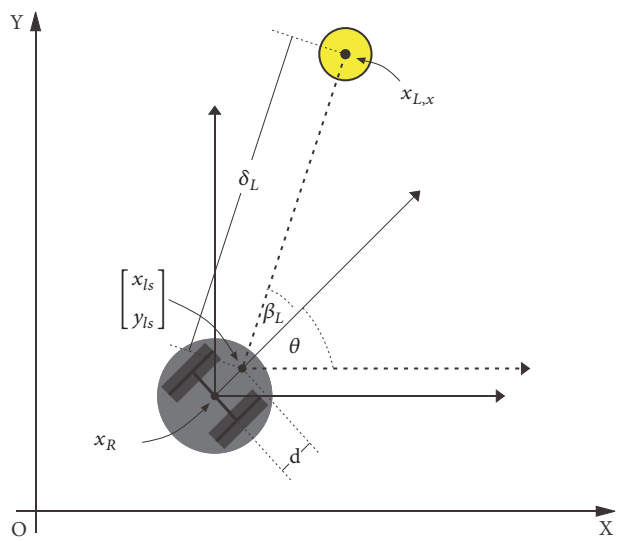

Figure 3: Graphical representation of landmark detection.

$$= \begin{bmatrix} x_r \\ y_r \end{bmatrix} + \delta_L^i \begin{bmatrix} \cos(\theta_r)\cos(\beta_L^i) - \sin(\theta_r)\sin(\beta_L^i) \\ \sin(\theta_r)\cos(\beta_L^i) + \cos(\theta_r)\sin(\beta_L^i) \end{bmatrix} \tag{105}$$

At this point, the derivation of SVSF and SLAM algorithm used for wheeled mobile robot are fully formulated. Therefore, by chaining them we have ASVSF-SLAM algorithm.

4.3. Adaptive SVSF Based SLAM Algorithm

begin algorithm

(i) ***Initialization***

$$\hat{x}_0, P_0, e_{z,0}, \hat{q}_0, \hat{Q}_0 \hat{r}_0, R_0, \gamma \tag{106}$$

(ii) ***Prediction update***

(one-step smoothing method)

$$\begin{aligned} \check{x}_{k|k-1} &= \hat{f}(\hat{x}_{k-1|k-1}) + \hat{q}_{k-1} \\ P_{k|k-1} &= F_x P_{k-1|k-1} F_x{}^T + \hat{Q}_{k-1} \end{aligned} \tag{107}$$

(iii) ***Observation and Update***

for *all feature observation*

if *correspondence is found before seen landmark*

calculate a posteriori measurement error

$$\begin{aligned} \hat{z}_{k|k-1} &= \hat{h}(\check{x}_{k|k-1}) + \hat{r}_k \\ \check{e}_{z,k|k-1} &= z_k - \hat{z}_{k|k-1} \end{aligned} \tag{108}$$

Covariance correction

$$\begin{aligned} v_k &= \check{e}_{z,k|k-1} \\ v_k^T v_k &\le S.tr\left[E\left(v_k^T v_k\right)\right] \\ C_{0,k} &= \begin{cases} v_k^T v_k & k = 1 \\ \dfrac{\rho C_{0,k} + v_k^T v_k}{1+\rho} & k > 1 \end{cases} \\ N_k &= tr\left(C_{0,k} - R\right)^T \\ M_k &= tr\left(P_{k,k-1}\right) \\ \lambda_0 &= tr\frac{[N_k]}{tr[M_k]} \\ \lambda_k &= \begin{cases} \lambda_0 & \lambda_0 \ge 1 \\ 1 & \lambda_0 < 1 \end{cases} \\ P_{k|k-1} &= \lambda_k . P_{k|k-1} \end{aligned} \tag{109}$$

Compute one-step smoothing SVSF gain

$$\begin{aligned} A &= \left(\left|\check{e}_{z,k|k-1}\right|_{abs} + \gamma \left|e_{z,k-1|k-1}\right|_{abs}\right) \\ S_k &= HP_{k|k-1}H^T + \hat{R}_k \\ \psi_k &= \left(\overline{\overline{A}}^{-1} HP_{k|k-1}H^T S_k^{-1}\right)^{-1} \\ & sat\left[\psi^{-1}\overline{\overline{\check{e}_{z,k|k-1}}}\right] \\ &= \begin{cases} 1 & \psi^{-1}\overline{\overline{\check{e}_{z,k|k-1}}} \ge 1 \\ \psi^{-1}\overline{\overline{\check{e}_{z,k|k-1}}} & if \ -1 < \psi^{-1}\overline{\overline{\check{e}_{z,k|k-1}}} < 1 \\ -1 & \psi^{-1}\overline{\overline{\check{e}_{z,k|k-1}}} \le -1 \end{cases} \\ K_k^{SVSF} &= H^+ \left\{\overline{\overline{A}} \circ sat\left[\psi^{-1}\overline{\overline{\check{e}_{z,k|k-1}}}\right]\right\} \left[\overline{\overline{\check{e}_{z,k|k-1}}}\right]^{-1} \end{aligned} \tag{110}$$

Calculate the smoothed state $\hat{x}_{k-1|k}$

$$\hat{x}_{k-1|k} = \hat{x}_{k-1|k-1} + K_k^{SVSF} \check{e}_{z,k|k-1} \tag{111}$$

Conduct the improved SVSF

$$\begin{aligned}
&\hat{x}_{k|k-1} = \hat{f}\left(\hat{x}_{k-1|k}\right) + \hat{q}_{k-1} \\
&\hat{z}_{k|k-1} = \hat{h}\left(\hat{x}_{k|k-1}\right) + \hat{r}_k \\
&e_{z,k|k-1} = z_k - \hat{z}_{k|k-1} \\
&S_k = HP_{k|k-1}H^T + \hat{R}_k \\
&A = \left(\left|e_{z,k|k-1}\right|_{abs} + \gamma\left|e_{z,k-1|k-1}\right|_{abs}\right) \\
&\psi_k = \left(\overline{\overline{A}}^{-1} HP_{k|k-1}H^T S_k^{-1}\right)^{-1} \\
&sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right] \\
&\quad = \begin{cases} 1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \geq 1 \\ \psi^{-1}\overline{\overline{e_{z,k|k-1}}} & if \ -1 < \psi^{-1}\overline{\overline{e_{z,k|k-1}}} < 1 \\ -1 & \psi^{-1}\overline{\overline{e_{z,k|k-1}}} \leq -1 \end{cases} \\
&K_k^{SVSF} = H^+\left\{\overline{\overline{A}} \circ sat\left[\psi^{-1}\overline{\overline{e_{z,k|k-1}}}\right]\right\}\left[\overline{\overline{e_{z,k|k-1}}}\right]^{-1}
\end{aligned} \tag{112}$$

Update state and covariance

$$\begin{aligned}
&\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k^{SVSF} e_{z,k|k-1} \\
&P_{k|k} = \left(I - K_k^{SVSF}H\right)P_{k|k-1}\left(I - K_k^{SVSF}H\right)^T \\
&\qquad + K_k^{SVSF}R_k K_k^{SVSF^T}
\end{aligned} \tag{113}$$

Estimate noise statistics

$$\begin{aligned}
&e_{z,k} = e_{z,k|k-1} \\
&\hat{q}_k = \hat{q}_{k-1} + d_k\left(K_k^{SVSF} e_{z,k}\right) \\
&\hat{r}_k = \hat{r}_{k-1} + d_k\left[\left(I - HK_k^{SVSF}\right)e_{z,k}\right] \\
&\hat{Q}_k = \left(1 - d_k\right)\hat{Q}_{k-1} + d_k\left[P_{k|k-1} - F_{k-1}P_{k-1|k-1}F_{k-1}{}^T\right] \\
&\hat{R}_k = \left(1 - d_k\right)\hat{R}_{k-1} \\
&\quad + d_k\left[2\left(\left(I - HK_k^{SVSF}\right)e_{z,k}e_{z,k}{}^T\left(I - HK_k^{SVSF}\right)^T\right)\right. \\
&\quad \left. - e_{z,k}e_{z,k}{}^T + HP_{k|k-1}H^T\right]
\end{aligned} \tag{114}$$

Update the prior measurement error

$$e_{z,k|k} = z_k - \hat{h}\left(\hat{x}_{k|k}\right) \tag{115}$$

end if

end for

(iv) ***Map Management***

for *all nonassociated new feature*

Initialize a new feature

$$x_L^{new} = h^{-1}\left(x_{R,k}, z_i\right) \tag{116}$$

Increment the state vector by adding x_L^{new} to the current state vector $\hat{x}_{current}$

$$\hat{x}_{increment} = \left[\hat{x}_{current}, x_L^{new}\right] \tag{117}$$

Jump $\hat{x}_{increment}$ to Observation and Update Process

Time Increment

$$k = k + 1 \tag{118}$$

end for

end begin

5. Result and Discussion

In order to verify the effectiveness and accuracy, the proposed method was simulated. It was compared with classical algorithm in term of RMSE of the estimated path and map. Ideally, the absence of noise in both process and measurement gives the reference trajectory as the base, while the presence of these noises following the process and measurement will obviously give the difference to both expected path and map. Henceforth this difference is termed as the residual values. Based on it the RMSE can be calculated to qualitatively represent the filter performance referring to definition of SLAM which are locating the robot current location (path) when a robot detects new feature in the environment and mapping all observed features in the environment (map). Therefore, on the other hand it can be defined that by knowing the different between the actual and predicted position of the path and map, the effectiveness and accuracy of the proposed method can be validated. Initially, some parameters related to the robot and the proposed algorithm were defined as can be seen below:

$$\begin{aligned}
w_r &= 33\,\text{cm}, \\
d_{ls} &= 14\,\text{cm}, \\
\gamma &= 15e-2 \\
\text{and } e_{z,0} &= \left[0.1; \frac{0.5\pi}{180}\right]
\end{aligned} \tag{119}$$

Furthermore, the initial state and its error covariance were defined as follows:

$$\begin{aligned}
\hat{x}_o &= \begin{bmatrix} 0 \\ 0 \\ \dfrac{35\pi}{180} \end{bmatrix}, \\
P_o &= \begin{bmatrix} 1.5^2 & 0 & 0 \\ 0 & 1^2 & 0 \\ 0 & 0 & \left(\dfrac{2.5\pi}{180}\right)^2 \end{bmatrix}
\end{aligned} \tag{120}$$

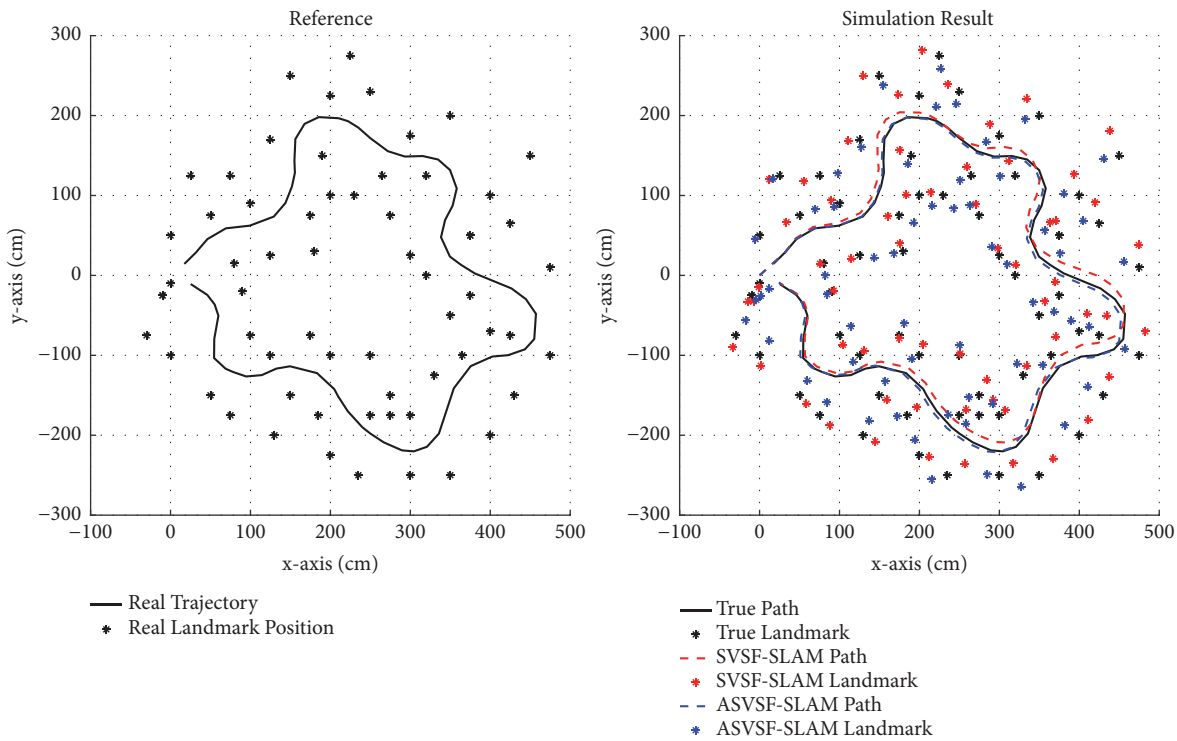

Figure 4: SVSF and ASVSF-SLAM algorithm.

According to the initial noise statistic there would be two different simulation cases presented in this paper. Both can be shown below.

(i) *1st Test*: the initial process and measurement noise are considered as follows:

$$\hat{q}_o = \begin{bmatrix} 0.03 \\ \dfrac{\pi}{180} \end{bmatrix},$$
$$\hat{Q}_o = \begin{bmatrix} (0.03)^2 & 0 \\ 0 & \left(\dfrac{\pi}{180}\right)^2 \end{bmatrix},$$
$$\hat{r}_o = \begin{bmatrix} 0.2 \\ \dfrac{3\pi}{180} \end{bmatrix},$$
$$\hat{R}_o = \begin{bmatrix} (0.2)^2 & 0 \\ 0 & \left(\dfrac{3\pi}{180}\right)^2 \end{bmatrix} \tag{121}$$

Then the result of SVSF-SLAM and SVSF-SLAM algorithm can be compared based on Figure 4.

Figure 4 illustrates the performance of SVSF and ASVSF-SLAM algorithm applied for autonomous robot. It depicts that the ASVSF-SLAM algorithm provides better solution proven by the successful in following the reference path. For more detail, it can be analysed by the result shown in Figure 5.

Figure 5 shows the RMSE of different algorithm performances in estimating the path. Comparing to the conventional approach, ASVSF-SLAM algorithm has better accuracy pointed by the smaller RMSE in all benchmarks. Furthermore, in an effort to provide more comparative result, the different RMSE of estimated map is also presented as shown in Figure 6.

Figure 6 shows the different quality of SVSF/ASVSF-SLAM algorithm in estimating the location of the landmark. According to this figure, the proposed method shows less optimal results in the estimated map for x-coordinate, but it still shows better results in estimating the y-coordinate landmark compared with SVSF-SLAM algorithm. Thus, ASVSF-SLAM algorithm can still guarantee the filter stability. To confirm this statement, Table 1 is presented.

In average, the proposed method gives higher quality in estimating path and landmark. It is significantly shown by the better achievement compared to SVSF-SLAM algorithm in almost all benchmark.

(ii) *2st Test*: the initial process and measurement noise are considered as follows:

$$\hat{q}_o = \begin{bmatrix} 0.08 \\ \dfrac{5\pi}{180} \end{bmatrix},$$
$$\hat{Q}_o = \begin{bmatrix} (0.08)^2 & 0 \\ 0 & \left(\dfrac{5\pi}{180}\right)^2 \end{bmatrix},$$

Table 1: RMSE of SVSF and ASVSF-SLAM algorithm.

Algorithm	x-Pose (cm)	y-Pose (cm)	Heading (rad)	Landmark x-Coordinate (cm)	Landmark y-Coordinate (cm)
SVSF-SLAM	5.5823	9.7952	0.1045	11.2296	14.4148
ASVSF-SLAM	4.6835	2.3872	0.0989	13.2414	11.5277

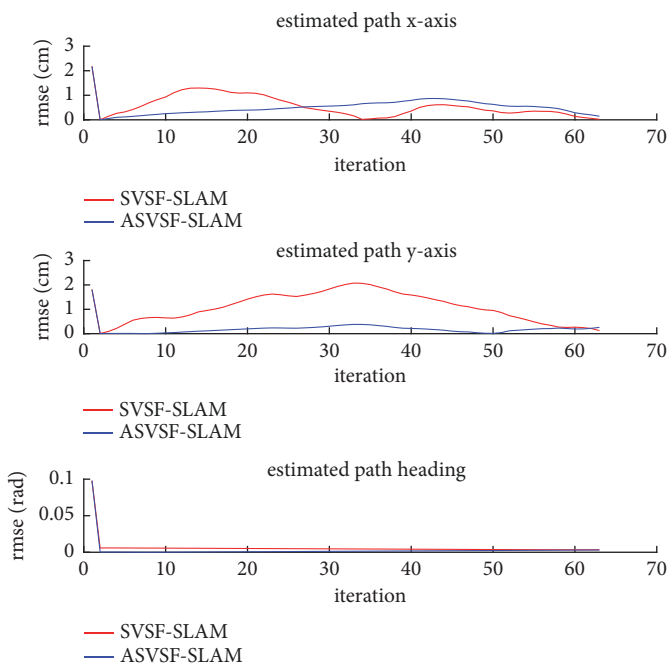

Figure 5: RMSE of estimated path.

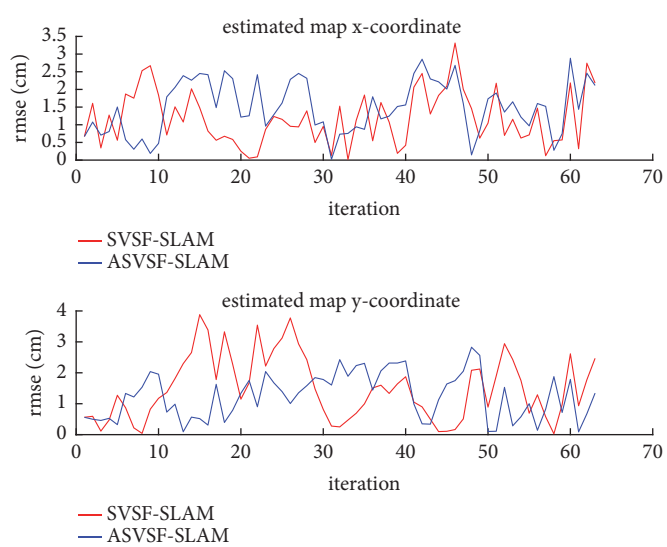

Figure 6: RMSE of estimated map.

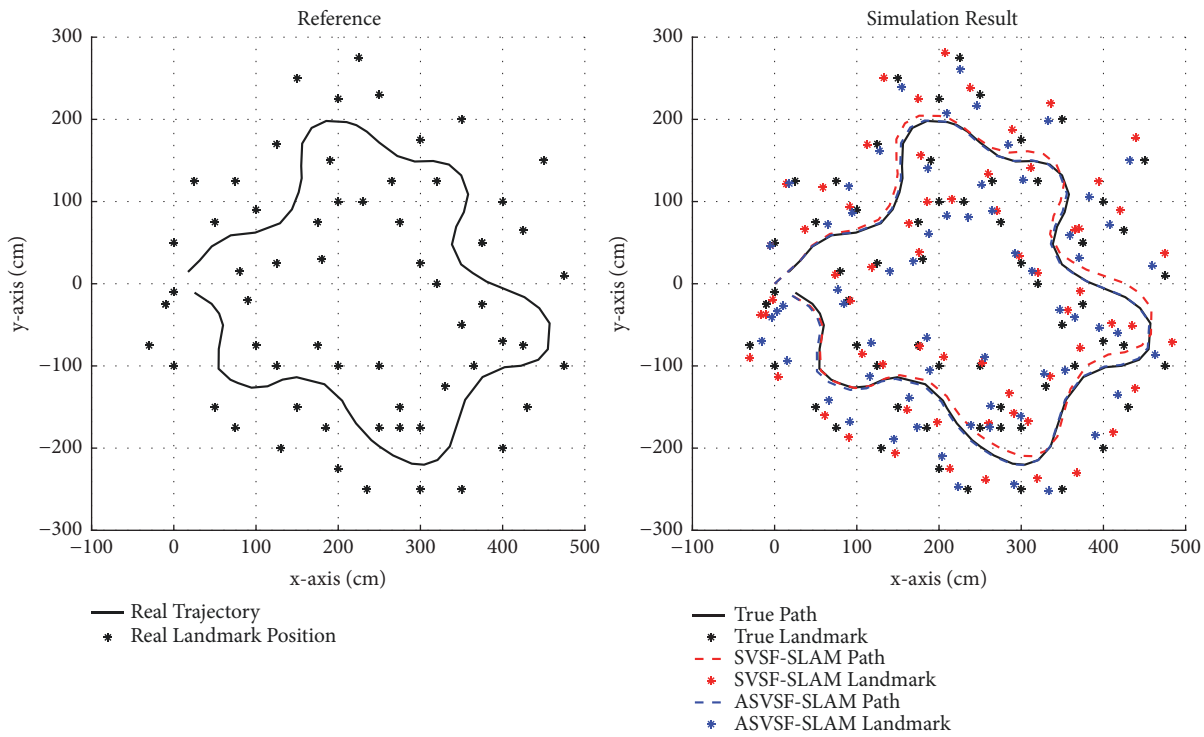

Figure 7: SVSF and ASVSF-SLAM algorithm.

$$\hat{r}_o = \begin{bmatrix} 0.8 \\ \dfrac{3\pi}{180} \end{bmatrix},$$
$$\hat{R}_o = \begin{bmatrix} (0.8)^2 & 0 \\ 0 & \left(\dfrac{3\pi}{180}\right)^2 \end{bmatrix} \tag{122}$$

Like the previous experiment, the general performance of SVSF and ASVSF-SLAM algorithm is evaluated from the graphical performance shown in Figure 7.

Figure 7 depicts that the increment of initial noise statistic does not affect the stability of ASVSF-SLAM. Therefore, it can be noted that the proposed method provides more stable filter compared with SVSF-SLAM algorithm. For the 2nd Test, the SVSF and ASVSF-SLAM performance are also compared in terms of RMSE. It is depicted by Figure 8.

According to Figure 8, ASVF-SLAM algorithm shows its effectiveness in locating current robot position. It is proven by the smaller RMSE for the all benchmark. Then, it can be noted that the accuracy of ASVSF-SLAM algorithm is guaranteed even though there exists a noise statistic increment. Additionally, the result of estimated map is also presented as shown in Figure 9.

Similarly, Figure 9 shows that the SVSF-SLAM has better solution of estimating map for x-coordinate, pointed by small different, while the proposed method shows better solution in estimating y-coordinate significantly. Therefore, ASVSF-SLAM performed well-enough in estimating the landmark. It is confirmed by Table 2.

This table shows clearly that the ASVSF-SLAM algorithm provides a stable filter. Additionally, according to the result of 1st and 2nd Test presented above, it can be noted that the ASVSF-SLAM has better quality in providing solution under noise statistics increment. The presence of one-step smoothing and time-varying of the noise statistic does not damage the characteristic of SVSF. But instead, it precisely increases the accuracy of SVSF proven by the achievement in Figures 5 and 8.

6. Conclusion

In this paper the ASVSF algorithm is proposed to solve the SLAM dynamic problem since there exist the inaccurate model, dynamic system uncertainty, and unknown noise statistics. The recursive time-varying noise estimator is derived based on MAP creation and weighted exponent which tunes the SVSF by utilizing the one-step smoothing method. It results in the enhanced SVSF. Additionally, to provide the ability of avoiding filter divergences, the ASVSF also applied implies a covariance correction process adopted from the divergence suppression concept. The ASVSF was implemented for solving SLAM problem as the proposed algorithm named as ASVSF-SLAM algorithm. It was simulated and compared to the SVSF-SLAM. The simulated results in terms of RMSE of estimated path and map were analysed

Table 2: RMSE of SVSF and ASVSF-SLAM algorithm.

Algorithm	x-Pose (cm)	y-Pose (cm)	Heading (rad)	Landmark x-Coordinate (cm)	Landmark y-Coordinate (cm)
SVSF-SLAM	5.9065	10.0448	0.1099	10.8078	13.6891
ASVSF-SLAM	4.6835	2.3872	0.0989	11.4657	11.0790

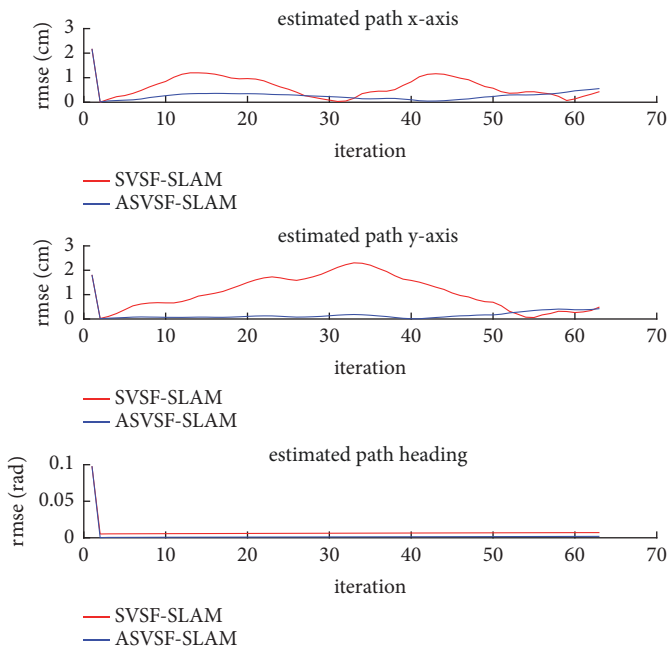

Figure 8: Estimated path.

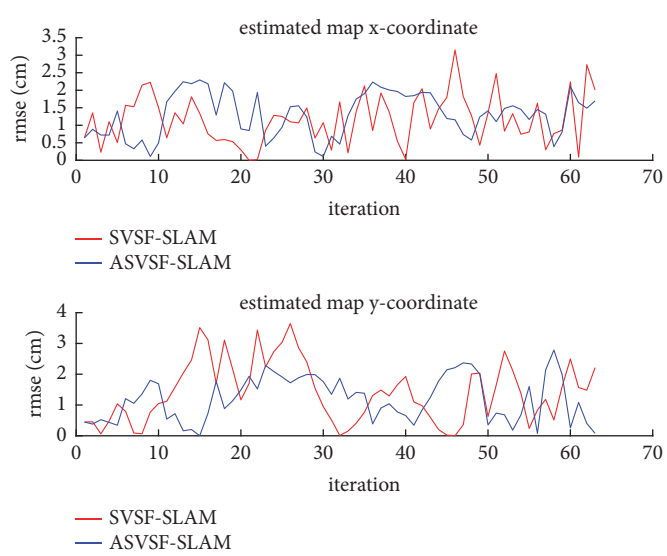

Figure 9: RMSE of estimated map.

and it has been showing the effectiveness and robustness of the ASVSF-SLAM algorithm.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors detected no conflicts of interest.

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